

# Hawking radiation and black hole entropy in a gravity's rainbow

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**Abstract** In the context of gravity's rainbow, Planck scale correction on Hawking radiation and black hole entropy in Parikh and Wilczk's tunneling framework is studied. We calculate the tunneling probability of massless particles in the modified Schwarzschild black holes from gravity's rainbow. In the tunneling process, when a particle gets across the horizon, the metric fluctuation must be taken into account, not only due to energy conservation but also to spacetime Planck scale effect. Our results show that the emission rate is related to changes of the black hole's quantum corrected entropies before and after the emission. In the same time, for the modified black holes, a series of correction terms including a logarithmic term to Bekenstein–Hawking entropy are obtained. Correspondingly, the spectrum of Planck scale corrected emission is obtained and it deviates from the thermal spectrum. In addition, a specific form of modified dispersion relation is proposed and applied.

**Keywords** Hawking radiation · Black hole entropy · Gravity's rainbow · Modified dispersion relations

## 1 Introduction

Based on Hawking's great discovery that black holes have thermal radiations [1,2], black hole thermodynamics get a solid fundament and then general relativity, quantum

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mechanics, and thermodynamics have a significant link. However, following this, the information loss paradox emerges [3,4]. It is that, since a thermal spectrum cannot bring out any information other than only the parameter of temperature, all information about the matter making up of the black hole will be lost, when a black hole evaporate away completely. In [3], Hawking argued that, the formation and evaporation of a black hole are not commanded by quantum mechanics. That is to say, the pure states of matter forming the black hole evolve into the mixed states of thermal radiation and then the underlying unitary theory of quantum mechanics is violated. Besides, in the light of Hawking's description, black hole evaporation is quantum tunneling effect [5]. It is to say, due to the vacuum fluctuation near the horizon of a black hole, when a pair of particles is spontaneously created just inside the horizon, the positive energy particle can tunnel out the horizon to the infinity. At the same time, the negative energy particle remains behind the horizon and effectively lowers the mass of the black hole—the negative energy orbit cannot exist outside the horizon. However, due to lack of the tunneling barrier, the actual derivation of Hawking radiation does not proceed in the way of Hawking's description [6,7]. And that, most of Hawking radiation are based upon quantum field theory on a fixed background spacetime without considering the gravity back-reaction of emitted particles and the quantum fluctuation of spacetime.

Recently Hawking has put forward that [4], the information hidden in a black hole could come out, if Hawking radiation was not exactly thermal through some corrections. And that, Parikh and Wilczek have presented a semi-classical method of deriving Hawking radiation by implementing Hawking radiation as a tunneling process from the horizon of a black hole [8–10], where the emission rate is related to the changes of the Bekenstein–Hawking (B-H) entropy of the black hole before and after the emission and the non-thermal spectrum of back-reaction corrected radiation is obtained. Thus, this method presents a quantum tunneling description on Hawking radiation and opens a way to a possible resolution of the information loss paradox. Taking energy conservation into account is the crucial point of the program and thus the background is allowed to fluctuate because of the particle's back-reaction. Following this method, many recent researches have extended this tunneling study to many cases including static or stationary black holes [11–15], cosmological horizons [16,17] and different kinds of tunneling particles such as massive and charged particles [12,13]. The results confirm that Hawking radiation is not purely thermal spectrum and the information conservation of black holes could be possible. However, the quantum effects of spacetime have not been considered in Parikh and Wilczek's original work [8–10] and much less attention in the literatures was paid to the particle's tunneling from the quantum horizon [18–20]. Thus, when the quantum correction terms of black hole entropy are taken into account, whether the tunneling probability is still related to the change of black hole entropy and the information conservation whether could still be possible are open problems.

It is remarkable that, for understanding Hawking radiation, black hole entropy, and the information loss paradox, the Planck scale physics should be useful. In fact, in a general way, the radiation spectrum of black hole has arbitrarily high frequencies and their energy can go below Planck energy [19,21,22]. Therefore, in Parikh and Wilczek's semi-classical method of investigating Hawking radiation, Planck scale effects should be taken into account. In this paper, by using Parikh and Wikczek's

tunneling framework, we investigate the particle's tunneling in a Planck scale corrected spacetime named as gravity's rainbow [23]. By incorporating Planck scale effects with Parikh and Wilczek's tunneling framework, our main aim is to study the relationship between tunneling probability and quantum corrected entropy of black holes. And that, Hawking radiation and information conservation of black holes are investigated in a Planck scale corrected spacetime. In the present tunneling, while a particle tunneling through the Planck scale corrected horizon, the metric fluctuation must be taken into account, not only due to energy conservation but also the Planck scale effects of spacetime. The results of the paper show that, in the gravity's rainbow, the emission rate of a massless particle is related to the changes of the quantum corrected entropy of black holes and information conservation of black hole is possible. In the same time, the logarithms corrected entropy of the gravity's rainbow is obtained and the emission spectrums with Planck scale corrections are presented. Compared with Parikh and Wilczek's non-thermal spectrum, the presented emission spectrum is further departure from pure thermal spectrum.

The paper is organized as follows. In Sect. 2, in the context of a deformed formalism of special relativity namely double special relativity (DSR), the modified Schwarzschild solution from gravity's rainbow is reviewed and its some thermodynamics quantities are presented. Then in Sect. 3, by using Parikh and Wilczek's tunneling framework, the emission rates of massless particles in the modified black holes are obtained. In Sect. 4, black hole entropy for the modified Schwarzschild spacetime is calculated and a series of correction items include a logarithmic item to B-H entropy are obtained. Accordingly, the deviation of the emission spectrum of the modified black hole to thermal spectrum is investigated. The last part is the summary and conclusion.

## 2 The modified black holes from the gravity's rainbow

As one generally believed viewpoint, the existence of a minimally observable length order of Planck length is a universal feature of quantum gravity [24–27]. Recently, such character has invoked many researches on the fate of Lorentz symmetry at Planck scale. The reason is that, the character in principle may contract any object to arbitrarily small size by Lorentz boost and seemly leads to a apparent confliction with Lorentz symmetry.

At present, when keeping Planck energy as an invariant scale, namely a universal constant for all inertial observers, to preserve the relativity of inertial frames, DSR as a deformed formalism of special relativity has been proposed [28–33]. DSR's starting point and main result is that the usual energy momentum relation in special relativity may be modified in term of the ratio of particle's energy to Planck energy. And that, the modified dispersion relations (MDR) can be expressed as [28–30]

$$E^2 f_1^2(E; \lambda) - p^2 f_2^2(E; \lambda) = m_0^2, \quad (1)$$

where  $f_1$  and  $f_2$  are two energy functions from which a specific formulation of boost generator can be defined, in which  $\lambda$  is a parameter of order Planck length. The equation indicates that, MDR is energy dependent. It is to say, particles with different

energies have different energy–momentum relations. In addition, it should be pointed out, MDR can be presented by different ways [34–36] and can be used to explain a rich and energetic phenomenology [29–32].

In the same time, great efforts also have been devoted to DSR and its implications [23, 32, 33, 37–49]. Where, the deformed spacetime geometry from DSR has been investigated by different proposals [23, 41–45]. In [23, 41], it has been put forward that flat spacetime has energy dependent metric, namely rainbow metric. In other words, the DSR spacetime is endowed with an energy dependent quadratic invariant [23, 41], namely

$$ds^2 = -\frac{dt^2}{f_1^2} + \frac{dr^2}{f_2^2} + \frac{r^2}{f_2^2} d\Omega^2. \quad (2)$$

Furthermore, DSR has been extended to deformed general relativity and the rainbow metric has been extended to gravity's rainbow [23]. Similar to rainbow metric, the geometry of gravity's rainbow is described by one parameter family of metric as a function of particle's energy observed by an inertial observer. And that, the modified Schwarzschild solution from the gravity's rainbow has been demonstrated in terms of energy independent coordinates and the energy independent mass parameter [23], namely

$$dS^2 = -\frac{(1-\frac{2GM}{r})}{f_1^2} dt_s^2 + \frac{1}{f_2^2 (1-\frac{2GM}{r})} dr^2 + \frac{r^2}{f_2^2} d\Omega^2. \quad (3)$$

The metric concretely indicates that, the spacetime of gravity's rainbow depends on the energy of particle moving in it. That is, if a given observer probes the spacetime using the quanta with different energies, he will conclude that spacetime geometries have different effective descriptions. Here, the particle's energy denotes the total energy measured at infinity from the black hole. By this, the present spacetime is endowed with Plank scale effects shown as energy dependence.

From the metric Eq. (3), it is seen the horizon  $r_+ = 2GM$  is universal for all observers and at the usual place as the usual Schwarzschild black hole. However, the horizon area

$$A = \frac{16\pi G^2 M^2}{f_2^2} \quad (4)$$

is different from the usual value and depends on particle's energy. This should have some modification on black hole thermodynamics.

Besides, the surface gravity on the horizon of the modified black hole can be defined by [46]

$$\kappa = -\frac{1}{2} \lim_{r \rightarrow r_+} \frac{1}{g^{tt}} \sqrt{\frac{-g^{rr}}{g^{tt}} \frac{\partial g^{tt}}{\partial r}}, \quad (5)$$

and obtained as

$$\kappa = \frac{f_2}{f_1} \frac{1}{4GM}. \quad (6)$$

Thus, the temperature of the modified black hole is obtained as

$$T = \frac{\kappa}{2\pi} = \frac{f_2}{f_1} \frac{1}{8\pi GM}. \quad (7)$$

It shows that the temperature of the gravity's rainbow has the dependence upon the energy of probe particle. That is, using the quanta with different energy, an observer at infinity will probe different effective temperature for the Planck scale corrected black hole.

In addition, from Eqs. (3) and (2), we can see the modified Schwarzschild solution is asymptotically DSR. And that, it has been pointed that the asymptotically DSR spacetime has equality with the usual asymptotically flat spacetime [47]. Then, using the Komar integrals, we define the total Arnowitt–Deser–Misner (ADM) mass  $M_{\text{ADM}}$  of the Planck scale corrected spacetime as

$$M_{\text{ADM}} = -\frac{1}{8\pi G} \int_s \varepsilon_{abcd} \nabla^c \xi^d = \frac{M}{f_1 f_2}. \quad (8)$$

We find that, for the modified Schwarzschild black holes, the ADM mass is not equal to the mass parameter  $M$ . And that, the total energy of the spacetime depend on the energy of probe particle.

It is easy verified that, in the modified Schwarzschild black holes from the gravity's rainbow, the energy dependence of thermodynamics quantities should arise from the energy dependence of the Planck scale corrected spacetime. In other word, the character of energy dependence should be the exhibition of the Planck scale effects of the spacetime. So, the analysis on the character is necessary for us to investigate Hawking radiation and black hole entropy in the Planck scale corrected spacetime.

### 3 Tunneling probability in modified black holes

DSR and gravity's rainbow can be seen as a low energy effect of quantum gravity. It is that, the modified black hole of Eq. (3) is a coarse grained model of quantum spacetime at semi-classical level. Here, we assume that Parikh and Wilczk's quantum tunneling program of investigating Hawking radiation still holds for the large modified black holes in the gravity's rainbow. Therefore, in this section, following Parikh and Wilczk's tunneling framework, we calculate the tunneling probability of massless particles in the modified black hole. The novel point of the present tunneling investigation is that the Planck scale effect of geometry is considered in the tunneling process.

In Parikh and Wilczk's tunneling scheme, the particles behind horizon can tunnel out along a classically forbidden trajectory and the tunneling probability is given by

means of WKB approximation. That is, the emission rate can be expressed as the imaginary part of the action for the trajectory [8–10], namely

$$\Gamma \sim \exp(-2 \operatorname{Im} I). \quad (9)$$

For calculating the action  $I$  in the modified black holes shown as Eq. (3), the coordinate singularity at the horizon must be removed. Here, following Painleve coordinate transformation [50], we introduce a new time coordinate  $t$ . Letting

$$dt_s = dt - F(r) dr, \quad (10)$$

and

$$\frac{1}{f_2^2 \left(1 - \frac{2GM}{r}\right)} - \frac{\left(1 - \frac{2GM}{r}\right)}{f_1^2} F^2(r) = 1, \quad (11)$$

then we have

$$\begin{aligned} ds^2 = & -\frac{\left(1 - \frac{2GM}{r}\right)}{f_1^2} dt^2 + \frac{2}{f_1 f_2} \sqrt{1 - f_2^2 \left(1 - \frac{2GM}{r}\right)} dt dr \\ & + dr^2 + \frac{r^2}{f_2^2} \left(d\theta^2 + \sin^2 \theta d\varphi^2\right). \end{aligned} \quad (12)$$

It is easy to find that, for us to implement the calculation on the emission rate of particles tunneling through the Planck scale corrected horizon, the Painleve-like metric of the modified black holes has some advantages. Firstly, none of the components or the inverse of the metric diverges at the horizon. Secondly, the coordinate system has Killing vector  $\partial/\partial t$ . In addition, as expected, the metric has Planck scale effects shown as the energy dependence. This denotes that, even if the black hole has a fixed mass parameter  $M$ , the emitted particles with different energy will be affected by different metric.

It is assumed that a massless particle with energy  $E = \frac{1}{f_1 f_2} \omega$  measured at infinity tunnels outside the horizon of the modified black hole. For the massless particle, its motion equation can be given by the radial null geodesics of the geometry Eq. (12). Let  $ds^2 = 0$ , in the presence of Planck scale effects, we have the radial null geodesic as

$$\dot{r} = \frac{dr}{dt} = \frac{1}{f_1 f_2} \left[ \pm 1 - \sqrt{1 - f_2^2 \left(1 - \frac{2GM}{r}\right)} \right], \quad (13)$$

where “+” corresponds to outgoing particles, “−” corresponds to ingoing particles.

However, if we enforce energy conservation, when the particle tunnels out the horizon, the mass of the modified black hole should vary. That is, the back-reaction of emitted particles should affect the background geometry. Here, the particle can be treated as an  $s$ -wave, i.e., an energy shell. In spherical symmetry spacetime, the

back-reaction effects of emitted shell have been investigated in [51]. So, we can obtain that, when the particle radiates outside the horizon of the modified black hole, due to the particle’s self-gravitation, the mass parameter  $M$  in the metric Eq. (12) should be replaced with  $M - \omega$  [8–10,51]. Therefore, we get the geometry between the horizon and the spherical shell as

$$\begin{aligned}
 ds^2 = & -\frac{\left(1 - \frac{2G(M-\omega)}{r}\right)}{f_1^2} dt^2 + \frac{2}{f_1 f_2} \sqrt{1 - f_2^2 \left(1 - \frac{2G(M-\omega)}{r}\right)} dt dr \\
 & + dr^2 + \frac{r^2}{f_2^2} \left(d\theta^2 + \sin^2 \theta d\varphi^2\right). \tag{14}
 \end{aligned}$$

In fact, the above back-reaction effect of particles is consistent with Birkhoff’s theorem. The theorem tells us that, in spherical symmetry spacetime, the only effect on geometry due to spherical shell is to provide a junction condition for matching the total mass inside and outside the shell.

Then, we can see the locations of the horizon before and after the particle’s emission are  $r_i = r_+(M) = 2GM$  and  $r_f = r_+(M-\omega) = 2G(M-\omega)$ , respectively. Thus, due to energy conservation and the shrinking of the black hole, the tunneling barrier is created by the emitted particle itself. And, the set of the potential barrier is not affected by the Planck scale effects of the black hole-emitted particle system. This should be investigated further.

Thus, considering the background dynamical effects arise from energy conservation and the Planck scale effects of the spacetime, the radial motion equation of the tunneling particle should be modified as

$$\dot{r} = \frac{dr}{dt} = \frac{1}{f_1 f_2} \left(1 - \sqrt{1 - f_2^2 \left(1 - \frac{2G(M-\omega)}{r}\right)}\right). \tag{15}$$

In addition, for the tunneling process, a canonical Hamiltonian treatment gives a simple result for the total action of the black hole-particle system [51], namely

$$I = \int dt \left(p_t + \frac{dr}{dt} p_r\right), \tag{16}$$

where  $p_t$  and  $p_r$  are the conjugate momentum corresponding to Painleve’s coordinates  $t$  and  $r$ , respectively. Here, only the second term in Eq. (16) contributes to the imaginary part of the action and it is

$$\text{Im } I = \text{Im} \int_{t_i}^{t_f} dt \frac{dr}{dt} p_r = \text{Im} \int_{r_i}^{r_f} p_r dr = \text{Im} \int_{r_i}^{r_f} \int_0^{p_r} dp'_r dr, \tag{17}$$

where  $t_i$  and  $t_f$  are the Painleve coordinate times corresponding  $r_i$  and  $r_f$ , respectively.

To proceed with an explicit computation, we now apply the Hamilton’s equation

$$\dot{r} = \frac{dH}{dp_r} = \frac{dM'_{ADM}}{dp_r}, \tag{18}$$

there  $M'_{ADM} = \frac{1}{f_1 f_2} M'$  is the ADM mass of the modified black hole after emitting a particle with energy  $E' = \frac{1}{f_1 f_2} \omega'$ . Substituting Eq. (18) into Eq. (17), and switching the order of the integral, we have

$$\begin{aligned} \text{Im } I &= \text{Im} \int_{r_i}^{r_f} \int_0^{p_r} \frac{dM'_{ADM}}{\dot{r}} dr = \text{Im} \int_M^{M-\omega} dM'_{ADM} \int_{r_i}^{r_f} \frac{dr}{\dot{r}} \\ &= \text{Im} \int_M^{M-\omega} \int_{r_i}^{r_f} \frac{f_1}{f_2} \frac{r \left( 1 + \sqrt{1 - f_2^2 \left( 1 - \frac{r'_+}{r} \right)} \right)}{r - r'_+} dr d \frac{M'}{f_1 f_2}, \end{aligned} \tag{19}$$

where  $r'_+ = 2GM'$  is the horizon location after emitting the particle. Here, for calculating the action, the radiation of particle with energy  $\omega$  is treated as a process of emitting energy from 0 to  $\omega$  [8–10]. Then, in Eq. (19),  $f_1$  and  $f_2$  should be treated as variables.

Considering the particle tunneling through the horizon, we can see that  $r'_+$  is a single pole in Eq. (19). Then the integral can be evaluated by deforming the contour around the pole. In this way, we finished the integral over  $r$  and get

$$\text{Im } I = -4\pi G \int_M^{M-\omega} \frac{f_1}{f_2} M' d \frac{M'}{f_1 f_2}. \tag{20}$$

Now, for the modified black hole in the tunneling process, we apply the first law of black hole thermodynamics

$$dM'_{ADM} = T' dS'. \tag{21}$$

In fact, many previous works [13–15] in the literature have confirmed that the first law of black hole thermodynamics holds in the tunneling process. Then, inserting the temperature expression Eq. (7) into Eq. (21), we have

$$4\pi G \frac{f_1}{f_2} M' d \frac{M'}{f_1 f_2} = \frac{1}{2} dS', \tag{22}$$



and

$$\text{Im } I = -\frac{1}{2} \int_S^{S+\Delta S} dS' = -\frac{1}{2} \Delta S, \tag{23}$$

where  $\Delta S = S(M - \omega) - S(M)$  is the difference of the black hole entropies before and after the emission.

Thus, Substituting Eq. (23) into Eq. (9), the tunneling probability of massless particle from the modified Schwarzschild black hole in gravity’s rainbow is obtained as

$$\Gamma = \exp(-2 \text{Im } I) = \exp(\Delta S). \tag{24}$$

We find that, in the Planck scale corrected spacetime, the tunneling probability is related to the change of the entropies of the modified black hole. This is consistent with the result obtained from the usual Schwarzschild black hole [8–10]. However, the present black hole entropy should have Planck scale correction to B-H entropy. This is a radical difference with Parikh and Wilczk’s original results, in which, black hole entropy is obtained and applied as B-H entropy. Accordingly, the emission spectrum of the modified black hole should has Planck scale corrections to the only back-reaction corrected emission spectrum obtained from the usual black hole. In the next section, by calculating the Eq. (22), we obtain the Planck scale corrected entropy and emission spectrum for the modified black hole.

#### 4 Entropy and radiation spectrum of the modified black holes

In the present tunneling investigation, to calculate the Eq. (22) and then to obtain the entropy and the radiation spectrum for the modified black hole, we need the explicit DSR, i.e., specific energy functions  $f_1$  and  $f_2$ . Some researches has been devoted to the investigation on the explicit MDR models and different correction functions  $f_1$  and  $f_2$  have been proposed [30,37]. In low energy realm, i.e.,  $E/E_p \ll 1$ , where  $E_p \equiv 1/\sqrt{8\pi G}$  is Planck energy, the correspondence principle requires that  $f_1$  and  $f_2$  both approach to unit. However, as so far, the standard form of  $f_1$  and  $f_2$  has not been given and the further investigations are necessary. Here, for convenience, we take

$$f_1 = f = \left(1 - E^2/E_p^2\right)^{-\frac{1}{2}}, \quad f_2 = 1. \tag{25}$$

Then, based on the specific MDR, from the Eqs. (4) and (7), the horizon area and the temperature of the modified black holes are, respectively,

$$A = 16\pi G^2 M^2, \tag{26}$$

$$T^2 = \frac{1}{f^2} \frac{1}{(8\pi GM)^2}. \tag{27}$$

And that, from Eq. (22), we can obtain the differential form of the black hole entropy. That is

$$dS = 8\pi GMfd \left( \frac{M}{f} \right) = 4\pi GdM^2 - 8\pi GM^2 \frac{df}{f}. \quad (28)$$

We find that, for the modified black hole in the gravity's rainbow, the entropy equation depends on particle's energy. It is to say, the effective black hole entropy have the dependence on the energy of probe particle. Now, for large modified black holes, we use the characteristic temperature by identifying the energy of particles emitted from the black holes with the black hole's temperature [46,49,52,53], namely

$$E = T. \quad (29)$$

This can be understood as a statistical treatment of obtaining the black hole entropy. Supposing all the emitted particles form an ensemble outside the black hole, then the average energy of the particles is equal to the temperature of the black hole. In other words, we use the particle with energy  $T$  to probe the black hole entropy and ascertain it as the intrinsic entropy, i.e., the black hole entropy. Thus, from Eqs. (25) and (29), we have

$$f = \left( 1 - T^2/E_p^2 \right)^{-\frac{1}{2}}, \quad \frac{df}{f} = \frac{1}{2} \frac{dT^2}{E_p^2 - T^2}. \quad (30)$$

And that, substituting Eq. (30) into Eq. (27), we have

$$T^2 = \frac{1}{f^2} \frac{1}{(8\pi GM)^2} = \left( 1 - \frac{T^2}{E_p^2} \right) \frac{1}{(8\pi G)^2 M^2} = \frac{1}{(8\pi G)^2 M^2} - \frac{1}{8\pi GM^2} T^2. \quad (31)$$

Solving Eq. (31) and considering Eq. (26), we obtain

$$T^2 = \frac{1}{4\pi A + 8\pi G}. \quad (32)$$

Then, substituting Eq. (32) into Eq. (30), we have

$$\frac{df}{f} = \frac{1}{2} \frac{dT^2}{E_p^2 - T^2} = -\frac{1}{2} \frac{dA}{E_p^2 (4\pi A + 8\pi G) A}. \quad (33)$$

Next, substituting Eq. (33) into Eq. (28), we obtain

$$dS = d \left( \frac{A}{4G} \right) + \frac{1}{2} d \ln \left( \frac{A}{4G} + \frac{1}{2} \right). \quad (34)$$

So, we have

$$S = \frac{A}{4G} + \frac{1}{2} \ln \left( \frac{A}{4G} + \frac{1}{2} \right) + \text{const} = \frac{A}{4G} + \frac{1}{2} \ln \frac{A}{4G} + \frac{1}{2} \ln \left( 1 + \frac{2G}{A} \right) + \text{const}. \tag{35}$$

For large black hole with  $A \gg 2G$ , we taking Taylor expansion on Eq. (34) and get

$$S = \frac{A}{4G} + c_0 \ln \frac{A}{4G} + c_n \left( \frac{A}{4G} \right)^{-n} + \text{const}, \tag{36}$$

where,  $c_0 = \frac{1}{2}$ ,  $c_n = (-1)^{n-1} \frac{1}{n} 2^{-(n+2)}$ .

It is worth to point out that, the present black hole entropy has a series of corrected terms to B-H entropy and the result is consistent with many other research’s results on quantum corrected entropy of black holes (for a review of the correspondence see [54]). That is to say, the leading order correction to B-H entropy goes as the logarithm of black hole area and the other terms go as the inverse of the area and high order area, respectively. Therefore, we may say that, in the modified black hole from the gravity’s rainbow, the emission rate of massless particle is related to the change of the quantum corrected entropy of black holes.

Next, substituting Eq. (36) into Eq. (24) and thinking of Eq. (26), we can obtain the radiation spectrum of the modified black holes as

$$\begin{aligned} \Gamma &\sim \exp(S(M - \omega) - S(M)) \\ &= \left( \frac{(1 - \frac{\omega}{M})^2 + (\frac{M_p}{M})^2}{1 + (\frac{M_p}{M})^2} \right)^{\frac{1}{2}} \times \exp \left( -8\pi GM\omega \left( 1 - \frac{\omega}{2M} \right) \right). \end{aligned} \tag{37}$$

Compared with the usual self-gravitation correction radiation spectrum from the usual black holes derived in [8–10], namely,

$$\Gamma \sim \exp \left( -8\pi GM\omega \left( 1 - \frac{\omega}{2M} \right) \right), \tag{38}$$

we find the present radiation spectrum has a series of Planck scale modification factors and it further depart from pure thermal spectrum. But, If we do not consider the corrections from the Planck scale effects of the spacetime, i.e., neglecting the logarithmic correction term and the inverse area items in the black hole entropy, the factor of the final exponential in Eq. (37) equals to unit and the radiation spectrum has the same type of non-thermal form shown as Eq. (38). And that, if we further overlook the effect of emitted particle’s back-reaction by neglecting  $\omega/M$  in the expression Eq. (37), the present tunneling rate takes the form of Boltzmann factor  $e^{-\beta\omega}$  ( $\beta \equiv 1/T = 8\pi GM$ ) and the Hawking’s thermal formula is obtained.

## 5 Summary and discussions

In the present work, in Parikh and Wikzek's tunneling framework [8–10], Hawking radiation and black hole entropy in the modified Schwarzschild black hole from gravity's rainbow are investigated. In the tunneling process, the Planck scale effects of spacetime shown as energy dependence are taken into account. Thus, while the particles tunnel across the horizon of the modified black hole, the background metric is dynamical, due to not only energy conservation but also the Planck scale effects of geometry. We find that, incorporating Planck scale effects with the tunneling program, the tunneling probabilities of massless particles are related to the changes of the quantum corrected entropy of black holes and information conservation in the gravity's rainbow is possible.

For the modified Schwarzschild black hole from the gravity's rainbow, by analyzing its some thermodynamics quantities and using the first law of black hole thermodynamics, the black hole entropy with a series of correction terms to B-H entropy is obtained. Here, the leading order correction item is the logarithm of the black hole area and the expression of black hole entropy is consistent with the standard form of quantum corrected black hole entropy [54]. Accordingly, the Planck scale corrected emission spectrum in the modified black hole is obtained and it deviates from thermal spectrum. Meanwhile, for calculating the black hole entropy in the gravity's rainbow, a specific MDR of Eq. (25) is proposed and the obtained result of entropy formula Eq. (36) support the choice. It is remarkable that, black hole entropy is an important landmark to Planck scale physics. As a low-energy quantum gravity effect, different models of MDR with some underlying meaning in the quantum gravity should be tested with black hole entropy.

The research here not only provides further evidence to support Parikh and Wikzek's tunneling program, which gives an explicit calculation to investigate Hawking radiation, but also gives an extension for the tunneling program from classical spacetime to a Planck scale corrected black hole. And, the work should be extended to other modified black holes from gravity's rainbow. On this issue, further work is in progress.

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