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Selection of a Representative Value Function for Robust Ordinal Regression in Group Decision Making

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Abstract In this paper, we introduce the concept of a representative value function in a group decision context. We extend recently proposed methods UTA^{GMS}-GROUP and UTADIS^{GMS}-GROUP with selection of a compromise and collective preference model which aggregates preferences of several decision makers (DMs) and represents all instances of preference models compatible with preference information elicited from DMs. The representative value function is built on results of robust ordinal regression, so its representativeness can be interpreted in terms of robustness concern. We propose a few procedures designed for multiple criteria ranking, choice, and sorting problems. The use of these procedures is conditioned by both satisfying different degrees of consistency of the preference information provided by all DMs, as well as by some properties of particular decision making situations. The representative value function is intended to help the DMs to understand the robust results, and to provide them with a compromise result in case of conflict between the DMs.

Keywords Group decision · Robust ordinal regression · Additive value function · Representative value function · Compromise

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1 Introduction

Decision aiding is defined as an activity of using some models which help answering questions asked by stakeholders in a decision making process (Roy 1985). Conclusions which stem only from the analysis of performances of considered alternatives on a set of criteria are usually too weak to significantly contribute to the elements of these answers. Therefore, working out a recommendation relies to a high degree on the preference information elicited from the stakeholders playing the role of decision makers (DMs). This information may be either direct or indirect. In the first case, it specifies directly values of some parameters used in the preference model. In the other case, the preference information is composed of some examples of holistic judgments on a subset of alternatives $A^R \subseteq A$, which are called reference alternatives and play the role of a training set. Indirect preference information is used in the disaggregation-aggregation paradigm (Jacquet-Lagrèze and Siskos 1982). The idea of inferring compatible instances of a preference model which restore the exemplary statements provided by a DM, has been employed in methods involving three main families of preference models: an outranking relation (Mousseau and Słowiński 1998), a set of decision rules (Greco et al. 1999), and a value function (e.g., the UTA method Jacquet-Lagrèze and Siskos 1982; Siskos et al. 2005).

Since there are many parameters of the preference model to be estimated, there are, typically, many alternative optimal solutions, i.e. compatible instances of a preference model. As a result, the output of the method may vary substantially depending on which solution is selected. Traditionally, existing approaches based their outcomes only on a single compatible instance of a preference model. Computing final recommendation, they, indeed, neglected the fact of possible existence of many alternative instances of a preference model inferred from the provided preference information. Thus, it seemed reasonable to take into account all instances of a preference model compatible with the preference information provided by the DM, and to work out a recommendation for the set of alternatives A taking into account all these compatible instances. As a result, the principle of robust ordinal regression has been proposed (Greco et al. 2010c), and two new methods based on this principle, UTA$^{\text{GMS}}$ (Greco et al. 2008) and GRIP (Figueira et al. 2009), have been presented. These methods consider the whole set of value functions compatible with the preference information provided by the DM, while traditional UTA-like methods are using only one such value function or a limited subset of such functions. In Kadziński et al. (2012a), one has extended these methods by an analysis of extreme ranking results. Having considered complete rankings that follow the use of all compatible value functions, one can determine the best and the worst ranks taken by each alternative $a \in A$. Further, robust ordinal regression has been adapted in the UTADIS$^{\text{GMS}}$ method to deal with sorting problems (Greco et al. 2010b), and in ELECTRE$^{\text{GKMS}}$, which is a general scheme implementing this paradigm to outranking methods (see Greco et al. 2011b, 2009). Robust ordinal regression has also been applied to preference model based on Choquet integral in order to handle interaction among criteria (Angilella et al. 2010). All these methods supply the DM with two kinds of results: necessary and possible. The necessary result (ranking or assignment) specifies the most certain recommendation worked out on the basis of all compatible instances of a preference model considered jointly, whereas the possible result identifies a possible recommendation which stems from the use of at least one compatible instance of a preference model.

The family of robust ordinal regression methods has been originally designed to deal with preferences expressed by a single DM. However, in Greco et al. (2011a) and Greco et al. (2009) this principle has been enlarged on group decision. The main multiple criteria decision methods to which robust ordinal regression has been applied are considered there, and corresponding methods for group decision have been obtained. Within the framework of UTA^{GMS}-GROUP, UTADIS^{GMS}-GROUP, and ELECTRE^{GKMS}-GROUP, several DMs can cooperate to make a collective decision. They share the same description of the decision problem, i.e. the set of alternatives, consistent family of criteria, and evaluation matrix. For each DM, who expresses her/his individual preference information, we use the respective GMS or GKMS method, and check whether the necessary and possible preference relations or assignments hold either for at least one or for all DMs. Thus, we consider two levels of certainty. The first level is related to verification of the necessary and possible consequences of preference information provided by each DM. The other level involves indication whether in the considered set of DMs the specific outcome is confirmed by at least one DM (possibly) or by all DMs (necessarily). In this way, we are reasoning in terms of the necessary and possible outcomes and coalitions of DMs, and we arrive to four types of results: necessary-necessary (i.e., result which is necessary for all DMs), necessary-possible (i.e., result which is necessary for at least one DM), possible-necessary (i.e., result which is possible for all DMs), and possible-possible (i.e., result which is possible for at least one DM).

The main aim of this paper is to introduce the concept of a representative value function in robust ordinal regression applied to multiple criteria group decision problems. Despite the interest of the robust rankings and assignments provided by the family of GROUP methods, for some decision-making situations we need to provide a univocal recommendation, and, e.g., assign a single score to each alternative. Moreover, the robust results might not be understandable to some DMs. Therefore, we propose to work out a single preference model and representative results which follow its use, without loosing advantage of knowing all compatible value functions for all DMs. We have already considered similar requirements with respect to a single DM (see Greco et al. 2011c; Kadziński et al. 2012b). The representative value function is about to highlight the most stable part of the robust results, i.e., the necessary-necessary preference relation for problems of ordering alternatives from the best to the worst, and possible-necessary assignments for sorting problems. In order to control the differences between values of alternatives we use a maximin rule. Moreover, we distinguish three types of decision making situations conditioned by different degrees of consistency of the preference information. Precisely, we consider cases where preference information provided by all DMs can or cannot be represented jointly by at least one value function, and a case where preference information of each DM is compatible when analyzed individually.

Till now, UTA-like methods have been used in several works for conflict resolution in multi-actor decision making (Jacquet-Lagrèze and Shakun 1984; Jarke et al. 1987; Matsatsinis and Samaras 2001). In the introduced approach, we extend the family of GROUP methods, not only supporting DMs with a very intuitive representation of the output of robust ordinal regression, but also providing an insight into an achieved consensus solution. Consequently, we combine the robustness analysis conducted within UTA^{GMS}-GROUP and UTADIS^{GMS}-GROUP with the clarity of classical UTA-like methods. The presented approach may be of interest to researchers in management and business with such applications as, e.g., evaluation of consumers' preferences, personnel selection, or allocation of priorities to projects. However, it is important to stress that following the assumption of the GROUP methods, the introduced procedure could be used only in these group decision problems where performances of all considered alternatives on the common family of criteria are the same for all DMs.

The paper is organized in the following way. In the next section, we recall the basic principles of robust ordinal regression methods within Multi-Attribute Utility Theory (MAUT), i.e. UTA^{GMS} and UTADIS^{GMS} along with their extensions dealing with preferences expressed by several DMs. Section 3 is devoted to presentation of the representative value function in the context of multiple criteria group decision. Section 4 presents two didactic examples relative to applications of the presented methodology aiming to recommend the best choice, or ranking, or sorting of alternatives. We provide the detailed results of the robust ordinal regression methods, and present the way these outcomes may be enriched with selection of the representative value function. Finally, Sect. 5 provides conclusions and avenues for future research.

2 Reminder on Robust Ordinal Regression Methods Within the Framework of Multi-Attribute Utility Theory

We are considering decision situations in which a finite set of alternatives $A = \{a_1, a_2, \ldots, a_i, \ldots, a_n\}$ is evaluated on a consistent family of criteria $G = \{g_1, g_2, \ldots, g_j, \ldots, g_m\}$. The evaluation of $a_i \in A$ on criterion $g_j \in G$ is denoted by $g_j(a_i)$. The set of all different evaluations on g_j is denoted by X_j, and ordered values of X_j are denoted as: $x_j^1, x_j^2, \ldots, x_j^{n_j}$, with $x_j^k < x_j^{k+1}$, $k = 1, 2, \ldots, n_j - 1$, $n_j \leq n$.

In order to represent DM's preferences, we use a preference model in the form of an additive value function $U(a) = \sum_{j=1}^{m} u_j(g_j(a)) \in [0, 1]$, where u_j is the marginal monotone value function for criterion g_j, $u_j(x_j^1) = 0$, for all $j \in J$, and $\sum_{j=1}^{m} u_j(x_j^{n_j}) = 1$. The comprehensive value serves as an index used to decide the position in the ranking, or presence in the subset of the best alternatives, or the assignment to the predefined ordered classes.

In this section, we recall two groups of robust ordinal regression methods in the framework of MAUT. One of them is designed to deal with ranking and choice problems, whereas the other is intended to support decision processes related to sorting problems. Note that we are considering the choice problem as a particular case of the ranking problem.

2.1 Robust Ordinal Regression for Ranking and Choice Problems

The idea of considering the whole set of compatible value functions to deal with ranking and choice problems was originally introduced in the UTA^{GMS} method

(Greco et al. 2008). The initial idea of selecting a value function representing the possible and the necessary preference relations was introduced in Figueira et al. (2008). In the same vein the procedure for selecting a representative value function among all compatible ones was proposed in Kadziński et al. (2012b). Finally, the approach was extended in UTA$^{\text{GMS}}$-GROUP to handle a group decision context (see Greco et al. 2011a, 2009).

The UTA$^{\text{GMS}}$ procedure starts with asking the DM for preference information in the form of pairwise comparisons for some reference alternatives. Then, it leads through the statement of appropriate ordinal regression problems which define the set of compatible value functions, i.e. such value functions that can restore all pairwise comparisons provided by the DM. The method results in calculation of two binary preference relations on the set of all alternatives:

- necessary weak preference relation $\succsim^N$, in case $U(a) \geq U(b)$ for all compatible value functions;
- possible weak preference relation $\succsim^P$, in case $U(a) \geq U(b)$ for at least one compatible value function.

From the two weak preference relations $\succsim^N$ and $\succsim^P$, one can get preference, indifference, and incomparability in a usual way.

Robust ordinal regression for ranking and choice problems has been applied to group decision in the UTA$^{\text{GMS}}$-GROUP method. Let us denote the set of DMs by $\mathcal{D} = \{d_1, \ldots, d_p\}$. For each $d_r \in \mathcal{D}' \subseteq \mathcal{D}$, who expresses her/his individual preferences as in UTA$^{\text{GMS}}$, we consider all compatible value functions, and, subsequently, we calculate the necessary and possible preference relations $\succsim^N_{d_r}$ and $\succsim^P_{d_r}$. With respect to all DMs, four situations are interesting for a pair $(a, b) \in A \times A$:

- $a \succsim^{N,N}_{\mathcal{D}'} b$: $a \succsim^N_{d_r} b$ for *all* $d_r \in \mathcal{D}'$,
- $a \succsim^{N,P}_{\mathcal{D}'} b$: $a \succsim^N_{d_r} b$ for *at least one* $d_r \in \mathcal{D}'$,
- $a \succsim^{P,N}_{\mathcal{D}'} b$: $a \succsim^P_{d_r} b$ for *all* $d_r \in \mathcal{D}'$,
- $a \succsim^{P,P}_{\mathcal{D}'} b$: $a \succsim^P_{d_r} b$ for *at least one* $d_r \in \mathcal{D}'$.

On the basis of these four relations, we can identify results reflecting sure and possible with respect to the preference information provided by at least one or all DMs.

2.2 Robust Ordinal Regression for Sorting Problems

The principle of robust ordinal regression has been applied to sorting problems in UTADIS$^{\text{GMS}}$ (Greco et al. 2010b). Then, the concept of a representative value function was presented in Greco et al. (2011c), and the method was extended to the case of several DMs in UTADIS$^{\text{GMS}}$-GROUP (see Greco et al. 2011a, 2009). Let us denote predefined preference ordered classes, each having a specific semantic definition, by $C_1, C_2, \ldots, C_p$, where C_{h+1} is preferred to C_h, $h = 1, \ldots, p-1$.

In UTADIS$^{\text{GMS}}$, the DM is asked for preference information in the form of a set of assignment examples, each one consisting of an alternative $a^* \in A^R \subseteq A$ and its desired assignment $a^* \to [C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$. The set of inferred compatible value functions consists of such value functions that are able to reproduce all assignments given by the DM for reference alternatives. The method returns two kinds of assignments for any alternative $a \in A$:

– the possible assignment $C^P(a)$ determines the set of indices of classes C_h for which there exists at least one compatible value function $U \in \mathcal{U}_{A^R}$ assigning a to C_h:

$$C^P(a) = [L_P^{\mathcal{U}}(a), R_P^{\mathcal{U}}(a)] = \{h \in H : \exists U \in \mathcal{U}_{A^R} : h \in [L^U(a), R^U(a)]\},$$

– the necessary assignment $C^N(a)$ specifies the set of indices of classes C_h for which all compatible value functions $U \in \mathcal{U}_{A^R}$ assign a to C_h:

$$C^N(a) = [L_N^{\mathcal{U}}(a), R_N^{\mathcal{U}}(a)] = \{h \in H : \forall U \in \mathcal{U}_{A^R} : h \in [L^U(a), R^U(a)]\},$$

where $L^U(a)$ and $R^U(a)$ are, respectively, the worst and the best class to which an alternative a is assigned by value function U.

Within UTADIS$^{\text{GMS}}$-GROUP method, for each $a \in A$ and for each $d_r \in \mathcal{D}' \subseteq \mathcal{D}$, we determine the necessary $C_{d_r}^N(a)$ and possible $C_{d_r}^P(a)$ assignments as in UTADIS$^{\text{GMS}}$. Then, for each subset of DMs, $\mathcal{D}' \subseteq \mathcal{D}$, we define the following four types of assignment:

– $C_{\mathcal{D}'}^{P,P}(a) = \bigcup_{d_r \in \mathcal{D}'} C_{d_r}^P(a)$, $C_{\mathcal{D}'}^{N,P}(a) = \bigcup_{d_r \in \mathcal{D}'} C_{d_r}^N(a)$,
– $C_{\mathcal{D}'}^{P,N}(a) = \bigcap_{d_r \in \mathcal{D}'} C_{d_r}^P(a)$, $C_{\mathcal{D}'}^{N,N}(a) = \bigcap_{d_r \in \mathcal{D}'} C_{d_r}^N(a)$.

By analogy to UTA$^{\text{GMS}}$-GROUP, these assignments reflect sure or possible results for at least one or all DMs.

3 Selection of a Representative Value Function in Group Decision Making

In this section, we introduce the concept of a representative value function in the context of multiple criteria group decision. The representative value function provides synthetic representation of all compatible value functions, it returns a numerical score for each alternative, and it permits assessing relative importance of criteria understood as their maximal share in a comprehensive value. In the proposed methodology, outcomes of all compatible preference models are represented by a single value function. When applied to group decision problems, the representative function serves moreover as a compromise group solution. Since the analysis of a single representative value function is less abstract than that of the whole set of compatible value functions, we facilitate understanding of the results of robust ordinal regression, and enable easy and intuitive comparison of consequences of one's preferences with the consensus representing the whole group of DMs. Thus, the motivation for extending the family of GMS and GROUP methods with this approach is even more sound than for MCDA approaches considering preferences of just a single DM.

The idea underlying our proposal for selection of the representative value function is to stress the differences between values of some alternatives taking into account the most stable part of the robust results, i.e. necessary-necessary preference relations for multiple criteria ranking problems, or possible-necessary assignments for multiple criteria sorting problems. Generally, we wish to emphasize the difference between values of alternatives if one can observe the evident advantage of one alternative over the other, or diminish this difference when it is impossible or very difficult to indicate the better alternative. Consequently, we use minimax and maximin rules to control the difference between values of alternatives. Whether we maximize or minimize the difference between $U(a)$ and $U(b)$ for a given pair of alternatives $(a, b) \in A \times A$, is conditioned by satisfying specific binary relations. Precisely, if one alternative is better than the other for all compatible value functions, i.e. its rank is higher or a class assigned to it is better, the difference between their values should be as large as possible. On the contrary, if two alternatives are necessarily indifferent or incomparable in terms of all compatible rankings or assignments, the difference between their values should be minimized. In this way, we are building the representative value function on the outcomes of robust ordinal regression. As a result, the selected value function is representative in the sense of robustness concern.

In the following sections, we consider different types of decision making situations. Firstly, if pieces of preference information provided by all DMs are complementary when considering jointly, then, in order to identify a representative value function, we suggest employment of a suitably adapted procedures for a single fictitious DM whose preferences consist of pairwise comparisons or assignments given by all DMs. Secondly, if exemplary statements of each DM are consistent when considered individually, but not necessarily consistent when considered jointly, we refer to the results of the corresponding GROUP method, and we make the desired difference between values of specific alternatives dependent on the number of DMs for whom the relation holds. Precisely, this intensity is connected with the inclusion relation on the set of all subsets of DMs. Note that distinction between these two cases may also be made on a different ground. If the set of alternatives is numerous, and the DMs serve as experts only with respect to its small disjoint subsets of alternatives, then it is reasonable to combine their knowledge into preference information of a single fictitious DM. On the other hand, if the DMs have outlook of the whole set of alternatives (usually, when it is small), and their exemplary decisions are interrelated, then it is more suitable to analyze their statements separately, and examine subsequently the spaces of agreement and disagreement. However, this remark should only be treated as a suggestion for the case of an automatic choice of the procedure. Nevertheless, in general, it is the matter of the analyst which procedure will be used.

3.1 Representative Value Function for the Case of Preference Information of All DMs Considered Jointly

In this section, we consider a decision making situation where pieces of preference information provided by all DMs can be represented together by at least one additive value function composed of general marginal value functions. To verify whether it is possible for the set of DMs $\mathcal{D}' \subseteq \mathcal{D}$, preference information given by all DMs from $\mathcal{D}'$ is translated into the set of constraints, $E^{A^R}_{\mathcal{D}',X}$, where for ranking problems $(E^{A^R}_{\mathcal{D}',X}) = (E^{A^R}_{\mathcal{D}',rank})$, such that:

$$\left.\begin{array}{l} U(a^*) \geq U(b^*) + \varepsilon \quad \text{if } a^* \succ_{d_r} b^* \\ U(a^*) = U(b^*) \quad \text{if } a^* \sim_{d_r} b^* \end{array}\right\} \forall (a,b) \in B^R_{d_r},\ d_r \in \mathcal{D}' \left.\vphantom{\begin{array}{l} a \\ b \end{array}}\right\} (E^{A^R}_{\mathcal{D}',rank}),$$

whereas for sorting problems $(E^{A^R}_{\mathcal{D}',X}) = (E^{A^R}_{\mathcal{D}',sort})$, such that:

$$\left.\begin{array}{c} U(a^*) \geq U(b^*) + \varepsilon, \quad \forall a^* \in A^R_{d_i},\ b^* \in A^R_{d_j} \\ \text{such that } L^{d_i}(a^*) > R^{d_j}(b^*),\ d_i, d_j \in \mathcal{D}' \end{array}\right\} (E^{A^R}_{\mathcal{D}',sort}).$$

Along with constraints ensuring monotonicity of the marginal value functions and normalization of the comprehensive values to the range between 0 and 1, this set forms the set of linear constraints $E^{A^R}_{\mathcal{D}'}$, which defines the set of compatible value functions. Then, we consider the following linear program:

$$\textit{Maximize: } \varepsilon$$

s.t.

$$\left.\begin{array}{l} (E^{A^R}_{\mathcal{D}',X}) \\ \left.\begin{array}{l} u_j(x_j^k) - u_j(x_j^{(k-1)}) \geq 0, \quad j = 1, \ldots, m,\ k = 2, ..., n_j \\ u_j(x_j^1) = 0,\ j = 1, \ldots, m,\ \ \sum_{j=1}^m u_j(x_j^{n_j}) = 1 \end{array}\right\} (E^{A^R}_{\mathcal{D}',\ base}) \end{array}\right\} (E^{A^R}_{\mathcal{D}'}).$$

Let us denote by ε^* the maximal value of ε obtained from the above program (i.e., $\varepsilon^* = max\ \varepsilon$, s.t. $E^{A^R}_{\mathcal{D}'}$). The set of compatible value functions $\mathcal{U}_{\mathcal{D}'}$ is not empty if $E^{A^R}_{\mathcal{D}'}$ is feasible and $\varepsilon^* > 0$. If there is no value function compatible with the preference information provided by all DMs, i.e. $\varepsilon^* \leq 0$ or $E^{A^R}_{\mathcal{D}'}$ is infeasible, one may either identify reasons of incompatibility in order to remove it, or determine the representative value function for the maximal number of DMs whose statements are not contradictory. Dealing with the incompatibility issue is discussed in Sect. 3.3.

If preference information provided by all DMs from $\mathcal{D}'$ is consistent, one may select the representative value function considering jointly all pieces of preference information supplied by these DMs. In this case, for multiple criteria ranking problems one obtains two preference relations, such that for any pair of alternatives $(a, b) \in A \times A$:

- $a \succsim^N_{\mathcal{D}'} b$: a is ranked at least good as b if and only if, $U(a) \geq U(b)$ for all compatible value functions $U \in \mathcal{U}_{\mathcal{D}'}$;
- $a \succsim^P_{\mathcal{D}'} b$: a is ranked at least good as b if and only if, $U(a) \geq U(b)$ for at least one compatible value functions $U \in \mathcal{U}_{\mathcal{D}'}$.

The truth and falsity of the necessary $\succsim^N_{\mathcal{D}'}$ and possible $\succsim^P_{\mathcal{D}'}$ preference relations is verified by solution of appropriate LP problems. To check whether $a \succsim^N_{\mathcal{D}'} b$, we calculate $\varepsilon_* = max\ \varepsilon$, subject to $E^{A^R}_{\mathcal{D}'}$ incremented with a constraint $U(b) \geq U(a) + \varepsilon$. If $\varepsilon_* \leq 0$, then $a \succsim^N_{\mathcal{D}'} b$. On the other hand, to check whether $a \succsim^P_{\mathcal{D}'} b$, we calculate

$\varepsilon^* = max\ \varepsilon$, subject to $E^{A^R}_{\mathcal{D}'}$ incremented with a constraint $U(a) \geq U(b)$. If $\varepsilon^* > 0$, then $a \succsim^P_{\mathcal{D}'} b$.

Necessary weak preference relation $\succsim^N_{\mathcal{D}'}$ is a partial preorder (i.e., it is reflexive ($a \succsim^N_{\mathcal{D}'} a$, since for all $a \in A,\ U(a) = U(a)$), and transitive (for all $a, b, c \in A$, if $a \succsim^N_{\mathcal{D}'} b$ and $b \succsim^N_{\mathcal{D}'} c$, then $a \succsim^N_{\mathcal{D}'} c$). Possible weak preference relation $\succsim^P_{\mathcal{D}'}$ is a strongly complete (i.e. for all $a, b \in A,\ a \succsim^P_{\mathcal{D}'} b$ or $b \succsim^P_{\mathcal{D}'} a$), and negatively transitive (i.e. $\forall a, b, c \in A$, if $not(a \succsim^P_{\mathcal{D}'} b)$ and $not(b \succsim^P_{\mathcal{D}'} c)$, then $not(a \succsim^P_{\mathcal{D}'} c)$) binary relation.
Binary relations $\succsim^N_{\mathcal{D}'}$ and $\succsim^P_{\mathcal{D}'}$ satisfy the following interesting properties.

Proposition 31 *For any subset of decision makers $\mathcal{D}' \subseteq \mathcal{D}$, if $\mathcal{U}_{\mathcal{D}'} \neq \emptyset$, then:*

1. $\succsim^{N,N}_{\mathcal{D}'} \subseteq \succsim^N_{\mathcal{D}'}$,
2. $\succsim^P_{\mathcal{D}'} \subseteq \succsim^{P,N}_{\mathcal{D}'}$,

Proof Notice that all compatible value functions of each DM from $\mathcal{D}'$ do not necessarily represent all preferences of all DMs from $\mathcal{D}'$. It is the case, because $\mathcal{U}_{\mathcal{D}'}$ constitutes intersection of the sets of compatible value functions of particular DMs from $\mathcal{D}'$. Since $\mathcal{U}_{\mathcal{D}'} = \cap_{d_r \in \mathcal{D}'} \mathcal{U}_{d_r} \subseteq \cup_{d_r \in \mathcal{D}'} \mathcal{U}_{d_r}$, then on condition that $\mathcal{U}_{\mathcal{D}'}$ is not empty, for all $a, b \in A$:

1. $a \succsim^{N,N}_{\mathcal{D}'} b \Rightarrow a \succsim^N_{\mathcal{D}'} b$, i.e. $\succsim^{N,N}_{\mathcal{D}'} \subseteq \succsim^N_{\mathcal{D}'}$.
2. $a \succsim^P_{\mathcal{D}'} b \Rightarrow a \succsim^{P,N}_{\mathcal{D}'} b$, i.e. $\succsim^P_{\mathcal{D}'} \subseteq \succsim^{P,N}_{\mathcal{D}'}$. □

Since $\succsim^N_{\mathcal{D}'} \subseteq \succsim^P_{\mathcal{D}'}$ and $\succsim^{P,N}_{\mathcal{D}'} \subseteq \succsim^{P,P}_{\mathcal{D}'}$, we also know that $\succsim^N_{\mathcal{D}'} \subseteq \succsim^{P,N}_{\mathcal{D}'}$ and $\succsim^P_{\mathcal{D}'} \subseteq \succsim^{P,P}_{\mathcal{D}'}$. Thus, the following inclusion relations are true:

$$\succsim^{N,N}_{\mathcal{D}'} \subseteq \succsim^N_{\mathcal{D}'} \subseteq \succsim^P_{\mathcal{D}'} \subseteq \succsim^{P,N}_{\mathcal{D}'} \subseteq \succsim^{P,P}_{\mathcal{D}'} .$$

Proposition 32 *Considering two subsets of DMs, $\mathcal{D}' \subseteq \mathcal{D}'' \subseteq \mathcal{D}$, if $\mathcal{U}_{\mathcal{D}''} \neq \emptyset$, then:*

1. $\succsim^N_{\mathcal{D}'} \subseteq \succsim^N_{\mathcal{D}''}$,
2. $\succsim^P_{\mathcal{D}''} \subseteq \succsim^P_{\mathcal{D}'}$.

Proof Notice that $\mathcal{D}' \subseteq \mathcal{D}''$ implies that $\mathcal{U}_{\mathcal{D}''} \subseteq \mathcal{U}_{\mathcal{D}'}$. If so:

1. $\succsim^N_{\mathcal{D}'} \Rightarrow \succsim^N_{\mathcal{D}''}$, i.e. $\succsim^N_{\mathcal{D}'} \subseteq \succsim^N_{\mathcal{D}''}$.
2. $\succsim^P_{\mathcal{D}''} \Rightarrow \succsim^P_{\mathcal{D}'}$, i.e. $\succsim^P_{\mathcal{D}''} \subseteq \succsim^P_{\mathcal{D}'}$. □

For multiple criteria sorting problems, one obtains two kinds of assignments for any alternative $a \in A$:

– $C^P_{\mathcal{D}'}(a) = \{h \in H, \exists U \in \mathcal{U}_{\mathcal{D}'} : h \in [L^U(a), R^U(a)]\}$, i.e. a set of indices of classes C_h for which there exists at least one value function $U \in \mathcal{U}_{\mathcal{D}'}$ assigning a to C_h;

- $C^N_{\mathcal{D}'}(a) = \{h \in H, \forall U \in \mathcal{U}_{\mathcal{D}'} : h \in [L^U(a), R^U(a)]\}$, i.e. a set of indices of classes C_h for which all value functions $U \in \mathcal{U}_{\mathcal{D}'}$ assign a to C_h.

Reasoning analogously to the conclusions drawn for ranking problems, we can state that assignments $C^N_{\mathcal{D}'}$ and $C^P_{\mathcal{D}'}$ satisfy the following interesting properties.

Proposition 33 *For any subset of decision makers $\mathcal{D}' \subseteq \mathcal{D}$ and for all $a \in A$, if $\mathcal{U}_{\mathcal{D}'} \neq \emptyset$, then:*

1. $C^{N,N}_{\mathcal{D}'}(a) \subseteq C^N_{\mathcal{D}'}(a)$,
2. $C^P_{\mathcal{D}'}(a) \subseteq C^{P,N}_{\mathcal{D}'}(a)$,

Proof Analogously to the proof of Proposition 31, with the proviso that we consider the assignment of alternative a to class $C(a)$ rather than the weak preference relation $a \succsim b$. □

Proposition 34 *Considering two subsets of DMs, $\mathcal{D}' \subseteq \mathcal{D}'' \subseteq \mathcal{D}$, if $\mathcal{U}_{\mathcal{D}''} \neq \emptyset$, then:*

1. $C^N_{\mathcal{D}'}(a) \subseteq C^N_{\mathcal{D}''}(a)$,
2. $C^P_{\mathcal{D}''}(a) \subseteq C^P_{\mathcal{D}'}(a)$.

Proof Analogously to the proof of Proposition 32. □

In order to translate the concept of representativeness into mathematical formulations, at first we need to define binary relations which build on the results of robust ordinal regression. They allow easy indication of the better alternative within a given pair of alternatives, or provide evidence about indifference or incomparability of the alternatives with respect to the outcomes of all compatible instances of the preference model. The truth of these relations for some specific pairs would condition the desired difference between their comprehensive values. With respect to multiple criteria ranking problems, let us consider the following binary relations for pairs $(a, b) \in A \times A$:

- $a \succ^N_{\mathcal{D}'} b$, i.e. $a \succsim^N_{\mathcal{D}'} b$ and $not(b \succsim^N_{\mathcal{D}'} a)$—in this case the advantage of a over b is evident, since for all compatible value functions $U \in \mathcal{U}_{\mathcal{D}'}$, a is at least good as b, while the opposite relation is not true;
- $not(a \succ^N_{\mathcal{D}'} b)$ and $not(b \succ^N_{\mathcal{D}'} a)$—in this case neither a is strongly necessarily preferred to b, nor b is strongly necessarily preferred to a; as a result, they are either necessarily incomparable, i.e. $aR^N_{\mathcal{D}'}b$, or necessarily indifferent, i.e. $a \sim^N_{\mathcal{D}'} b$; in the former case, for some $U_1 \in \mathcal{U}_{\mathcal{D}'}$, a is preferred to b, but for some other $U_2 \in \mathcal{U}_{\mathcal{D}'}$, b is preferred to a; in the other case ($a \sim^N_{\mathcal{D}'} b$), the comprehensive values of a and b are equal for all compatible value functions.

Considering the above relations, in a representative case, we wish to maximize the difference between $U(a)$ and $U(b)$ for (a, b), such that $a \succ^N b$, and to minimize this difference for (c, d), such that $not(c \succ^N d)$ and $not(d \succ^N c)$.
With respect to multiple criteria sorting problems, let us consider the following binary relations:

- $a \succ^{\rightarrow}_{\mathcal{D}'} b$, which means that for all compatible value functions $U \in \mathcal{U}_{\mathcal{D}'}$, a is assigned to a class better than the class of b, i.e. for all $U \in \mathcal{U}_{\mathcal{D}'}$, such that $L^U(a) > R^U(b)$;

– $a \sim^{\rightarrow}_{\mathcal{D}'} b$ or $a \succ\prec^{\rightarrow}_{\mathcal{D}'} b$, where the former relation means that for any compatible value function $U \in \mathcal{U}_{\mathcal{D}'}$, a is assigned to the same range of classes as b, i.e. for all $U \in \mathcal{U}_{\mathcal{D}'}$, such that $L^U(a) = L^U(b)$ and $R^U(a) = R^U(b)$, and the latter relation means that the order of classes for alternatives a and b is not univocal (i.e. for some compatible value functions alternative a is classified better than b whereas for some other compatible value function it is classified worse: there exist $U_1, U_2 \in \mathcal{U}_{\mathcal{D}'}$, such that $L^{U_1}(a) > R^{U_1}(b)$ and $L^{U_2}(b) > R^{U_2}(a)$).

Analogously to the interpretation presented for the case of ranking problems, in a representative case, we aim to maximize the difference between $U(a)$ and $U(b)$ for $a, b \in A$, such that $a \succ^{\rightarrow} b$, and to minimize this difference for $c, d \in A$, such that $c \sim^{\rightarrow} d$ or $c \succ\prec^{\rightarrow} d$. Notice that we would like to achieve these targets for a single value function. However, according to the experiments we have conducted, if we first maximized the difference between values of some specific pairs of alternatives, there would not be much space for its minimization with respect to some other pairs of alternatives. Therefore, within the proposed framework, we look for the best possible compromise between these two targets. If after the proper procedure there is still more than one optimal value function, we choose the one for which the sum of elementary components is maximal.

Obviously, it is possible to conduct optimization of these differences taking advantage of other rules that have been proposed in the literature (see, e.g., Bous et al. 2010; Despotis et al. 1990). However, we have decided to apply a maximin rule, and search for such a function in the feasible space, which maximizes the smallest distance to any introduced constraint. The selected solution is equivalent to the center of the largest circle that can be inscribed into this space. Note that in this case, the obtained results could be easily interpreted. For example, for ranking problems, we can observe what is the minimal difference between comprehensive values of alternatives related by the necessary $\succ^N_{\mathcal{D}}$ relation, and what is the maximal difference between comprehensive values of alternatives which are related by necessary incomparability $R^N_{\mathcal{D}}$. In this way, we could examine the extent to which it was possible to emphasize the results of robust ordinal regression.

Procedure for Selection of a Representative Value Function for Group Ranking Problems

In order to select a representative value function for group ranking problems one may use the following procedure:

1. Ask all DMs from $\mathcal{D}'$ for pairwise comparisons of reference alternatives.
2. Verify if the set of compatible value functions $\mathcal{U}_{\mathcal{D}'}$ is not empty.
3. Check the truth of the necessary $\succsim^N_{\mathcal{D}'}$ and the possible $\succsim^P_{\mathcal{D}'}$ weak preference relations for all $a, b \in A$.
4. For all quadruples of alternatives $(a, b, c, d) \in A \times A \times A \times A$, such that $a \succ^N_{\mathcal{D}'} b$ and $not(c \succ^N_{\mathcal{D}'} d)$ and $not(d \succ^N_{\mathcal{D}'} c)$, add the following constraints to the linear programming (LP) constraints $E^{A^R}_{\mathcal{D}'}$ for ranking problems:

$$\begin{cases} U(a) - U(b) \geq U(c) - U(d) + \varepsilon(a,b,c,d), \\ \varepsilon(a,b,c,d) \geq \varepsilon. \end{cases}$$

5. Maximize ε, subject to the set of LP constraints from point 4). This amounts to the following optimization:

$$max_{U \in \mathcal{U}_{\mathcal{D}'}} \{min_{a,b,c,d \in A} \{[U(a) - U(b)] - [U(c) - U(d)],$$
$$\text{for } a,b,c,d \in A, \text{ such that } a \succ^N_{\mathcal{D}'} b \text{ and } not(c \succ^N_{\mathcal{D}'} d) \text{ and } not(d \succ^N_{\mathcal{D}'} c)\}\}.$$

6. Add the constraint $\varepsilon = \varepsilon^*$, with $\varepsilon^* = \max \varepsilon$ from the previous point, to the set of LP constraints considered in point 5).
7. Maximize $\sum_{a,b,c,d \,:\, a \succ^N_{\mathcal{D}'} b \text{ and } not(c \succ^N_{\mathcal{D}'} d) \text{ and } not(d \succ^N_{\mathcal{D}'} c)} \varepsilon(a,b,c,d)$, subject to the set of LP constraints from point 6).

After optimizations, for all alternatives in A we are able to read off the representative comprehensive values and their marginal components from the solution of the LP problem. On this basis, we are able to draw representative marginal and comprehensive $U^R_{\mathcal{D}'}$ value functions. The comprehensive values assigned to considered alternatives can be used to rank the alternatives.

Procedure for Selection of a Representative Value Function for Group Sorting Problems

In case of group sorting problems, the following procedure for selection of a representative value function corresponds to the previously discussed concept of representativeness:

I. Ask all DMs for possibly imprecise assignment examples of reference alternatives.
II. Verify if the set of compatible value functions $\mathcal{U}_{\mathcal{D}'}$ is not empty.
III. Determine the possible sorting $C^P_{\mathcal{D}'}(a)$ and the necessary sorting $C^N_{\mathcal{D}'}(a)$ for each considered alternative $a \in A$. For all pairs of alternatives $(a,b) \in A \times A$, check the truth or falsity of relations $a \succ^{\rightarrow}_{\mathcal{D}'} b$, $a \sim^{\rightarrow}_{\mathcal{D}'} b$, and $a \succ\prec^{\rightarrow}_{\mathcal{D}'} b$ (the corresponding algorithms are described in Greco et al. 2011c).
IV. For all quadruples of alternatives $(a,b,c,d) \in A \times A \times A \times A$, such that $a \succ^{\rightarrow}_{\mathcal{D}'} b$ and $c \sim^{\rightarrow}_{\mathcal{D}'} d$ or $c \succ\prec^{\rightarrow}_{\mathcal{D}'} d$, add the following constraints to the LP constraints $E^{A^R}_{\mathcal{D}'}$ for sorting problems:

$$\begin{cases} U(a) - U(b) \geq U(c) - U(d) + \varepsilon(a,b,c,d), \\ \varepsilon(a,b,c,d) \geq \varepsilon. \end{cases}$$

V. Maximize ε, subject to the set of LP constraints from point IV). This amounts to the following optimization:

$$max_{U \in \mathcal{U}_{\mathcal{D}'}}\{min_{a,b,c,d \in A}\{[U(a) - U(b)] - [U(c) - U(d)],$$
$$\text{for } a, b, c, d \in A, \text{ such that } a \succ_{\overrightarrow{\mathcal{D}'}} b \text{ and } (c \sim_{\overrightarrow{\mathcal{D}'}} d \text{ or } c \succ\prec_{\overrightarrow{\mathcal{D}'}} d)\}\}.$$

To save space, we do not provide similar explanations for the optimizations conducted in the procedures presented in the following sections. They would be analogous to these which are presented here.

VI. Add the constraint $\varepsilon = \varepsilon^*$, with $\varepsilon^* = \max \varepsilon$ from the previous point, to the set of LP constraints considered in point V).

VII. Maximize $\sum_{a,b,c,d \;:\; a \succ_{\overrightarrow{\mathcal{D}'}} b \text{ and } (c \sim_{\overrightarrow{\mathcal{D}'}} d \text{ or } c \succ\prec_{\overrightarrow{\mathcal{D}'}} d)} \varepsilon(a, b, c, d)$, subject to the set of LP constraints from point VI).

After the above optimizations, for all alternatives in A we are able to read off the representative comprehensive and marginal values. The preference model selected according to the above principles, can be used along with the assignment examples supplied at the beginning by all DMs to drive an autonomous example-based sorting procedure (see Greco et al. 2011c, 2010b), which would deliver representative assignments to classes for all considered alternatives.

In the above procedures we account for two targets. One of them consists in maximization of the difference between values of alternatives satisfying a specific binary relation which confirms the advantage of one alternative over the other alternative. The other target consists in minimization of the difference between values of alternatives satisfying some other binary relations which indicate that the result of the comparison of a pair of alternatives is not univocal. Although in a general case, we recommend selection of a value function that would reflect the best compromise between these two targets, one can also imagine iterative optimization of these targets. Then, in the following iterations we would account for the values of already optimized targets. In this case, the selected value function would be the most discriminant with respect to pairs of alternatives related by the necessary relation for ranking problems, or pairs for which the intervals of assigned classes are always disjoint for sorting problems. On the other hand, if one would perform minimization in the initial step, the function would account first and foremost for equalization of the values of alternatives not related by the necessary preference, or alternatives being in the same class for all compatible value functions or those for which the order of classes is not univocal.

3.2 Representative Value Function for the Case of Preference Information of Each DM Considered Individually

In this section, we consider a decision making situation where preference information provided by each DM is consistent when analyzed separately, i.e. $\mathcal{U}_{A^R, d_h} \neq \emptyset$, for all $d_h \in \mathcal{D}$, while preference information of all DMs considered jointly is not necessarily consistent. If the set of compatible value functions for each DM is not empty, we compute results of a corresponding GROUP method, and we employ an adapted procedure for selection of the representative value function. It consists in consideration of the same conditions as in procedures presented in Sect. 3.1, but the desired intensity of difference between comprehensive values of specific alternatives depends on the inclusion relation on the set of all subsets of DMs for whom these relations hold. Precisely, the greater the number of DMs for whom the specific relation holds, the greater or the less the difference between values of alternatives depending whether originally we wanted to maximize the minimal difference or minimize the maximal difference, respectively.

Procedure for Selection of a Representative Value Function for Group Ranking Problems

The procedure for selection of a representative value function for group ranking decision problems is based on the exploitation of the necessary-necessary preference relation and weighting the desired difference between values of alternatives by the number of DMs for whom the specific conditions hold. The general idea is to maximize the difference between values of alternatives which are related by the necessary-necessary preference relation for any subset of DMs. The more DMs for whom this condition holds, the greater the difference should be. Simultaneously, we minimize the difference between values of alternatives which are not necessary-possibly preferred one to another. By analogy, the more DMs for whom aforementioned relation holds, the less the desired difference. Note that applying these rules is conditioned by the truth of the inclusion relation between the subsets of DMs for whom we observe specific results. The following procedure for selecting a representative value function for ranking and choice problems corresponds to the above interpretation of representativeness:

1. Determine the necessary and the possible weak preference relation in the considered set of alternatives for each DM. Combine them into necessary-necessary, necessary-possible, possible-necessary, and possible-possible results.
2. For all quadruples of alternatives $(a, b, c, d) \in A \times A \times A \times A$, such that $a \succ^{N,N}_{\mathcal{D}''} b$ and $c \succ^{N,N}_{\mathcal{D}'''} d$ and $\mathcal{D}''' \subset \mathcal{D}'' \subseteq \mathcal{D}'$, and neither $b \succ^{N,P}_{\mathcal{D}'} a$ nor $d \succ^{N,P}_{\mathcal{D}'} c$, add the following constraints to the LP constraints of UTA$^{\text{GMS}}$:

$$\begin{cases} U(a) - U(b) \geq U(c) - U(d) + \varepsilon(a, b, c, d), \\ \varepsilon(a, b, c, d) \geq \varepsilon. \end{cases}$$

3. Maximize ε, subject to the set of LP constraints from point 2), and conduct the subsequent steps to optimize the sum of elementary components $\varepsilon(a, b, c, d)$ analogously to steps 6 and 7 of the procedure from Sect. 3.1.

Let us explain more clearly the idea underlying consideration of the condition specified in point 2 of the above procedure. We identify quadruples $(a, b, c, d) \in A \times A \times A \times A$, such that $a \succ^{N,N}_{\mathcal{D}''} b$ and $c \succ^{N,N}_{\mathcal{D}'''} d$ with $\mathcal{D}''' \subset \mathcal{D}'' \subseteq \mathcal{D}'$. Since more DMs confirm the truth of relation $\succ^N$ for a pair (a, b) than for (c, d), it is reasonable to require the difference between $U(a)$ and $U(b)$ to be greater than between $U(c)$ and $U(d)$.

Notice that at the same time for all $d_r \in \mathcal{D}' \setminus \mathcal{D}''$, we have $not(a \succ^N_{d_r} b)$, whereas for all $d_h \in \mathcal{D}' \setminus \mathcal{D}'''$, we have $not(c \succ^N_{d_h} d)$. Therefore, from another perspective, since $(\mathcal{D}' \setminus \mathcal{D}'') \subset (\mathcal{D}' \setminus \mathcal{D}''')$, we require the difference between $U(c)$ and $U(d)$ to be less than between $U(a)$ and $U(b)$.

Viewing all conditions defined in point 2 as a logical whole, we wish the difference between $U(a)$ and $U(b)$ for $a, b \in A$, such that for all $d_r \in D$, $a \succ^N_{d_r} b$, to be the greatest, the difference between $U(c)$ and $U(d)$ for $c, d \in A$, such that for all $d_r \in D' \subset \mathcal{D}$, $c \succ^N_{d_r} d$, to be slightly less, and the difference between $U(e)$ and $U(f)$, such that for all $d_r \in \mathcal{D}'$, $not(e \succ^N_{d_r} f)$, to be the least. We exclude from consideration in this point such pairs $a, b \in A$, for which we can observe the contradictory strict necessary preference relations with respect to the set of DMs $\mathcal{D}$, i.e. $a \succ^{N,P}_{\mathcal{D}'} b$ and $b \succ^{N,P}_{\mathcal{D}'} a$. In this case, we can simply say nothing about the desired difference between $U(a)$ and $U(b)$, because for some DMs a is preferred to b, whereas for some other DMs b is preferred to a.

Note that in the above procedure, we do not associate the desired intensity of difference between comprehensive values of alternatives directly with the number of DMs confirming the specific relation, but rather with inclusion relation on the set of all subsets of DMs. To explain this proposal, let us consider the set of DMs $\mathcal{D} = \{d_1, d_2, d_3, d_4, d_5\}$, and its subsets $\mathcal{D}' = \{d_1, d_2, d_3\}$ and $\mathcal{D}'' = \{d_4, d_5\}$. In case, $a \succ^{N,N}_{\mathcal{D}'} b$ and $c \succ^{N,N}_{\mathcal{D}''} d$, one may claim that the desired difference between $U(a)$ and $U(b)$ should be greater than the difference between $U(c)$ and $U(d)$, only because more DMs support the advantage of a over b than the advantage of c over d. However, subsets $\mathcal{D}'$ and $\mathcal{D}''$ are not related by the inclusion relation, and thus they are incomparable in terms of the partial order determined by $\subseteq$ on the set of all subsets of $\mathcal{D}$. If so, one should not formulate any direct requirements with respect to comparison of $U(a) - U(b)$ and $U(c) - U(d)$. This example proves why, in general, it is not justified to relate the desired intensity of difference with the cardinalities of subsets of DMs only. Instead, we consider such pairs of alternatives for which the specific results are confirmed by proper (strict) subsets of DMs. In other words, when considering a pair (a, b), such that $a \succ^{N,N}_{\mathcal{D}'} b$, with $\mathcal{D}' = \{d_1, d_2, d_3\}$, we may wish that $U(a) - U(b)$ is greater than $U(e) - U(f)$ for pairs (e, f), such that $e \succ^{N,N}_{\mathcal{D}'''} f$, with $\mathcal{D}''' \in \{\emptyset, \{d_1\}, \{d_2\}, \{d_3\}, \{d_1, d_2\}, \{d_1, d_3\}, \{d_2, d_3\}\}$, and less that $U(g) - U(h)$ for pairs (g, h), such that $g \succ^{N,N}_{\mathcal{D}''''} h$, with $\mathcal{D}'''' \in \{\{d_1, d_2, d_3, d_4\}, \{d_1, d_2, d_3, d_5\}, \{d_1, d_2, d_3, d_4, d_5\}\}$. In this case, one can easily compare arguments supporting the advantage of one alternative over the other.

Procedure for Selection of a Representative Value Function for Group Sorting Problems

The procedure for selection of a representative value function for group sorting problems maximizes the difference between values of alternatives, such that one of them is always assigned to a class better than the other, and minimizes the difference between values of alternatives which do not have this property. Again, the intensity of difference between values of alternatives depends on the inclusion relation on the subsets of DMs who confirm the specific observation for different pairs $(a, b) \in A \times A$. The transposition of the above idea into optimization terms is done in a similar way as for ranking problems. The procedure for selection of a representative value function for group sorting problems is presented below:

I. Determine the possible sorting $C^P_{d_r}(a)$ and the necessary sorting $C^N_{d_r}(a)$ for each considered alternative $a \in A$ for each DM, $d_r \in \mathcal{D}'$. For all pairs of alternatives $(a, b) \in A \times A$, check the truth or falsity of relations $a \succ^{\rightarrow}_{d_r} b$, $a \sim^{\rightarrow}_{d_r} b$, and $a \succ\prec^{\rightarrow}_{d_r} b$ for each $d_r \in \mathcal{D}'$.

II. For all quadruples of alternatives $(a, b, c, d) \in A \times A \times A \times A$, such that $a \succ^{\rightarrow}_{d_h} b$, for all $d_h \in \mathcal{D}''$, and $c \succ^{\rightarrow}_{d_k} d$, for all $d_k \in \mathcal{D}'''$, and $\mathcal{D}''' \subset \mathcal{D}'' \subseteq \mathcal{D}'$, and neither $b \succ^{\rightarrow}_{d_r} a$ nor $d \succ^{\rightarrow}_{d_r} c$ for any $d_r \in \mathcal{D}'$, add the following constraints to the LP constraints of UTADIS$^{\mathrm{GMS}}$:

$$\begin{cases} U(a) - U(b) \geq U(c) - U(d) + \varepsilon(a, b, c, d), \\ \varepsilon(a, b, c, d) \geq \varepsilon. \end{cases}$$

III. Maximize ε, subject to the set of LP constraints from point II), and conduct the subsequent steps to optimize the sum of elementary components $\varepsilon(a, b, c, d)$ analogously to steps VI and VII of the procedure from Sect. 3.1.

Notice that one can generalize the above formulations in order to deal with the coalitions of DMs, and not just single DMs. This might be required, if preference information is supplied by experts from different companies or agencies, e.g., $\mathcal{D}' = \{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_k\}$, where $\mathcal{D}_1 = \{d_1, \ldots, d_i\}$, $\mathcal{D}_2 = \{d_j, \ldots, d_m\}$, ..., $\mathcal{D}_k = \{d_p, \ldots, d_s\}$. Then, it would be reasonable to treat each coalition of DMs in the way a single DM is treated in the standard formulation of the procedure, and check the truth of inclusion relation in the second point of the above procedures with respect to the coalitions $\mathcal{D}_1, \ldots, \mathcal{D}_k$, rather than DMs $d_1, \ldots, d_s$, considered individually.

3.3 Analysis of Incompatibility

In case of incompatibility, there is no value function U compatible with the preference information provided by all DMs. Notice that $\mathcal{U}_{\mathcal{D}'}$ corresponds to the intersection of sets of compatible value functions for all $d_r \in \mathcal{D}'$ (each one being non-empty). This means that exemplary decisions of at least two DMs are contradictory. Such a situation may occur if some pieces of preference information do not match the additive model, or most often if their decision policy differs substantially. Treating the problem, one may identify some reasons for either removing it, or searching for a representative value function for a minimal magnitude of the error, or a minimal number of contradictory statements, or a maximal number of DMs whose reference statements are not contradictory.

Notice that in the first case, one needs to extend the set of constraints $E^{A^R}_{\mathcal{D}'}$ with an additional inequality $\varepsilon \geq \varepsilon^*$, such that $\varepsilon^* = \max \varepsilon$, s.t. $E^{A^R}_{\mathcal{D}'}$, so that the resulting new constraints $E^{A^R}_{\mathcal{D}',ext}$ are feasible. Let us rather focus on the latter case, which is more specific for group decision problems. At first, we need to identify the troublesome pieces of preference information responsible for incompatibility. There may exist several sets of pairwise comparisons or exemplary assignments which, once removed, make the set of compatible value functions non-empty. Performing these computations for all $\mathcal{D}'' \subset \mathcal{D}'$ enables to identify coalitions of convergent DMs, for which the necessary and possible consensus rankings or assignments exist. Main steps of the procedure which identifies these coalitions are outlined below. This procedure is inspired by the general schemes for analysis of incompatibility introduced in Mousseau et al. (2003) and Mousseau et al. (2006).

Identification of the contradictory statements requires solving a mixed integer programming (MIP) problem. Using binary variables v_{a^*,b^*} associated with each couple of reference alternatives (a^*, b^*), such that $a^* \succsim b^*$ for any DM in case of ranking problems, or $L^{d_i}(a^*) > R^{d_j}(b^*)$ for sorting problems (with the proviso that the assignments for a^* and b^* may have been provided by different DMs), one rewrites the set of constraints $(E^{A^R}_{\mathcal{D}'})$ into $(E^{A^R}_{\mathcal{D}'})'$. Subsequently, to identify a minimal subset of troublesome exemplary decisions, we solve the following MIP problem:

$$min\ f = \sum_{a^*,b^* \in A^R,\ condition(a^*,b^*)} v_{a^*,b^*}$$

s.t.

$$\left.\begin{array}{l}(E^{A^R}_{\mathcal{D}',X})' \\ (E^{A^R}_{\mathcal{D}',base})\end{array}\right\} (E^{A^R}_{\mathcal{D}'})'$$

where for ranking problems $condition(a^*, b^*)$=$a^* \succsim b^*$, and $(E^{A^R}_{\mathcal{D}',X})' = (E^{A^R}_{\mathcal{D}',rank})'$, such that:

$$\left.\begin{array}{l}U(a^*) + Mv_{a^*,b^*} \geq U(b^*) + \varepsilon \ \text{ if } \ a^* \succ_{d_r} b^*,\ d_r \in \mathcal{D}' \\ \left.\begin{array}{l}U(a^*) + Mv_{a^*,b^*} \geq U(b^*) \\ U(b^*) + Mv_{a^*,b^*} \geq U(a^*)\end{array}\right\} \ \text{if } \ a^* \sim_{d_r} b^*,\ d_r \in \mathcal{D}' \\ v_{a^*,b^*} \text{ is a binary variable}\end{array}\right\} (E^{A^R}_{\mathcal{D}',rank})',$$

and for sorting problems $condition(a^*, b^*) = L^{d_i}(a^*) > R^{d_j}(b^*)$ and $(E^{A^R}_{\mathcal{D}',X})' = (E^{A^R}_{\mathcal{D}',sort})'$, such that:

$$\left.\begin{array}{l}U(a^*) + Mv_{a^*,b^*} \geq U(b^*) + \varepsilon, \ \text{for all } a^* \in A^R_{d_i},\ b^* \in A^R_{d_j} \\ \text{such that } L^{d_i}(a^*) > R^{d_j}(b^*),\ d_i, d_j \in \mathcal{D}' \\ v_{a^*,b^*} \text{ is a binary variable}\end{array}\right\} (E^{A^R}_{\mathcal{D}',sort})',$$

where ε is a small positive value and $M > 1$.

If $v_{a^*,b^*} = 1$, then the corresponding constraint is satisfied whatever the value function is, which is equivalent to elimination of this constraint. The optimal solution indicates one of the minimal subsets of conditions being the cause of incompatibility. Other subsets can be identified by solving this problem with additional constraint that forbids finding again the same solution:

$$\sum_{(a^*,b^*)\in S_i} v_{a^*,b^*} \leq f_i^* - 1,$$

where f_i^* is the optimal value of the objective function of $(E_{\mathcal{D}'}^{A^R})'$ in the i-th iteration, $S_i = \{(a^*, b^*) \in A^R \times A^R,\ a^* \succsim_{d_r} b^*$ for ranking or $L^{d_i}(a^*) > R^{d_j}(b^*)$ for sorting and $v_{a^*,b^*}^{*i} = 1\}$, and v_{a^*,b^*}^{*i} are the corresponding values of the binary variables at the optimum.

The solutions identified in this way may still be rather similar to each other. To ensure their greater diversity, we could maximize k indicating the number of pieces of preference information presented as the subset underlying incompatibility in the current iteration that should not be included in the next proposal. Additionally, we should require that the cardinality of the new subset of pieces of preference information is not too great, i.e. it could be greater than f_i^* by a small t ($t \geq 0$). Proceeding in this way, requires considering the following MIP problem:

$$Maximize : k$$

s.t.

$$\left.\begin{array}{l} (E_{\mathcal{D}'}^{A^R}) \\ \sum_{(a^*,b^*)\in S_i} v_{a^*,b^*} \leq f_i^* - k \\ \sum_{a^*,b^*\in A^R,\ condition(a^*,b^*)} v_{a^*,b^*} \leq f_i^* + t \\ v_{a^*,b^*} \text{ is a binary variable} \end{array}\right\} (E_{\mathcal{D}'}^{A^R})_{i+1}.$$

Note that identification of the minimal subsets of inconsistent pieces of preference information in the context of multiple DMs, could lead to removal of a significant subset of preferences of a particular DM, while preserving all statements of all other DMs. Such solution would not be perceived as "fair" by all DMs. To address this problem, one could account for minimization of the maximal number of pieces of preference information of each DM that should be removed. This could be achieved by consideration of the sum of binary variables associated with statements provided by each DM. For example, in case of ranking problems, for each $d_r \in \mathcal{D}'$ we would introduce the following variable:

$$v_{d_r} = \sum_{a^*\{\succ_{d_r},\sim_{d_r}\}b^*} v_{a^*,b^*}, \text{ for each } d_r \in \mathcal{D}'.$$

Subsequently, we would minimize $v_{\mathcal{D}'}$ defined as follows:

$$v_{\mathcal{D}'} = \max_{d_r \in \mathcal{D}'} v_{d_r}, \text{ i.e., } v_{\mathcal{D}'} \geq v_{d_r} \text{ for each } d_r \in \mathcal{D}'.$$

Outcomes of such a procedure may be presented to the DMs as possible ways for removing incompatibility. In this case, the DMs are asked to review their statements. Alternatively, once all the minimal subsets of pieces of preference information causing incompatibility are identified, one may proceed to identification of the minimal subset of DMs who provided contradictory statements. To achieve this, we need to associate pairwise comparisons or exemplary assignments coded as v_{a^*,b^*} with identifiers of the DMs who provided a particular piece of preference information. Knowing the set of identifiers of DMs corresponding to $\{(a^*, b^*) \in A^R \times A^R : v_{a^*,b^*} = 1\}$ for all minimal subsets of constraints being the cause of incompatibility, we can easily point out the minimal subset of DMs whose preferences cannot be represented together. Subsequently, we eliminate all pieces of preference information and respective constraints corresponding to the DMs for whom $v_{a^*,b^*}=1$, and pursue the analysis searching for a representative value function for the maximal number of DMs whose preference statements were consistent. Such a function may constitute a good support for generating reactions from the part of the "excluded" DMs, who may modify the previously supplied preference statements. It may also be used to work out the final recommendation, especially in these decision making situations which involve a single DM and several experts who provide their preferences.

Notice that all presented ways of dealing with incompatibility for group decision, i.e. accounting for the minimal magnitude of error, or for the minimal number of contradictory statements, or for the maximal number of DMs whose statements are consistent, may be applied to different formulations of the procedures discussed in Sects. 3.1 and 3.2. Moreover, for the case of considering preference information of each DM individually, we may simply assume that the best compromise is reflected by the value function for which the optimized target is maximal. Obviously, a representative value function as well as the necessary and possible outcomes resulting from these computations will not fully respect all exemplary decisions initially provided by the DMs.

4 Case Studies

In this section, we report results of numerical experiments with two case studies. In the first case study, we reconsider the problem which has been originally presented in Greco et al. (2008) to illustrate the UTA$^{\text{GMS}}$ method. We analyze it in the context of several DMs who cooperate to make a collective ranking. We show representative value functions identified according to the different procedures introduced in this paper. The second case study is devoted to the problem of sorting different countries to one of four types of regimes. We use data published by the Economist Intelligence Unit in 2007 for estimation of the democracy index (EIU 2007), and we reconsider the stated problem for 40 countries from different regions of the world. We discuss the necessary and possible assignments for preference information of all DMs considered individually or jointly. Further, we show representative value functions identified for these results by different procedures.

4.1 Assessment of Sales Managers

The CEO of a medium size firm wants to hire new international sales managers. A recruitment agency has interviewed 15 potential candidates which have been evaluated on 3 criteria (sales management skills (g_1), international experience (g_2), and human qualities (g_3)) with an increasing direction of preference. Each of the five workers of the agency has attended a few interviews and is able to express a confident judgment about some candidates. The evaluations of candidates are given in Table 1, and preference information in form of pairwise comparisons provided by all DMs, $\mathcal{D} = \{d_1, d_2, d_3, d_4, d_5\}$, is summarized in Table 2.

Representative Value Function for the Case of Preference Information of All DMs Considered Jointly

Pairwise comparisons provided by all DMs are consistent, and, consequently, a set of compatible value functions compatible with reference statements of all DMs in not empty, i.e. $\mathcal{U}_{\mathcal{D}} \neq \emptyset$. For this preference information, the necessary and possible weak preference relations are computed using UTA$^{\text{GMS}}$. The Hasse diagram of the necessary relation $\succsim^N_{\mathcal{D}}$ for this particular problem is presented in Fig. 1. Note that the necessary preference relation is transitive. In this perspective, let us remind that arrows that can be obtained by transitivity are not represented in the Hasse diagram.

Table 1 Performance matrix and representative comprehensive values resulting from four different procedures for the problem of ranking sales managers

	Name	g_1	g_2	g_3	$U^R_{\mathcal{D}}$	$U^R_{\mathcal{D}_1}$ (pos)	$U^R_{\mathcal{D}_2}$ (pos)	$U^R_{\mathcal{D},GMS}$ (pos)
I	Alexievich	4	16	63	0.538	0.533 (6)	0.538 (9)	0.478 (7)
II	Bassama	28	18	28	0.615	0.600 (5)	0.654 (5)	0.609 (4)
III	Calvet	26	40	44	0.731	0.622 (4)	0.885 (1)	0.696 (1)
IV	Dubois	2	2	68	0.308	0.289 (14)	0.308 (14)	0.391 (12)
V	El Mrabat	18	17	14	0.385	0.378 (12)	0.500 (11)	0.435 (11)
VI	Feeret	35	62	25	0.846	0.844 (1)	0.885 (1)	0.696 (1)
VII	Fleichman	7	55	12	0.538	0.533 (6)	0.577 (7)	0.478 (7)
VIII	Fourny	25	30	12	0.538	0.511 (8)	0.692 (4)	0.478 (7)
IX	Frechet	9	62	88	0.692	0.689 (3)	0.769 (3)	0.652 (3)
X	Martin	0	24	73	0.308	0.311 (13)	0.308 (14)	0.391 (12)
XI	Petron	6	15	100	0.423	0.400 (11)	0.538 (9)	0.478 (7)
XII	Psorgos	16	9	0	0.231	0.222 (15)	0.423 (12)	0.348 (15)
XIII	Smith	26	17	17	0.500	0.489 (9)	0.577 (7)	0.522 (6)
XIV	Varlot	62	43	0	0.846	0.844 (1)	0.615 (6)	0.609 (4)
XV	Yu	1	32	64	0.423	0.422 (10)	0.385 (13)	0.391 (12)

Table 2 Preference infromation provided by each DM for the problem of ranking sales managers

	Preference information
d_1	Frechet $\succ$ Fleichman $\succ$ Martin, Fleichman $\succ$ Yu
d_2	Frechet $\succ$ Fourny $\succ$ Petron, Varlot $\succ$ Petron
d_3	Fleichman $\succ$ Yu $\succ$ Martin, Varlot $\succ$ Yu
d_4	Varlot $\succ$ Calvet, Varlot $\succ$ Fleichman
d_5	Petron $\succ$ Martin, Fourny $\succ$ Petron

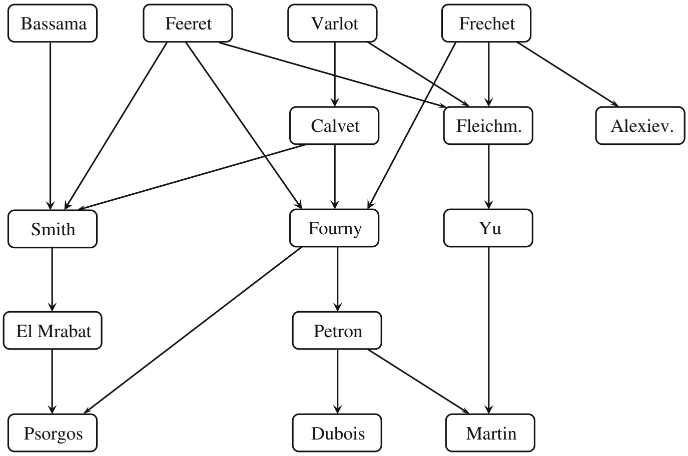

Fig. 1 Hasse diagram of the necessary preference relation $\succsim^N_{\mathcal{D}}$ for the problem of ranking sales managers

The valid necessary relation means that a definite pair of alternatives is ranked in the same way whatever the compatible additive value function. As a result, the DMs may recognize the necessary relations as the certain preference statements. The necessary results reflect also the ability of the inferred model to reproduce all pairwise comparisons of reference alternatives. In order to choose a representative value function, we analyze the necessary weak preference relation. In the considered problem, there are 49 ordered pairs of alternatives (a, b), for which $a \succsim^N_{\mathcal{D}} b$ [e.g., (Frechet, Alexievich)]. Since for these pairs the advantage of a over b is evident, we would like to emphasize it, maximizing the difference between $U(a)$ and $U(b)$. On the other hand, there are also 88 pairs of alternatives $c, d \in A$, for which *not*$(c \succ^N_{\mathcal{D}} d)$ and *not*$(d \succ^N_{\mathcal{D}} c)$ (e.g., Alexievich and Fleichman). These pairs of alternatives are necessarily incomparable or necessarily indifferent, so analyzing robust results one cannot indicate the better alternative among them. Therefore, we wish to minimize the difference between $U(c)$ and $U(d)$. All pairs (a, b) and (c, d) satisfying these specific conditions contribute to the definition of a representative value function according to the procedure presented in Sect. 3.1.

In a sense, the employed procedure "flattened" the consequences of applying all compatible value functions to set A by using a single instance of the preference model. The representative value function permits the users to see the score of each

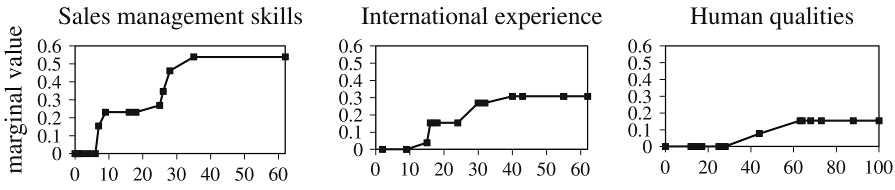

Fig. 2 Representative marginal value functions for the case of considering preference information of all DMs jointly

alternative, and to assess relative importance of the criteria, extending capacity of the robust method in explaining its final output. Figure 2 illustrates the representative marginal value functions. They form a very intuitive representation of the output of the ordinal regression method, which helps to understand the results of the robust ranking. The characteristic points marked in the figure correspond to the marginal values of the considered alternatives. Although in the figure connections between these points are linear, it would be sufficient if they reflected the monotonic character. For this case, sales management skills criterion has the greatest share in the comprehensive values, whereas human qualities criterion has the least share. The same order can be observed with respect to the variation of marginal values. Such distribution of the shares along with the differentiation of the values by marginal value functions reflect consequences of the preferences of the DMs in the best way.

The resulting representative comprehensive values are presented in Table 1 (see column $U^R_{\mathcal{D}}$). Notice that relations between these values represent all pieces of preference information which have been provided by the DMs. Moreover, analyzing the specific pairs of alternatives, one can notice that the difference between their values is either relatively great, because one of them is necessarily preferred to the other [e.g., (Bassama, Psorgos), (Frechet, Alexievich)], or intentionally neglected as they are possibly indifferent (there are a few pairs or triples of alternatives which are assigned equal comprehensive values [e.g., (Alexievich, Fleichman), (Petron, Yu)].

Note that the highest representative comprehensive values are assigned to Feeret and Varlot, which are not necessarily outranked by any other alternative. On the other hand, for Psorgos, Dubois, and Martin the comprehensive values are the smallest. These alternatives are not necessarily better than any other alternative, while being necessarily worse than numerous other candidates. Finally, let us note that although Calvet is necessarily outranked by Varlot, its representative comprehensive value is greater than $U^R_{\mathcal{D}}(a)$ for Bassama, which could be possibly ranked at the top of the ranking for some compatible value functions. It is the case, since Calvet is necessarily preferred to a majority of other alternatives, while Bassama is incomparable in terms of the necessary relation even with candidates that could be perceived as the potential worst options, i.e. Dubois and Martin. These observations prove that the representative value function emphasizes the most certain recommendation, and at the same finds a reasonable compromise between all possible recommendations. In this way, the clarity of classical UTA-like methods is combined with the analysis conducted within the framework of robust ordinal regression. Consequently, the ranking which stems from ordering the alternatives with respect to the values $U^R_{\mathcal{D}}(a)$ should be perceived as representative in the sense of robustness of the provided solutions.

Representative Value Functions for the Maximal Subset of DMs Whose Preference Statements are Consistent

Let us assume that in addition to preference information given in Table 1, d_5 provided preference statement: Fourny $\succ$ Varlot. In this case, analysis of compatibility reveals that there is no additive function which could reproduce partial preorders supplied by all DMs. This means that pairwise comparisons of two (or more) DMs are contradictory. Indeed, after their identification, we get to know that statements Varlot $\succ$ Calvet and Fourny $\succ$ Varlot cannot be represented together. The previous has been provided by d_4, whereas the other by d_5. Therefore, there are two maximal subsets of DMs whose statements can be reproduced by a consistent collective model, i.e.: $\mathcal{D}_1 = \{d_1, d_2, d_3, d_4\}$ and $\mathcal{D}_2 = \{d_1, d_2, d_3, d_5\}$.

Having removed preference information either of d_4 or d_5, we obtain preference models which are representative for coalitions of convergent DMs whose statements are not contradictory. One can notice some slight differences when looking at the representative marginal value functions obtained for $\mathcal{D}_1$ and $\mathcal{D}_2$ (see Fig. 3). In the other case, the share of sales management skills in the comprehensive values is less than in the first case in favor of human qualities criterion. The resulting representative comprehensive values are presented in Table 1 (see columns $U^R_{\mathcal{D}_1}$ and $U^R_{\mathcal{D}_2}$). Since they reflect preferences of the same three DMs and differ with respect to only one expert, the correlation (measured using Kendall's coefficient) between the representative rankings which follow those values is high (nearly 0.7). The main differences result from the explicit preferences of the included (excluded) DMs. For coalition $\mathcal{D}_1$, Varlot is higher in the ranking than for $\mathcal{D}_2$, since d_4 strongly opts for its high position. On the other hand, for coalition $\mathcal{D}_2$, Fourny and Calvet occupy higher positions than for $\mathcal{D}_1$, because the previous is preferred by d_5 over two other alternatives, and we

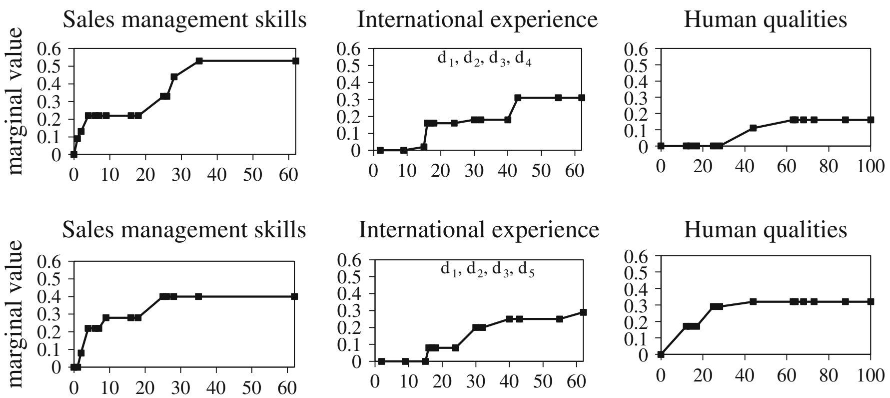

Fig. 3 Representative marginal value functions for the maximal subsets of DMs whose statements are not contradictory

have excluded from consideration the statements of d_4 who claimed that Calvet is worse than Varlot.

Analysis of the representative values and corresponding rankings may convince the DMs whose preferences were incompatible with pairwise comparisons of the others, to reconsider their statements. In particular, d_5 should reconsider the statement: Fourny $\succ$ Varlot, since when neglecting her/his preferences, in a representative case Varlot is ranked first, while Fourny is ranked eighth. In the same spirit, d_4 may be encouraged to remove statement: Varlot $\succ$ Calvet, because for the subset $\mathcal{D}_2$, Calvet is ranked at the very top. Finally, all DMs should pay special attention to the alternatives which are ranked either high (e.g., Feeret, Frechet, Calvet) or low (e.g., Psorgos, Dubois, Martin, Yu) for both analyzed subsets of DMs, and possibly refer to these candidates when providing pairwise comparisons in the next iteration.

Representative Value Function for the Case of Preference Information of Each DM Considered Individually

Although the preference information provided by all DMs in the previously discussed case is inconsistent when considered jointly, the statements of each DM are consistent when analyzed individually. Consequently, in order to select a representative value function and obtain results which follow its use, we can take advantage of the UTA$^{\text{GMS}}$-GROUP method, and subsequently make the difference between values of specific alternatives dependent on the inclusion relation on the set of all subsets of DMs for whom the specific relations hold. The results concerning the "necessary-necessary" preference relation $\succsim_{\mathcal{D}}^{N,N}$ for the subset of alternatives are presented in Table 3. Its cells are filled with indices (1, 2, 3, 4, or 5) standing for the DMs for whom the necessary relation $\succsim_{d_r}^{N}$ holds [e.g., (VII, X) = (Fleichman, Martin) = {1, 3}, means that Fleichman $\succsim_{d_r}^{N}$ Martin holds for d_1 and d_3, and does not hold for d_2, d_4, and d_5].

In this case, the representative value function is identified with the use of the procedure presented in Sect. 3.2. Explaining its main rule with respect to the considered example, one may say that the desired difference between values of alternatives for which the strict necessary preference relation holds for all DMs [e.g., (VI, VII), (IX, X)] should be greater than for such pairs of alternatives, for which this relation

Table 3 A part of the matrix of the necessary $\succsim_{\mathcal{D}}^{N,N}$ relation for the problem of ranking sales managers

$\succsim_{\mathcal{D}}^{N,N}$	.	VI	VII	VIII	IX	X	.
.	.	.	.	.	.	.	.
VI	.	1, 2, 3, 4, 5	1, 2, 3, 4, 5	1, 2, 3, 4, 5	–	1, 3, 5	.
VII	.	–	1, 2, 3, 4, 5	3	–	1, 3	.
VIII	.	–	–	1, 2, 3, 4, 5	–	3, 5	.
IX	.	–	1, 2, 3, 4, 5	2, 3	1, 2, 3, 4, 5	1, 2, 3, 4, 5	.
X	.	–	–	–	–	1, 2, 3, 4, 5	.
.	.	.	.	.	.	.	.

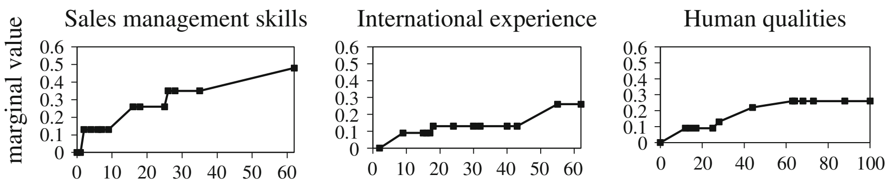

Fig. 4 Representative marginal value functions for the case of considering preference information of all DMs individually

holds for the proper subsets of $\mathcal{D}$ [e.g., (VI, X), (VII, VIII)]. Further, the difference between values of alternatives for which the relation $\succ_{d_r}^{N}$ holds for d_1, d_3, and d_5 [e.g., (VI, X)] should be greater than for pairs of alternatives for which the considered relation holds only for d_1 and d_3 [e.g., (VII, X)], or d_3 and d_5 [e.g., (VIII, X)], or only d_3 [e.g., (VII, VIII)], etc. Finally, if for the considered pair of alternatives the relation $\succ_{d_r}^{N}$ holds for at least one DM, the desired difference between their comprehensive values should be greater than between values of alternatives for which the relation $\succ_{d_r}^{N}$ does not hold for any expert.

Figure 4 illustrates the obtained marginal value functions for the case of considering preference information of each DM individually. They are similar to those which were identified in the previous case with sales management skills having the greatest share in the comprehensive values.

Table 1 summarizes comprehensive values obtained for the representative value function (see column $U_{\mathcal{D},GMS}^{R}$). The ranking determined by these values can be considered as a synthetic representation of the results of the UTA^{GMS}-GROUP method. Its correlation with the rankings determined for $\mathcal{D}_1$ and $\mathcal{D}_2$ is equal to 0.72 and 0.79, respectively. However, one has to notice that it does not reproduce all pairwise comparisons of all DMs, since, as it was previously proved, it is impossible when modeling preference using a general additive value function (e.g., relations Varlot $\succ$ Calvet and Fourny $\succ$ Varlot are not reflected). As already mentioned in Sect. 3.3, a possible extension of this approach would be to require reproduction of the maximal number of consistent statements of the DMs, or to guarantee fulfillment of pairwise comparisons provided by the subset of DMs who are included in all possible maximal subsets of DMs whose statements are complementary.

4.2 Democracy Index: Assigning Countries to the Types of Regimes

There is no consensus on how to measure democracy. The Economist Intelligence Unit (EIU) proposed to consider 60 indicators grouped in five categories which form a coherent conceptual whole: electoral process and pluralism (g_1), the functioning of government (g_2), political participation (g_3), political culture (g_4), and civil liberties (g_5). The index they work out, which is the simple average of the five main criteria, provides a snapshot of the current state of democracy worldwide for 165 independent states and two territories. Then, index values are used to place countries within one of

Table 4 Preference information provided by each DM for the problem of assigning countries to different types of regimes

	Preference information
d_1	C_4—USA, Costa Rica, C_3—Honduras, C_2—Nicaragua, C_1—Cuba
d_2	C_3—Taiwan, Indonesia, Malaysia, Philippines, C_2—Singapore, Cambodia, Fiji
d_3	C_4—Japan, C_3—South Korea, Hong Kong, C_2—Kyrgyzstan, C_1—Uzbekistan, China

four types of regimes: full democracies (C_4), flawed democracies (C_3), hybrid regimes (C_2), and authoritarian regimes (C_1).

Let us reconsider a subset of 40 countries from the data set provided by EIU. These countries can be grouped into 3 regions: 14 from Northern and Central America, 13 from Australasia and Southeast Asia, and 13 from Central and East Asia. Let us assume that there are 3 DMs ($\mathcal{D}=\{d_1, d_2, d_3\}$), each one being an expert of the political issues of the specific region. Thus, they are able to provide some exemplary assignments concerning countries they are familiar with (see Table 4).

Representative Value Function for the Case of Preference Information of Each DM Considered Jointly

Since every DM serves as an expert only for a subset of countries, it is reasonable to conduct the comprehensive analysis, considering preference information of all DMs jointly. The possible assignments $[L^P_{\mathcal{D}}(a), R^P_{\mathcal{D}}(a)]$, for all $a \in A$, are given in Table 5. The average width of the range of possible classes is equal to 1.375. For 29 alternatives for which the possible assignment is precise, the necessary range of classes is not empty, $[L^N_{\mathcal{D}}(a), R^N_{\mathcal{D}}(a)] \neq \emptyset$. For other alternatives the result of sorting depends on which compatible value function would underly the proper assignment procedure. Since the preference information provided by the DMs agrees with the actual classes assigned by EIU, and the weighted sum aggregation model used by them is a particular case of the additive value function, one can notice that the class determined by EIU (see column $C(a)$ in Table 5) is in the range of possible assignments for each country. In order to formulate targets which should be optimized when selecting a representative value function, we check the truth of relations $\succ^{\rightarrow}_{\mathcal{D}}$, $\sim^{\rightarrow}_{\mathcal{D}}$, and $\succ\prec^{\rightarrow}_{\mathcal{D}}$ for all pairs of alternatives. As a result, there are 426 pairs (a, b), such that $a \succ^{\rightarrow}_{\mathcal{D}} b$, and 582 pairs $c, d \in A$, such that $c \sim^{\rightarrow}_{\mathcal{D}} d$ or $c \succ\prec^{\rightarrow}_{\mathcal{D}} d$.

Figure 5 illustrates the obtained representative marginal value functions (see the continuous lines). The number of characteristic points for these functions is close to the number of alternatives, so we have marked only those points for which the marginal value changes. Electoral process and pluralism criterion (g_1) has slightly greater share in the comprehensive values than other criteria, but the maximal share of all criteria is greater than 0.15. It is interesting in the context of equal weights of all impacts assumed by EIU in their analysis. Table 6 summarizes values and assignments computed using the representative value function (see columns U^R and $L^R_{\mathcal{D}} - R^R_{\mathcal{D}}$). Analysis of the distribution of the representative comprehensive values allows

Table 5 Performance matrix and actual and possible assignments for the problem of assigning countries to different types of regimes

	Country	g_1	g_2	g_3	g_4	g_5	$C(a)$	$L_{\mathcal{D}}^{P}(a)$	$R_{\mathcal{D}}^{P}(a)$
1	Australia	10.00	8.93	7.78	8.75	10.00	4	4	4
2	Canada	9.17	9.64	7.78	8.75	10.00	4	4	4
3	New Zealand	10.00	8.57	8.33	8.13	10.00	4	4	4
4	USA	8.75	7.86	7.22	8.75	8.53	4	4	4
5	Japan	9.17	7.86	5.56	8.75	9.41	4	4	4
6	Costa Rica	9.58	8.21	6.11	6.88	9.41	4	4	4
7	South Korea	9.58	7.14	7.22	7.50	7.94	3	3	3
8	Taiwan	9.58	7.50	6.67	5.63	9.71	3	3	3
9	Panama	9.58	7.14	5.56	5.63	8.82	3	3	3
10	Jamaica	9.17	7.14	5.00	6.25	9.12	3	3	4
11	Trin. & Tob.	9.17	6.79	6.11	5.63	8.24	3	3	3
12	Mexico	8.75	6.07	5.00	5.00	8.53	3	2	3
13	Mongolia	9.17	6.08	3.89	5.63	8.24	2	3	3
14	Papua New G.	7.33	6.43	4.44	6.25	8.24	3	2	4
15	Philippines	9.17	5.36	5.00	3.75	9.12	3	3	3
16	Indonesia	6.92	7.14	5.00	6.25	6.76	3	3	3
17	Timor Leste	7.00	5.57	5.00	6.25	8.24	3	2	4
18	Honduras	8.33	6.43	4.44	5.00	7.06	3	3	3
19	Salvador	9.17	5.43	3.89	4.38	8.24	3	2	4
20	Dom. Rep.	9.17	4.29	3.33	5.63	8.24	3	2	3
21	Guatemala	8.75	6.79	2.78	4.38	7.65	3	2	4
22	Hong Kong	3.50	5.71	5.00	6.25	9.71	3	3	3
23	Malayasia	6.08	5.71	4.44	7.50	6.18	3	3	3
24	Singapore	4.33	7.50	2.78	7.50	7.35	2	2	2
25	Nicaragua	8.25	5.71	3.33	3.75	7.35	2	2	2
26	Thailand	4.83	6.43	5.00	5.63	6.47	2	2	4
27	Fiji	6.50	5.21	3.33	5.00	8.24	2	2	2
28	Cambodia	5.58	6.07	2.78	5.00	4.41	2	2	2
29	Haiti	5.58	3.64	2.78	2.50	6.47	2	1	2
30	Kyrgyzstan	5.75	1.86	2.78	5.00	5.00	2	2	2
31	Kazkhstan	2.67	2.14	3.33	4.38	5.59	1	1	2
32	Cuba	1.75	4.64	3.89	4.38	2.94	1	1	1
33	China	0.00	4.64	2.78	6.25	1.18	1	1	1
34	Vietnam	0.83	4.29	2.78	4.38	1.47	1	1	1
35	Tajikistan	1.83	0.79	2.22	6.25	1.18	1	1	2
36	Laos	0.00	3.21	1.11	5.00	1.18	1	1	1
37	Uzbekistan	0.08	0.79	2.78	5.00	0.59	1	1	1
38	Turkmenistan	0.00	0.79	2.78	5.00	0.59	1	1	1
39	Myanmar	0.00	1.79	0.56	5.63	0.88	1	1	1
40	North Korea	0.83	2.50	0.56	1.25	0.00	1	1	1

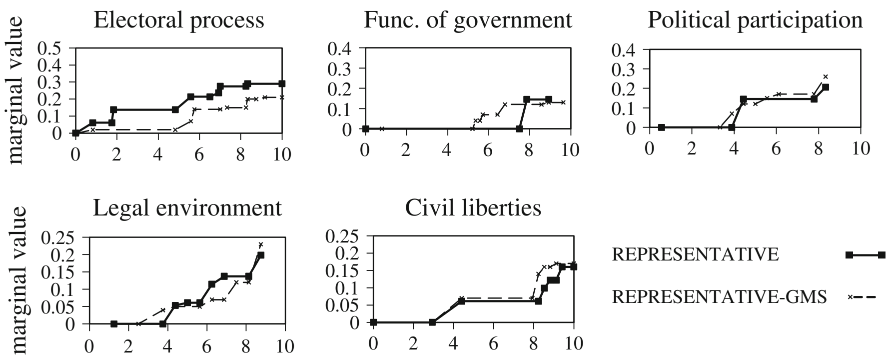

Fig. 5 The marginal value functions resulting from two different procedures of selection of a single compatible value function for the problem of assigning countries to different types of regimes

Table 6 Comprehensive values and assignments stemming from two different procedures for selection of the representative value function for the problem of assigning countries to different types of regimes

	Country	$U_{\mathcal{D}}^{R}$	$L_{\mathcal{D}}^{R} - R_{\mathcal{D}}^{R}$	$U_{\mathcal{D},GMS}^{R}$	$L_{\mathcal{D},GMS}^{R} - R_{\mathcal{D},GMS}^{R}$
1	Australia	0.939	4	0.915	4
2	Canada	0.939	4	0.915	4
3	New Zealand	0.939	4	0.880	4
4	USA	0.878	4	0.880	4
5	Japan	0.939	4	0.880	4
6	Costa Rica	0.878	4	0.744	4
7	South Korea	0.634	3	0.692	3
8	Taiwan	0.656	3	0.726	3
9	Panama	0.618	3	0.692	3
10	Jamaica	0.672	3 − 4	0.692	3
11	Trin. & Tob.	0.567	3	0.692	3
12	Mexico	0.586	3	0.603	3
13	Mongolia	0.412	2 − 3	0.538	3
14	Papua New G.	0.586	3	0.538	3
15	Philippines	0.567	3	0.513	3
16	Indonesia	0.567	3	0.506	3
17	Timor Leste	0.586	3	0.504	3
18	Honduras	0.567	3	0.513	3
19	Salvador	0.405	2 − 3	0.513	3
20	Dom. Rep.	0.412	2 − 3	0.402	2 − 3
21	Guatemala	0.405	2 − 3	0.444	3
22	Hong Kong	0.567	3	0.444	3
23	Malayasia	0.567	2 − 3	0.513	3
24	Singapore	0.336	2	0.325	2
25	Nicaragua	0.336	2	0.325	2
26	Thailand	0.405	2 − 3	0.325	2

Table 6 Continued

	Country	$U_{\mathcal{D}}^{R}$	$L_{\mathcal{D}}^{R} - R_{\mathcal{D}}^{R}$	$U_{\mathcal{D},GMS}^{R}$	$L_{\mathcal{D},GMS}^{R} - R_{\mathcal{D},GMS}^{R}$
27	Fiji	0.336	2	0.325	2
28	Cambodia	0.336	2	0.256	2
29	Haiti	0.275	$1-2$	0.137	1
30	Kyrgyzstan	0.336	2	0.256	2
31	Kazkhstan	0.252	$1-2$	0.137	1
32	Cuba	0.115	1	0.137	1
33	China	0.115	1	0.068	1
34	Vietnam	0.115	1	0.068	1
35	Tajikistan	0.252	$1-2$	0.085	1
36	Laos	0.061	1	0.051	1
37	Uzbekistan	0.061	1	0.051	1
38	Turkmenistan	0.061	1	0.051	1
39	Myanmar	0.061	1	0.051	1
40	North Korea	0.061	1	0.017	1

distinction of four ranges of main concentration of the respective alternatives. These are: $0.061 - 0.115$, $0.336 - 0.336$, $0.567 - 0.656$, and $0.878 - 0.939$. Each of these ranges aggregates alternatives for which there is no doubt about their assignment (their possible assignments are precise). The width of such a single range is significantly less than the distance between extreme values of two consecutive ranges. It stems from the requirements we have stated with respect to the representative value function. Precisely, given a pair of alternatives, the one which is certainly in a better class in the context of all compatible functions, should have significantly greater value. On the other hand, if two alternatives are always in the same class or the order of their classes is not univocal, the difference between their values should be reasonably small. Outside of the above listed four ranges of comprehensive values, there are values of alternatives for which the possible assignments are not precise.

As far as representative assignments are concerned, they are more precise than possible ones, and more general than necessary assignments. In fact, since the representative value function $U_{\mathcal{D}}^{R} \in \mathcal{U}_{\mathcal{D}}$, for all $a \in A$, it always holds:

$$[L_{\mathcal{D}}^{N}(a), R_{\mathcal{D}}^{N}(a)] \subseteq C_{\mathcal{D}}^{R}(a) = [L_{\mathcal{D}}^{R}(a), R_{\mathcal{D}}^{R}(a)] \subseteq [L_{\mathcal{D}}^{P}(a), R_{\mathcal{D}}^{P}(a)].$$

Among 22 non-reference alternatives, there are 12 for which the outcomes of the example-based procedure driven by the representative value function are precise. For this particular case, these assignments agree with the classes determined by EIU. The evaluation profiles of the remaining 10 alternatives which are assigned to imprecise ranges of classes, are most often not typical for a given political regime, i.e. they are either among the best or the worst countries in a particular class with respect to a democracy index computed by EIU (see, e.g., Jamaica, Malaysia, Guatemala, Dominican Republic, Thailand, Haiti, Kazakhstan). In this context, the hesitation in the final recommendation seems to be justified.

Comparison with Results of UTADIS^{GMS}

In order to provide empirical evidence of some implications presented in Sect. 3.1, we will discuss results obtained for this particular problem with UTADIS^{GMS}-GROUP. In this case, we consider preference information of each DM individually, we determine its possible and necessary consequences, and only then we combine them into necessary-necessary, necessary-possible, possible-necessary, and possible-possible results.

Since necessary assignments are often empty, especially when preference information supplied by the DMs is not rich, $C_{\mathcal{D}}^{N,N}(a),\ a \in A$, are empty as well. As far as $C_{\mathcal{D}}^{N,P}(a),\ a \in A$, are concerned, for this particular problem, they are non-empty for 23 countries, which means that at least one DM from $\mathcal{D}$ is either sure about their desired class and expresses it directly in her/his preference statements, or all instances of a preference model compatible with her/his preference information confirm the same resulting assignment. However, as it was claimed in Greco et al. (2011a), in connection with emptiness of the "necessary-" assignments for numerous alternatives, for real-world decision problems the analysis should be rather focused on $C_{\mathcal{D}}^{P,P}$ and $C_{\mathcal{D}}^{P,N}$. The possible-possible assignments review all possible consequences of preference information of all DMs on sorting of the whole set of countries (see Table 7). For each $a \in A$, $C_{\mathcal{D}}^{P,P}(a)$ is not precise, which means that in the context of all compatible instances of a preference model, there is a hesitation with respect to their desired class. Precisely, there are 16 countries which can be possibly assigned to all four classes $C_1 - C_4$, 18 countries assigned to the range of three contiguous classes, and 6 countries which are assigned to the range $C_2 - C_3$ or $C_3 - C_4$.

If the ranges of classes representing the possible-possible assignments are too wide to be decisive enough, one should analyze the possible-necessary assignments. They are formed by the intersection of the possible ranges of classes for all DMs (see Table 8). With respect to $C_{\mathcal{D}}^{P,N}$, there are no countries possibly assigned to four classes.

Table 7 Possible-possible $C_{\mathcal{D}}^{P,P}$ assignments for the problem of assigning countries to different types of regimes

$C_{\mathcal{D}}^{P,P}$	Assigned countries
$C_3 - C_4$	Australia, Canada, New Zealand, Japan, South Korea
$C_2 - C_4$	USA, Costa Rica, Taiwan, Panama, Jamaica, Trinidad, Mexico, Mongolia
$C_1 - C_4$	Papua, Philippines, Indonesia, Timor, Salvador, Dominican Rep., Guatemala, Hong Kong, Malaysia, Singapore, Thailand, Fiji, Kyrgyzstan, China, Tajikistan, Myanmar
$C_2 - C_3$	Honduras
$C_1 - C_3$	Nicaragua, Cambodia, Haiti, Kazakhstan, Cuba, Vietnam, Laos, Uzbekistan, Turkmenistan, North Korea

Table 8 Possible-necessary $C_{\mathcal{D}}^{P,N}$ assignments for the problem of assigning countries to different types of regimes

$C_{\mathcal{D}}^{P,N}$	Assigned countries
C_4	Australia, Canada, New Zealand, USA, Japan, Costa Rica
$C_3 - C_4$	Jamaica
$C_2 - C_4$	Salvador, Guatemala
C_3	South Korea, Taiwan, Panama, Trinidad, Honduras
$C_2 - C_3$	Mexico, Mongolia, Dominican Rep.
$C_1 - C_3$	Papua, Timor, Thailand
C_2	Philippines, Indonesia, Hong Kong, Malaysia, Singapore, Nicaragua, Fiji, Cambodia, Kyrgyzstan
$C_1 - C_2$	Haiti, Kazakhstan, Tajikistan, Laos, Uzbekistan, Turkmenistan, Myanmar, North Korea
C_1	Cuba, China, Vietnam

In fact, for 23 of them the possible-necessary assignments are precise, 12 countries are assigned to two contiguous classes, and only for 5 alternatives each DM admits assignment to the same range of three classes. Analyzing results of this problem, we can find confirmation of the following general relation: for all $a \in A$, $C_{\mathcal{D}}^{P}(a) \subseteq C_{\mathcal{D}}^{P,N}(a)$, and, more generally, for all $a \in A$ and $\mathcal{D}' \subseteq \mathcal{D}$:

$$C_{\mathcal{D}'}^{N,N}(a) \subseteq C_{\mathcal{D}'}^{N}(a) \subseteq C_{\mathcal{D}'}^{R}(a) \subseteq C_{\mathcal{D}'}^{P}(a) \subseteq C_{\mathcal{D}'}^{P,N}(a) \subseteq C_{\mathcal{D}'}^{P,P}(a),$$

which stems from the propositions presented in Sect. 3.1 and in Greco et al. (2011a).

Representative Value Function for the Case of Preference Information of Each DM Considered Individually

For illustration, let us also present the representative value function for the case of considering exemplary assignments of each DM individually. In order to formulate the targets which should be optimized in this case, we check the truth of relations $\succ^{\rightarrow}$, $\sim^{\rightarrow}$, and $\succ\prec^{\rightarrow}$ for all pairs of alternatives for each DM, and run the procedure presented in Sect. 3.2. The representative comprehensive values are presented in Table 6, and denoted by $U_{\mathcal{D},GMS}^{R}$. Note that these values are characterized by greater differentiation than results obtained for the case of considering preference information of all DMs jointly. Since consequences of preferences of all DMs are analyzed separately, there are more pairs of alternatives to which we would like to assign different intensity of the difference between their comprehensive values. Consequently, one could indicate several ranges of main concentration of the comprehensive values, and not only four such ranges as in case of $U_{\mathcal{D}}^{R}$. Nevertheless, the correlation between the orders determined by $U_{\mathcal{D}}^{R}$ and $U_{\mathcal{D},GMS}^{R}$ is very high. There is also a great similarity in the shape of the marginal value functions (see dashed lines in Fig. 5). The main differences concern slightly less share of electoral process and pluralism criterion (g_1) in the comprehensive value, and greater differentiation of marginal values. As for the representative assignments (see column $L^R_{\mathcal{D},GMS} - R^R_{\mathcal{D},GMS}$ in Table 6), they are precise for 39 out of 40 alternatives. When comparing them to the classes determined by EIU, the provided recommendation differs only in case of Haiti. It is also interesting to note that the intersection of ranges $L^R_{\mathcal{D}} - R^R_{\mathcal{D}}$ and $L^R_{\mathcal{D},GMS} - R^R_{\mathcal{D},GMS}$ is non-empty for all considered alternatives.

5 Conclusions

In this paper, we considered decision situations with multiple stakeholders, each having her/his own preferences. We have introduced the concept of the representative value function for group decision concerning multiple criteria ranking, choice, and sorting problems. The presented approach adopts features of UTA$^{\text{GMS}}$, UTADIS$^{\text{GMS}}$, UTA$^{\text{GMS}}$-GROUP, and UTADIS$^{\text{GMS}}$-GROUP, as it takes into account the set of all additive value functions composed of monotonic general marginal value functions compatible with the preference information provided by all DMs. It is an innovative approach, because we optimize the targets which refer to the robust results, i.e. to the consequences of preference information supplied by the DMs considered in terms of compatible value functions, rather than only to preference information itself. Since every compatible value function contributes to the necessary and possible results, it has a direct impact on the shape of the representative value function. In this way, the introduced procedure refers to "one for all, all for one" motto, and does not contradict the rationale of robust ordinal regression. It is the case, because we do not loose the advantage of knowing all compatible instances of the preference model of all DMs. The representative instance should be rather perceived as continuation of the trend of taking into account all compatible value functions, which allows to address potential problems with interpretation of the outcomes of the GROUP methods.

Moreover, our proposal is innovative also with respect to consideration of general additive value functions, which are composed of monotonic marginal value functions, instead of linear or piecewise-linear ones. Besides, when comparing the introduced approach with other UTA-like procedures (see, e.g., Siskos and Yanacopoulos 1985; Beuthe and Scannella 2001; Bous et al. 2010), one has to stress the sole context of group decision, which raises importance of our methodology. Indeed, group decision-making is among the most crucial and frequently encountered processes within companies and organizations.

We suggest several uses of a representative value function:

- a complementary use along with the family of GROUP methods in order to help the DMs to understand results of the robust ordinal regression,
- assessment of a share of a given criterion in the comprehensive value,
- an autonomous use in order to supply the DM with the scores obtained by alternatives for the representative value function as well as with the representative univocal recommendation,
- estimation of the similarities and disagreements between consequences of one's preferences with a compromise results representative for all DMs.

As far as future developments of the presented methodology are concerned, we intend to introduce the concept of the representative set of parameters in robust ordinal regression for outranking methods. Its definition would be based on the exploitation of the matrices of necessary and possible outranking relations. This would be done for a single DM (an initial idea of this approach has been presented in Greco et al. 2010a) as well as for multiple decision makers. Moreover, dealing with incompatibility of preference information in a group decision context and proposing some new procedures for ensuring "fairness" (understood in different ways) of the solutions suggested for removing inconsistency constitutes envisaged future development. In particular, these procedures may account for differentiation of the weights of the DMs.

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