Simulated Negotiation Outcomes Through Recommendation Crowding

Tomasz Szapiro · Przemysław Szufel

Abstract The goal of the paper is to propose a method for finding outcomes of negotiations of multiple parties with a multi-criteria decision outcomes evaluation. Firstly, the procedure for reduction of continuous set of options to large but finite crowd of compromise recommendations is presented. Next, the crowd is integrated to a single benchmark which is treated as a proposal of artificial auxiliary player called a lone wolf as presented by Kersten and Szapiro (Proceedings of the international conference of the society of systems and science. Budapest, 1987). The strategy of negotiation with this artificial player is proposed to generate a benchmark evaluations scheme for real negotiation. Finally, a multi-agent simulation approach for multi-lateral negotiation process modeling is presented in order to estimate possible negotiation outcomes.

Keywords Negotiation support procedure · Reference point · Multi-agent simulation · Negotiation strategy

1 Introduction

Outcomes of negotiations are constant challenge for researchers of market economics since they determine transactions' results. The most natural approach—the game theory—requires to abandon attempts to model dynamics but investigates states of equilibrium. If an equilibrium exists then other states appear unstable—conflicting goals of players can be satisfied achieving higher than current utilities of players. This classical view inspired L. Hurwicz to question the silent assumption on stability of economic mechanism creating players' payoffs. In his seminal papers, Hurwicz raised

T. Szapiro · P. Szufel (✉)
Division of Decision Analysis and Support, Warsaw School of Economics, Warsaw, Poland
e-mail: pszufe@gmail.com

two questions—on optimality of economic mechanism with respect to social goals and on incentives for players to behave accordingly to their rationalities and achieve social goals.

Sound theoretical results of game theory and of mechanism design theory deepen our understanding of markets but do not provide applicable recipes for players. One can prove existence of analytical solutions of equations describing future states, but they are hardly computable and difficult to calibrate using real data. Wider discussion of this issue is presented in survey of Axtell (2000) who advocates another approach—multi-agent simulation (MAS). The MAS is based on modeling heterogeneously described populations (including relations among individuals) in order analyze social and economic behaviors of individuals and populations. This approach—provides skyrocketing number of applications when describing measurable features of individuals (agents) and their interactions. From point of view of negotiation modeling this approach has important omission—it doesn't model conflicts of agents' rationalities.

The aim of the paper is to provide benchmarks for players in negotiation. The concept of benchmark under consideration is similarly to equilibrium in game theory, but it results from modeling dynamics of conflict resolution. The benchmarks are constructed using assumptions on strategies of players thus simulation of artificial players' decisions leads to results—compromise or fallacy of negotiation process. Knowledge of the of benchmarks allows player to locate the current position of other players in resulting frame and verify and evaluate her own assumptions on unknown strategies of others.

Each benchmarks is obtained through iterations simulating possible course of negotiation as a way to forecast outcomes of negotiations. One considers all courses of negotiations arriving thus to set of benchmarks. Different benchmarks can be viewed either as result of different changes of preference of one player or as result of constant preference of different players. We assume the later perspective and consider increasing (in each round of negotiation) population of artificial players. Involvement of growing number of negotiation courses into consideration corresponds to observation of growing number artificial players. The dynamics of negotiation assumes use of procedure bireference interactive procedure (BIP) to provide each such player with compromise recommendation. Consequently, in paper the term "recommendation crowding" refers to this process.

Hence, the direct objective of the paper—creation of benchmarks - requires computational tool capable to follow all possible courses of negotiation using the knowledge preference dynamics and to use this tool to forecast outcomes of negotiations.

In this paper the importance of distinguished states is appreciated like in game theoretical argument, however unstable dynamics of preference is included to the model. On the other hand, heterogeneity of MAS inspired the presented approach to consider simultaneously all possible subjective negotiation starting points and preference changes. Thus we crowd negotiation recommendations through introducing artificial, auxiliary players who are related with all possible outcomes of negotiations with the different starting point and preference changes. Hence the dynamics of preference is described using a control variable elicited from players. The knowledge on dependence of their reactions to current situations enables simulation of results of a negotiation process.

The presented approach offers general procedure to present possible results of negotiations. To perform the simulations one has to assume some arbitrarily accepted assumptions. Without loss of generality, for the sake of clarity of presentation, in the paper, a simplified version of assumptions is used. The procedure provides players with interpretable benchmarks which can influence the way of persuading, communicating and submitting statements on real compromise proposals.

An analysis of a multidimensional negotiation process here is restricted to intersection of couple only perspectives as in e.g. Harvard Business School Press (2000), Kilgour and Eden (2010) and Zartman (1993). Firstly, it is a theoretical perspective. While there exist in practice processes which possess features described here, we do not address such specific processes but consider the general problem which is defined by a collection of features which are treated as axioms. Secondly, we assume the support scenario where the interaction among parties is assisted by negotiation analysts who provide parties with recommendations. A party in negotiation is not obliged to use a recommendation since an optimality concept is used for the technical data processing in partial individual decisions and not as a rationality principle. Thirdly we assume, that the problem is complex enough to justify a formal description which is needed to for a computer supported recommendation construction and that parties agree to use such framework.

The support procedure considered in the paper assumes four axioms: bounded rationality of players, asymmetry of information, agreement of players on use of computer support as mediating tool providing reference as presented by Polak and Szapiro (1997, 2000), and finally—MCDM-type framing of the negotiation problem.

More precisely, the bounded rationality axiom (Simon 1986) means that players can be satisfied with solution if optimization fails. The asymmetry of information axiom means that players do not have knowledge on other players' utilities and criteria. The agreement of players on use of computer support as mediating tool axiom means that players accept use of the negoBIP method (see Szapiro 1997 and Kuszewski et al. 1998) to support group decision making and the role of mediator is played by computer software, see e.g. Szapiro and Matysiewicz (2001). Finally, the MCDM-type framing of the negotiation problem axiom means that decisions can be characterized by vectors and criteria—by linear vector valued mappings, while individual preferences are described by cone dominance (Kersten and Szapiro 1986) .

Usually there are considered three modes for assistance of negotiation parties. In the arbitrage mode the gain for parties results from the fact that the outcome of a negotiation provided by an arbiter halts costs of the negotiation process. In the advising mode advantage of advisor's competence over party's one, results in a better data processing and a resultant outcome. In the mediator mode, the advantage raise from the previous reason, but also is amplified due to use of all parties data. In the latter case information in preserving confidentiality way and only proposal of compromise is recommended. In the paper we consider the mediator perspective. Since the considered procedure can be used also in advisor mode and the presentation is simpler in this case, we start with advisor mode and expand this for mediator setting.

Next, in the procedure a population of recommendations is created. These recommendations for each party are constructed as sets of all possible reactions to offers of other parties. They are called individual crowds. Since the complexity of the crowd

Fig. 1 The dynamics of negotiation can be simulated through a process which involves two subsequent aggregations. In the first one, virtual answers of one party is are aggregated, while the second considers positions of different parties. Obtained in this way reference point (Lone Wolf) is considered to represent adversary to negotiate with
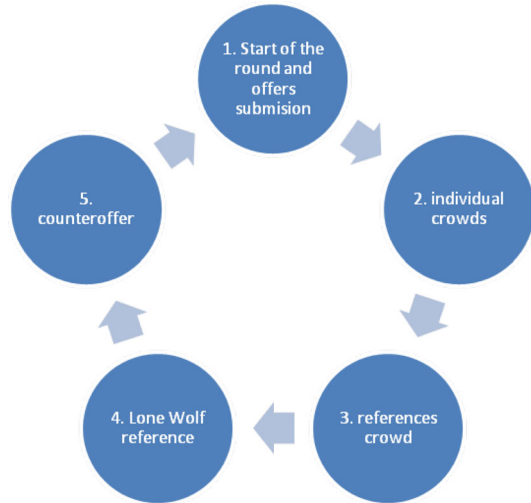
impedes optimization, the party under consideration is assumed to limit analytic rationality and simplify her model of situation. The simplification has dual characteristics. Firstly, for each party the individual crowd is perceived as an aggregated offer evaluation that balances all potential recommendations. Secondly, these aggregated individual evaluations are integrated in one which is treated as an offer evaluation of an auxiliary adversary called Lone Wolf. Given individual recommendation crowds, in each round the Lone Wolf reference points are generated to provide recommendation. The Lone Wolf represents alone the resultant position of all adversaries. The negotiation is thus reduced to two party bargaining.

The negotiation therefore is transformed to sequence of interaction loops, where in each loop consists of the phases shown on the Fig. 1.

In the next section the mathematical representation of negotiation is presented.

2 The Negotiation Model-Statics

The negotiation problem considered in the paper is owned by m parties who attempt to compromise on a selected from the common (for all parties) set of options the final compromise. To this end of all parties evaluate options and formulate offers.

We assume that options and evaluation can be expressed by mathematical objects. We assume that realistic options form the feasible decision set X which is nonempty, constrained by linear functions and thus it is convex.

Let m, $m \in N$, $m > 1$ be the number of negotiating parties decision makers (DM). Let x, $x \in X \subset R^n$ represent an offer (decision) that is evaluated by all negotiating parties. We assume that the decision variables are known by all negotiating parties in each round.

Let us moreover assume that m^i, $m^i \in N$, where $i = 1, \ldots, m$, is the number of outcomes evaluated by each negotiating party i, $i = 1, \ldots, m$. The number of
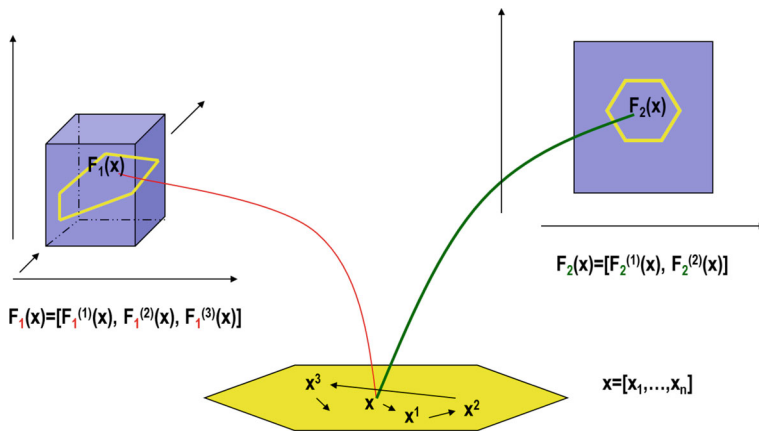
Fig. 2 A negotiation problem can be framed as an m-tuple of multiple criteria decision problems with the same decision space. Above, one considers two parties with three and two criteria, respectively

outcomes for each DM is not known by the other DM—but it is presented to the mediator. Let the mapping $f_i : X \rightarrow Y_i \subset R^{mi}$, here $y_i (x) = f_i (x) \in Y_i \subset R^{mi}$ represent outcomes for a particular decision x evaluated by the DM i. As mentioned earlier, the outcome functions are revealed only to the mediator they are linear by assumption (Fig. 2).

Let us assume that functions $y_i (x) = f_i (x) \in Y_i \subset R^{mi}$ representing outcomes for a particular decision x evaluated by the decision maker i are linear. Thus for i = 1, ..., m. we consider the m problems of maximization of the vector objective functions $\mathbf{f_i} : \mathbf{X} \rightarrow R^{mi}, f_i (x) = C_i x, \mathbf{C}_i \in M_{mi \times n}, \mathbf{C}_i = [c_{kl}]_{mi \times n}$ are matrices with m_i rows representing m_i criteria of the ith party.

Let us assume, that the feasible decision set X is linearly constrained and thus can be defined as:

$$\mathbf{X} = \left\{ \mathbf{x} \in \mathbf{R^k} \middle| \mathbf{Ax} \leq \mathbf{b} \right\}, \quad \mathbf{b} = \left[b^1, \dots, b^{m)} \right] \in \mathbf{R}, \mathbf{A} \in M(k \times n). \quad (1)$$

The option $\mathbf{x}'$ is preferred to $\mathbf{x}''$ if its evaluation $\mathbf{f}(\mathbf{x}')$ dominate $\mathbf{f}(\mathbf{x}'')$. The solution of problem of maximization requires determination the set $\mathbf{Y}_{ND}$ of non-dominated outcomes and is defined by the triplet $< \mathbf{A}, \mathbf{b}, \mathbf{C} >$.

The collections of goal functions $f_i (x)$, conveys all information known to parties on their preference, as the mappings fi define as above cone dominance—partial orders in outcomes spaces, see Yu (1985). Consequently there in no utility description. Parties negotiate a decision from the set of feasible decision x, $x \in X \subset R^n$ and try to find a compromise that will maximize their unknown utilities accordingly to their preference revealed by cone dominance in outcome space, i.e. if outcome fi(x') is dominated by fi(x'') then x'' is preferred to x', otherwise preference (utility) is not observable but it can be revealed in decisions of players which are made but not elicited.

The incomplete preference description forces us to consider interactive interviewing in search for control on decision making process.

In general, eight major goals have been discussed for using information technology to suport negotiations. These goals include: enabling asynchronous negotiations, offering advice, providing checklists, reducing transaction costs, providing a rationale for bargaining positions, structuring offers, managing negotiation data, and prescribing a negotiation process or protocol. To meet all of these goals it is necessary to create multiple subsystems in a negotiation support system (NSS).

On the other hand, process-oriented NSS focus on providing general support of the give-and-take process of negotiation (Chaudhury 1995; Kersten and Szapiro 1986).

3 The BiReference Procedure

There is a bundle of methods *Based on the assumption the that search for information should investigate subjective views of parties on their choices and should not phrased in technical terms*.

The lone wolf aggregation requires integration with a multi-criteria decision procedure meeting the following two assumption. Firstly, the procedure needs to be interactive—i.e. allowing DM to interactively input information in the decision process. The lone wolf aggregation enables to aggregate input from multiple negotiating parties. Hence, an aggregated information becomes the input for an interactive process. Secondly the interactive procedure must enable a DM in each step to select a decision from a finite set of available options. In each step of the negotiation process offers placed by negotiating parties are being aggregated to a decision that becomes the input for the interactive procedure.

Any multi-criteria decision support method meeting the above two criteria (interactivity and decision making from a finite set of possible choices) can be utilized to support for support of negotiation process with lone wolf aggregation. As an example implementation we use the BIP presented by Michalowski and Szapiro (1989) to this end (other procedures can be also used as the procedure has modular form—for interactive procedures with instructive comments see e.g. classical surveys of Evans 1984 and Laritchev and Nikiforov 1987). Thus using BIP Procedure one can generate an individual recommendation. Before we continue with idea of negotiation model description let us recall the BiP Procedure and apply it to the abovementioned negotiation context.

The BiReference Procedure uses the ideal point and the worst outcome as dual reference which determines improvement direction and which is displaced accordingly to DM requests. The DM controls this restructuring (set of feasible options, the ideal and worst outcomes) qualitatively through the criteria split in the group of those which are met, those which are to be improved and those to be worsened. Advantages of this procedure result from decrease of random influence of the structure of a problem on recommendations. It also allows preference reversals as presented by Reilly (1982).

The procedure starts with the definition of the vector of worst outcomes $\mathbf{y_W}(0) \in \mathbf{R^m}$, representing the pessimistic expectation of a DM. Moreover she defines the collection $\boldsymbol{\varepsilon} \in \mathbf{R^m}$ of tolerances which serve as indifference subjective criteria. We neglect here the index "i", labeling the decision maker in order to clarify the presentation.

In the rth iteration, $r \geq 1$, the BIP Procedure defines the improvement direction using the displaced worst outcome $\mathbf{y_W}$ (r) and the displaced recommendation $\mathbf{y_T}$ (r − 1) from the previous iteration, (if r = 1 the $\mathbf{y_T}(0) = \mathbf{y_U}$, where $\mathbf{y_U}$ is the ideal point of $\mathbf{Y}$, $\mathbf{y_U} = [y_U^1, \ldots, y_U^m]^T$ where $y_U^j = \max y^j|[y^1, \ldots, y^m]^T \in \mathbf{Y}, j \in \{1, \ldots, m\}$). The recommendation $\mathbf{y_T}$ (r − 1) is communicated to DM who states whether she is willing to keep, improve or worsen levels $\mathbf{y_T}$ (r − 1)^j representing partial goal achievement by components of the outcome evaluation $\mathbf{y_T}$ (r − 1). The split $\{I^+ (r), I^0 (r), I^- (r)\}$ of the set of all criteria in the rth iteration which corresponds to this information is denoted by π_r.

Given the split π_r (the change in preference when facing the recommendation) the procedure modifies problem structure (displaces the set of feasible evaluations, worst outcome and the ideal point). The resultant set of feasible outcomes is of the form:

$$\mathbf{Y_r} = \left\{ \mathbf{y} \in \mathbf{Y} | \forall j \in I^0 (r) \; y^j = y_T (r-1)^j \wedge \forall j \in I^+ (r) \; y^j \geq y_T (r-1)^j \right\}, \quad (2)$$

Its ideal point is consequently denote by $\mathbf{y_U}(r)$. The vector of worst outcomes is given as

$$y_w(r)^i = \begin{cases} y_T(r-1)^i & \text{dla } i \in I^0(r) \cup I^+(r) \\ y_w(0)^i & \text{dla } i \in I^-(r) \end{cases} \quad (3)$$

with natural interpretation. Displaced worst outcome and ideal define displaced improvement direction: $\mathbf{d}(r) = \mathbf{y_U}(r) - \mathbf{y_W}(r)$. The procedure is halted by the following stop rule:

$$\forall i \in \{1, \ldots, m\} \left| y_T (r)^i - y_T (r-1)^i \right| < \varepsilon^i. \quad (4)$$

Then the last trial solution $\mathbf{y_T}(r)$ becomes the recommendation.

In the two parties negotiation context, the natural definition of worst outcome is given as other party offer. The BIP Procedure serves as the vehicle to assist the DM who therefore is equipped with qualitatively expressed control variables—splits of criteria set.

4 The Negotiation Model-Dynamics Simulation

Splits of the set of outcome criteria are used to describe the dynamics model for the negotiation under consideration.

4.1 The Recommendation Crowding

Using BIP Procedure to assist parties one can simulate all negotiation trajectories. To this end let us consider for the party "i" the space Π_i of all triplets consisting of three subsets of the set of criteria indices $I = \{1, \ldots, m_i\}$ splits $\pi = \{I^+, I^0, I^-\}$ of the set of all criteria. Let us also consider the set $S(\Pi_i)$ of all sequences $\{\pi_r\}$ consisting

of elements of π_r, $\pi_r \in \Pi_i$, $r \in N$ (natural numbers). The space $S(\Pi_i)$ consist of many artificial negotiation trajectories. Let's consider the subspace $(P\Pi_i) = P_i$ of the space $S(\Pi_i)$ which consists of elements which are constant from some place (stop rule effect) and satisfy other conditions characterizing negotiation processes compatible with BIP support. The elements the subspace P_i are called protocols as they describe the whole process of decision making of the ith party.

Although some general theorems on existence of convergent protocols can be shown, the analytical investigation of the space of protocols and providing descriptions of compromises and strategies is difficult since the space itself is complicated $(S\Pi_i)$, see Szapiro (1993). For that reason, in order to analyze negotiation strategies we simulate all negotiation i.e. we generate the whole space of feasible protocols and negotiation outcomes. This procedure is further referred to as *individual recommendations' crowding*.

The crowd of recommendations which we obtain in this way is too complex for informal human analyses, therefore—accordingly to bounded rationality assumption—we simplify the information and arrive to the frame of the aggregate of crowding recommendations, see Tversky and Kahneman (1985).

There are several methods to build the above mentioned aggregate. We follow the *firm negotiation rule* which recommends slow concessions and only with respect one issue simultaneously. Thus we consider the space of P^f of *firm protocols* (which correspond to concessions with respect to one only objective—the first order concessions). Firm protocols by definition are of the form $\pi = \{I^+, I^0, I^-\}$, where I^- consists of one element only, I^+ is empty and thus I^0 consists of remaining indices of objectives. Firm protocols describe all permutations of concessions with respect to one objective made in belief that others will proceed in the same way.

In the simulation of party "1" dynamics, we assume that for each of her firm protocol, her evaluations $y^1(x_r^i)$ of other parties offers x_r^i, $i \neq 1$, are considered to be worst outcomes and a crowd of recommendations $x_\pi^1(x_r^i)$ is generated following the BIP Procedure. Next, the party "1" submits the counteroffer x_{r+1}^1 which is defined as randomly selected option the non-dominated outcome in the set $F_1(X^{crowd})$ where $X^{crowd} = \{x_\pi^1(x_r^i) : \pi \in P^f\}$. Next this procedure is performed for other parties and we arrive to the set of all counteroffers. The set of counteroffers is then transformed with use of firm protocols (excluding the one just used) into the next set of crowds and provide next counteroffers. We proceed until the last firm protocol is used.

Theorem 4.1 *The measure of the convex span of counteroffers obtained with use of firm protocols decreases compared to initial set of offers.*

The proof results from the fact that concessions increase the feasible regions and non-dominated outcomes of different parties become closer.

Conclusion 4.1 *If the measure of the convex span $Z = Conv\_span(x_{\pi\ r}^i)$ is smaller, then $|\varepsilon|^m$ then the procedure is halted since differences do not exceed tolerance and any member of this set is accepted by stop rule.*

If the condition from the Conclusion 4.1 cannot be met then second order concessions are to be made. When for all protocols the condition is not met then the procedure cannot provide solution recommendation.

4.2 Lone Wolf Aggregation

If processing of the crowding procedure does not lead to the result, then the aggregation of the second type (Lone Wolf aggregation) is performed. In this case, counter-offers are aggregated in the outcome space to the center of gravity y^{LW} of the span Z. This outcome is said to represent the Lone Wolf strategy which represents the result of forming coalition of adversaries of the party labeled "1" and presenting by them a common counteroffer x^{LW} evaluated as y^{LW} by the party under consideration.

The interval $B = [y^1, y^{LW}]$ represents then the bargaining area and the power proportion can approximated as the quotient $\gamma = {}^1/m - 1$. Thus the offer reflecting the concession of the party "1" splits at a proportion γ the interval B.

Theorem 4.2 *Let us assume that the measure of the convex span of counteroffers does not satisfy the condition from the Conclusion 4.1 and Lone Wolf Aggregation was used. Then the procedure is halted in a finite number of steps.*

The proof results from monotonicity of lengths of bargaining intervals which converge to point and therefore fire the stop rule.

4.3 The Algorithm

The following simulation algorithm is proposed:

1. The parties agree on the feasible decision set X, $X \subset R^n$ (not evaluating decision outcomes at this point).
2. Each party constructs her multi-criteria decision evaluation mappings (individual outcome functions) and submits them to mediator (computer system).
3. The mediator creates an aggregated multi-criteria decision problems using outcome functions of all negotiating parties.
4. The mediator determines the ideal and the worst outcomes by maximizing and minimizing respectively individual functions $y_i(x)$, $i = 1, \ldots, m$.
5. The individual improvement directions (defined by ideal and worst outcomes) and a feasible trial solution are determined.
6. Negotiating party's behavior is simulated iteratively according to procedure described above i.e. for all parties the ideal point, worst outcome and improvement direction are displaced based on protocols which simulate decisions on one step concessions until recommendation is achieved.
7. Crowd of recommendation and their counteroffers is generated
8. If the stop rule halts then the negotiation compromise is achieved.
9. Otherwise a coalition—Lone Wolf—is considered aggregated evaluation of adversaries offers. This aggregate is used to simulate two party negotiation process based on split of bargaining area in proportion to power of both parties.
10. A simulation of *lone wolf strategy* with parameter γ (*resistance*) is used to create a reference curve x_γ, which is presented to players as the reference to evaluate current offers and counteroffers to supported parties.

The algorithm uses Bi-Reference Procedure as presented by Michałowski and Szapiro (1992) to simulate the individual decision making processes and the Lone Wolf strategy in simulation of negotiation dynamics.

4.4 Practical Applications—Simulated Negotiation Dynamics

The goal of this section is to present practical applications of the lone wolf approach. The results presented in this have been obtained through numerical simulation. We analyze group negotiations with three parties and a mediator. We present how the lone aggregation can be utilized to analyze possible negotiation trajectories. The first simulation (Sect. 4.4.1) has been implemented as a spreadsheet model, while the second simulation (Sect. 4.4.2) has been implemented as multi-agent negotiation model with the MASON library in Java (see Luke et al. 2004). We take the point of view of the first negotiator who decision point

4.4.1 Three Decision Variables and a Single Criteria

The goal of this subchapter is to present Lone Wolf aggregation in a simulated negotiation process.

Let's consider a company offering information technology outsourcing services. The company has three independent branches based in three different European countries. All three branches share a common budget for internet advertising and web site positioning. However in each of the three countries a different language is used and a separate search engine optimization is required.

Branch directors are discussing on how the advertising budget should be divided between them. Let x_1, x_2, x_3, $x_i \in [0, 1]$, be the budget shares for the first branches for branches 1, 2 and 3 respectively. Hence, three decision variables are considered x_1, x_2, x_3 such as $x_i \in [0, 1]$ and $x_1 + x_2 + x_3 = 1$.

Each branch of the considered outsourcing company differently benefits from the advertising budget. Let $y_1 = a_1 x_1$, $y_2 = a_2 x_2$ and $y_3 = a_3 x_3$ represent benefits of each of the branches i, $i = 1, 2, 3$ arising from an advertising campaign. For simplicity it has been assumed that each negotiating party has only one criteria evaluation for a given negotiation outcome.

In each negotiation round r, $r \in N$ each party i, $i = 1, 2, 3$ makes a compromise offer $x^{(i)}(r) = \left[x_1^{(i)}(r), x_2^{(i)}(r), x_3^{(i)}(r) \right]^{\mathrm{T}}$ where $x_1^{(i)}(r)$, $x_2^{(i)}(r)$, $x_3^{(i)}(r)$ are offers of the budget share for the first, second and third player respectively.

We analyze the negotiation from the perspective of the player I. Let's assume that the player one can start with one of three initial bids: low, medium and high being, where for the purposes of numerical analysis $x^{(1)}(1)_{\mathrm{LOW}} = [0.4, 0.3, 0.3]^{\mathrm{T}}$, $x^{(1)}(1)_{\mathrm{MEDIUM}} = [0.6, 0.2, 0.2]^{\mathrm{T}}$ and $x^{(1)}(1)_{\mathrm{HIGH}} = [0.8, 0.1, 0.1]^{\mathrm{T}}$. After the proposal the first player is made each of opponents can makes a counter proposal. The counter proposals can be again be one of three natures : low, medium and high.

Let's assume that in each negotiation opponents (2 and 3) use the offering strategy (they both place an offer of either LOW, MEDIUM or HIGH variety. The player

I does not know what counter offers will be played by the opponents. Hence, player I creates artificial opponents pairs $(x^{(2)}(1)_{LOW}, x^{(3)}(1)_{LOW})$, $(x^{(2)}(1)_{MEDIUM}, x^{(3)}(1)_{MEDIUM})$, $(x^{(2)}(1)_{HIGH}, x^{(3)}(1)_{HIGH})$. Next, the player 1 performs a lone wolf (LW) aggregation for each opponent pair considering three possible biddings (low, medium and high):

$$
\begin{aligned}
x^{LW}(r)_{LOW} &= \Big[(x_1^{(2)}(r)_{LOW} + x_1^{(3)}(r)_{LOW})/2, (x_2^{(2)}(r)_{LOW} \\
&\quad + x_2^{(3)}(r)_{LOW})/2, (x_3^{(2)}(r)_{LOW} + x_3^{(3)}(r)_{LOW})/2 \Big]^{T}, \\
x^{LW}(r)_{MEDIUM} &= \Big[(x_1^{(2)}(r)_{MEDIUM} + x_1^{(3)}(r)_{MEDIUM})/2, (x_2^{(2)}(r)_{MEDIUM} \\
&\quad + x_2^{(3)}(r)_{MEDIUM})/2, (x_3^{(2)}(r)_{MEDIUM} + x_3^{(3)}(r)_{MEDIUM})/2 \Big]^{T}, \\
x^{LW}(r)_{HIGH} &= \Big[(x_1^{(2)}(r)_{HIGH} + x_1^{(3)}(r)_{HIGH})/2, (x_2^{(2)}(r)_{HIGH} \\
&\quad + x_2^{(3)}(r)_{HIGH})/2, (x_3^{(2)}(r)_{HIGH} + x_3^{(3)}(r)_{MEDIUM})/2 \Big]^{T},
\end{aligned}
$$

In result in the first step we have three possible trajectories for the negotiation. Subsequently we assume that player's I decision are deterministic—i.e. calculated as

$$
\begin{aligned}
x^{(1)}(r) &= \Big[(x_1^{LW}(r-1) + x_1^{(1)}(r-1))/2, (x_2^{LW}(r-1) \\
&\quad + x_2^{(1)}(r-1))/2, (x_3^{LW}(r-1) + x_3^{(1)}(r-1))/2 \Big]^{T},
\end{aligned}
$$

while offers placed by the opponents are being treated as unknown – and for each subsequent offer again three trajectories are analyzed. Hence, at a the step r there are 3^r possible simulation trajectories.

The Fig. 3. presents compromises achieved in six steps depending on different initial offers by the player I. The presented analyses can be extended by adding probabilities of making LOW, MEDIUM and HIGH bids by players 1 and 2.

If we assume that all bids are equally possible (i.e. $p_{LOW} = p_{MEDIUM} = p_{HIGH} = 1/3$) than the probabilities of negotiation outcome for the player I presents as on Fig. 4a). If we rather assume that opponents are more likely to request more for them (i.e. $p_{LOW} = 0.2 \, p_{MEDIUM} = 0.2 \, p_{HIGH} = 0.6$) than the distribution of probability of negotiation outcomes looks like presented on the Fig. 4b). Finally if the opponents are very reluctant to compromise (i.e. $p_{LOW} = 0.05 \, p_{MEDIUM} = 0.1 \, p_{HIGH} = 0.85$) than the negotiation result might be much less profitable for the Player I as presented in the Fig. 4c).

4.4.2 Two Decision Variables and a Multiple Criteria

The goal of this Section is to propose a simulation scenario that allows analyses possible negotiation outcomes. The proposed negotiation dynamics simulation scenario will be based on the multi-agent approach (e.g. Axelrod 1997; Macal and North 2006).
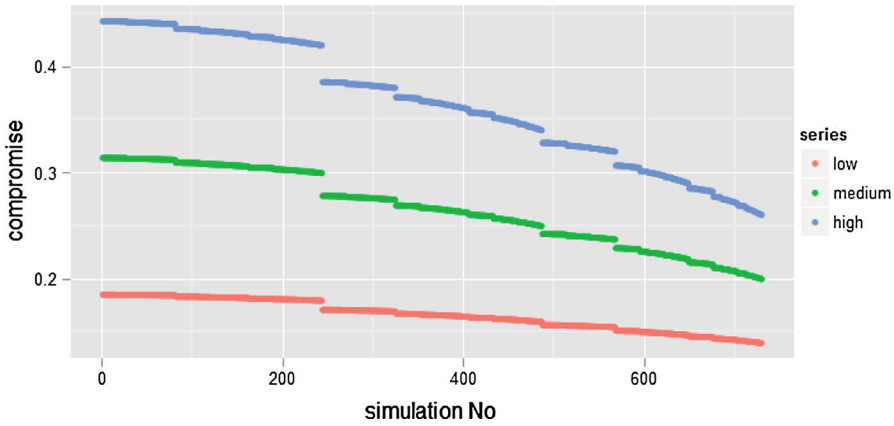
Fig. 3 Each series represents $3^6 = 729$ possible simulation outcomes. The outcomes (simulation numbers) are sorted by opponents willingness to compromise. The final compromise strongly depends on the first bid made by the negotiator 1
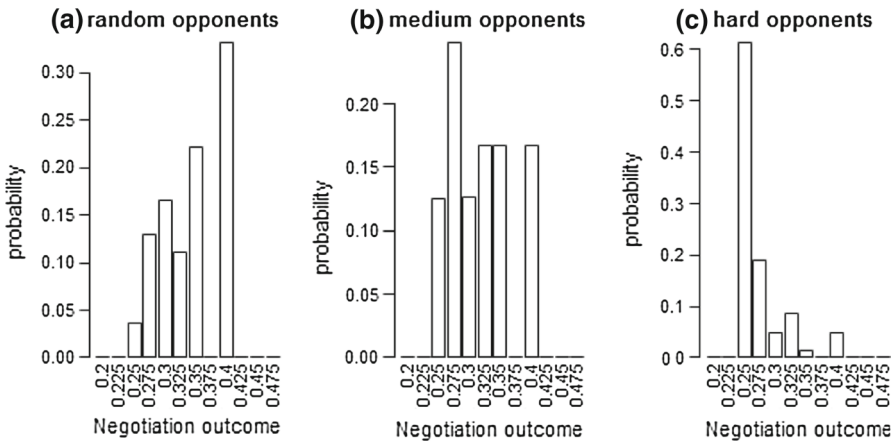
Fig. 4 Simulated distribution of possible negotiation outcomes from the point of view of negotiator 1. The outcomes depend on whether opponents—players 2 and 3 make offers randomly or aggressively

After justifying the need for multi-agent approach we propose a scenario for an experiment scenario that will allow to benchmark the full-search approach versus simulation approach.

Let us consider three companies interested in running for a government project. The has high formal requirements regarding the annual income and number of employees. None of the considered companies can apply independently for the contract. However, the companies can form a consortium. The consortium will be big enough to apply for the public tender and will have a reasonable chance to place a winning bid. Completing the project requires (among other tasks) to provide the government customer with a tailor made software product. However, in the public tender announcement the software requirements have not been described in enough detail. The consortium's

participants need to decide what information to place in their offer—however each of three participants has slightly different goals.

Two decision variables are considered x_1—number of months devoted to product development and x_2—number of features to be implemented in the software product. The consortium participants have agreed that the product needs to be delivered in no later than 25 months ($x_1 \leq 25$). Due to communication issues within the consortium the software product cannot be delivered sooner than in 10 months ($x_1 \geq 10$). The minimum number of features that will make the bid acceptable for the government customer is 5 ($x_2 \geq 5$). However as the bid price will depend on the number of implemented features the maximum number of features is 20 ($x_2 \leq 20$). Implementing each additional feature over 10 requires to extend the project timeline for at least half month ($2x_1 - x_2 \geq 10$). Each feature and each month of development increases project's cost and there is a budget constraint ($x_1 + x_2 \leq 40$). For the purposes of numerical feasible decision set has been discretized and is presented on the Fig. 6a.

We assume that the negotiator 1 considers three outcomes of the bid y_1, y_2, y_3 where each outcome can be presented by a linear function. Firstly, the negotiator being a software development company is interested in a large amount of features having an income from each feature ($y_1 = 4x_2$, $y_1 \rightarrow$ max). Secondly, extending the project timeline influences the profit where the profit can be calculated as ($y_2 = -2x_1 + 5x_2$, $y_2 \rightarrow$ max). Lastly, the project's risk depends on number of features and decreases with the project timeline ($y_3 = -3x_1 + x_2$, $y_3 \rightarrow$ min).

In the presented algorithm as it was stated earlier at each step each negotiating party can make three decisions regarding a negotiated criteria $\pi = \{I^+, I^0, I^-\}$. It means that, assuming that at each step negotiating party is agreeing to worsen at least one criteria, at each negotiation turn 3^{n-m} possible decisions can be made, where n is the number of criteria and m is the number of negotiating parties. Requirement for simulation of several negotiation steps further adds substantial computational complexity and makes the problem computationally infeasible when a large number decision criteria needs to be analyzed.

For supporting negotiation problems with a large number of dimensions *n* describing offer evaluations we propose a multi-agent approach. The multi-agent approach is a way of a system simulation where the system is divided into independent elements—agents. Macal and North (2006) define an agent as an independent entity in an environment for which goals can be specified and that react to environmental changes to satisfy those goals.

We postulate that in the multi-agent model each negotiating party is represented, while the NSS is modeled as an environment. The mediator is also modeled as a part of the environment. Agent's goals can be defined as utility maximization. We assume that for each simulation run, each agent has a set of linear goal functions with parameters randomly chosen for each simulation run. In the negotiation environment an agent reacts to offers placed by other agents and makes decisions leading to maximize its utility function—thus for each negotiation step decisions by each agents are made. The negotiating agents make their decisions in subsequent negotiation steps (see Fig. 3). The final result of simulated negotiation process becomes a part of simulated crowded values.

The presented scenario is tested for 3 negotiating parties. In order to test the two presented simulation scenarios (full-search and multi-agent) firstly for each negotiating party a number of criteria (2 or 3) is randomly chosen and goal functions are generated. Secondly a discrete random set of feasible decisions is generated. For the generated simulation environment simulations can be carried out and allow to compare the two presented approaches.

For each simulation a feasible decision space X, $X \subset R^2$ is generated. Decision space is represented as a set of points with integer values within a hexagon with side length of 8 (see Fig. 4a).

For each of negotiators goal function parameters C_1, C_2, C_3 are randomly generated where linear goal functions of negotiators can be presented as $Y_1 = C_1x$, $Y_2 = C_2x$, $Y_3 = C_3x$. It is assumed that the first negotiator has three goal functions, the second has four goal functions and the third has five goal functions. Under above assumptions for each negotiator her goal functions can be represented as gradients in decision space X, $X \subset R^2$. Figure 4b represents example set of goal function gradients for each negotiating party.

In the simulation it is assumed that negotiating parties present their goal functions to the mediator. After the goals are presented, the mediator aggregates goal functions calculates the initial trial solution that takes into consideration the goal functions of all negotiating parties. This process of achieving the initial trial solution consists of three steps. Firstly, the goal multi-criteria goal functions Y_1 Y_2, Y_3 are aggregated and a new mult-criteria goal function Y_{Agg} is created where $Y_{Agg} = [Y_1 Y_2 Y_3]^T$. In the presented simulation scenario the considered aggregated goal function has $m = 3 + 4 + 5 = 12$ outcomes i.e. $Y_{Agg} \in \mathbf{R}^m$. Secondly, the ideal and worst points are calculated in the outcome space $\mathbf{R}^m$. As described in the BIP method the ideal point y_u, $y_u \in \mathbf{R}^m$ is calculated by independently maximizing each of m goal functions in regard to feasible decision set X, while the worst point y_w, $y_w \in \mathbf{R}^m$ is calculated by minimizing those functions. Thirdly, the trial solution y_t, $y_t \in \mathbf{R}^m$ is calculated as the point within feasible decision set that is as close as possible to ideal outcome y_u and as far as possible from worst outcome y_w. More specifically a difference between Euclidean distance from y_t to y_u and Euclidean distance from y_t to y_w is minimized i.e. $y_t = arg\,min_{y*}(d(y*, y_u) - d(y*, y_w))$, where $d(a, b)$ is Euclidean distance between a and b. The trial solution can be also represented in decision space as $x_t \in X, X \subset R^2$. The feasible decision set is assumed to be concave (in the presented example it is a hexagon) and therefore the trial solution and subsequent solution lie on one of the feasible decision set's borders as in Fig. 4c.

The software implementation of simulation algorithm can be divided into two parts: setup (model preparation) and simulation, that are portrayed respectively in Figs. 5 and 6.

The model setup includes generation of its parameters and performing pre-calculations (see Fig. 5). Parameters include a feasible decision set (that is constant across simulations) and negotiators' goal functions that are different in each simulation run. The pre-calculations are performed in order to speed-up the simulation process. As it was mentioned earlier the feasible decision set is represented as a hexagon for all simulations (as in Fig. 4a). The goal function parameters are randomly generated as integer numbers from the range $< -8; 8 >$. Afterwards the values of goal
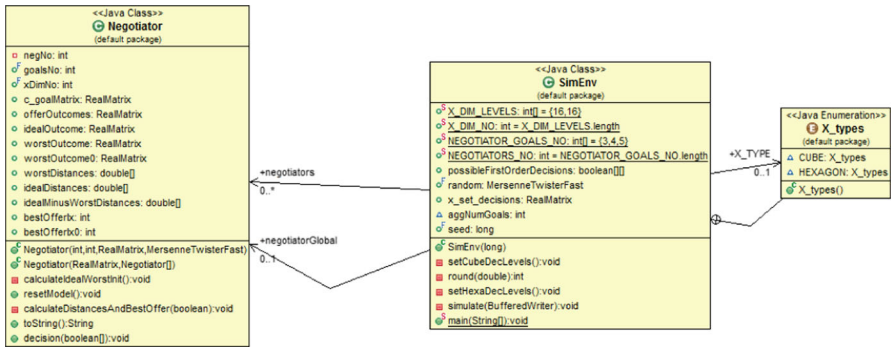
Fig. 5 Representation of a discrete feasible decision set, randomly generated goal functions and an initial trial solution (compromise) in a simulated negotiation process dynamics
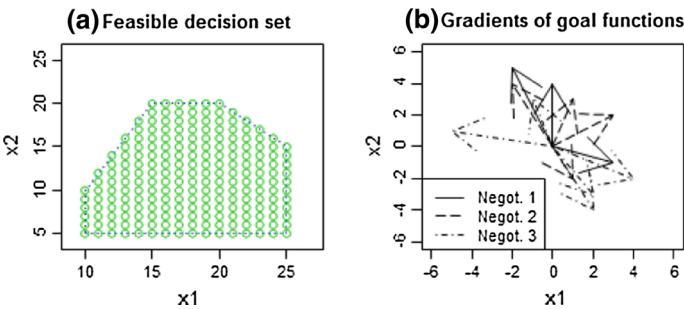
Fig. 6 The multi-agent simulation model consists of one agent class—Negotiating party

functions for each negotiator are calculated across feasible decision set. This allows later for faster simulation processing as the results of these calculations are used several times.

After the simulation model is set up—possible simulation process trajectories are analyzed. It is assumed that at each point each negotiating party is presented a current trial solution y_T and can make one of two decisions regarding each criteria: (1) worsen the criteria (i.e. make a concession) or (2) keep the criteria at the same (or higher) level. Further in the simulation it is assumed that in each negotiating step each party makes exactly one concession (regarding one criteria) while requesting to keep other criteria at the same level. As it was assumed that the first negotiating party has three negotiation assessment criteria she can make one of 3 possible decision (eg. worsen first criteria, worsen second, worsen third). Respectively the second party has four criteria and can make one of four decisions at any point and the third party has five criteria and can make one out of five decisions. In conclusion under above assumptions, after a trial solution is presented there are 3*4*5=60 possible decisions of the negotiating parties. Each possible decision leads to a new trial solution. However some of decisions could lead to the same trial solution while other lead to different ones.

The number of possible negotiation trajectories grows exponentially at the rate of 60^n. As in this rate of growth it is impossible to simulate every possible simulation

```
simulation_model_setup() {
  //model initialization
  generate discrete set of decisions;
  randomly generate goal functions;
  //initial pre-calculations
  for each (feasible_decisions) {
    calculate goal function value;
  }
  generate an array of all possible negotiation proposals
      with one concession;
}
```

Fig. 7 Simulation model setup includes generation of parameters for a particular simulation run and pre-calculation performed in order speed up the later simulation.

```
simulate () {
  for (negotiation_process_depth in 1:5) {
    randomly   select   up   to   10000   possible   negotiation
trajectories;
    for each (negotiation trajectory in possible trajectories) {
      set the initial trial solution;
      for each (proposal in trajectory) {
        estimate next trial solution;
      }
      save last trial solution recommendation;
    }
    calculate percentage of different recommendations;
  }
}
```

**Fig. 8** The multi-agent simulation model consists of one agent class—negotiating party

trajectory. Therefore in order to simulate negotiation lasting 3 turns or more we randomly choose 10,000 possible negotiation trajectories and simulate for them 10,000 negotiation outcomes (e.g. new trial solutions that these decisions lead to). The algorithm for the described process is presented in the Fig. 8.

An example simulation results are presented on Fig. 9. Firstly, in the left upper corner the model setup (feasible decision set, goal function) is presented together with the initial trial solution. For readability the goal function gradients are attached to the middle of hexagon rather than in coordinate system origin. At the graph the initial trial recommendation $x_T = (5, 0)$ is represented. For this recommendation goal function values of all negotiating parties can be calculated (Figs. 8, 9).

For the given feasible decision set, goal functions and an initial trial solution negotiation path trajectories have been simulated. Firstly, in the step 1, all possible recommendations for all possible 60 concessions are presented on a sunflower plot. The amount of leaves depends on the percentage of decisions that lead to a particular recommendation. It can be seen that among 60 possible concessions

- 35 lead to a trial solution with recommendation $x_T^1 = (5, 0)$
- 18 lead to a trial solution with recommendation $x_T^1 = (2, 5)$
- 5 lead to a trial solution with recommendation $x_T^1 = (7, 0)$
- 2 lead to a trial solution with recommendation $x_T^1 = (4, 1)$
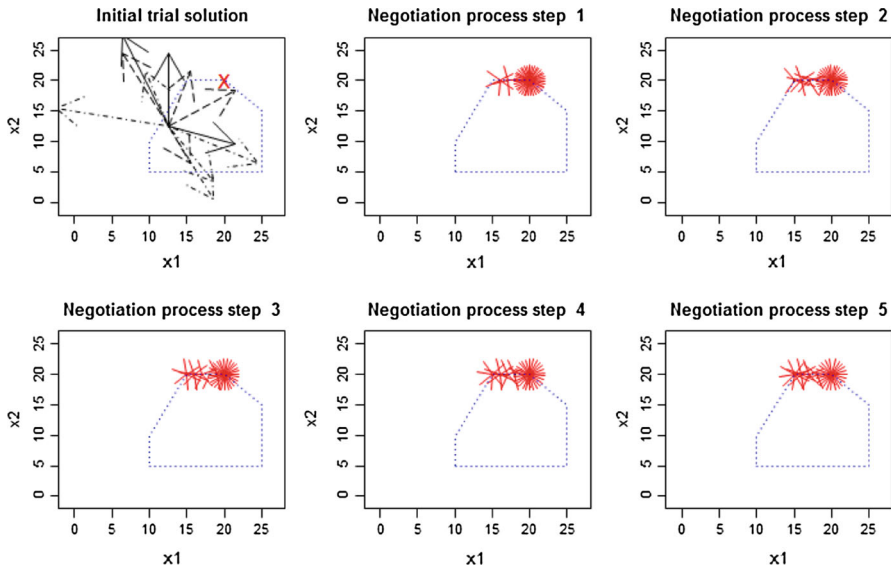
Fig. 9 After an initial trial solution (compromise) is negotiation process stages

After two negotiation turn the number of possible simulation trajectories amounts to $60^2 = 3,600$. For each of 3,600 possible negotiation scenarios trial solutions have been simulated. The results are presented as a sunflower plot—again the amount of sunflower leaves corresponds to the percentage of possible negotiation scenarios that lead to a particular outcome. In the example presented for the step 2. It turns out that out of 3,600 possible simulation trajectories around 70 % (=2,500/3,600) lead to the trial recommendation of $x_T^1 = (5, 0)$, around 27.5 % (=1,000/3,600) lead to the trial recommendation of $x_T^1 = (2, 5)$, around 1.5 % (=50/3,600) lead to the trial recommendation of $x_T^1 = (8, 0)$, around 1 % (35/3,600) lead to the trial recommendation of $x_T^1 = (4, 1)$, around 0.5 % (15/3,600) lead to the trial recommendation of $x_T^1 = (7, 0)$. The number of leave in the sunflower plot directly depends on the numbers above.

After three negotiation turn the number of possible simulation trajectories grows to $60^3 = 3,600$. For each of 60^3 possible negotiation scenarios trial solutions could be simulated. As the amount of needed simulation would be very large 10,000 random simulation trajectories have been selected (see Fig. 8). Again the number of leaves in three steps simulation is related to percentage of simulation trajectories that lead to a particular recommendation—one of {(5,0), (2,5), (8,0), (4,1),(7,0)}.

Further plots show possible negotiation outcomes after four and five steps. We can see that the recommendation crowd stabilizes with a similar percentage of negotiation trajectories leading to a particular compromise.

The results show that despite of a large number of possible simulation trajectories the simulation process ends in a limited number of points. The simulation results can be useful twofold. Firstly, the presented approach can allow to estimate possible negotiation outcomes together with possibility of reaching them. Secondly, when a limited rationality is assumed in an extreme case negotiators make random concessions

an each negotiation turns. In that case we can talk about probability of reaching a particular compromise in a limited rationality negotiations.

5 Concluding Remarks

The paper presents a procedure to assist parties in a negotiation using their behavior agent simulation. As the result one obtains a benchmark curve composed of points representing such compromise proposals (recommendations) which respect a firm party behavior (possibly small concessions) and the power of a coalition. The benchmark curve is obtained for arbitrary values of two control variables. The first arbitrary choice is related to the equal treatment of concessions (consideration of all permutations without weighting their values). The second one is the power measure as count of coalition members. Introduction of this variables in the simulation allows to more flexibly model and assist the negotiation problem.

It is shown in the paper that the benchmarking curve exists for linearly constrained problems with multiple linear criteria and bounded feasible set. This properties allow to design a multi agent system to determine and visualize the benchmarking curve. Here agents are representing negotiation parties and their interactions are modeled using BIP Procedure. This visualization may serve to evaluate current positions in real life negotiation and to modify their dynamics.

In order to validate the presented approach we have created simulation models: a simulation for a single and a agent-based simulation for multi-criteria negotiation. The simulation model allows to predict possible negotiation outcomes.

The proposed approach has several limitations. Firstly, we made the assumption the negotiating parties were able to agree the feasible decision set and that the set is linearly constraint. Secondly, we have assumed the multi-criteria decision evaluation function is linear. Thirdly, we have assumed that the parties will decide to negotiate with a mediator in the middle and they will follow a strict negotiation protocol enforced by the interactive method.

However, the simulation results show that for the given negotiation scenario the propose method allow to estimate a group a simulation results and see how they depend on the attitude of negotiating parties.

The further research will focus on relaxing the assumptions on decision problem modeling and simulation scenario. Also empirical experiments including human-in-the loop simulation could be considered.

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