Hyperfine Interact (2009) 194:93–98
DOI 10.1007/s10751-009-0035-4

Developments for the direct determination of the *g*-factor of a single proton in a Penning trap

C. C. Rodegheri · K. Blaum · H. Kracke · S. Kreim · A. Mooser · C. Mrozik · W. Quint · S. Ulmer · J. Walz

Published online: 26 August 2009

Abstract The measurement and comparison of the magnetic moment (or *g*-factor) of the proton and antiproton provide a stringent experimental test of the CPT-theorem in the baryonic sector (Quint et al., Nucl Instrum Methods Phys Res, B 214:207, 2004). We present an experimental setup for the first direct high-precision measurement of the *g*-factor of a single isolated proton in a double cylindrical Penning trap. The application of the continuous Stern-Gerlach effect to detect quantum jumps between the two spin states of the particle, together with a novel trap design specially developed for this purpose, offers the possibility of measuring the magnetic moment not only of a single proton but also of a single antiproton. It is aimed to achieve a relative uncertainty of 10^{-9} or better. Preliminary results including mass spectra of particle clouds as well as single proton preparation and detection are shown.

Keywords Penning trap · Magnetic moment · Antiproton · CPT

1 Introduction

A self-contained and direct high precision measurement of the *g*-factor on a single proton has never been done before even though the *g*-factor has been already

C. C. Rodegheri (✉) · H. Kracke · S. Kreim · A. Mooser · C. Mrozik · S. Ulmer · J. Walz
Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany
e-mail: carvalho@uni-mainz.de

K. Blaum
Max-Planck-Institut für Kernphysik, 69117 Heidelberg, Germany

K. Blaum · W. Quint · S. Ulmer
Ruprecht-Karls-Universität, 69047 Heidelberg, Germany

W. Quint
GSI Darmstadt, 64291 Darmstadt, Germany

determined indirectly in many previous experiments [2–5]. The so far most precise measurement of the magnetic moment of the proton was performed by Kleppner et al. in 1972 [5]. In this experiment the ratio of the electron magnetic moment to the proton magnetic moment was determined by observing simultaneously electron and proton transitions with a hydrogen maser operating in a 0.35 T magnetic field. Thus the g-factor of the proton could be calculated to an accuracy of 10^{-8} regarding the necessary corrections due to the binding in the hydrogen atom.

The magnetic moment of the antiproton on the other hand is currently known to a relative precision of only 10^{-3}. In this case the magnetic moment was derived from measurements of the fine structure splitting in an X-ray transition of $\bar{p}^{208}Pb$ atoms [6]. Another proposal to determine the g-factor of the antiproton by means of spectroscopy of antiprotonic helium [7] aims for an uncertainty of 10^{-5}.

We propose the determination of the magnetic moment by means of the non-destructive measurement of two frequencies of a single proton in a Penning trap. The measuring process and the experimental setup can be applied for protons as well as for antiprotons as described in [8]. The advantage of performing the measurement on an isolated particle is that the achievable precision will be basically limited by the modifications in the eigenfrequencies of the particle in a real Penning trap compared to that in an ideal trap.

2 Single proton in a Penning trap

Charged particles can be confined in a Penning trap by means of the superposition of an axial homogeneous magnetic field $\mathbf{B} = B_0\mathbf{e_z}$ and an electrostatic quadrupole potential $\phi(z,\rho) = \frac{U_0}{2d^2}\left(z^2 - \frac{\rho^2}{2}\right)$. The parameter d is a trap dimension [9] and U_0 a dc-voltage applied between the endcaps and the ring electrode of a hyperbolical Penning trap. The resulting field will be ideally quadrupolar, so that a charged particle on the trap axis will see a harmonic trapping potential. The magnetic field in the axial direction provides the radial confinement. The motion of the particle inside an ideal trap can be seen as the superposition of three harmonic oscillations, the modified cyclotron, the magnetron and the axial motion with the eigenfrequencies $\omega_\pm = \frac{\omega_c}{2} \pm \sqrt{\left(\frac{\omega_c}{2}\right)^2 - \frac{\omega_z^2}{2}}$ and $\omega_z = \sqrt{\frac{eU_0}{md^2}}$, respectively. Typical values for the proton eigenfrequencies in our setup are $\nu_+ = 28.9$ MHz, $\nu_- = 8$ kHz and $\nu_z = 692$ kHz.

The Penning trap designed for this experiment has a cylindrical geometry [10, 11]. Ideally, i.e. if the electric field is perfectly quadrupolar and the magnetic field is constant, the motion of a charged particle in a cylindrical Penning trap is the same as in the case of the hyperbolical trap. The potential, however, will not be ideal and can be expanded in a Taylor series:

$$\phi(r,z) = \sum_{k=0}^{\infty}\left(\sum_{i=0}^{k} C_{i,k} r^i z^{k-i}\right), \tag{1}$$

with $C_{i,k} = \binom{k}{i}\frac{\partial^{i+k}\phi}{\partial r^i \partial z^k}\Big|_{(0,0)}$. To overcome the imperfections in the potential shape two additional electrodes, the correction electrodes, are placed between the ring and the endcaps. Applying a dc-voltage to these electrodes one can emend the lack of harmonicity of the electric potential. The “tunning ratio” is the ratio of the voltage applied to the correction electrode to that applied to the ring. The expression for the axial frequency in the case of an orthogonal trap [10] takes the form: $\omega_z = \sqrt{\frac{2eC_2}{m}}$.

3 Determination of the *g*-factor

The determination of the *g*-factor results from the accurate measurement of two frequencies of the proton in the double trap setup [12], as it can be calculated as

$$g = 2\frac{\omega_L}{\omega_c} = 2\frac{\nu_L}{\nu_c}, \tag{2}$$

where $\omega_c = \frac{e}{m}B$ is the free cyclotron frequency and $\omega_L = g\frac{e}{2m}B$ the Larmor frequency.

The motional frequencies ω_+, ω_- and ω_z are measured non-destructively by means of the image-current technique [13] in the so-called precision trap where the magnetic field is homogeneous. The oscillating motion of the proton in the trap induces image currents in the electrodes. Using tuned circuits with high quality factors a voltage drop across the resonant resistance can be detected. This tiny detectable signal, $U_{ind} = IQ\omega_r L$ with I of the order of fA, has the same frequency as that of the ion and has to be amplified. With a fast Fourier transform the desired frequency spectrum is then obtained. The free cyclotron frequency ω_c can be determined by the so-called "invariance theorem" [14]: $\omega_c = \sqrt{\omega_+^2 + \omega_-^2 + \omega_z^2}$.

The energy eigenstates of a spin carrying charged particle in the trap are Zeeman-split due to the external homogeneous magnetic field. The Larmor frequency ω_L can be determined by inducing radio frequency transitions between the two spin states in the precision trap. The resulting spin state can be detected via the continuous Stern-Gerlach effect [15] in the analysis trap. This effect is based on a coupling of the magnetic moment μ of a particle to its axial frequency ω_z in the trap, achieved by a "magnetic bottle" B_2 superimposed to the homogeneous magnetic field B_0. A magnetic term given by

$$\phi_z^{mag} = \pm\mu_z\left[B_0 + B_2\left(z^2 - \frac{\rho^2}{2}\right)\right] \tag{3}$$

is added to the potential energy of the particle. The effective trapping force is thus modified by the magnetic interaction, and the axial frequency is shifted upwards or downwards depending on the sign of μ_z:

$$\omega_z^{'} \cong \omega_z \pm \frac{\mu_z B_2}{m\omega_z} = \omega_z \pm \delta_{\omega_z}. \tag{4}$$

The axial frequency jump between the two spin directions will be $\omega_z^{'}(\uparrow) - \omega_z^{'}(\downarrow) = 2\frac{\mu_z B_2}{m\omega_z}$.

The inhomogeneous magnetic field component is produced by a ferromagnetic ring electrode. In order to increase the frequency jump to a detectable range a novel trap design, the hybrid Penning trap [11] was developed. A CoFe ring electrode with a saturation magnetization of 2.35 T and toroidal geometry was introduced into the cylindrical electrode stack, as illustrated in Fig. 1. With this configuration a B_2 term of 400 mT/mm^2 and a frequency jump of 252 mHz can be achieved.

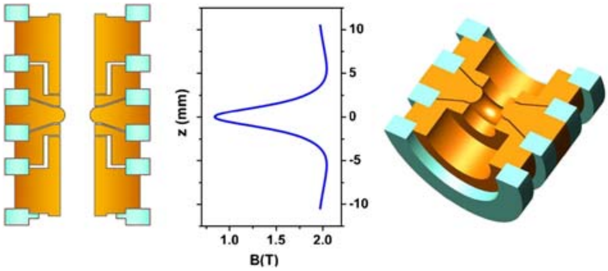

Fig. 1 Hybrid Penning trap design with toroidal ring electrode and achieved "magnetic bottle" in the 1.898 T homogeneous magnetic field [11]

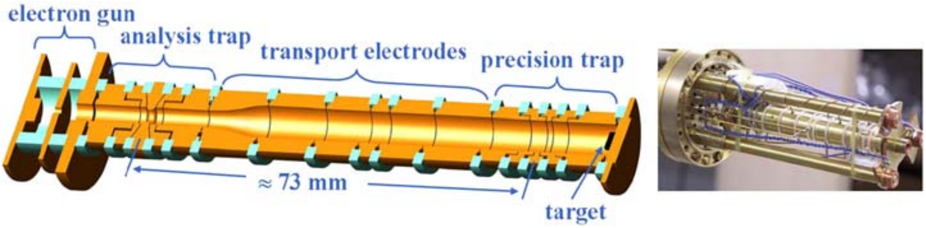

Fig. 2 The double Penning trap. *Left*: technical drawing of the stack of electrodes. *Right*: Picture of the assembled trap

3.1 Measurement procedure with the double Penning trap

In Fig. 2 the complete electrode stack of the double Penning trap setup is shown. The two traps are separated by transport electrodes to about 73 mm, because otherwise the magnetic inhomogeneity would severely limit the measurement accuracy. The eigenfrequencies of the proton are measured in the precision trap in which the magnetic field is spatially homogeneous. By calculating the free cyclotron frequency ω_c the magnetic field strength at the position of the proton can be determined. At the same time spin transitions are induced by applying a radio-frequency field in the range of the proton Larmor frequency $\nu_L \cong 80$ MHz. Therefore fluctuations in the magnetic field can be cancelled to a high degree in the ratio of ω_L/ω_c. The spin state is detected at the strong inhomogeneous magnetic field of the analysis trap. Thus the proton has to be transported adiabatically between both traps. This process is repeated for different values of ν_L. The Larmor frequency can be obtained from the maximum of the spin-flip probability vs. the frequency of the excitation field.

4 The experimental setup

The experiment is performed in a closed setup at cryogenic temperatures yielding extremely low background pressure, which allows long storage times of the proton in the trap. The cryogenic environment also allows for increasing the signal-to-noise ratio. Using a low vibration Gifford MacMahon pulse tube cooler a temperature of 3.9 K is achieved at the position of the trap. The protons are created by electron impact ionization inside the trap chamber—a sealed system consisting of the trap

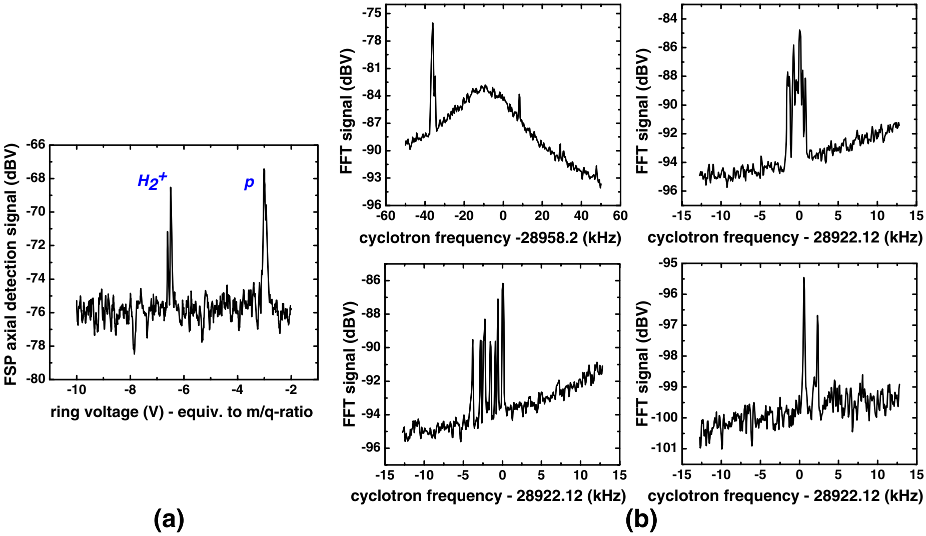

Fig. 3 **a** Mass spectrum: particle clouds in resonance with the axial tank circuit at $\nu_z = 688.18$ kHz. **b** Single proton preparation: by multiple excitation of cyclotron motion individual protons ca be removed from the trap. In the *first graph* one can see the noise spectrum of the resonance circuit with the proton cyclotron signal. In the *lower right graph* the individual particles can be recognized

electrodes, a black polyethylene target, a field emission electron source and the cyclotron detection system located outside the trap region.

The highly sensitive detection system basically consists of a helical resonator with a high Q-value ($Q = 5500$ for the superconducting axial resonatror and $Q = 1000$ for the cyclotron resonator), a low noise cryogenic FET amplifier ($e_n = 1.9$ nV/$\sqrt{\text{Hz}}$ for axial and $e_n = 0.7$ nV/$\sqrt{\text{Hz}}$ for cyclotron) and a room temperature amplifier. The voltage supply for the trap electrodes, the UM 1-14 from the company Stahl Electronics, with a stability of 1 ppm/K constitutes an extremely stable and accurate voltage source, a fundamental requirement for the ultrahigh precision measurement proposed in this work.

5 Preliminary results

In Fig. 3 some of our first results are shown. A mass spectrum of particle clouds measured right after loading the trap is presented. The ions are put in resonance with the axial tank circuit by varying the voltage applied to the ring electrode in the precision trap. The undesired species, products of the in-trap ion creation process, are removed from the trap by applying a strong dipolar excitation at the axial frequencies of the ions and lowering the trap potential.

The resulting proton cloud is successively reduced up to one single proton by exciting the cyclotron motion in a detuned trap. By setting a not optimized tuning ratio the potential in (1) becomes anharmonic which means that the frequency of the ion will not be independent of its energy. Thus with a broadband excitation one can spread the ion cloud and then remove the protons individually with a stronger and well directed further excitation. The storage time of the isolated proton in the trap is longer than several weeks.

6 Conclusions and outlook

We presented an overview of an experiment for the first direct high-precision measurement of the magnetic moment of the proton, just as the first successful preparation and confinement of a single proton in a Penning trap. Further progress is being made towards the optimization of the potential in the precision trap as well as the adiabatic transport to the analysis trap. The next crucial step consists in successfully inducing spinflip transitions and detecting the axial frequency shift in the magnetic field inhomogeneity of the analysis trap. In the future the same measurement will be performed on an antiproton at the present Antiproton Decelerator (AD) ring at CERN or at the low-energy antiproton area at the future GSI accelerator complex FLAIR.

Acknowledgements Financial support by the Programme Alan, the European Union Programme of High Level Scholarships for Latin America, scholarship no. (E06D101305BR), the DFG-QU122/3 and the Helmholtz Association for National Research Centers (HGF) under contract VH-NG-037 is acknowledged.

References

1. Quint, W., Alonso, J., Djekic, S., Kluge, H.J., Stahl, S., Valenzuela, T., Verdu, J., Vogel, M., Werth, G.: Nucl. Instrum. Methods Phys. Res., B **214**, 207 (2004)
2. Frisch, R., Stern, O.: Zeitsch. Phys. A **85**, 4 (1933)
3. Bloch, F., Hansen, W.W., Packard, M.: Phys. Rev. **70**, 474 (1946)
4. Collington, D.J., Dellis, A.N., Sanders, H., Turberfield, K.C.: Phys. Rev. **99**, 1622 (1955)
5. Winkler, P.F., Kleppner, D., Myint, T., Walther, F.G.: Phys. Rev., A **5**, 83 (1972)
6. Kreissl, A., Hancock, A.D., Koch, H., Köhler, T., Poth, H., Raich, U., Rohmann, D., Wolf, A.: Zeitsch. Phys., C **37**, 557 (1988)
7. Bakalov, D., Widmann, E.: Phys. Rev., A **76**, 012512 (2007)
8. Verdu, J., Kreim, S., Alonso, J., Blaum, K., Djekic, S., Quint, W., Stahl, S., Ulmer, S., Vogel, M., Walz, J., Werth, G.: Proc. of the LEAP Conference **796**, 260 (2005)
9. Brown, L.S., Gabrielse, G.: Rev. Mod. Phys. **58**, 233 (1986)
10. Gabrielse, G., Haarsma, L., Rolston, S.: Int. J. Mass Spectrom. Ion Process. **88**, 319 (1989)
11. Verdu, J., Kreim, S., Blaum, K., Kracke, H., Quint, W., Ulmer, S., Walz, J.: New J. Phys. **10**, 103009 (2008)
12. Häffner, H., Beier, T., Djekic, S., Hermanspahn, N., Kluge, H.J., Quint, W., Stahl, S., Verdu, J., Valenzuela, T., Werth, G.: Eur. Phys. J., D **22**, 163 (2003)
13. Dehmelt, H.G., Walls, F.L.: Phys. Rev. Lett. **21**, 127 (1968)
14. Brown, L.S., Gabrielse, G.: Phys. Rev., A **25**, 2423 (1982)
15. Dehmelt, H.G.: Proc. Natl. Acad. Sci. USA **83**, 2291 (1986)