General Relativity and Gravitation (2023) 55:26
https://doi.org/10.1007/s10714-023-03070-2

RESEARCH ARTICLE

Conformal Cyclic Cosmology, gravitational entropy and quantum information

Michał Eckstein[1]

Received: 26 June 2022 / Accepted: 13 January 2023 / Published online: 28 January 2023

Abstract

We inspect the basic ideas underlying Roger Penrose's Conformal Cyclic Cosmology from the perspective of modern quantum information. We show that the assumed loss of degrees of freedom in black holes is not compatible with the quantum notion of entropy. We propose a unitary version of Conformal Cyclic Cosmology, in which quantum information is globally preserved during the entire evolution of our universe, and across the crossover surface to the subsequent aeon. Our analysis suggests that entanglement with specific quantum gravitational degrees of freedom might be at the origin of the second law of thermodynamics and the quantum-to-classical transition at mesoscopic scales.

Keywords Conformal Cyclic Cosmology · Entropy · Quantum information · Arrow of time

Contents

✉ Michał Eckstein
michal.eckstein@uj.edu.pl

1 Institute of Theoretical Physics, Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, ul. Łojasiewicza 11, 30–348 Kraków, Poland

1 Introduction

Within general relativity the universe is modelled by a spacetime manifold equipped with a Lorentzian metric solving the Einstein field equations with suitable matter fields. The observational data support the standard model of cosmology (ΛCDM) with cold dark matter and a positive cosmological constant Λ, on top of the ordinary baryonic matter. The universe manifold should admit a global time function, hence it must be stably causal. Typically, one assumes a stronger property of global hyperbolicity, which guarantees the well-posedness of the Cauchy problem [1].

The existence of a cosmic time facilitates an evolutionary viewpoint on the universe. In this picture the universe begins with the Big Bang, a state characterised by an infinite matter density and temperature, and then expands causing the matter to dilute and cool down. The observed large-scale structure is commonly assumed to result from the initial density inhomogeneities. In the remote future, the cosmological constant will prevail over the matter content, resulting in an exponential expansion. This "dark era" is expected to be dominated by the Hawking radiation from evaporating black holes.

Despite the unquestionable success of the ΛCDM model in explaining the cosmological data, it suffers from several problems calling for resolution. The most pressing one is connected with the unknown nature of dark matter and dark energy (or the *cosmological constant*). Another riddle, known as the *horizon problem* [2], is the observed high-degree homogeneity of the cosmic microwave background from causally disconnected regions of the universe. A farther mystery—the *flatness problem* is associated with the fact that the present matter–energy density in the universe is close to a critical value ρ_c needed for a spatially-flat universe. The Friedmann equations imply that the ratio of the actual density to the critical one ρ/ρ_c increases in cosmic time and hence the matter density must have been much closer to the critical value, $|\rho_0/\rho_c - 1| \sim 10^{-27}$, in the electroweak epoch some 10^{-11} sec after the Big Bang [3].

On the mathematical side, the Big Bang is a naked singularity, which marks the past geodesic incompleteness of the universe [4, 5]. Under suitable assumptions about isotropy one can, via a conformal transformation, include it as spacetime boundary $\mathscr{B}^-$ and prove the well-posedness of the Cauchy problem for a variety of matter models [6–10]. At the other end, Helmut Friedrich has shown [11–17] that conformal initial data can also be specified on the future conformal boundary $\mathscr{I}^+$, which is a spacelike 3-hypersurface in asymptotically de Sitter spacetimes. However, the admissible initial data on $\mathscr{B}^-$ is much more constrained than that on $\mathscr{I}^+$ [17, 18]. This fact could be seen as yet another instance of cosmic fine-tuning and related to the global growth of entropy in the universe, aka the *arrow of time*. While the growth of entropy is consistent with the general picture of a thermalising expanding universe, its microscopic origins remain fundamentally unapprehended.

In 2005 Roger Penrose put forward an "outrageous" proposal to resolve some of the cosmic riddles [19]. His Conformal Cyclic Cosmology (CCC) bases upon an imaginative identification of the Big Bang hypersurface $\mathscr{B}^-$ with the conformal infinity $\mathscr{I}^+$ of a previous aeon [20]. This new cosmic paradigm is still in its infancy, with quite a few conceptual and technical issues waiting to be resolved [21]. Nevertheless, Penrose and co-authors have already been able to confront the predictions of CCC with the WMAP and Planck CMB data [22–26]. While suggestive and certainly intriguing, the claimed evidence in favour of CCC has so far been treated with reservation [27–29] (see [26] for a rebuttal).

The purpose of this note is to confront the CCC, along with its auxiliary hypotheses, with modern understanding of quantum information. We start in Sect. 2 with a brief outline of CCC, focused on the entropic aspects. Then, in Sect. 3, we pass on to the setting of quantum information and its implications for CCC. We begin with a recollection on the basic notions of mixed quantum states, purification and von Neumann entropy. We then argue that the information loss of degrees of freedom, as described by Penrose in [20], leads to the *increase* of global entropy. This is consistent with the Second Law of thermodynamics, but undermines the idea of 'entropy renormalisation' needed to match the degrees of freedom on the crossover hypersurface between the aeons.

To overcome this problem we propose, in Sect. 4, a unitary variant of CCC. It is based upon two independent hypotheses: H1 asserts that gravitational clumping induces the activation of quantum gravitational degrees of freedom, while H2 assumes that the quantum information trapped in the black hole region is eventually restored in the correlations between modes of Hawking radiation. We then describe the scenario in which quantum information is preserved during the entire evolution of the universe, and throughout the crossover surface. We study the flow of information between different sectors—matter, Hawking radiation and gravity—following our recent works on black hole evaporation [30, 31].

Then, we discuss the motivation behind the two hypotheses and their implications. In Sect. 4.2 we focus on gravitational entropy, which we interpret as the quantum von Neumann entropy. Our analysis suggests that the origin of the Second Law might be attributed to the entanglement of ordinary matter with the quantum degrees of freedom of the gravitational field. Intriguingly, this mechanism might explain the lack of quantum superposition in macroscopic objects. Next, in Sect. 4.3, we pass on to hypothesis H2, which was put forward in 1980 by Don Page [32]. We discuss the issues related to the problems of information cloning on time-slices, violation of entanglement monogamy and superluminal signalling. We argue that whereas the unitary evaporation involving quantum gravitational degrees of freedom must entail a new type of nonlocality—possibly beyond-quantum [33, 34]—, it need not imply the possibility of causal loops.

Finally, in Sect. 5, we summarise our study and point towards some of the challenges for the unitary Conformal Cyclic Cosmology.

2 Thermodynamic entropy and information in CCC

Within CCC the universe manifold consists of a, possibly infinite, sequence of aeons separated by spacelike crossover hypersurfaces—see Fig. 1. Each aeon is equipped with a Lorentzian metric, which solves the Einstein equations with some matter fields and a positive cosmological constant Λ. The physical metric $\check{g}$ in one aeon is determined, through the formula $\check{g} = \hat{\Omega}^{-4}\hat{g}$, by the physical metric $\hat{g}$ in the previous aeon

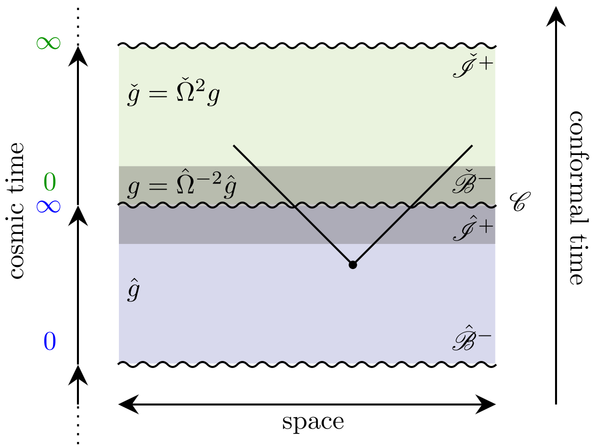

Fig. 1 The spacetime diagram of CCC. An aeon (shaded blue region) is a spacetime $(\hat{\mathcal{M}}, \hat{g})$, with past and future conformal boundaries—the Big Bang, $\hat{\mathscr{B}}^-$, and the future conformal infinity, $\hat{\mathscr{I}}^+$, respectively. The CCC proposal consists in identifying the $\hat{\mathscr{I}}^+$ with the Big Bang of a subsequent aeon, $\check{\mathscr{B}}^-$, as a single *crossover hypersurface*, $\mathscr{C} := \hat{\mathscr{I}}^+ \equiv \check{\mathscr{B}}^-$. The matching is achieved through a conformal rescaling of the metric in some neighbourhood of $\mathscr{C}$ (gray shaded region), $g = \hat{\Omega}^{-2}\hat{g} = \check{\Omega}^{-2}\check{g}$, with $\hat{\Omega} \to \infty$ and $\check{\Omega} \to 0$ towards $\mathscr{C}$. Penrose's *Reciprocal Hypothesis* [20, 21] assumes the relation $\hat{\Omega}\check{\Omega} = -1$, so that the metric $\check{g}$ is completely determined by $\hat{g}$ and $\hat{\Omega}$. Within CCC, the aeons are causally connected so that the information carried by massless particles can flow through the crossover hypersurface

and a conformal factor $\hat{\Omega}$. The latter is a key new element, which facilitates a smooth transition through the crossover hypersurface.

Here we shall focus on the entropic aspects of Penrose's model and the Second Law, which was actually the key motivation for the introduction of CCC [19]. For a detailed exposition about the mathematical structure of CCC the Reader is invited to consult [18, 20, 35–40]. The discussion on possible empirical evidence in favour of CCC can be found in [22–26].

The Second Law of thermodynamics is often regarded as one of the most fundamental and universal laws of physics. It says that the entropy of an isolated system cannot decrease in time, which is clearly visible in the perceived phenomena. On the other hand, because of its general universality, the precise formulation of the Second Law depends on the physical context and the definition of the entropy itself. The standard Boltzmann *thermodynamic entropy*, $\mathcal{S}_B(\rho) = k_B \log V$, is proportional to the logarithm of the volume V containing the state ρ in the phase space of the system, associated with the chosen coarse-graining.

If we assume the universal validity of the Second Law and apply it to the universe as a whole we come to the conclusion that the Big Bang must have been a state with a very low entropy. In dynamical terms, it means that the initial state of the universe must have been very special in contrast to the final state, which is expected to be typical. However, this picture seemingly contradicts the empirical data from the CMB, which is homogeneous and isotropic to an accuracy of the order of 10^{-5}. In other words, the matter in the universe was rather close to a thermal equilibrium already during the last scattering epoch some 360'000 years after the Big Bang.

This puzzle can be resolved—according to Penrose [41]—if one takes into account the gravitational degrees of freedom. Although there is no standard notion of entropy of a general gravitational field (see, however, [42]), it is commonly accepted that one can associate entropy with a black hole. The Bekenstein–Hawking formula gives $\mathcal{S}_{\mathrm{BH}} = 4\pi\, m^2/m_{\mathrm{Pl}}^2$ for a Schwarzschild black hole of mass m, with the Planck mass $m_{\mathrm{Pl}} \approx 2.12 \cdot 10^{-8}$ kg. Now, as a matter of fact, the amount of entropy presently associated with supermassive black holes outweighs that of ordinary matter by a factor 10^{15} (see e.g. [43]). Consequently, the total entropy, $\mathcal{S}_{\mathrm{matter}} + \mathcal{S}_{\mathrm{BH}}$, at the present moment is indeed much larger than during the last scattering epoch.

Penrose argues [20, 41] that the growth of entropy can in general be associated with gravitational clumping. This is because the gravitational attraction activates the gravitational degrees of freedom, which are otherwise idle. Hence, a uniform distribution of matter has a low value value of $\mathcal{S}_{\mathrm{grav}}$, while a bunch of compact objects in empty space has a high value of $\mathcal{S}_{\mathrm{grav}}$. In this picture, the black holes can be though of as states of 'gravitational thermal equilibrium', as they maximise the gravitational entropy. On the other hand, with the Big Bang one should associate an extremely low value, as it was almost perfectly homogeneous and isotropic.

More precisely, Penrose proposed to associate the gravitational entropy with the Weyl tensor [41], arguing that the latter corresponds to genuine gravitational degrees of freedom, in contrast to the Ricci tensor, which is determined by the matter content via Einstein equations. This lays at the foundation of his *Weyl Curvature Hypothesis*, which says that the Weyl tensor must vanish on the Big Bang hypersurface $\mathscr{B}^-$. More recently, there were several attempts to define the gravitational entropy, with the guiding principle that it should reproduce the Bekenstein–Hawking formula for the Schwarzschild spacetime [42, 44–48]. The most popular proposal [42] is based on the Bel–Robinson tensor, which is determined by the Weyl tensor, and produces reasonable results for cosmic solutions to Einstein equations. It increases as the structure formation occurs and harmonises with the Weyl Curvature Hypothesis.

These considerations provide a justification, at the macroscopic level, for the global growth of entropy in our universe, an hence for the cosmic arrow of time [41]. However, it seems to clash with the basic idea behind CCC, which requires a low value of entropy at the beginning of each new aeon. Penrose argues (see [20, Chapter 3.4]) that in order to resolve this conundrum one needs to assume that information is irreversibly lost during the black hole evaporation. More precisely, Penrose posits that a "loss of degrees of freedom" occurs, which would induce a dimension drop in the global phase space of the universe. This would inflict a 'renormalisation of entropy' to a much smaller value, i.e. a subtraction of a large constant value, which would facilitate the matching of active degrees of freedom on the crossover hypersurface $\mathscr{C}$. Penrose emphasises that such a phenomenon should not be interpreted as a violation of the Second Law of thermodynamics, because it has a global effect on the phase space and not on the specific phase trajectory followed by our universe. Consequently, the loss of degrees of freedom would not affect the evolution in the phase space of any local system.

To complete the picture it should be mentioned that Penrose postulates that (quantum) information is actually routinely lost in the quantum measurement process. In his model [49, 50], the standard time-reversible unitary evolution of a quantum system is intertwined with random irreversible projections. Penrose assumes that the latter are a natural phenomenon caused by the spontaneous collapse of the superposition of spacetime metrics.

It is clear that the incarnation of Penrose's CCC, as outlined in [20], requires a rather radical departure from the standard quantum theory. In the next section we will show that such a theoretical scheme would not be compatible with the quantum notion of entropy based on entanglement.

3 Quantum information and entropy

In quantum mechanics an isolated system is described through a pure state $|\psi\rangle$ in a suitable Hilbert space $\mathcal{H}$. It represents the maximal available knowledge about the system. If $\mathcal{H}$ has a finite dimension, i.e. $\mathcal{H} \simeq \mathbb{C}^N$ for some N, then N is the number of (quantum) degrees of freedom. If $\mathcal{H}$ is infinite-dimensional, then one typically defines N to be the number of active modes in the system [51]. For sake of simplicity, we shall work with finite-dimensional Hilbert spaces.

If the quantum system at hand is not perfectly isolated, then one should describe it in terms of a density operator ρ, which is a positive semi-definite operator on $\mathcal{H}$ with $\mathrm{Tr}_{\mathcal{H}}\, \rho = 1$. Such a density operator is called a mixed state if it cannot be written as a projection on a single pure state, i.e. there is no $|\psi\rangle \in \mathcal{H}$ such that $\rho = |\psi\rangle\langle\psi|$. Every mixed state admits a (non-unique) ensemble decomposition, $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$, in terms of some basis states $|\psi_i\rangle \in \mathcal{H}$ and probabilities p_i summing up to 1, $p_i \geq 0$, $\sum_i p_i = 1$.

Every mixed state can be *purified* by extending the Hilbert space $\mathcal{H}$ into a larger one $\mathcal{H} \otimes \mathcal{H}'$. A purification of ρ is a pure state $|\Psi\rangle \in \mathcal{H} \otimes \mathcal{H}'$ such that $\rho = \mathrm{Tr}_{\mathcal{H}'} |\Psi\rangle\langle\Psi|$, where $\mathrm{Tr}_{\mathcal{H}'}$ is the partial trace over the auxiliary Hilbert space $\mathcal{H}'$. If ρ is decomposed as $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ then its purification $|\Psi\rangle \in \mathcal{H} \otimes \mathcal{H}'$ has the form

$$|\Psi\rangle = \sum_i \sqrt{p_i} |\psi_i\rangle_{\mathcal{H}} |\phi_i\rangle_{\mathcal{H}'}, \tag{3.1}$$

for some orthonormal system $\{|\phi_i\rangle\}$.

Mixed states correspond to the situation when we do not (or cannot, for some reason) have the full knowledge about a given quantum system. The purification theorem tells us that subjective uncertainty arises because the system is entangled with some environment, which we ignore in our description. The studied system S and its environment E together form a closed quantum system, the state of which is a vector in the product Hilbert space

$$|\Psi\rangle_{S+E} = \sum_i a_i |\psi_i\rangle_S |\phi_i\rangle_E, \quad \text{with } a_i \in \mathbb{C}, \ \sum_i |a_i|^2 = 1. \tag{3.2}$$

The system's state is then obtained by tracing out the environmental degrees of freedom,

$$\rho_S = \mathrm{Tr}_E\, |\Psi\rangle\langle\Psi|_{S+E} = \sum_i |a_i|^2 |\psi_i\rangle\langle\psi_i|_S\,. \tag{3.3}$$

Note that the full state (3.2) contains more information than (3.3), because the latter ignores the complex phases, $\arg a_i$, between the quantum degrees of freedom of the system and the environment.

A measure of the subjective uncertainty in a given quantum state is provided by the *von Neumann entropy*

$$\mathcal{S}(\rho) := -\operatorname{Tr} \rho \log \rho. \tag{3.4}$$

It equals to 0 if ρ is a pure state, what harmonises with the fact that pure states correspond to the maximal knowledge about the system. For a mixed state of the form (3.3) we have

$$\mathcal{S}(\rho_S) = -\sum_i |a_i|^2 \log |a_i|^2. \tag{3.5}$$

The latter is the *information-theoretic* Shannon entropy with the index i playing the role of a random variable. Formula (3.5) can also be given a thermodynamical interpretation (we set $k_B = 1$ from now on) as the Gibbs entropy: The states $|\psi_i\rangle_S$ are the microstates of the system and $|a_i|^2$ are their corresponding probabilities. Let us stress, however, that because the decomposition of a density operator ρ into statistical ensembles is not unique, a different decomposition would lead to a different set of microstates with different probabilities. Nevertheless, the von Neumann entropy does not depend upon the chosen ensemble decomposition and is a well-defined functional on density operators. Finally, the connection with the thermodynamical Boltzmann formulation arises if one assumes that the microstates belong to a coarse-grained region V of the system's phase space and they are all equally probable, i.e. $|a_i|^2 = 1/V$ for all i.

The Boltzmann entropy of two independent systems is additive, while if the systems are correlated a general subadditivity relation holds, $\mathcal{S}_{A+B} \leq \mathcal{S}_A + \mathcal{S}_B$. In the quantum case, the subadditivity can achieve an extreme form, with the total entropy vanishing and the individual entropies being maximal. The archetypical example is a two-qubit maximally entangled Bell state, $|\psi\rangle_{A+B} = \frac{1}{\sqrt{2}}\big(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\big)$, which is pure—hence $S_{A+B} = 0$, while $\rho_A = \rho_B = \frac{1}{2}\mathbb{1}$ are maximally mixed states, hence $\mathcal{S}_A = \mathcal{S}_B = \log 2$. Consequently, the von Neumann entropy of the total system can be smaller than the individual entropy of any of its subsystems, which is impossible in the classical setting [52].

In summary, with a quantum system S embedded in an environment E one can associate the entropy,

$$\mathcal{S}(\rho_S) = -\operatorname{Tr} \rho_S \log \rho_S, \quad \text{where} \quad \rho_S = \operatorname{Tr}_E |\Psi\rangle\langle\Psi|_{S+E}, \tag{3.6}$$

determined by the global pure state $|\Psi\rangle_{S+E}$. The non-zero entropy of the system arises because of its correlations with the environment.

Such a perspective provides an explanation of the *local* Second Law of thermodynamics at the microscopic level: Indeed, suppose that the system is initially prepared in a pure state, $|\Psi^i\rangle_{S+E} = |\psi^i\rangle_S|\phi^i\rangle_E$. Then, it will interact with the environment and get entangled with its degrees of freedom, $|\Psi^f\rangle_{S+E} = U_{S+E}|\Psi^i\rangle_{S+E} =$

$\sum_n a_n |\psi_n\rangle_S |\phi_n\rangle_E$. If the dimension of $\mathcal{H}_E$ is large then a typical unitary evolution U_{S+E} would quickly drive the effective state of the system towards the maximally mixed state, $\rho_S^f \approx \frac{1}{\dim \mathcal{H}_S} \mathbb{1}$ (see e.g. [53]). The latter is a finite-dimensional counterpart of the thermal state [54], which maximises the von Neumann entropy, $\mathcal{S}_{\max} = \log(\dim \mathcal{H}_S)$.

Let us stress that the unitary evolution, however complicated, is in principle reversible. If one had access to the environmental degrees of freedom one could distentangle the system from its environment and *prepare* it in a pure state uncorrelated with the state of the environment. Consequently, the von Neumann entropy of the system would drop to 0. But the price to pay (in terms of resources) would be to interact with the environment E with the help of a suitable quantum apparatus A, which gets entangled with E. Consequently, the initial state $|\Psi^i\rangle_{S+E+A} = \big(\sum_j a_j |\psi_j\rangle_S |\phi_j\rangle_E\big)|\xi\rangle_A$ evolves unitarily into $|\Psi^f\rangle_{S+E+A} = |\psi^f\rangle_S \big(\sum_k b_k |\phi_k\rangle_E |\xi_k\rangle_A\big)$. Hence, the von Neumann entropy of the system and its environment grows during the process of the system's state preparation. On the other hand, quantum entropy of the total system $S + E + A$ vanishes throughout the process, because the evolution is globally unitary.

Now, if we apply the notion of von Neumann entropy to the universe as a whole we are forced into the conclusion that the total entropy vanishes and remains idle during the cosmic evolution. This is because the universe is a closed quantum system, and therefore it should be described by a pure state, which evolves unitarily.

This picture would change if we assumed that unitarity is violated in some physical processes, as postulated by Penrose [20, 49, 50]. In terms of quantum information the *information loss* means that an initial pure state evolves into a statistical mixture [55]. Such an effect is predicted by the quantum field theory in curved spacetimes to occur during the black hole evaporation [56, 57]. The objective loss of information in a closed quantum system would *increase* its von Neumann entropy, as its state evolves from a definite pure state into a statistical mixture. Consequently, if one assumes that evaporating black holes process all the infallen quantum information into a thermal state of Hawking radiation, then the global von Neumann entropy of the universe would grow.

Let us analyse the latter situation more carefully. Let $\mathcal{H}_M$ be the Hilbert space associated with all matter which would eventually end up in a black hole. The remaining matter is encompassed by $\mathcal{H}_A$, while $\mathcal{H}_H$ accommodates the modes of the Hawking radiation. Suppose that the black hole wipes out all the incoming quantum information and dies out by emitting purely thermal Hawking radiation. This corresponds to a map Φ on the space of density operators on $\mathcal{H}_M \otimes \mathcal{H}_H$, which takes any state of the form $|\psi\rangle\langle\psi|_M \otimes |0\rangle\langle 0|_H$ to $|0\rangle\langle 0|_M \otimes \phi_H$, where $|0\rangle$ denotes the vacuum and ϕ_H is the global thermal state of the Hawking radiation. By the no-hair theorem [58] the latter would depend only upon the black holes' initial masses, charges and angular momenta. The map Φ must be linear, for otherwise it would induce troubles with superluminal communication [59, 60]. This implies that $\Phi\big(\rho_M \otimes |0\rangle\langle 0|_H\big) = |0\rangle\langle 0|_M \otimes \phi_H$, for any mixed state ρ_M of the matter.

Initially, the universe is in a global pure state

$$\rho_U^i = |\Psi\rangle\langle\Psi|_U, \quad \text{with} \quad |\Psi\rangle_U = \sum_k a_k |\xi_k\rangle_A |\psi_k\rangle_M |0\rangle_H,$$

which encodes the fact that the matter M can be correlated with the matter A. Then, the black holes form and evaporate, while the auxiliary matter undergoes some quantum evolution. Eventually, the universe evolves into the state

$$\begin{aligned}\rho_U^f &= \sum_{k,\ell} a'_k \overline{a'_\ell} \, |\xi'_k\rangle\langle\xi'_\ell|_A \otimes \Phi\big(|\psi'_k\rangle\langle\psi'_\ell|_M \otimes |0\rangle\langle 0|_H\big) \\ &= |\xi''\rangle\langle\xi''|_A \otimes |0\rangle\langle 0|_M \otimes \phi_H,\end{aligned} \tag{3.7}$$

with $|\xi''\rangle_A = \sum_k a'_k |\xi'_k\rangle$.

The matter A, which did not take part in the gravitational collapse, ends up in a pure state completely decorrelated from the Hawking radiation. If one focuses solely on the matter A, then one would conclude that its von Neumann entropy has actually dropped to zero.[1] Indeed, we have

$$\mathcal{S}\big(\operatorname{Tr}_{M+H} \rho_U^i\big) = \sum_k |a_k|^2 \log |a_k|^2 \gg \mathcal{S}\big(\operatorname{Tr}_{M+H} \rho_U^f\big) = \mathcal{S}\big(|\xi''\rangle\langle\xi''|_A\big) = 0.$$

However, the total von Neumann entropy of the universe includes also that of the Hawking radiation:

$$\mathcal{S}(\rho_U^f) = \mathcal{S}\big(|\xi''\rangle\langle\xi''|_A\big) + \mathcal{S}\big(\rho_H\big) = \mathcal{S}\big(\rho_H\big) \ggg \mathcal{S}\big(\rho_U^i\big) = 0.$$

In the scenario with purely thermal Hawking radiation $\mathcal{S}\big(\rho_H\big)$ can be computed as the thermodynamic entropy of a radiating black body. It turns out to be larger by a factor $\sim$1.5 than the Bekenstein–Hawking entropy of the initial black holes [62–64]. Notably, $\mathcal{S}\big(\rho_H\big)$ is very much larger than the initial entropy quantifying correlations between matter in sectors M and A. This is because $\mathcal{S}_{\mathrm{BH}}$ increases faster with mass than the entropy of any known matter [65].

The punch line is that the information loss, understood as an evolution of a pure state into a statistical mixture, induces the growth of the von Neumann entropy, hence it is justified to connect it with the *global* Second Law of thermodynamics. Clearly, the same argument applies if one assumes, as suggested by Penrose [20, 49, 50], that information is irreversibly lost during the quantum measurement process.

However, this is in direct opposition to Penrose's assertion that information loss induces an effective *decrease* of global entropy. Indeed, the "loss of degrees of freedom" of a quantum system would mean that the system's state ρ, initially defined on a Hilbert space $\mathcal{H} = \mathcal{H}' \otimes \mathcal{H}''$, is projected onto a state ρ' on a subspace $\mathcal{H}'$ of smaller dimension. But a typical state (pure or mixed) on $\mathcal{H}$ would exhibit correlations (classical or quantum) between the sectors $\mathcal{H}'$ and $\mathcal{H}''$, which are also lost and hence contribute to the lack of knowledge encoded in the final state ρ'. Formally, this is implemented by taking the partial trace over the lost degrees of freedom, $\rho' = \operatorname{Tr}_{\mathcal{H}''} \rho$, with the result that $\mathcal{S}(\rho') > \mathcal{S}(\rho)$. On the contrary, the purification theorem shows that the effective decrease of entropy is associated with the *increase* of the number

[1] In more realistic astrophysical situations the Hawking radiation cannot be purely thermal [61] and one should expect some residual correlations between sectors A and H. However, such correlations would bring far less entropy than the Hawking radiation itself.

of degrees of freedom. Indeed, any mixed state ρ on a Hilbert space with a positive von Neumann entropy can be reinterpreted as resulting from a pure state on a *larger* Hilbert space.

4 Gravity and quantum information

In the previous section we exhibited a major conceptual problem of compatibility between the 'renormalisation of entropy' required by CCC and the actual implications of quantum information loss. We shall now propose a modified—*unitary*—scenario of CCC, which overcomes this issue while keeping in line with the key ideas involved in CCC.

Our proposal is based on two hypotheses:

H1 *Gravitational clumping induces the activation of quantum gravitational degrees of freedom.*
H2 *During the black hole evaporation all quantum information associated with the black hole region is eventually transferred to the Hawking radiation.*

The two hypotheses are logically independent. The former is an extension of the scheme presented by the author in collaboration with Erik Aurell and Paweł Horodecki in [30, 31]. As it will turn out, it also offers a new viewpoint on the quantum-to-classical transition problem. On the other hand, H2 was put forward by Don Page over 40 years ago [32, 66] and developed within various quantum gravity scenarios (see e.g. [67–69] and references therein). We shall first present the general scheme of unitary CCC and then discuss the details and implications of H1 and H2 independently.

4.1 Unitary conformal cyclic cosmology

As previously, we describe the universe in terms of a pure state in an appropriate Hilbert space. We divide the latter into four sectors, $\mathcal{H} = \mathcal{H}_M \otimes \mathcal{H}_G \otimes \mathcal{H}_H \otimes \mathcal{H}_A$. The first one, $\mathcal{H}_M$, involves all matter (including radiation and dark matter), which will eventually end up in a black hole. The second one, $\mathcal{H}_G$, accommodates the (quantum) gravitational degrees of freedom activated by the clumping of matter M. The third one, $\mathcal{H}_H$, encompasses the Hawking radiation. Finally, $\mathcal{H}_A$, is needed to take into account the carriers of quantum information, which evade the trapping horizons. These include free matter (again, including radiation and dark matter) outside of the black holes, together with its gravitational degrees of freedom, as well as the gravitational waves.

Let us now fix a time-slicing of the universe and inspect the main stages of the global unitary evolution—see Fig. 2. At the Big Bang hypersurface the gravitational degrees of freedom are inactive and there is no Hawking radiation, hence the primordial matter must be in a pure state

$$|\Psi(t_0 \gtrsim 0)\rangle_U = \sum_i a_i |\psi_i\rangle_M |0\rangle_G |0\rangle_H |\chi_i\rangle_A, \tag{4.1}$$

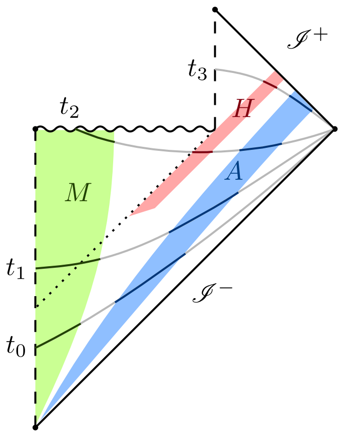

Fig. 2 The spacetime diagram of a static evaporating black hole. The green region M depicts matter undergoing the gravitational collapse, the blue region A shows the auxiliary systems, while the red region H stands for the Hawking radiation. (The latter is widened for sake of readability, as the Hawking radiation is emitted strictly at the horizon.) The dotted line is the trapping horizon and the curvy one is the singularity. The unitary evolution of quantum information is analysed in a chosen time-slicing marked by the gray curves. In the picture we have incorporated Penrose's "mass fade-out" hypothesis, which implies that the matter in sector A would eventually become conformal. Within the cosmological context $\mathscr{I}^+$ would become a spacelike hypersurface (see [70, Fig. 1]), and matter A and M originate at the Big Bang, which is also a spacelike hypersurface

which may involve entanglement between matter in sectors M and A. As matter begins to clump, the gravitational degrees of freedom get activated and correlated with other degrees of freedom,

$$|\Psi(t_1)\rangle_U = \sum_{i,j} a_i' \Big(b_{ij} |\psi_j'\rangle_M |\phi_j\rangle_G \Big) |0\rangle_H |\chi_i'\rangle_A. \tag{4.2}$$

The latter state has the form of "entangled entanglement", which means that the entanglement between any two subsystems from the tripartite system $M + G + A$ is a property, which itself is entangled with the third subsystem [71, 72].

Later on, black holes form and start evaporating through quanta of Hawking radiation. The latter will be entangled with both the matter under trapping horizons and the gravitational degrees of freedom,

$$|\Psi(t_2)\rangle_U = \sum_{i,j,k} a_i'' \Big(c_{ijk} |\psi_j''\rangle_M |\phi_k'\rangle_G |\xi_k\rangle_H \Big) |\chi_i''\rangle_A.$$

On the strength of Hypothesis H2, as the evaporation proceeds the quantum information encoded in sectors M and G is gradually transferred into the global state of the Hawking radiation. Eventually, near $\mathscr{I}^+$, there will only be Hawking radiation on top of the information-carriers in sector A, which did not take part in the gravitational collapse,

$$|\Psi(t_3 \lesssim \infty)\rangle_U = \sum_i a_i'''|0\rangle_M|0\rangle_G|\xi_i'\rangle_H|\chi_i'''\rangle_A. \quad (4.3)$$

The final state of the unitary evaporation (4.3) is very different from the state (3.7) predicted by the semi-classical analysis [57]. The state of the Hawking radiation in (4.3) is pure and involves multi-mode entanglement, which encodes all quantum information previously carried by matter M. Furthermore, the initial correlations between primordial matter in sectors A and M are not lost, but are eventually transferred into correlations between the residual matter (and gravitational waves) and the Hawking radiation.

Now, the final state of one aeon (4.3) could be glued with the initial state of the subsequent one (4.1), provided that the quantum information encoded in the Hawking radiation is somehow "rewritten" onto the primordial matter. This is conceivable within the scheme of CCC, as matter fields and gravitational waves would distort the crossover hypersurface [20]. Such an effect due to gravitational radiation from the previous aeon is at the basis of the empirical predictions of CCC [73, 74]. We note, however, that the (classical) equations of motion for the matter fields can be prolonged through the crossover hypersurface only if they involve massless particles.[2] Penrose suggests [20] that in asymptotically de Sitter spacetimes the rest mass could "fade away", as a consequence of the Λ-dependence of the Casimir operator associated with the group of symmetries of de Sitter spacetime. This is an independent problem within CCC [21, 37], which is beyond the scope of the present note.

At the level of quantum information, the extension through $\mathscr{C}$ can be implemented by a simple SWAP unitary operation. Concretely, if we decompose the Hilbert space into sectors in subsequent aeons,

$$\mathcal{H} = \mathcal{H}_{\hat{M}} \otimes \mathcal{H}_{\hat{G}} \otimes \mathcal{H}_{\hat{H}} \otimes \mathcal{H}_{\hat{A}} = \mathcal{H}_{\check{M}} \otimes \mathcal{H}_{\check{G}} \otimes \mathcal{H}_{\check{H}} \otimes \mathcal{H}_{\check{A}},$$

and match the relevant (effective) dimensions,

$$\dim \mathcal{H}_{\hat{H}} + \dim \mathcal{H}_{\hat{A}} = \dim \mathcal{H}_{\check{M}} + \dim \mathcal{H}_{\check{A}},$$

then

$$|\Psi(\check{t} = 0)\rangle_{\check{U}} = U_{\text{SWAP}}\Big(|\Psi(\hat{t} = \infty)\rangle_{\hat{U}}\Big) = \sum_i \hat{a}_i'''|\check{\psi}_i\rangle_{\check{M}}|0\rangle_{\check{G}}|0\rangle_{\check{H}}|\check{\chi}_i\rangle_{\check{A}}. \quad (4.4)$$

Such a SWAP gate is instantaneous, but one can smoothen it and incorporate within the cosmic evolution. Namely, let $\tau \in \mathbb{R}$ be the conformal time parameter, with $\tau_{\mathscr{C}} = \check{t}_0 = \hat{t}_\infty$, and let H_{SWAP} be the Hermitian generator of U_{SWAP}, i.e. $U_{\text{SWAP}} = e^{iH_{\text{SWAP}}}$.

[2] Curiously, such an extension through the crossover hypersurface is also possible if the particle's mass is fine-tuned to the cosmological constant, $m = \sqrt{2\Lambda/3}$, as shown in [17].

Let us pick two smooth functions f, g such that

$$f(\tau) = \begin{cases} 1, & \text{for} \quad \tau \leq \tau_{\mathscr{C}} - 2\varepsilon, \\ 0, & \text{for} \quad \tau \geq \tau_{\mathscr{C}} - \varepsilon, \end{cases} \qquad g(\tau) = \begin{cases} 0, & \text{for} \quad \tau \leq \tau_{\mathscr{C}} - \varepsilon, \\ 1, & \text{for} \quad \tau = \tau_{\mathscr{C}}, \\ 0, & \text{for} \quad \tau \geq \tau_{\mathscr{C}} + \varepsilon, \end{cases}$$

with some $\varepsilon > 0$, which can be chosen arbitrarily small. Then, we can write the global unitary evolution as

$$U(\tau) = \exp\left\{ i\tau \left(f(\tau)\hat{H} + g(\tau)H_{\text{SWAP}} + (1 - f(\tau - 3\varepsilon))\check{H} \right) \right\}. \tag{4.5}$$

The generators of the cosmic evolution in subsequent aeons, $\hat{H}$ and $\check{H}$, need not commute neither with each other nor with H_{SWAP}, but formula (4.5) is sound because the functions f, g and $1 - f(\cdot - 3\varepsilon)$ do not overlap.

In summary, under hypotheses H1 and H2, one can construct a global unitary evolution of the universe's state and continue it smoothly through the crossover hypersurface. The quantum information in the universe is globally preserved, though it flows between the matter and gravitational sectors during the cosmic evolution. One can quantify this flow by computing the von Neumann entropy associated with the sector G,

$$\mathcal{S}_G(t) = \mathcal{S}\big(\text{Tr}_{M+H+A}\, |\Psi(t)\rangle\langle\Psi(t)|_U\big) = \mathcal{S}\big(\text{Tr}_G\, |\Psi(t)\rangle\langle\Psi(t)|_U\big). \tag{4.6}$$

At the Big Bang Eq. (4.1) yields $\mathcal{S}_G(t_0 = 0) = 0$. Then, $\mathcal{S}_G$ grows monotonically as gravitational clumping proceeds. Once the black holes have formed $\mathcal{S}_G$ reaches its peak value equal to the total Bekenstein–Hawking entropy. Then, $\mathcal{S}_G$ decreases as quantum information is transferred into the correlations between the modes of Hawking radiation. Eventually, we have $\mathcal{S}_G(t_3 = \infty) = 0$ and a smooth transition to the subsequent aeon takes place. Hence, $\mathcal{S}_G$ essentially follows the *Page curve* [66].

Recall that the sector A also includes some gravitational degrees of freedom associated with the clumping of matter in A and the gravitational radiation. The former can be safely assumed to be much smaller than $\mathcal{S}_G$ once the black holes have formed. The latter is of a different nature than the one associated with the "Coulomb-like" gravitational sources and would require separate attention (cf. [42]).

4.2 Gravitational clumping and decoherence

Let us now discuss the motivation and some consequences of hypothesis H1. In [30] we argued that the Bekenstein–Hawking entropy can be interpreted as von Neumann entropy measuring the entanglement of matter with the (quantum) gravitational degrees of freedom. The price to pay is that the gravitational collapse activates a huge number of new degrees of freedom

$$\dim \mathcal{H}_G \approx e^{4\pi (m/m_{\text{Pl}})^2}. \tag{4.7}$$

For a solar mass black hole this number is immense, $\dim \mathcal{H}_G \approx 10^{10^{70}}$.

Hypothesis H1 assumes that the activation of quantum gravitational degrees of freedom is universally associated with the gravitational entropy rather than with the black hole horizons. As mentioned before, there is no generally accepted notion of gravitational entropy, but it is commonly assumed that such an entropy should be associated with the Weyl degrees of freedom and increase in course of gravitational clumping [42].

It is instructive to analyse a static point-like quantum system of mass m. Its gravitational field is then described by the Schwarzschild metric [75], hence the prescription of [42] yields the formula $\dim \mathcal{H}_G = e^{4\pi(m/m_{\mathrm{Pl}})^2}$, in coherence with (4.7). As it turns out, for $m \lesssim \frac{1}{10} m_{\mathrm{Pl}}$, we have $\dim \mathcal{H}_G \approx 1$, which essentially means that the quantum gravitational degrees of freedom are inactive. On the other hand, this number grows very rapidly with the mass of the system. Already for $m = m_{\mathrm{Pl}}$ we have $\dim \mathcal{H}_G = e^{4\pi} > 280'000$.

This observation suggests that quantum systems with a mass of the order of the Planck mass would rapidly decohere because of the activation of a large number of new quantum degrees of freedom. This is consistent with the current limits on the quantum behaviour of macroscopic objects [76–78]. It would provide an explanation of the quantum-to-classical transition in terms of an effective decoherence caused by entanglement with the "gravitational environment". Interestingly, such a scenario leads to empirical predictions, which are qualitatively equivalent to the ones involving objective information loss [49, 50]. Indeed, an experiment showing that the state of a quantum system has freely evolved from a coherent superposition into a statistical mixture would admit two completely equivalent explanations—in terms of objective collapse or entanglement with unknown degrees of freedom.

It should also be mentioned that such a mechanism would normally induce heating of the quantum system at hand—which effect sets rather stringent bounds on the collapse models [78–80]—, unless the involved gravitational degrees of freedom do not carry energy. Notably, this harmonises with the assumption that gravitational degrees of freedom shall be associated with the Weyl tensor, which does not affect the energy–momentum conservation [42]. In terms of quantum information it means that the selfadjoint generator, H_{collapse}, of the unitary transformation governing the transition must exhibit substantial degeneration[3]. That is,

$$H_{\text{collapse}} |\psi_E\rangle_S |\phi_i\rangle_G = E |\psi_E\rangle_S |\phi_i\rangle_G,$$

for an energy level E and a family of orthonormal vectors $\{|\phi_i\rangle\}_i$, with $i \lesssim \dim \mathcal{H}_G$.

Clearly, this is but a qualitative picture as quantum objects considered to be point-like, as e.g. the electron, have masses several orders of magnitude below the Planck mass. Developing a more precise mechanism would require a careful study of the gravitational entropy associated with *extended* massive objects. This will be discussed in a forthcoming work.

For sake of establishing the unitary CCC model it is sufficient to assume that the quantum gravitational degrees of freedom are only activated during the gravitational

[3] We thank Paweł Horodecki for drawing our attention to this fact.

collapse. It should, however, be stressed that in this picture the gravitational entropy is *not* associated with the existence of a horizon, but rather with the gravitational clumping. Consequently, the development of a cosmological horizon, induced by the positive cosmological constant, does not lead to the activation of any new degrees of freedom. This is coherent with the argumentation provided by Penrose [20], but it clashes with the proposal for gravitational entropy, put forward in [42], based on a formal analogy with the classical laws of thermodynamics.

4.3 Unitary evaporation of black holes

The case for unitary evaporation of black holes has been thoroughly discussed in the literature—see [67–69] and references therein. Any such scenario would be at odds with the basic principles of quantum field theory, as discussed at the end of Sect. 4.1 (cf. also [55] and [67]). Furthermore, the unitary evaporation seems to conflict with the basics theorems of quantum mechanics [67], such as no-cloning and entanglement monogamy [81], which in turn puts threat on the relativistic causal structure [82]. We shall argue, however, that while unitary evaporation does violate the principles of *local* quantum mechanics (as laid out e.g. in [83]), it does not necessarily induce troubles with causality.

Let us inspect a pure quantum state imprinted on a test mass freely falling into a Schwarzschild black hole—see Fig. 3. Assume first that the state is completely shielded from its environment, also the gravitational one. As the test mass follows a timelike geodesic, it will reach the central singularity in finite proper time. The state is protected as long as its carrier exists and is not lost, hence it must eventually be 'swapped-out' into the correlations between the modes of Hawking radiation (cf. [82]). Clearly, this would constitute an instance of (instantaneous) superluminal signalling, as the quantum information is transmitted into a spacelike-separated location. Note, however, that it does not lead to causal paradoxes, provided that the state is sent outside of the past causal cone of its worldline. Indeed, in such a scenario the information is only transmitted superluminally from inside of the horizon, so that no two observers could effectuate a causal loop.

If, however, one now adopts a different slicing (dashed lines in Fig. 3), then the quantum state is not swapped, but *cloned*—the same quantum information exists in two different locations on one time-slice. Such an effect cannot be implemented with any quantum channel [84, 85] hence, in particular, with any unitary operation. At the same time, even if cloning does take place, it still does not facilitate the completion of a causal loop. This is because any local observer able to read out the information from the Hawking radiation cannot influence the causal past of the original quantum information carrier.

Clearly, the assumption of the state's perfect shielding is unrealistic. One would rather expect that the quantum information infalling into the black hole gets "scrambled", that is thoroughly mixed with other degrees of freedom [86]. On the other hand, such scrambling must be associated with a physical interaction. Let us focus solely on the gravitational interaction, since in the presented scenario the gravitational degrees of freedom are expected to dominate during the collapse. While one expects strong

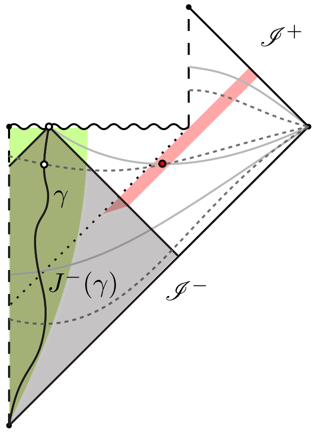

Fig. 3 The spacetime diagram of a static evaporating black hole. A test mass, which carries a shielded quantum state, marked with a white dot, follows a time-like geodesic γ, which ends at the central singularity. Upon the encounter with the singularity, the quantum state is transferred nonlocally to the correlations between the modes of Hawking radiation (red dot). Using a time-slicing with solid gray lines, such an effect would be described by a SWAP gate. On the other hand, in a different slicing, marked with dashed dark lines, the same process appears as information cloning. Yet, this does not facilitate causal loops, if we assume that the region to which quantum information is transferred out from the black hole (red dot) lays outside of the causal past of the initial state's worldline, $J^-(\gamma)$

tidal forces operating near the singularity, the gravitational interaction at the horizon of an astrophysical black hole is rather weak. Indeed, the (Newtonian) acceleration experienced by a test mass at the trapping horizon of a Schwarzschild black hole,

$$a_{\mathrm{BH}} = \frac{GM}{r_S^2} = \frac{c^4}{4GM},$$

is of the order of 10^{-9} m/s^2 for a solar-mass black hole. Hence, one can safely assume that the infalling quantum states are, for the most part, indeed protected from the gravitational mixing. The gravitational scrambling effectively starts only near the singularity and not the horizon. Consequently, the picture arising from Fig. 3 remains qualitatively valid.

It is clear from Fig. 3 that the description of the dynamics involving the quantum information scrambling and transfer into the Hawking radiation depends on the chosen time-slicing of spacetime. The same is true for the scenario involving information loss: In one slicing the information is lost gradually, while in another one it vanishes abruptly in the last burst of Hawking radiation—see [20, Fig. 3.15]. However, the choice of the slicing is a passive operation. It is completely irrelevant for physics, both outside and inside the trapping horizon. In particular, an observer decoding the quantum information from Hawking modes has no operational means to determine whether it was swapped out, cloned or transferred from the gravitational registers.

Other arguments—most notably the *black hole complementarity principle* [87]—, not involving explicitly the gravitational degrees of freedom, have been invoked to assure the consistency of the unitary evaporation scenario [67]. The presented scheme has an additional advantage of avoiding the *AMPS (firewall) paradox* [88]. The latter is a statement that for a sufficiently old black hole the outgoing Hawking modes must be (nearly maximally) entangled both with their ingoing partners and the early Hawking radiation, which would violate the fundamental property of quantum mechanics – the *entanglement monogamy* [81]. In the presented scenario the monogamy is not violated, because the early Hawking radiation is entangled with the gravitational degrees of freedom.

An important conclusion from this discussion is that the unitary evaporation mediated through the quantum gravitational sector must involve some new type of nonlocality, which goes beyond the standard quantum theory. This is the price to pay for avoiding the AMPS paradox [88]. More directly, it is well-known that quantum entanglement cannot increase through LOCC (local operations and classical communication) [81]. But Hawking modes need to get entangled 'at a distance' without any local interaction. We emphasise, however, that the violation of the LOCC principle does not necessarily imply the *operational* violation of relativistic causality. In fact, there exist probabilistic theories exhibiting a beyond-quantum form of nonlocality, while not allowing for any superluminal transfer of information [33]. In this context, a new information-theoretic framework was recently proposed [34], in which correlations can change at a distance and the monogamy of correlations is violated, and yet it is impossible to form a causal loop in spacetime.

5 Summary

The Conformal Cyclic Cosmology put forward by Roger Penrose offers a valuable alternative to the much more popular inflationary models. It is founded on rigorous mathematical results concerning the conformal extensions of spacetime manifolds and yields concrete testable predictions. Distinctively, CCC assumes that quantum gravity effects do not play any significant role in the very early universe, while they are eventually important in the description of black holes. This is because Penrose associates the gravitational degrees of freedom with the Weyl (and not Ricci) curvature, which is expected to vanish at the Big Bang and diverge at the black hole singularities.

The central notion in CCC and the main motivation behind it is that of gravitational entropy, which extends the concept of Bekenstein–Hawking black hole entropy. Conceptualised by Penrose at an intuitive level [20, 41, 89], it recently acquired a more concrete shape within the general-relativistic framework [42, 44–48]. In order to match the available degrees of freedom at the crossover hypersurface between the aeons, Penrose assumes that information is inevitably lost during the black hole evaporation and argues that such a phenomenon would effectively 'renormalise' the total entropy in the dark epoch of an aeon to a much smaller value.

However, the notion of entropy adopted by Penrose is inherently classical and does not take into account the quantum nature of phenomena. When analysed at the quantum level, the information loss would induce an *increase* of entropy. This is a universal conclusion basing on the standard von Neumann notion of entropy in quantum systems. It could be applied in the context of evaporating black holes, in which the quantum nature of gravity is expected to play a significant role. But it can equally well be employed in the study of objective collapse models, as the one put forward by Penrose [49, 50]. The bottom line is clear: Whenever a pure quantum state evolves into a statistical mixture, its von Neumann entropy grows.

Undoubtedly, Penrose's proposal for an objective wave function collapse of massive quantum systems requires a major departure from the standard quantum theory. One might thus expect that such a new theory would involve completely new notions of information and entropy. However, the latter would have to be consistent with the quantum notions when the mass of the system is small. In the same vein as the quantum information becomes effectively classical if coherence is negligible and von Neumann entropy reproduces Boltzmann's formula for large systems in thermal equilibrium.

The scenario for gravitation-induced decoherence proposed in this article is in fact not so different from the original vision put forward by Penrose. If one assumes that the quantum gravitational degrees of freedom are not accessible to any local observer, then the entanglement with these effectively amounts to a loss of information. Such an assumption seems plausible as gravitational clumping occurs spontaneously and one needs to invest energy (and entropy) to reverse it. Consequently, one could still claim that some information is hopelessly concealed in massive objects.

The difference becomes important only in the late stages of the universe, when the information is eventually revealed from the black holes in quantum correlations between the Hawking modes. Nevertheless, the thermal spectrum of individual Hawking modes is perfectly consistent with the Bekenstein–Hawking formula [31]. Ultimately, it is the global state of Hawking radiation, which determines, for the most part, the initial state of the subsequent aeon. Yet, the macroscopic effect of deformation of the crossover hypersurface—which is basis of CCC's empirical predictions—would still be dominated by the gravitational waves from black hole mergers in the previous aeon. This is because the latter carry much more energy than the Hawking radiation, whether in a thermal or pure state.

Both in the original and in the present version of CCC it is unclear what happens with the information concealed in massive particles, which evaded the gravitational collapse or were emitted as Hawking quanta. Within the presented scheme, the 'mass fadeout' effect which, to our best knowledge, does not so far have a dynamical implementation (see, however, [37]) would have to be unitary.

Another pressing problem is that of dark matter. In Penrose's original proposal, a primordial form of dark matter emerges upon the transition to the next aeon from a "phantom" (i.e. non-dynamical) field related to the conformal factor $\hat{\Omega}$ utilised for the rescaling [20, 90]. From the unitary point of view, the phantom field would then have to constitute yet another sector of the total Hilbert space, which would remain completely shielded until the beginning of a new aeon. More generally, it is presently not clear how to uniquely fix the conformal rescaling factor given the matter content in one aeon [21].

All these pertinent questions call for a further theoretical study of the 'Unitary Conformal Cyclic Cosmology'.

Acknowledgements We acknowledge the support of Foundation for Polish Science under the Team-Net project no. POIR.04.04.00-00-17C1/18-00. We thank Erik Aurell, Paweł Horodecki and Tomasz Miller for inspiring discussions and comments on the manuscript.

Data availability The manuscript has no associated data.

Declarations

Conflict of interest The author declares no competing interests.

References

1. Ringström, H.: The Cauchy Problem in General Relativity. ESI Lectures in Mathematics and Physics, vol. 6. European Mathematical Society, Zürich (2009). https://doi.org/10.4171/053
2. Misner, C.W.: The isotropy of the universe. Astrophys. J. **151**, 431 (1968)
3. Liddle, A.R.: In: Masiero, A., Senjanović, G., Smirnov, A. (eds.) An Introduction to Cosmological Inflation, p. 269. https://doi.org/10.1142/9789814527538
4. Hawking, S.W., Ellis, G.F.R.: The Large Scale Structure of Space-time. Cambridge University Press, Cambridge (1973)
5. Wald, R.M.: General Relativity. University of Chicago Press, Chicago (1984)
6. Anguige, K., Tod, K.P.: Isotropic cosmological singularities: I. Polytropic perfect fluid spacetimes. Ann. Phys. **276**(2), 257–293 (1999). https://doi.org/10.1006/aphy.1999.5946
7. Anguige, K., Tod, K.P.: Isotropic cosmological singularities II. The Einstein–Vlasov system. Ann. Phys. **276**(2), 294–320 (1999). https://doi.org/10.1006/aphy.1999.5947
8. Anguige, K.: Isotropic cosmological singularities III. The Cauchy problem for the inhomogeneous conformal Einstein–Vlasov equations. Ann. Phys. **282**(2), 395–419 (2000). https://doi.org/10.1006/aphy.2000.6037
9. Tod, K.P.: Isotropic cosmological singularities: other matter models. Class. Quant. Gravity **20**(3), 521–534 (2003). https://doi.org/10.1088/0264-9381/20/3/309
10. Tod, P.: Isotropic cosmological singularities in spatially homogeneous models with a cosmological constant. Class. Quant. Gravity **24**(9), 2415–2432 (2007). https://doi.org/10.1088/0264-9381/24/9/017
11. Friedrich, H.: The asymptotic characteristic initial value problem for Einstein's vacuum field equations as an initial value problem for a first-order quasilinear symmetric hyperbolic system. Proc. Roy. Soc. Lond. A Math. Phys. Sci. **378**(1774), 401–421 (1981). https://doi.org/10.1098/rspa.1981.0159
12. Friedrich, H.: On purely radiative space-times. Commun. Math. Phys. **103**(1), 35–65 (1986). https://doi.org/10.1007/BF01464281
13. Friedrich, H.: Existence and structure of past asymptotically simple solutions of Einstein's field equations with positive cosmological constant. J. Geom. Phys. **3**(1), 101–117 (1986). https://doi.org/10.1016/0393-0440(86)90004-5
14. Friedrich, H.: On the existence of n-geodesically complete or future complete solutions of Einstein's field equations with smooth asymptotic structure. Commun. Math. Phys. **107**(4), 587–609 (1986). https://doi.org/10.1007/BF01205488
15. Friedrich, H.: On the global existence and the asymptotic behavior of solutions to the Einstein–Maxwell–Yang–Mills equations. J. Differ. Geom. **34**(2), 275–345 (1991). https://doi.org/10.4310/jdg/1214447211

16. Friedrich, H.: Gravitational fields near space-like and null infinity. J. Geom. Phys. **24**(2), 83–163 (1998). https://doi.org/10.1016/S0393-0440(97)82168-7
17. Friedrich, H.: Smooth non-zero rest-mass evolution across time-like infinity. Ann. Henri Poincaré **16**(10), 2215–2238 (2015). https://doi.org/10.1007/s00023-014-0368-7
18. Tod, P.: The equations of conformal cyclic cosmology. Gen. Relativ. Gravit. **47**(3), 1–13 (2015). https://doi.org/10.1007/s10714-015-1859-7
19. Penrose, R.: Before the Big Bang: an outrageous new perspective and its implications for particle physics. In: Proceedings of EPAC, Edinburgh, pp. 2759–2763 (2006). http://accelconf.web.cern.ch/AccelConf/e06/PAPERS/THESPA01.PDF
20. Penrose, R.: Cycles of Time: An Extraordinary New View of the Universe. The Bodley Head, London (2010)
21. Tod, P.: Some questions about conformal cyclic cosmology. Preprint arXiv:2202.10864 (2022)
22. Gurzadyan, V.G., Penrose, R.: On CCC-predicted concentric low-variance circles in the CMB sky. Eur. Phys. J. Plus **128**(2), 1–17 (2013). https://doi.org/10.1140/epjp/i2013-13022-4
23. Gurzadyan, V.G., Penrose, R.: CCC and the Fermi paradox. Eur. Phys. J. Plus **131**, 11 (2016). https://doi.org/10.1140/epjp/i2016-16011-1. arXiv:1512.00554 [astro-ph.CO]
24. Meissner, K.A., Nurowski, P., Ruszczycki, B.: Structures in the microwave background radiation. Proc. Roy. Soc. A Math. Phys. Eng. Sci. **469**(2155), 20130116 (2013). https://doi.org/10.1098/rspa.2013.0116
25. An, D., Meissner, K.A., Nurowski, P.: Ring-type structures in the Planck map of the CMB. Mon. Not. R. Astron. Soc. **473**(3), 3251–3255 (2017). https://doi.org/10.1093/mnras/stx2299
26. An, D., Meissner, K.A., Nurowski, P., Penrose, R.: Apparent evidence for Hawking points in the CMB sky. Mon. Not. R. Astron. Soc. **495**(3), 3403–3408 (2020). https://doi.org/10.1093/mnras/staa1343
27. DeAbreu, A., Contreras, D., Scott, D.: Searching for concentric low variance circles in the cosmic microwave background. J. Cosmol. Astropart. Phys. **2015**(12), 031–031 (2015). https://doi.org/10.1088/1475-7516/2015/12/031
28. Jow, D.L., Scott, D.: Re-evaluating evidence for hawking points in the CMB. J. Cosmol. Astropart. Phys. **2020**(03), 021–021 (2020). https://doi.org/10.1088/1475-7516/2020/03/021
29. Lopez, M., Bonizzi, P., Driessens, K., Koekoek, G., de Vries, J., Westra, R.: Searching for ring-like structures in the cosmic microwave background (2021). arXiv preprint arXiv:2105.03990
30. Aurell, E., Eckstein, M., Horodecki, P.: Quantum black holes as solvents. Found. Phys. **51**(2), 1–13 (2021). https://doi.org/10.1007/s10701-021-00456-7
31. Aurell, E., Eckstein, M., Horodecki, P.: Hawking radiation and the quantum marginal problem. J. Cosmol. Astropart. Phys. **2022**(01), 014 (2022). https://doi.org/10.1088/1475-7516/2022/01/014
32. Page, D.N.: Is black-hole evaporation predictable? Phys. Rev. Lett. **44**, 301–304 (1980). https://doi.org/10.1103/PhysRevLett.44.301
33. Popescu, S.: Nonlocality beyond quantum mechanics. Nat. Phys. **10**(4), 264 (2014). https://doi.org/10.1038/nphys2916
34. Horodecki, P., Ramanathan, R.: The relativistic causality versus no-signaling paradigm for multi-party correlations. Nat. Commun. **10**(1), 1701 (2019). https://doi.org/10.1038/s41467-019-09505-2
35. Lübbe, C.: Conformal scalar fields, isotropic singularities and conformal cyclic cosmologies (2013). preprint arXiv:1312.2059
36. Tod, P.: Conformal methods in general relativity with application to conformal cyclic cosmology: a minicourse given at the IXth IMLG Warsaw 2018 (2021). preprint arXiv:2102.02701
37. Bachelot, A.: Propagation of massive scalar fields in pre-big bang cosmologies. Commun. Math. Phys. **380**(2), 973–1001 (2020). https://doi.org/10.1007/s00220-020-03880-4
38. Nurowski, P.: Poincaré–Einstein approach to Penrose's conformal cyclic cosmology. Class. Quant. Gravity **38**(14), 145004 (2021). https://doi.org/10.1088/1361-6382/ac0237
39. Nurowski, P.: Conformally flat models in Penrose's conformal cyclic cosmology (2021). preprint arXiv:2102.11823
40. Kopiński, J., Kroon, J.A.V.: The Bach equation and the matching of spacetimes in the conformal cyclic cosmology models. Phys. Rev. D **106**, 084034 (2022) https://doi.org/10.1103/PhysRevD.106.084034 preprint arXiv:2201.10875
41. Penrose, R.: Singularity and time-asymmetry. In: Hawking, S.W., Israel, W. (eds.) General Relativity: An Einstein Centenary Survey, pp. 581–638. Cambridge University Press, Cambridge (1979)
42. Clifton, T., Ellis, G.F.R., Tavakol, R.: A gravitational entropy proposal. Class. Quant. Gravity **30**(12), 125009 (2013). https://doi.org/10.1088/0264-9381/30/12/125009

43. Egan, C.A., Lineweaver, C.H.: A larger estimate of the entropy of the universe. Astrophys. J. **710**(2), 1825–1834 (2010). https://doi.org/10.1088/0004-637x/710/2/1825
44. Li, N., Li, X.-L., Song, S.-P.: Kullback–Leibler entropy and Penrose conjecture in the Lemaître–Tolman–Bondi model. Eur. Phys. J. C **75**(3), 1–9 (2015). https://doi.org/10.1007/s10701-018-0162-3
45. Marozzi, G., Uzan, J.-P., Umeh, O., Clarkson, C.: Cosmological evolution of the gravitational entropy of the large-scale structure. Gen. Relativ. Gravit. **47**(10), 1–19 (2015). https://doi.org/10.1007/s10714-015-1955-8
46. Belgiorno, F., Catino, G.: A Weyl entropy of pure spacetime regions. Class. Quant. Gravity **37**(22), 225014 (2020). https://doi.org/10.1088/1361-6382/abb958
47. Gregoris, D., Ong, Y.C., Wang, B.: Thermodynamics of shearing massless scalar field spacetimes is inconsistent with the Weyl curvature hypothesis. Phys. Rev. D **102**, 023539 (2020). https://doi.org/10.1103/PhysRevD.102.023539
48. Gregoris, D., Ong, Y.C.: Understanding gravitational entropy of black holes: a new proposal via curvature invariants. Phys. Rev. D **105**, 104017 (2022). https://doi.org/10.1103/PhysRevD.105.104017
49. Penrose, R.: On gravity's role in quantum state reduction. Gen. Relativ. Gravit. **28**(5), 581–600 (1996). https://doi.org/10.1007/BF02105068
50. Penrose, R.: On the gravitization of quantum mechanics 1: quantum state reduction. Found. Phys. **44**(5), 557–575 (2014). https://doi.org/10.1007/s10701-013-9770-0
51. Braunstein, S.L., van Loock, P.: Quantum information with continuous variables. Rev. Mod. Phys. **77**, 513–577 (2005). https://doi.org/10.1103/RevModPhys.77.513
52. Cerf, N.J., Adami, C.: Negative entropy and information in quantum mechanics. Phys. Rev. Lett. **79**, 5194–5197 (1997). https://doi.org/10.1103/PhysRevLett.79.5194
53. Zurek, W.H.: Decoherence, einselection, and the quantum origins of the classical. Rev. Mod. Phys. **75**, 715–775 (2003). https://doi.org/10.1103/RevModPhys.75.715
54. Weedbrook, C., Pirandola, S., García-Patrón, R., Cerf, N.J., Ralph, T.C., Shapiro, J.H., Lloyd, S.: Gaussian quantum information. Rev. Mod. Phys. **84**, 621–669 (2012). https://doi.org/10.1103/RevModPhys.84.621
55. Unruh, W.G., Wald, R.M.: Information loss. Rep. Prog. Phys. **80**(9), 092002 (2017). https://doi.org/10.1088/1361-6633/aa778e
56. Hawking, S.W.: Black hole explosions? Nature **248**(5443), 30–31 (1974). https://doi.org/10.1038/248030a0
57. Wald, R.M.: On particle creation by black holes. Commun. Math. Phys. **45**(1), 9–34 (1975). https://doi.org/10.1007/BF01609863
58. Heusler, M.: Black Hole Uniqueness Theorems. Cambridge University Press, Cambridge (1996)
59. Gisin, N.: Stochastic quantum dynamics and relativity. Helv. Phys. Acta **62**(4), 363–371 (1989)
60. Simon, C., Bužek, V., Gisin, N.: No-signaling condition and quantum dynamics. Phys. Rev. Lett. **87**, 170405 (2001). https://doi.org/10.1103/PhysRevLett.87.170405
61. Visser, M.: Thermality of the Hawking flux. J. High Energy Phys. **2015**(7), 1–15 (2015). https://doi.org/10.1007/JHEP07(2015)009
62. Zurek, W.H.: Entropy evaporated by a black hole. Phys. Rev. Lett. **49**, 1683–1686 (1982). https://doi.org/10.1103/PhysRevLett.49.1683
63. Page, D.N.: Comment on entropy evaporated by a black hole. Phys. Rev. Lett. **50**, 1013–1013 (1983). https://doi.org/10.1103/PhysRevLett.50.1013
64. Page, D.N.: Hawking radiation and black hole thermodynamics. New J. Phys. **7**, 203–203 (2005). https://doi.org/10.1088/1367-2630/7/1/203
65. 't Hooft, G.: Dimensional reduction in quantum gravity (1993). preprint gr-qc/9310026
66. Page, D.N.: Information in black hole radiation. Phys. Rev. Lett. **71**, 3743–3746 (1993). https://doi.org/10.1103/PhysRevLett.71.3743
67. Harlow, D.: Jerusalem lectures on black holes and quantum information. Rev. Mod. Phys. **88**, 015002 (2016). https://doi.org/10.1103/RevModPhys.88.015002
68. Marolf, D.: The black hole information problem: past, present, and future. Rep. Prog. Phys. **80**(9), 092001 (2017). https://doi.org/10.1088/1361-6633/aa77cc
69. Almheiri, A., Hartman, T., Maldacena, J., Shaghoulian, E., Tajdini, A.: The entropy of Hawking radiation. Rev. Mod. Phys. **93**, 035002 (2021). https://doi.org/10.1103/RevModPhys.93.035002
70. Ashtekar, A.: Implications of a positive cosmological constant for general relativity. Rep. Prog. Phys. **80**(10), 102901 (2017). https://doi.org/10.1088/1361-6633/aa7bb1

71. Krenn, G., Zeilinger, A.: Entangled entanglement. Phys. Rev. A **54**, 1793–1797 (1996). https://doi.org/10.1103/PhysRevA.54.1793
72. Walther, P., Resch, K.J., Brukner, V., Zeilinger, A.: Experimental entangled entanglement. Phys. Rev. Lett. **97**, 020501 (2006). https://doi.org/10.1103/PhysRevLett.97.020501
73. Nelson, W., Wilson-Ewing, E.: Pre-big-bang cosmology and circles in the cosmic microwave background. Phys. Rev. D **84**, 043508 (2011). https://doi.org/10.1103/PhysRevD.84.043508
74. Tod, P.: Penrose's circles in the CMB and a test of inflation. Gen. Relativ. Gravit. **44**(11), 2933–2938 (2012). https://doi.org/10.1007/s10714-012-1431-7
75. Balasin, H., Nachbagauer, H.: The energy-momentum tensor of a black hole, or what curves the Schwarzschild geometry? Class. Quant. Gravity **10**(11), 2271–2278 (1993). https://doi.org/10.1088/0264-9381/10/11/010
76. Arndt, M., Hornberger, K.: Testing the limits of quantum mechanical superpositions. Nat. Phys. **10**(4), 271–277 (2014). https://doi.org/10.1038/nphys2863
77. Fein, Y.Y., Geyer, P., Zwick, P., Kiałka, F., Pedalino, S., Mayor, M., Gerlich, S., Arndt, M.: Quantum superposition of molecules beyond 25 kDa. Nat. Phys. **15**(12), 1242–1245 (2019)
78. Carlesso, M., Donadi, S., Ferialdi, L., Paternostro, M., Ulbricht, H., Bassi, A.: Present status and future challenges of non-interferometric tests of collapse models. Nat. Phys. **18**(3), 243–250 (2022). https://doi.org/10.1038/s41567-021-01489-5
79. Bassi, A., Lochan, K., Satin, S., Singh, T.P., Ulbricht, H.: Models of wave-function collapse, underlying theories, and experimental tests. Rev. Mod. Phys. **85**, 471–527 (2013). https://doi.org/10.1103/RevModPhys.85.471
80. Donadi, S., Piscicchia, K., Curceanu, C., Diósi, L., Laubenstein, M., Bassi, A.: Underground test of gravity-related wave function collapse. Nat. Phys. **17**(1), 74–78 (2021). https://doi.org/10.1038/s41567-020-1008-4
81. Horodecki, R., Horodecki, P., Horodecki, M., Horodecki, K.: Quantum entanglement. Rev. Mod. Phys. **81**, 865–942 (2009). https://doi.org/10.1103/RevModPhys.81.865
82. Grudka, A., Hall, M.J., Horodecki, M., Horodecki, R., Oppenheim, J., Smolin, J.A.: Do black holes create polyamory? J. High Energy Phys. **2018**(11), 1–25 (2018). https://doi.org/10.1007/JHEP11(2018)045
83. Haag, R.: Local Quantum Physics: Fields, Particles Algebras Theoretical and Mathematical Physics. Springer, Berlin (1996)
84. Dieks, D.: Communication by EPR devices. Phys. Lett. A **92**(6), 271–272 (1982). https://doi.org/10.1016/0375-9601(82)90084-6
85. Wootters, W.K., Żurek, W.H.: A single quantum cannot be cloned. Nature **299**(5886), 802–803 (1982)
86. Hayden, P., Preskill, J.: Black holes as mirrors: quantum information in random subsystems. J. High Energy Phys. **2007**(09), 120–120 (2007). https://doi.org/10.1088/1126-6708/2007/09/120
87. Susskind, L., Thorlacius, L., Uglum, J.: The stretched horizon and black hole complementarity. Phys. Rev. D **48**, 3743–3761 (1993). https://doi.org/10.1103/PhysRevD.48.3743
88. Almheiri, A., Marolf, D., Polchinski, J., Sully, J.: Black holes: complementarity or firewalls? J. High Energy Phys. **2013**(2), 1–20 (2013). https://doi.org/10.1007/JHEP02(2013)062
89. Penrose, R.: The Road to Reality: A Complete Guide to the Laws of the Universe. Jonathan Cape, London (2004)
90. Penrose, R.: The Big Bang and its dark-matter content: whence, whither, and wherefore. Found. Phys. **48**(10), 1177–1190 (2018). https://doi.org/10.1007/s10701-018-0162-3