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GOLDEN OLDIE EDITORIAL

Editorial note to: Brandon Carter, Black hole equilibrium states Part II. General theory of stationary black hole states

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Brandon Carter's lectures on the general theory of stationary black holes, reprinted here as a Golden Oldie, belong to the vivid legend of the 1972 Les Houches Summer School.[1] The School was a marvelous and unforgettable experience for its youngest student participants, like me. Although Carter and a few other lecturers were just slightly older than most of us, they already enjoyed the fame of deep thinkers, great physicists and masterly mathematicians, and were well-known for their fundamental discoveries in the black hole theory.

Carter's black hole lectures at Les Houches concentrated around the characterization of the Kerr geometry as a solution of the black hole boundary problem for the vacuum Einstein equations, and the question of whether, as such, it is unique.[2] This problem had occupied his mind for some time, and he already made several significant contributions that paved the road to its solution.

[1] Legends are on high demand and price. Amazon.com sells the single-volume proceedings of the 1972 Les Houches Summer School for 699$ apiece.
[2] The term *uniqueness* depends on the breadth of the category concerned; for example the uniqueness will definitely be lost if exotica such as Yang Mills fields are admitted.

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Prior to the les Houches school, Carter had already established uniqueness in the astrophysically "natural" but restricted vacuum category characterized by stationarity, axisymmetry, continuous accessibility (by spin up) from the spherical Schwarzschild limit, and causality violation only inside a well behaved horizon.

Subsequent work by many authors has successively removed most of these restrictions except for the stationarity condition, which is clearly indispensable, and the causality condition, whose mathematical necessity remains an open question even today. In a particularly noteworthy step, the previously established black hole "no hair" theorem—meaning the impossibility of getting other vacuum solutions by continuous variation from the Kerr solution—was extended by David Robinson [1] to a stronger uniqueness theorem excluding the existence even of discontinuously related vacuum solutions.

The situation shortly after the Les Houches school was succinctly described (in "A brief History of Time") by Stephen Hawking [2] as follows: *It was conjectured that any rotating body that collapsed to form a black hole would settle down to a stationary state described by the Kerr solution. In* 1970 *a colleague and fellow research student of mine at Cambridge, Brandon Carter, took the first step toward proving this conjecture. He showed that, provided a stationary rotating black hole had an axis of symmetry, like a spinning top, its size and shape would depend only on its mass and rate of rotation. Then in* 1971*, I proved that any stationary rotating black hole would indeed have such an axis of symmetry. Finally in* 1973 *David Robinson at King's college, London used Carter's and my results to show that the conjecture had been correct: such a black hole had indeed to be the Kerr solution.*

Carter himself emphasises that, in his own words, *a more obstinately difficult step, to which the most important contribution was due to Stephen Hawking* (*using prior results due mainly to Werner Israel*) *has been the demonstration that* (*unlike what occurs in the case of perfect fluids!*) *the assumption of axisymmetry can also be dispensed with. Hawking was able to prove this only in a category that remained restricted, notably by the condition of analyticity, and by a condition on the location of the "ergosphere" region. This latter restriction has since been removed by much more recent work due to Bob Wald and others, but I do not know of any proof yet of the redundancy of Hawking's analyticity postulate.*

As well as the successive removal of restrictions on the validity of the theorem for the Kerr geometry in the pure vacuum case, work in more recent years, particularly by David Robinson [3], Paweł Mazur [4,5], and Gary Bunting [6] has obtained analogous results for the charged Kerr–Newman geometry in the Einstein-Maxwell case, and in doing so has also provided more elegant methods for deriving what was previously known about the pure vacuum case.[3]

At Les Houches Carter was not only lecturing, but also conducting with James Bardeen and Stephen Hawking an intense research that resulted in a publication (next year) of their seminal paper [10] on the four laws of the black hole mechanics. "Mechanics", and not "thermodynamics" as we would say today, because the true thermodynamical nature of the *four laws* was not initially appreciated even by their

[3] For a few historical accounts of the subject see, e.g. [7–9].

three discoverers. Indeed, Carter himself wrote in his Les Houches article reprinted here: *"The four laws collected together are clearly of fundamental importance in their own right. Although they correspond closely to the classical laws of thermodynamics, it is to be emphasized that this is only an analogy whose significance should not be exaggerated. Although they are analogous, $\mathcal{T}$ and $\mathcal{A}$ play a quite distinct role from the temperature and entropy with which they should be not confused. The real effective temperature $\bar{\Theta}^H$ of a black hole is well defined and unambiguously zero, as is the chemical potential $\bar{\mu}^{(i)H}$. The ordinary particle conservation law, and the ordinary second law of thermodynamics, are unquestionably transcended by a black hole, in the sense that particles and entropy can be lost without trace from an external point of view. It is not possible to mitigate this transcendence by somehow relating the amount of entropy* (*or the number of particles*) *which have gone in the subsequent increase in the surface area $\mathcal{A}$."* While this view on black hole entropy and temperature started to change soon after the 1972 Les Houches Summer School due to the brilliant insight provided by Jacob Bekenstein [11] and Hawking's subsequent epoch-making discovery [12,13], a fully satisfactory physical interpretation of the Bekenstein–Hawking black hole entropy is still today a matter of an interesting debate, to which string theory contributes most forcefully.[4]

The obvious question that a practical mind must ask is, what do these mathematically powerful results imply (if anything) for the real astrophysical black holes that have already been discovered in our Galaxy? Should one care about the enormously large Bekenstein–Hawking entropy of the astrophysical black holes when calculating their observable properties? Could one use observations of compact astrophysical objects to examine the validity of the black hole uniqueness theorem?

Hawking radiation occurs whenever there is an "event horizon". Bill Unruh suggested [15] that this may happen, e.g. for an observer in the Minkowski spacetime with a constant acceleration a. This is called the *Unruh effect*—the observer finds himself in a thermal bath with a temperature $T = ha/2\pi\,\kappa\, c$. It was claimed that this effect has been measured in the electron storage ring [16]. Unruh suggested also the hydrodynamical analogy [17], in which the "hydrodynamical Hawking radiation" emerges due to phonons emitted in a moving fluid in which a sound horizon exists. However, although a quarter of century has already passed, the hydrodynamical Hawking radiation has not yet been experimentally detected [18].

It is still not clear whether there could be a practical method to examine the Bekenstein–Hawking entropy in astrophysical phenomena. The best known suggestion was a possible connection of the observed gamma ray bursts with the final explosive bursts of Hawking's radiation that should presently occur for primordial black holes (if they exist) with masses $\sim 10^{15}$ g; see, e.g. [19]. It seems that this possibility is now observationally excluded. Below, I give examples of two less known suggestions.

In a certain parameter range, for the same boundary conditions two stable solutions for black hole accretion flows exist: the "Shakura-Sunyaev" solution in which matter radiates away most of its internal energy before it is swallowed by the black

[4] At the web address http://www.stringwiki.org/wiki/Black_Hole_Entropy, the "string wikipedia" lists about twenty authoritative reviews and articles that discuss the string theory interpretation of the Bekenstein–Hawking entropy. The list starts with an excellent review by Thibault Damour [14].

hole, and the “adaf” solution in which matter radiates very little and thus most of its internal energy is advected into the hole. In the parameter range of interest, Nature always picks up the adaf solution, and the question is why. I have noticed [20] that if one accounts for the Bekenstein–Hawking entropy increase due to accretion, the adaf solution is favored by the Prigogine Entropy Principle (see, e.g. [21] for a discussion of the Prigogine Principle and references).

As noticed recently by Dimitrios Psaltis [22], *“in braneworld gravity models with a finite AdS curvature in the extra dimension, the AdS/CFT correspondence leads to a prediction for the lifetime of astrophysical black holes that is significantly smaller than the Hubble time, for asymptotic curvatures that are consistent with current experiments.”* Psaltis used the recent measurement of the velocity of the black hole XTE J1118+480 to calculate a lower limit 11×10^9 years on the time needed for this black hole to move from the Galaxy plane, where it was born, to its present position in the Galaxy. The travel time gives a lower limit for the black hole’s age, which implies an upper limit of 0.08 mm for the asymptotic AdS radius of curvature in the extra dimensions. The limit deduced by Psaltis from observations of the astrophysical black hole XTE J1118+480 is significantly stronger than any limit obtained so far from table-top experiments on sub-millimeter gravity.

Some recent discussions of observational properties of black holes may eventually become useful as an experimental back-up of the black hole uniqueness theorem. Ramesh Narayan argues [23] that specific differences in observed properties of black holes and neutron stars may be explained by the fact that the black holes have event horizons, and neutron stars have rigid surfaces that reflect back some radiation and matter. Many astrophysicists take this argument as a direct proof of the existence of the event horizon, while a few point out [24] that the “gravastars”, i.e. Bose–Einstein condensate stars suggested by Paweł Mazur and Emil Mottola [25], do have rigid surfaces, but are observationally indistinguishable from black holes.

One should also be aware of the fact that astrophysicists have recently developed new methods of measuring the Kerr parameter a. While all measurements to date are consistent with $a < m$ within errors, in the case of the Galactic black hole GRS 1915+105, it is $a > 0.98\,\mathrm{m}$ [26].

Comment by the Golden Oldie Editor: Several obvious misprints of the original article have been corrected in this reprinting, with the approval of the author. The obvious corrections are not marked by editorial footnotes. I am grateful to Marie-Noëlle Célérier for being an efficient intermediary between me and B. Carter when health problems on both sides made direct communication difficult.

Brandon Carter’s brief autobiography was printed together with Part 1 of this article, published in *Gen. Relativ. Gravit.* **41**, 2870 (2009), doi:10.1007/s10714-009-0887-6 [editor].

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