Effective Neutrino Masses from Four Flavor Neutrino Mixing Matrix

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Abstract

We consider the four neutrino oscillation that accommodate the all neutrino oscillation data. We consider the range of the corresponding mixing parameters by the result of neutrino oscillation experiments. Implication of the neutrino oscillation search for the neutrino mass square difference and mixing are discussed. We determine the possible values of the effective majorana neutrino mass $| \langle m \rangle | = | \sum_j U_{ej}^2 m_j |$ in the four neutrino scenario. In the four-neutrino scheme there is an upper bound on $| \langle m \rangle |$ of the normal mass order is 2.0074eV for $\alpha = 0^\circ$, $\beta = 0^\circ$ and $\gamma = 0^\circ$. In the case of inverted mass order the upper bound on $| \langle m \rangle |$ is 2.0069eV for $\alpha = 0^\circ$, $\beta = 0^\circ$ and $\gamma = 0^\circ$.

Keywords Neutrino masses · Four flavor neutrino mixing

1 Introduction

The phenomenon of neutrino oscillation, impressive advance have been made to understand the phenomenology of neutrino oscillation through solar neutrino, atmospheric neutrino, reactor neutrino and accelerator neutrino experiment. The experimental research on the nature of neutrinos through terrestrial as well as extra-terrestrial approaches has finally confirmed the neutrino oscillation in atmospheric [1–7] solar [8–14], reactor [15–17] and accelerator [18, 19] neutrino sources, establishing that neutrinos have mass. Furthermore, it is generally agreed that oscillations among three neutrino species are sufficient to explain the atmospheric, solar, reactor and accelerator neutrino puzzle. The neutrino oscillation experiments provide us with neutrino mass square differences, mixing angles and a possible hierarchy in the neutrino mass spectrum. The main physical goal in future experiment are the determination of the unknown parameter θ_{13} . In particular, the observation of δ is

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quite interesting from the point of view that δ is related to the origin of the matter in the universe. One of the most important parameter in neutrino physics is the magnitude of mixing angle θ_{13} and CP phase δ . The oscillation data also suggests that the neutrinos may belong to either a normal hierarchy ($m_1 < m_2 < m_3$) or an inverted hierarchy ($m_3 < m_1 < m_2$). The data do not exclude the possibility that the mass of the light neutrino could be much larger than $\sqrt{\Delta_{31}}$, which would imply the possible existence of a quasi-degenerate neutrino mass spectrum ($m_1 \approx m_2 \approx m_3$). On the other hand, the actual mass of neutrinos cannot be extracted from these data, only the study of tritium single β decay and nuclear double beta decay together can provide sharpest limits on the mass and nature of neutrinos. Neutrino oscillations, which only depend on mass square difference, give no information about the absolute value of the neutrino mass squared eigenvalues. Hence, there are various possibilities of neutrino hierarchy spectrums consistent with solar and atmospheric neutrino oscillation data. The mass eigenstates with eigen values m_i can be determined if the absolute value of effective mass of neutrino is exactly known. The current neutrinoless double beta decay experiments only provide the upper limit on effective Majorana neutrino mass $< m_{ee} >$ so that absolute scale of neutrino mass is not determined yet. Therefore, in the

Table 1 For $\alpha = 0^\circ$ and normal mass order, the effective Neutrino Mass for different value of Majorana phases

β	γ	$m_{ee}(\text{eV})$
0°	0°	2.0074
0°	45°	1.9944
0°	90°	1.9626
0°	135°	1.9301
0°	180°	1.9167
45°	0°	1.9905
45°	45°	1.9781
45°	90°	1.9467
45°	135°	1.9141
45°	180°	1.8998
90°	0°	1.9491
90°	45°	1.9371
90°	90°	1.9057
90°	135°	1.8730
90°	180°	1.8585
135°	0°	1.9068
135°	45°	1.8945
135°	90°	1.8631
135°	135°	1.8305
135°	180°	1.8162
180°	0°	1.8890
180°	45°	1.8760
180°	90°	1.8442
180°	135°	1.8119
180°	180°	1.7984

present work we have attempted to present a effective neutrino neutrino mass in the case of normal and inverted neutrino mass order.

In this paper, we will discuss the effective neutrino masses for different neutrino mass spectrum, namely normal mass hierarchy ($m_1 < m_2 < m_3$), inverted mass hierarchy ($m_3 < m_1 < m_2$). The present work is organized as follows. In Section 2, we outline the neutrino oscillation parameters. In Section 3, we have given the theoretical formalism to calculate the effective neutrino masses above mentioned hierarchies. In Section 4, we present the numerical results and Section 5 is devoted to the conclusions.

2 Mixing Angles and Neutrino Mass Squared Differences

The first evidence is the observation of zenith-angle dependence of atmospheric neutrino defect [20] dependent of the atmospheric neutrino $\nu_\mu \rightarrow \nu_\mu$ transition with the mass difference and the mixing as

$$\Delta_{31} = (1 - 2) \times 10^{-3} eV^2, \sin^2 2\theta_{23} = 1.0. \quad (1)$$

Table 2 For $\alpha = 45^\circ$ and normal mass order, the effective Neutrino Mass for different value of Majorana phases

β	γ	$m_{ee}(\text{eV})$
0°	0°	1.8808
0°	45°	1.8753
0°	90°	1.8475
0°	135°	1.8132
0°	180°	1.7925
45°	0°	1.8738
45°	45°	1.8691
45°	90°	1.8418
45°	135°	1.8074
45°	180°	1.7860
90°	0°	1.8377
90°	45°	1.8336
90°	90°	1.8066
90°	135°	1.7721
90°	180°	1.7503
135°	0°	1.7928
135°	45°	1.7886
135°	90°	1.7616
135°	135°	1.7270
135°	180°	1.7054
180°	0°	1.7656
180°	45°	1.7607
180°	90°	1.7333
180°	135°	1.6988
180°	180°	1.6777

From the recent three neutrino analysis of the Super-Kamiokande data [29] the following 90% CL were found for the normal (inverted) neutrino mass spectrum

$$1.9(1.7) \times 10^{-3} eV^2 \leq \Delta_{31} \leq 2.6(2.7) \times 10^{-3} eV^2 \quad (2)$$

The second evidence is the solar neutrino deficit [21], which is consistent with $\nu_\mu \rightarrow \nu_\tau/\nu_e$ transition. The SNO experiments [22] are consistent with the standard solar model [23] and strongly suggest the LMA solution.

$$\Delta_{21} = 7 \times 10^{-5} eV^2, \sin^2 2\theta_{12} = 0.8. \quad (3)$$

Solar neutrino experiments (Super-K, GALLEX, SAGE, SNO and GNO) show the neutrino oscillations, neutrino oscillation provide the most elegant explanation of all the data [24].

$$\Delta_{solar} = 7_{-1.3}^{+5} \times 10^{-5} eV^2, \quad (4)$$

$$\tan^2 \theta_{solar} = 0.4_{-0.1}^{+0.14}. \quad (5)$$

From the three neutrino global analysis of the solar and reactor KamLAND data was found [27],

$$\Delta_{21} = 7.5_{-20}^{+19} \times 10^{-3} eV^2, \quad \tan^2 \theta_{12} = 0.452_{-0.032}^{+0.035}. \quad (6)$$

Table 3 For $\alpha=90^\circ$ and normal mass order, the effective Neutrino Mass for different value of Majorana phases

β	γ	$m_{ee}(\text{eV})$
0°	0°	1.5326
0°	45°	1.5332
0°	90°	1.5096
0°	135°	1.4749
0°	180°	1.4495
45°	0°	1.5335
45°	45°	1.5351
45°	90°	1.5123
45°	135°	1.4777
45°	180°	1.4515
90°	0°	1.5030
90°	45°	1.5054
90°	90°	1.4832
90°	135°	1.4487
90°	180°	1.4219
135°	0°	1.4578
135°	45°	1.4603
135°	90°	1.4381
135°	135°	1.4036
135°	180°	1.3768
180°	0°	1.4242
180°	45°	1.4260
180°	90°	1.4033
180°	135°	1.3687
180°	180°	1.3424

Atmospheric neutrino experiments (Kamiokande, Super-K) also show the neutrino oscillation. The most excellent fit to the all data [24],

$$\Delta_{atmo} = 2.0_{-0.92}^{+1.0} \times 10^{-3} eV^2, \quad (7)$$

$$\sin^2 2\theta_{atmo} = 0.4_{-0.10}^{+0.14}. \quad (8)$$

The CHOOZ reactor experiment [25] gives the upper bound on the third mixing angle θ_{13} as

$$\sin^2 \theta_{13} < 0.20 \quad \text{for} \quad |\Delta_{31}| = 2.0 \times 10^{-3} eV^2, \quad (9)$$

$$\sin^2 \theta_{13} < 0.16 \quad \text{for} \quad |\Delta_{31}| = 2.5 \times 10^{-3} eV^2, \quad (10)$$

$$\sin^2 \theta_{13} < 0.14 \quad \text{for} \quad |\Delta_{31}| = 3.0 \times 10^{-3} eV^2, \quad (11)$$

at the 90% CL. The CP phase δ has not been constrained. From the two neutrino analysis of the MINOS data obtained was [26],

$$\sin^2 \theta_{13} < 0.90 \quad \text{for} \quad |\Delta_{31}| = (2.43 \pm 0.13) \times 10^{-3} eV^2, \quad (12)$$

The future neutrino experiments plan to measure the oscillation parameters precisely. The combined analysis of Day Bay, MINOS and Bugey-3 data [28] has excluded most

Table 4 For $\alpha=135^\circ$ and normal mass order, the effective Neutrino Mass for different value of Majorana phases

β	γ	$m_{ee}(\text{eV})$
0°	0°	1.0772
0°	45°	1.0780
0°	90°	1.0548
0°	135°	1.0202
0°	180°	0.9943
45°	0°	1.0785
45°	45°	1.0808
45°	90°	1.0587
45°	135°	1.0242
45°	180°	0.9974
90°	0°	1.0487
90°	45°	1.0521
90°	90°	1.0309
90°	135°	0.9967
90°	180°	0.9690
135°	0°	1.0036
135°	45°	1.0071
135°	90°	0.9861
135°	135°	0.9518
135°	180°	0.9240
180°	0°	0.9693
180°	45°	0.9719
180°	90°	0.9501
180°	135°	0.9157
180°	180°	0.8885

Table 5 For $\alpha = 180^\circ$ and normal mass order, the effective Neutrino Mass for different value of Majorana phases

β	γ	$m_{ee}(\text{eV})$
0°	0°	0.8173
0°	45°	0.8047
0°	90°	0.7733
0°	135°	0.74064
0°	180°	0.7267
45°	0°	0.8011
45°	45°	0.7902
45°	90°	0.7597
45°	135°	0.7264
45°	180°	0.7106
90°	0°	0.7604
90°	45°	0.7504
90°	90°	0.7204
90°	135°	0.6868
90°	180°	0.6701
135°	0°	0.7175
135°	45°	0.7069
135°	90°	0.6766
135°	135°	0.6432
135°	180°	0.6270
180°	0°	0.6990
180°	45°	0.6864
180°	90°	0.6552
180°	135°	0.6224
180°	180°	0.6083

of the parameter space of the mass squared difference $\Delta_{41} = m_4^2 - m_i^2$ and the mixing angle $\sin^2\theta_{14}$ for the sterile neutrino, the latest global analysis of neutrino oscillation data indicates that a small region around the best-fit value of [29]

$$\Delta_{41} = 1.7eV^2. \quad \text{and} \quad \sin^2\theta_{14} = 0.019 \quad (13)$$

3 Effective Majorana Mass of Electron Neutrino

In the presence of one sterile neutrino, in four flavour neutrino mixing the electron neutrino is combination of mass eigenstate, ν_i with eigenvalue m_i

$$\nu_e = \sum U_{ej}\nu_{ij} \quad i = 1, 2, 3, 4. \quad (14)$$

Here U_{ei} are the elements of the 4×4 mixing matrix, which relates the flavour states to the mass eigenstates. The $\beta\beta_{0\nu}$ decay rate is determined by the effective Majorana mass of

the electron neutrino m_{ee} . Under the assumption of four flavour neutrino mixing of neutrino, the effective Majorana neutrino mass m_{ee} [30] is

$$|m_{ee}|_{4\nu} = \left| \sum |U_{ej}|^2 e^{i\phi_j} m_j \right| \\ = |c_{13}^2 c_{12}^2 c_{14}^2 m_1 + c_{13}^2 s_{12}^2 c_{14}^2 e^{i\alpha} m_2 + s_{13}^2 c_{14}^2 e^{i\beta} m_3 + s_{14}^2 e^{i\gamma} m_4|. \quad (15)$$

We consider that the sterile neutrino with mass m_4 is heavier than light active neutrinos. Therefore, the effective Majorana mass in case of normal ordering of active neutrino is given as

$$m_{ee} = |c_{13}^2 c_{12}^2 c_{14}^2 m_1 + c_{13}^2 s_{12}^2 c_{14}^2 e^{i\alpha} \sqrt{m_1^2 + \Delta_{21}} + s_{13}^2 c_{14}^2 e^{i\beta} \sqrt{m_1^2 + \Delta_{31}} + s_{14}^2 e^{i\gamma} \sqrt{m_1^2 + \Delta_{41}}|. \quad (16)$$

Similarly for the case of inverted hierarchy of active neutrinos, the expression in (15) can be rewritten as

$$m_{ee} = |c_{13}^2 c_{12}^2 c_{14}^2 \sqrt{m_3^2 + \Delta_{23} - \Delta_{21}} + c_{13}^2 s_{12}^2 c_{14}^2 e^{i\alpha} \sqrt{m_3^2 + \Delta_{23}} \\ + s_{13}^2 c_{14}^2 e^{i\beta} m_3 + s_{14}^2 e^{i\gamma} \sqrt{m_3^2 + \Delta_{43}}|. \quad (17)$$

Table 6 For $\alpha = 0^\circ$ and inverted mass order, the effective Neutrino Mass for different value of Majorana phases

β	γ	$m_{ee}(\text{eV})$
0°	0°	2.0069
0°	45°	1.9939
0°	90°	1.9621
0°	135°	1.9298
0°	180°	1.9162
45°	0°	1.9899
45°	45°	1.9777
45°	90°	1.9462
45°	135°	1.9136
45°	180°	1.8994
90°	0°	1.9486
90°	45°	1.9366
90°	90°	1.9053
90°	135°	1.8726
90°	180°	1.8580
135°	0°	1.9063
135°	45°	1.8940
135°	90°	1.8626
135°	135°	1.8300
135°	180°	1.8157
180°	0°	1.8885
180°	45°	1.8755
180°	90°	1.8438
180°	135°	1.8115
180°	180°	1.7979

Table 7 For $\alpha = 45^\circ$ and inverted mass order, the effective Neutrino Mass for different value of Majorana phases

β	γ	$m_{ee}(\text{eV})$
0°	0°	1.8803
0°	45°	1.8748
0°	90°	1.8471
0°	135°	1.8127
0°	180°	1.7921
45°	0°	1.8733
45°	45°	1.8687
45°	90°	1.8414
45°	135°	1.8069
45°	180°	1.7856
90°	0°	1.8372
90°	45°	1.8331
90°	90°	1.8062
90°	135°	1.7717
90°	180°	1.7499
135°	0°	1.7923
135°	45°	1.7881
135°	90°	1.7611
135°	135°	1.7266
135°	180°	1.7049
180°	0°	1.7652
180°	45°	1.7602
180°	90°	1.7328
180°	135°	1.6984
180°	180°	1.6773

The PMNS matrix $U_{4 \times 4}$ depends on the mixing angles and CP violating phases. Using the parameterization in (13)

$$U = R_{34}R_{24}R_{14}R_{23}R_{13}R_{12}P, \quad (18)$$

where the matrices R_{ij} are rotations in ij space,

$$R_{14} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14}e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix} \quad \text{and} \quad R_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix}, \quad (19)$$

Table 8 For $\alpha = 90^\circ$ and inverted mass order, the effective Neutrino Mass for different value of Majorana phases

β	γ	$m_{ee}(\text{eV})$
0°	0°	1.5322
0°	45°	1.5328
0°	90°	1.5092
0°	135°	1.4745
0°	180°	1.4491
45°	0°	1.5331
45°	45°	1.5348
45°	90°	1.5119
45°	135°	1.4773
45°	180°	1.4512
90°	0°	1.5026
90°	45°	1.5050
90°	90°	1.4828
90°	135°	1.4483
90°	180°	1.4216
135°	0°	1.4574
135°	45°	1.4599
135°	90°	1.4377
135°	135°	1.4032
135°	180°	1.3765
180°	0°	1.4239
180°	45°	1.4256
180°	90°	1.4029
180°	135°	1.3684
180°	180°	1.3421

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$. The diagonal matrix P contains the three Majorana phases α , β and γ :

$$P = \text{diag}(1, e^{i\alpha/2}, e^{i(\beta+\delta_{13})}, e^{i(\gamma/2+\delta_{14})}). \quad (20)$$

Note that there are in total three Dirac CP-violating phase δ_{ij} . P is constructed in such a way that only Majorana phases show up in the effective mass expression.

4 Numerical Results

Note from (16) and (17) that the effective neutrino depends on new physics involving sterile neutrino mixing angle θ_{14} and the mass square difference Δ_{41} . Also crucially the type of neutrino mass spectrum, normal or inverse hierarchical spectrum. Hence, we consider a degenerated neutrino Mass spectrum and take the common neutrino mass to be 2 eV, which is upper limit of tritium beta decay spectrum [31].

Table 9 For $\alpha = 135^\circ$ and inversed mass order, the effective Neutrino Mass for different value of Majorana phases

β	γ	$m_{ee}(\text{eV})$
0°	0°	1.0769
0°	45°	1.0778
0°	90°	1.0546
0°	135°	1.012
0°	180°	0.9941
45°	0°	1.0782
45°	45°	1.0806
45°	90°	1.0585
45°	135°	1.0240
45°	180°	0.9971
90°	0°	1.0484
90°	45°	1.0519
90°	90°	1.0307
90°	135°	0.9963
90°	180°	0.9687
135°	0°	1.0033
135°	45°	1.0069
135°	90°	0.9859
135°	135°	0.9516
135°	180°	0.9238
180°	0°	0.9691
180°	45°	0.9716
180°	90°	0.9498
180°	135°	0.9154
180°	180°	0.8883

In Tables 1, 2, 3, 4 and 5 for normal mass order and Tables 6, 7, 8, 9 and 10 for inverted mass order, we list the effective neutrino mass Vs majorana phases for CP phase conserved case. Using the best fit value [24–26, 29] of oscillation parameter. For the computational purpose, the Majorana phase are taken as α , β and γ . We have taken these values from 0 – 180 degrees.

5 Conclusions

Future and present search for neutrinoless double beta decay purpose at probing lepton number violation and the Majorana nature of neutrinos with remarkable precession. Several experimental programs is currently under discussion.

Table 10 For $\alpha = 180^\circ$ and inverted mass order, the effective Neutrino Mass for different value of Majorana phases

β	γ	$m_{ee}(\text{eV})$
0°	0°	0.8171
0°	45°	0.8045
0°	90°	0.7731
0°	135°	0.7404
0°	180°	0.7265
45°	0°	0.8009
45°	45°	0.7899
45°	90°	0.7595
45°	135°	0.7262
45°	180°	0.7104
90°	0°	0.7603
90°	45°	0.7502
90°	90°	0.7202
90°	135°	0.6867
90°	180°	0.6699
135°	0°	0.7173
135°	45°	0.7067
135°	90°	0.6764
135°	135°	0.6430
135°	180°	0.6269
180°	0°	0.6988
180°	45°	0.6863
180°	90°	0.6550
180°	135°	0.6222
180°	180°	0.6081

In this paper, we have explored the impact of a majorana phases α , β and γ on the effective neutrino masses. Taking the best-fit value $\Delta_{41} = 1.7eV^2$ and $\sin^2\theta_{14} = 0.019$. We find the effective neutrino mass m_{ee} , for normal neutrino mass order is $m_{ee}^{upper-limit} = 2.0074eV$ for $\alpha = 0^\circ, \beta = 0^\circ$ and $\gamma = 0^\circ$ and $m_{ee}^{lower-limit} = 0.6083eV$ for $\alpha = 180^\circ, \beta = 180^\circ$ and $\gamma = 180^\circ$. The effective neutrino mass m_{ee} , for inverted neutrino mass order is $m_{ee}^{upper-limit} = 2.0069eV$ for $\alpha = 0^\circ, \beta = 0^\circ$ and $\gamma = 0^\circ$ and $m_{ee}^{lower-limit} = 0.6081eV$ for $\alpha = 180^\circ, \beta = 180^\circ$ and $\gamma = 180^\circ$. We have found due to nonzero value of majorana phases, effective neutrino masses suppresses.

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