Int J Theor Phys (2011) 50:3375–3384
DOI 10.1007/s10773-011-0842-7

Storage of Maximal Wigner-Yanase Skew Information of Two-Qubit System Using Nonlinear Interactions with Decay

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Received: 12 November 2010 / Accepted: 16 May 2011 / Published online: 24 June 2011

Abstract The decay mechanism is considered in some nonlinear interaction models for two-qubit system. Exact expression of the final states of two-qubit are given for different model. We find that the maximal Wigner-Yanase skew information can be modulated and stored via using decay mechanism and nonlinear interaction. Due to the maximal Wigner-Yanase skew information being equivalent to the concurrence, the conditions of generating the maximally entangled state (i.e., the Bell state) are obtained.

Keywords Storage · Maximal Wigner-Yanase skew information · Two-qubit system · Nonlinear interaction

1 Introduction

Wigner and Yanase have introduced the so-called Wigner-Yanase skew information [1–5] in the study of measurement theory. It refers to be an amount of information on the values of an observable A, and can be defined as [1]

$$I(\rho, A) = -\frac{1}{2}\,\mathrm{tr}[\rho^{1/2}, A]^2. \tag{1}$$

in terms of the density matrix ρ and the operator A. It is obvious that $I(\rho, A)$ may be interpreted as a degree of non-commutativity between ρ and A [6]. $I(\rho, A)$ can also be rewritten as

$$I(\rho, A) = \mathrm{tr}\,\rho A^2 - \mathrm{tr}\,\rho^{1/2} A \rho^{1/2} A. \tag{2}$$

For pure state, considering the property $\rho^k = \rho$, then $I(\rho, A)$ has the following form

$$I(\rho, A) = \langle\psi|A^2|\psi\rangle - \langle\psi|A|\psi\rangle^2. \tag{3}$$

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Actually, it is the fluctuation of the operator A.

On the other hand, spin squeezing is closely connected the spin fluctuation of the plane perpendicular to the mean spin direction [7–10]. To generate spin squeezing and spin squeezed states, Kitagawa and Ueda firstly introduced the one-axis twisting and the two-axis countertwisting model [7]. Subsequently, the one-axis twisting model was widely studied [9, 11–14]. Some works show that one-axis twisting model with an external transverse field and two-axis countertwisting model with an external transverse field can improve spin squeezing and entanglement [15, 16]. Interestingly, the spin squeezing can be effectively stored by rapidly turning-off the external field at a time that the maximal spin squeezing appears [17, 18], but this mechanism is invalid for small particles. However, a quantum system used in quantum information processing inevitably interacts with other quantum systems or with the surrounding environment, which induces decoherence phenomena. Thus, the environment is a important factor for quantum information processing. Recently, Yu and Eberly discovered a finite-time disentanglement in the general framework of two qubits-plus-environment, also known as entanglement sudden death (ESD) [19]. Motivated by the ESD, the spin squeezing sudden death was studied under the different decoherence channel [20]. In this paper, decay mechanism is considered in some classical nonlinear interaction models of two-qubit system. We find the exact states of two-qubit within the Schrödinger picture. We show that, with the nonlinear interaction and the decay mechanism, the maximal Wigner-Yanase skew information (MSI) can be stored like the storage of spin squeezing.

Our work is organized as follows. In Sect. 2, we study the MSI for symmetric pure states of two-qubit and demonstrate that the MSI is equivalent to the concurrence. In Sect. 3, we extend the one-axis twisting and the two-axis countertwisting model of two-qubit system and present the exact states dynamically generated from a superposition of the collective ground state $|11\rangle$ and the second excited state $|00\rangle$ via these models. Then we observe the storage time of the MSI of two-qubit system. Finally, a conclusion is given in Sect. 4.

2 Maximal Wigner-Yanase Skew Information of Two-Qubit Symmetric Pure State

We restrict states of pair qubits to the triplet sub-space. The collective angular momentum operator belonging to it can be expressed as follows

$$S_\mu = \sum_{i=1}^{2} S_{i,\mu}, \quad \mu \in \{x, y, z\}, \tag{4}$$

where $S_{i,\mu}$ are the spin operators for the ith qubit, and the cyclic commutation relations satisfy

$$[S_{i,\mu}, S_{i,\nu}] = i\epsilon_{\mu\nu\gamma} S_{i,\gamma}, \tag{5}$$

$$[S_\mu, S_\nu] = i\epsilon_{\mu\nu\gamma} S_\gamma, \quad \mu, \nu, \gamma \in \{x, y, z\}. \tag{6}$$

For the pure states, because the Wigner-Yanase skew information equals to the spin fluctuation for the pure states, it means that the MSI is identical to the maximal spin fluctuation denoted as I_{m}.

Lemma 1 *For the symmetric state*

$$|\psi\rangle = \sin\alpha|11\rangle + \cos\alpha|00\rangle, \tag{7}$$

there exist the relation between the MSI and the concurrence

$$I_{\mathrm{m}} = \frac{1}{2}(1 + C), \tag{8}$$

where $|0\rangle$ *and* $|1\rangle$ *are the eigenstates of the spin operator* $S_{i,z}$ *with the eigenvalue* $1/2$ *and* $-1/2$, *respectively.*

Proof For the entangled state

$$|\psi\rangle = \sin\alpha|11\rangle + \cos\alpha|00\rangle, \tag{9}$$

with the concurrence $C = |\sin 2\alpha|$ [10, 21, 22], the mean spin direction $\vec{n}_1$ defined as $(\langle S_x\rangle + \langle S_y\rangle + \langle S_z\rangle)/\sqrt{\langle S_x\rangle^2 + \langle S_y\rangle^2 + \langle S_z\rangle^2}$ is along the z-axis and its collective angular momentum operators satisfy the condition

$$\langle [S_z, S_x]_+\rangle = \langle [S_z, S_y]_+\rangle = 0, \tag{10}$$

then the maximal spin fluctuation has the following form [14]

$$\max(\Delta S_{\vec{n}})^2 = \max(\Delta S_z^2, \max \Delta S^2(x, y)), \tag{11}$$

where the operator $S_{\vec{n}}$ is defined as $S_{\vec{n}} = \sin\Theta\cos\Phi S_x + \sin\Theta\sin\Phi S_y + \cos\Theta S_z$ and $\max \Delta S^2(x, y)$ indicates the maximum fluctuation in the plane (x, y) [14]. It is easy to calculate the fluctuations ΔS_z^2 and $\Delta S^2(x, y)$. Exactly, we can obtain the fluctuation in the z direction

$$\Delta S_z^2 = \sin^2(2\alpha). \tag{12}$$

Furthermore, we can use the method in Refs. [9, 14, 23, 24] to get

$$\max \Delta S^2(x, y) = \frac{1}{2}[\langle S_x^2 + S_y^2\rangle + |\langle S_+^2\rangle|] = \frac{1 + |\sin 2\alpha|}{2}. \tag{13}$$

From the two equations above, it readily indicates that $\max \Delta S^2(x, y)$ was just the maximal spin fluctuation over the whole coordinate space. Finally, the MSI can be reduced to a simple form, namely,

$$I_{\mathrm{m}} = \frac{1}{2}(1 + |\sin 2\alpha|) = \frac{1}{2}(1 + C). \tag{14}$$

This is just our expected result.

It is shown that when (4) is the maximal entangled state, i.e. Bell state, $I_{\mathrm{m}} = 1$ for C reaches the maximum 1. From the above equation, it is noted that I_{m} entirely determined by the concurrence value in the two-qubit case. □

Lemma 2 *For an arbitrary symmetric state of two-qubit*

$$|\tilde{\psi}\rangle = \sin\alpha|\tilde{1}\tilde{1}\rangle + \cos\alpha|\tilde{0}\tilde{0}\rangle, \tag{15}$$

the relation given by (8) *is still valid, where states* $|\tilde{0}\rangle$ *and* $|\tilde{1}\rangle$ *are eigenstates of the operator* $\vec{\sigma}_i \cdot n$ *with an arbitrary direction* $n = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

$$|\tilde{0}\rangle = e^{\frac{-i\phi}{2}}\cos\frac{\theta}{2}|0\rangle + e^{\frac{i\phi}{2}}\sin\frac{\theta}{2}|1\rangle, \tag{16}$$

$$|\tilde{1}\rangle = e^{\frac{-i\phi}{2}} \sin\frac{\theta}{2}|0\rangle - e^{\frac{i\phi}{2}} \cos\frac{\theta}{2}|1\rangle \tag{17}$$

with eigenvalue 1 *and* −1, *respectively.*

Proof We begin with the eigenstates $|0\rangle$ and $|1\rangle$ of $\sigma_{i,z}$. Firstly, we perform a rotation of states around the y axis. In terms of the local operations, this rotation can be achieved by unitary transformation $U(S_{i,y}) = e^{-i\theta S_{i,y}}$. Then, the two eigenstates become

$$|0'\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle, \tag{18}$$

$$|1'\rangle = \sin\frac{\theta}{2}|0\rangle - \cos\frac{\theta}{2}|1\rangle. \tag{19}$$

Next, according to a similar procedure, if we perform a rotation of states $|0'\rangle$ and $|1'\rangle$ around the z axis, i.e., unitary transformation $U(S_{i,z}) = e^{-i\phi S_{i,z}}$, then the states $|0'\rangle$ and $|1'\rangle$ will be transformed to the states $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$ given by (16) and (17). It should be noted that operators $S_{i,\mu}$ act only on the states belonging to its own Hilbert space. Therefore, after performing the collective unitary transformation

$$U = e^{-i\phi S_z} e^{-i\theta S_y} \tag{20}$$

on the state in (9), we can transform the state $|\psi\rangle$ to the state

$$|\tilde{\psi}\rangle = \cos\alpha|\tilde{0}\tilde{0}\rangle + \sin\alpha|\tilde{1}\tilde{1}\rangle. \tag{21}$$

Now we perform the rotations of the local and collective operators. Under the rotations, we easily obtain the new local and global operators

$$\tilde{S}_\mu = \sum_{i=1}^{2} \tilde{S}_{i,\mu}, \quad \mu \in \{x, y, z\} \tag{22}$$

with

$$\tilde{S}_{i,\mu} = U_i S_{i,\mu} U_i^+, \quad \mu \in \{x, y, z\}, \tag{23}$$

where $U_i^+ = e^{i\theta S_{i,y}} e^{i\phi S_{i,z}}$. It is evident that the cyclic commutation relations satisfy

$$[\tilde{S}_{i,\mu}, \tilde{S}_{i,\nu}] = i\epsilon_{\mu\nu\gamma}\tilde{S}_{i,\gamma}, \tag{24}$$

$$[\tilde{S}_\mu, \tilde{S}_\nu] = i\epsilon_{\mu\nu\gamma}\tilde{S}_\gamma, \quad \mu, \nu, \gamma \in \{x, y, z\}. \tag{25}$$

For the all equations referred to the expectation values in Lemma 1, after a simply replacing S by $\tilde{S}$, they are still valid. Therefore, Lemma 1 is invariant under the above rotation. In other words, we need only to perform a corresponding transformation on the collective operators when the state $|\psi\rangle$ is transformed to the state $|\tilde{\psi}\rangle$. □

Proposition *For an arbitrary symmetric state of pair qubits*

$$|\Psi\rangle = a_1|00\rangle + a_2/\sqrt{2}(|01\rangle + |10\rangle) + a_3|11\rangle, \tag{26}$$

where coefficients a_1, a_2 and a_3 are arbitrary constants, and satisfy the normalizing condition, one can find that $I_{\mathrm{m}} > 1/2$ implies the entanglement and vice versa. There exists the relation between the MSI and the concurrence

$$I_{\mathrm{m}} = \frac{1}{2}(1 + C), \tag{27}$$

Proof In view of exact expression of states $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$, the symmetric state $|\tilde{\psi}\rangle$ can be reduced to a simple form, namely,

$$|\tilde{\psi}\rangle = A_1|00\rangle + A_2/\sqrt{2}(|01\rangle + |10\rangle) + A_3|11\rangle \tag{28}$$

with coefficients

$$\begin{aligned} A_1 &= e^{-i\phi}\left(\cos\alpha\cos^2\frac{\theta}{2} + \sin\alpha\sin^2\frac{\theta}{2}\right), \\ A_2 &= \sqrt{2}\cos\frac{\theta}{2}\sin\frac{\theta}{2}(\cos\alpha - \sin\alpha), \\ A_3 &= e^{i\phi}\left(\cos\alpha\sin^2\frac{\theta}{2} + \sin\alpha\cos^2\frac{\theta}{2}\right). \end{aligned} \tag{29}$$

By appropriate choice of α, θ and ϕ, one can achieve that A_1, A_2 and A_3 are equal to a_1, a_2 and a_3, respectively. From Lemma 2, we immediately have Proposition. □

3 Nonlinear Interaction Models of Two-Qubit System

The collective angular momentum operator of N qubits can be expressed as follows

$$S_\mu = \sum_{i=1}^{N} S_{i,\mu}, \quad \mu \in \{x, y, z\}, \tag{30}$$

where $S_{i,\mu}$ are the spin operators for the ith qubit. In order to generate the spin squeezing in an ensemble of N spin-1/2 particles, the one-axis twisting model with Hamiltonian $H = \chi S_x^2$ and initial collective state $\psi(0) = \prod_i^N |1\rangle_i$ was proposed as a building block, where χ is the correlation strength between the individual elementary spins. As a natural extension of one-axis twisting model, Authors introduced two axis countertwisting model with Hamiltonian $H = \frac{\chi}{2i}(S_+^2 - S_-^2)$ [7]. In the following, based on the generalized version of the two models, we study the storage of the MSI.

3.1 One Axis Twisting Model with Decay of Two-Qubit System

For the one axis twisting model with two qubits, considering the decay mechanism, the corresponding Hamiltonian is

$$H = 2\chi s^2 S_x^2, \tag{31}$$

where $s^2 = \exp[-\gamma t]$ and γ is the decay rate. Since the total spin of the two-qubit system is 1, such a system can be represented as a spin-1 particle. Then in the standard basis

$\{|\mathbf{1},1\rangle, |\mathbf{1},0\rangle, |\mathbf{1},-1\rangle\}$, the Hamiltonian above can be expressed as

$$H = \chi s^2 \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}. \tag{32}$$

We suppose initial state of two-qubit is the superposition state

$$\psi(0) = a_1|11\rangle + a_3|00\rangle, \tag{33}$$

where the probability amplitude be written as

$$a_1 = \frac{1+k}{\sqrt{2(1+k^2)}}, \tag{34}$$

$$a_3 = \frac{1-k}{\sqrt{2(1+k^2)}}, k \in [-1, 1]. \tag{35}$$

Via straight solving the time-dependent Schrodinger equation, we get the state vector at any time t

$$\psi(t) = a_1(t)|11\rangle + a_3(t)|00\rangle, \tag{36}$$

where the probability amplitudes have the following exact form (set $\hbar = 1$)

$$a_1(t) = \frac{(1+k)q(t)}{\sqrt{2(1+k^2)}} + \frac{(1-k)p(t)}{\sqrt{2(1+k^2)}}, \tag{37}$$

$$a_3(t) = \frac{(1-k)q(t)}{\sqrt{2(1+k^2)}} - \frac{(1+k)p(t)}{\sqrt{2(1+k^2)}}, \tag{38}$$

$$p(t) = \frac{e^{-\frac{i2\chi}{\gamma}} e^{\frac{i2\chi}{\gamma}\exp[-rt]} - 1}{2}, \tag{39}$$

$$q(t) = \frac{e^{-\frac{i2\chi}{\gamma}} e^{\frac{i2\chi}{\gamma}\exp[-rt]} + 1}{2}. \tag{40}$$

Now we consider the nonlinear dynamic of a general symmetric state of two-qubit with the dissipation process. We find that the state that possess arbitrary degree of the entanglement, can be obtained via adjusting the interaction strength. For example, if $\chi = (2n+1)\pi\gamma/4$ (n is integer), the evolved state must be a maximally entangled state no matter what initial state. In Fig. 1(a), the curves show the time evolution of the MSI for different χ. To further illustrate the observation, the time evolutions of the MSI with various k by setting $\chi = 3\pi\gamma/4$ was plotted as shown in Fig. 1(b). If $\chi = n\pi\gamma/2$, we find that the MSI of the final state will return to its initial value. Physically, when the nonlinear interaction vanishes as time increase due to the effect of decay mechanism, the evolved state will no longer change. So that the values of physical quantities will be invariable.

In a case of $k = 0$, the initial state not only is a eigenstate of the Hamiltonian with eigenvalue 1 but also is a maximally entangled. It is easy to check that the final state is different from the initial state, and the final state still is maximally entangled. It implies that the MSI of the maximally entangled state above can not be modulated by nonlinear interaction.

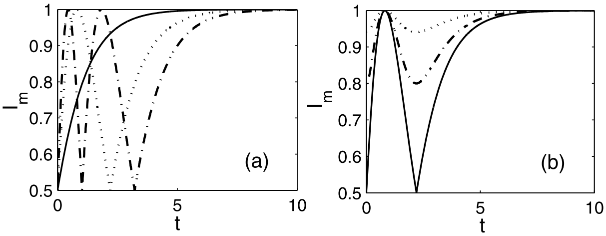

Fig. 1 I_m versus t. Parameter $\gamma = 0.5$. In (**a**), Parameter $k = 1$. $\chi = \pi\gamma/4$, $3\pi\gamma/4$ and $5\pi\gamma/4$ correspond the *solid line*, the *dotted line* and the *dashed dotted line*, respectively. In (**b**), $\chi = 3\pi\gamma/4$. $k = 0.25$, 0.5 and 1 correspond the *dotted line*, the *dashed dotted line*, and the *solid line*, respectively

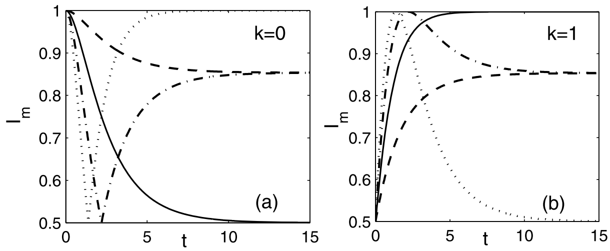

Fig. 2 I_m versus t for different k and χ. Parameters $\gamma = 0.5$. Parameter $\chi = 0.5\pi\gamma/4$, $\pi\gamma/4$, $1.5\pi\gamma/4$, $2\pi\gamma/4$ correspond the *dashed line* and the *solid line*, the *dashed dotted line* and the *dotted line*, respectively. In (**a**), $k = 0$. In (**b**), $k = 1$

3.2 Two Axis Countertwisting Model with Decay of Two-Qubit System

Considering the decay mechanism, the Hamiltonian of two-qubit system read

$$H = \frac{\chi s^2}{2i}(S_+^2 - S_-^2) = \chi s^2 \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}. \tag{41}$$

For the arbitrary initial state given by (33), we find that the state vector at any time t has the form $\psi(t) = a_1(t)|11\rangle + a_3(t)|00\rangle$, where

$$a_1(t) = \frac{(1+k)q(t)}{\sqrt{2(1+k^2)}} + \frac{(1-k)p(t)}{\sqrt{2(1+k^2)}}, \tag{42}$$

$$a_3(t) = \frac{(1-k)q(t)}{\sqrt{2(1+k^2)}} - \frac{(1+k)p(t)}{\sqrt{2(1+k^2)}}, \tag{43}$$

$$p(t) = i[e^{i\frac{\chi}{\gamma}(1-e^{-\gamma t})} - e^{-i\frac{\chi}{\gamma}(1-e^{-\gamma t})}]/2, \tag{44}$$

$$q(t) = [e^{i\frac{\chi}{\gamma}(1-e^{-\gamma t})} + e^{-i\frac{\chi}{\gamma}(1-e^{-\gamma t})}]/2. \tag{45}$$

Making an analysis, we find that the MSI with value 1 can be stored only if the equation $\tan\frac{2\chi}{\gamma} = \frac{-2k}{(1-k^2)}$ be established. However, the initial state is maximally entangled for $k = 0$, but it is not the eigenstate of the Hamiltonian, and the MSI of the final state can be controlled through nonlinear interaction. This is different from the case of the one-twisting model. The corresponding condition of the maximal entanglement is $\chi = n\pi\gamma/2$, as shown by the dotted line in Fig. 2(a). $k = \pm 1$, i.e., the state initially is in ground state $|11\rangle$ or exited state $|00\rangle$, the condition become $\chi = (2n+1)\pi\gamma/4$. For comparison, setting $k = 1$, we examine the evolution of the MSI, the condition $\chi = \pi\gamma/4$ is confirmed, as shown by the solid line in Fig. 2(b). These analysis shows that using the decay mechanism, the storage of the MSI always can be realized.

If two-qubit state initially is in the following entangled state

$$\psi(0) = a_1|11\rangle + a_3|00\rangle, \tag{46}$$

$$a_1 = i\frac{1+k}{\sqrt{2(1+k^2)}}, \tag{47}$$

$$a_3 = \frac{1-k}{\sqrt{2(1+k^2)}}, \quad k \in [-1, 1], \tag{48}$$

then, the corresponding probability amplitudes $a_1(t)$ and $a_3(t)$ can be solved

$$a_1(t) = \frac{i(1+k)q(t)}{\sqrt{2(1+k^2)}} + \frac{(1-k)p(t)}{\sqrt{2(1+k^2)}}, \tag{49}$$

$$a_3(t) = \frac{(1-k)q(t)}{\sqrt{2(1+k^2)}} - \frac{i(1+k)p(t)}{\sqrt{2(1+k^2)}}. \tag{50}$$

We find that the discussion of the MSI of the one-twisting model are completely valid here. Setting $\chi = (2n+1)\pi\gamma/4$, the final state will become the maximally entangled state for arbitrary a_1 and a_3. For the case with $k = 0$, the initial state is the eigenstate of the Hamiltonian. From (49) and (50), one can demonstrate that final state is the initial state itself, except for a global phase $q(t) - ip(t)$.

3.3 Mixed Model of the One-Axis Twisting and Two-Axis Countertwisting with Decay of Two-Qubit

As a natural supplement, we further consider a general model and call it as mixed model of one-axis twisting and two-axis countertwisting interaction. The corresponding Hamiltonian can be described by

$$H = 2\chi_1 s^2 S_x^2 + \frac{\chi_2 s^2}{2}(S_+^2 - S_-^2). \tag{51}$$

By straightforward calculation, we can obtain the exact solution. Considering (33) as initial state, the evolving state is determined by

$$a_1(t) = \frac{(1+k)q(t)}{\sqrt{2(1+k^2)}} + \frac{(1-k)p(t)}{\sqrt{2(1+k^2)}}, \tag{52}$$

$$a_3(t) = \frac{(1-k)q(t)}{\sqrt{2(1+k^2)}} - \frac{(1+k)p(t)}{\sqrt{2(1+k^2)}}, \tag{53}$$

$$p(t) = \frac{-(\chi_1 - i\chi_2)}{2\sqrt{\chi_1^2+\chi_2^2}}\left[e^{i\{\frac{\chi_1+\sqrt{\chi_1^2+\chi_2^2}}{\gamma}\}(1-e^{-\gamma t})} - e^{i\{\frac{\chi_1-\sqrt{\chi_1^2+\chi_2^2}}{\gamma}\}(1-e^{-\gamma t})}\right], \tag{54}$$

$$q(t) = \frac{1}{2}\left[e^{i\{\frac{\chi_1+\sqrt{\chi_1^2+\chi_2^2}}{\gamma}\}(1-e^{-\gamma t})} + e^{i\{\frac{\chi_1-\sqrt{\chi_1^2+\chi_2^2}}{\gamma}\}(1-e^{-\gamma t})}\right]. \tag{55}$$

From (52)–(55), we also find that when $\tan\frac{2\sqrt{\chi_1^2+\chi_2^2}}{\gamma} = \frac{-2k\sqrt{\chi_1^2+\chi_2^2}}{(1-k^2)\chi_2}$ be established, then the final state is the maximally entangled state. Evidently, this condition can be reduced to that of one twisting model (or two-countertwisting model) for $\chi_2 = 0$ (or $\chi_1 = 0$).

3.4 One Axis Twisting Model with a Transverse Field

In order to improve and control the spin squeezing, the one-axis twisting model with a transverse field was proposed [15]. In general, for the two-, three- and four-particle cases, the exact solutions of this model have been solved, but for many particles case, it is hard to be dealt analytically. Here, we make the nonlinear term and linear term along the same direction. Under the decay mechanism, the Hamiltonian of two-qubit system can be written as

$$H = 2\chi s^2 S_x^2 + \Omega' s S_x. \tag{56}$$

It can also be rewritten in the standard basis $\{|\mathbf{1}, 1\rangle, |\mathbf{1}, 0\rangle, |\mathbf{1}, -1\rangle\}$ as

$$H = \chi s^2 \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \Omega s \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{57}$$

where $\Omega = \Omega'/\sqrt{2}$, for convenience of calculation. Considering an arbitrary initial state

$$\psi(0) = a_1|11\rangle + a_3|00\rangle, \tag{58}$$

the final state can be obtained

$$\psi(t) = a_1(t)|11\rangle + a_2(t)[|10\rangle + |01\rangle]/\sqrt{2} + a_3(t)|00\rangle$$

with three coefficients

$$a_1(t) = \frac{p(t)+q(t)}{2\sqrt{2(1+k^2)}} + \frac{2k}{\sqrt{2(1+k^2)}}, \tag{59}$$

$$a_2(t) = \frac{1}{2\sqrt{(1+k^2)}}[p(t)-q(t)], \tag{60}$$

$$a_3(t) = \frac{p(t)+q(t)}{2\sqrt{2(1+k^2)}} - \frac{2k}{\sqrt{2(1+k^2)}}. \tag{61}$$

The exact expressions of $p(t)$ and $q(t)$ are

$$p(t) = \exp\left[\frac{i2\{\chi(e^{-\gamma t}-1) - \sqrt{2}\Omega(e^{-\gamma t/2}-1)\}}{\gamma}\right], \tag{62}$$

$$q(t) = \exp\left[\frac{i2\{\chi(e^{-\gamma t}-1) + \sqrt{2}\Omega(e^{-\gamma t/2}-1)\}}{\gamma}\right]. \tag{63}$$

We only consider the $\Omega \neq 0$ case here, for (59), and (61) can be reduced to (37) and (38) when $\Omega = 0$. We observe that although the probabilities $|a_1(t)|^2$, $|a_1(t)|^2$ and $|a_1(t)|^2$ do not vanish, the time evolution of the MSI is independent of Ω, as shown in Fig. 3(a). Meanwhile, the condition to generate the maximally entangled state is identical with the one-twisting model. To further confirm above observation, by setting $t = 2$, we plot the evolution of the MSI as a function of Ω, as shown in Fig. 3(b).

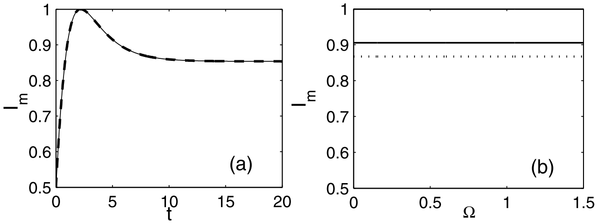

Fig. 3 I_m versus t and Ω. Parameters $\gamma = 0.5$ and $\chi = \pi\gamma/4$, $k = 1$. In (**a**), $\Omega = 0.5, 1.5$ correspond the *solid line* and *dashed line*, respectively. In (**b**), $t = 1$ and 10 correspond the *solid line* and the *dotted line*, respectively

4 Conclusion

In conclusion, we have studied the relation between the MSI and the concurrence in two-qubit symmetric pure states. Furthermore, for a two-qubit system, we introduced one-axis twisting model with decay mechanism and two countertwisting model with decay mechanism. Especially, the exact expression of state vectors for two-qubit system within Schrödinger picture and the time evolution of MSI were given. We found that due to the decay mechanism and the nonlinear interaction, the MSI can be modulated and stored.

Acknowledgements This work is supported by College Young Talents Foundation of Anhui Province with Grant no. 2010SQRL107, and the Natural Science Foundation of the Education Department of Anhui Province with Grant no. KJ2011Z234.

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