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CORRECTION

Correction to: Structure of attractors for boundary maps associated to Fuchsian groups

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The paper [2] studies the dynamics of a class of circle maps and their two-dimensional natural extensions built using the generators of a given cocompact and torsion-free Fuchsian group Γ. If $\mathbb{D}$ denotes the Poincaré unit disk model endowed with the standard hyperbolic metric, then $\Gamma\backslash\mathbb{D}$ is a compact surface of constant negative curvature and of a certain genus $g > 1$. Most of the considerations and proofs in the paper were done for a special case of surface groups, those that admit a fundamental domain $\mathcal{F}$ given by a *regular* $(8g-4)$-sided polygon. On p. 172 in the Introduction, we neglected to explain in the paragraph below equation (1.2) that, although not all surface groups admit such a fundamental domain, it is possible to reduce the general case to this special situation without affecting the results of the paper (see also [1, Appendix A]).

More precisely, given $\Gamma'\backslash\mathbb{D}$ a compact surface of genus $g > 1$, there exists a Fuchsian group Γ such that:

(i) $\Gamma\backslash\mathbb{D}$ is a compact surface of the same genus g;
(ii) Γ has a fundamental domain $\mathcal{F}$ given by a regular $(8g-4)$-sided polygon;
(iii) By the Fenchel–Nielsen theorem [3] there exists an orientation preserving homeomorphism h from $\bar{\mathbb{D}}$ onto $\bar{\mathbb{D}}$ such that $\Gamma' = h \circ \Gamma \circ h^{-1}$.

The original article can be found online at https://doi.org/10.1007/s10711-017-0251-z.

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One can now extend the considerations described in the introductory section of the paper to any compact surface $\Gamma'\backslash\mathbb{D}$, using the orientation preserving homeomorphism h and the setting for $\Gamma\backslash\mathbb{D}$. Let

$$T_i' = h \circ T_i \circ h^{-1},\ P_i' = h(P_i) \text{ and } Q_i' = h(Q_i).$$

Then the set $\{T_i'\}$ satisfies relations (1.3)–(1.5) and the order of the points $\{P_i'\} \cup \{Q_i'\}$ will be the same as for the set $\{P_i\} \cup \{Q_i\}$. The geodesics $P_i'Q_{i+1}'$ will produce the $(8g-4)$-sided polygon $\mathcal{F}'$ whose sides are identified by transformations T_i'. Adler and Flatto [1, Appendix A] conclude that region $\mathcal{F}'$ satisfies all the conditions of Poincaré's theorem, hence it is the fundamental domain for Γ'.

The main object of study in our paper is the generalized Bowen-Series circle map $f_A : S \to S$ given by (1.8)

$$f_{\bar{A}}(x) = T_i(x) \quad \text{if } A_i \le x < A_{i+1}\,,$$

with the set of jump points $\bar{A} = \{A_1, A_2, \ldots, A_{8g-4}\}$ satisfying the condition that $A_i \in (P_i, Q_i)$, $1 \le i \le 8g-4$. The corresponding two-dimensional extension map given by (1.9) is

$$F_{\bar{A}}(x, y) = (T_i(x), T_i(y)) \quad \text{if } A_i \le y < A_{i+1}\,.$$

Even though the main results of the paper (Theorems 1.2 and 1.3) were proved for the special situation of a genus g compact surface $\Gamma\backslash\mathbb{D}$ that admits a regular $(8g-4)$-sided fundamental region, the results remain true in full generality for an arbitrary genus g compact surface $\Gamma'\backslash\mathbb{D}$ with the set of $(8g-4)$ generators $\{T_i'\}$, the set of jump points $\bar{A}' = \{A_1', A_2', \ldots, A_{8g-4}'\}$ with $A_i' = h(A_i) \in (P_i', Q_i')$ and the corresponding maps:

$$f_{\bar{A}'}(x) = T_i'(x) \quad \text{if } A_i' \le x < A_{i+1}'; \quad F_{\bar{A}'}(x, y) = (T_i'(x), T_i'(y)) \quad \text{if } A_i' \le y < A_{i+1}'\,.$$

The orientation preserving homeomorphism $h : \bar{\mathbb{D}} \to \bar{\mathbb{D}}$ and the relations

$$f_{\bar{A}'} = h \circ f_{\bar{A}} \quad \text{and} \quad F_{\bar{A}'} = (h \times h) \circ F_{\bar{A}}$$

allow us to conclude that:

(a) A partition point $A_i' \in (P_i', Q_i')$, $1 \le i \le 8g-4$, satisfies the cycle property, i.e., there exist positive integers m_i, k_i such that

$$f_{\bar{A}'}^{m_i}(T_i' A_i') = f_{\bar{A}'}^{k_i}(T_{i-1}' A_i')$$

if and only if the corresponding partition point $A_i = h^{-1}(A_i') \in (P_i, Q_i)$ satisfies the cycle property

$$f_{\bar{A}}^{m_i}(T_i A_i) = f_{\bar{A}}^{k_i}(T_{i-1} A_i).$$

(b) A partition point A_i' satisfies the short cycle property

$$f_{\bar{A}'}(T_i' A_i') = f_{\bar{A}'}(T_{i-1}' A_i')$$

if and only if the corresponding partition point $A_i = h^{-1}(A_i')$ satisfies the short cycle property:

$$f_{\bar{A}}(T_i A_i) = f_{\bar{A}}(T_{i-1} A_i).$$

(c) If $\Omega_{\bar{A}} = \bigcap_{n=0}^{\infty} F_{\bar{A}}^{n}(\mathbb{S} \times \mathbb{S} \setminus \Delta)$ is the global attractor of the map $F_{\bar{A}}$, then $\Omega_{\bar{A}'} = (h \times h)(\Omega_{\bar{A}})$ is the global attractor of the map $F_{\bar{A}'}$. Also, if $\Omega_{\bar{A}}$ has finite rectangular structure, then $\Omega_{\bar{A}'}$ has finite rectangular structure, since $h \times h$ preserves horizontal and vertical lines.

We would like to use this opportunity to also correct some misprints: on p. 173, last paragraph, the text "of the fundamental domain $\mathcal{F}$" should read "of $\mathbb{D}$"; on p. 193, in equation (7.2), the term "$A_i + 1$" should read "A_{i+1}"; on p. 193, Proposition 7.1, the relations "$B_i = T_i A_i$, and $C_i = T_{i-1} A_i$" should read "$B_i = T_{\sigma(i-1)} A_{\sigma(i-1)}$, and $C_i = T_{\sigma(i+1)} A_{\sigma(i+1)+1}$."

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