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ERRATUM

Erratum to: An Entropy Functional for Riemann-Cartan Space-Times

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The original version of this article unfortunately contained a mistake. The authors rectified the errors and are shown below:

We correct the entropy functional constructed in Int. J. Theor. Phys. 51:362 (2012). The 'on-shell' functional one obtains from this correct functional possesses a holographic structure without imposing any constraint on the spin-angular momentum tensor of matter, in contrast to the conclusion made in the above paper.

The error made in [1] was the missing torsion trace $Q_i = Q_{ij}{}^j$ in the evaluation of the divergence terms. Indeed, the integral of a divergence in Riemann-Cartan space-times is of the form $\int_M \nabla_i V^i = \int_{\partial M} n_i V^i - \int_M 2Q_i V^i$. Taking this into account, the entropy functional that should be considered is actually the following more 'economical' one

$$S = \int d^4x\sqrt{-g}\Big[\alpha\big((\nabla_i u^j)(\nabla_j u^i) - (\nabla_i u^i)^2\big) + (\lambda g_{ij} + T_{ij})u^i u^j + (\Sigma_{ijk} + \Sigma_{ikj} + \Sigma_{kji})u^i \nabla^j u^k\Big]. \tag{1}$$

The additional terms introduced in [1] arise automatically when extracting total divergences. The variation of this functional with respect to the field u^i reads

$$\delta S = \int d^4x\sqrt{-g}\Big[2\alpha\big((\nabla_i \delta u^j)(\nabla_j u^i) - (\nabla_i \delta u^i)(\nabla_j u^j)\big) + 2(\lambda g_{ij} + T_{ij})\delta u^i u^j + (\Sigma_{ijk} + \Sigma_{ikj} + \Sigma_{kji})\delta u^i \nabla^j u^k + (\Sigma_{ijk} + \Sigma_{ikj} + \Sigma_{kji})u^i \nabla^j \delta u^k\Big]. \tag{2}$$

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Extracting from the total divergences the boundary integrals which do not contribute to the variation, the condition $\delta S = 0$ reads

$$\int \mathrm{d}^4x\sqrt{-g}\bigg(2\alpha\nabla_{[i}\nabla_{j]}u^j + (2\alpha Q_i g_{jk} - 2\alpha Q_k g_{ij} + \Sigma_{ikj})\nabla^j u^k + \bigg[\lambda g_{ij} + T_{ij} + \frac{1}{2}\big(\nabla^k + 2Q^k\big)(\Sigma_{ijk} + \Sigma_{kij} + \Sigma_{kji})\bigg]u^j\bigg)\delta u^i = 0. \tag{3}$$

From the identity $2\nabla_{[i}\nabla_{j]}u^j = -R_{ij}u^j - 2Q_{ij}{}^k\nabla_k u^j$, it follows that Eq. (3) is satisfied for arbitrary variations of u^i if

$$(2\alpha Q_{kij} + 2\alpha Q_i g_{jk} - 2\alpha Q_k g_{ij} + \Sigma_{ikj})\nabla^j u^k - \bigg[\alpha R_{ij} - \lambda g_{ij} - T_{ij} - \frac{1}{2}\big(\nabla^k + 2Q^k\big)(\Sigma_{ijk} + \Sigma_{kij} + \Sigma_{kji})\bigg]u^j = 0. \tag{4}$$

This in turn is satisfied for all u^i if and only if the content of each of the two square brackets vanishes identically, leading straightforwardly as in [1] to the Cartan-Sciama-Kibble field equations, but containing more correctly than in [1] the metric g_{ij} instead of the Kronecker-delta δ_{ij} in front of the torsion traces $Q_i g_{jk}$ and $Q_k g_{ij}$. The arguments used in [1] to prove the uniqueness of the functional Eq. (1) remain unchanged. When the field equations are substituted in Eq. (1) after integration by parts, however, the 'on-shell' entropy functional becomes simply

$$S = \frac{1}{8\pi G}\int_{\partial M} \mathrm{d}^3x\sqrt{|h|}n_i\big(u^j\nabla_j u^i - u^i\nabla_j u^j\big). \tag{5}$$

So contrary to the conclusion made in [1], the holographic structure emerges without imposing any constraint on the spin-angular momentum tensor of matter. Using Eq. (5), the application and conclusions made in [1] for the case of Dirac fields and black holes with intrinsic spin remain unchanged. For spin fluids obeying the Frenkel condition, however, the affine connection ∇ in Eq. (5) reduces as in [1] to the Levi-Civita connection $\overset{\circ}{\nabla}$ only for an isotropic deformation $u^i u^j = \frac{1}{4}u^2 g^{ij}$.

References

1. Hammad, F.: Int. J. Theor. Phys. **51**, 362 (2012)