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ORIGINAL PAPER

Effectiveness of Stone and Deep Mixing Lime Columns on Stability of Embankments Constructed on Soft Consolidating Soil

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Abstract One of the most common, effective and innovative ways to improve soft soil is to use stone columns or lime columns. An extensive study has been conducted to find the effectiveness of lime or stone columns to improve the settlement of embankments constructed over soft consolidating soil. There are very few studies to evaluate the effects of columns on embankment safety factor during foundation soil consolidation. This study examined the effectiveness of stone and lime columns on the embankment stability, both in terms of safety factor and settlement, which are determined at different intervals of time, during the foundation soil consolidation. Further, it was also investigated the effectiveness of stone and lime column composite system. In this numerical study, two-dimensional plane strain finite element method is used. This study shows that, stone columns are more effective than lime columns in reducing post-construction settlement and to accelerate consolidation process, whereas lime columns are observed to be more effective than stone columns in improving the safety factor after the embankment construction indicating that the usage of stone or lime columns is significantly affected by the purpose for which columns are used. In addition, it is also observed that the column effectiveness on safety factor is strongly influenced by the relative shear strength of embankment and foundation soils. Further, provision of stone and lime columns as a composite system performs better than providing only stone or lime columns.

Keywords Embankment · Safety factor · Settlement · Stone columns · Lime columns · Finite element method · Soft consolidating soil

1 Introduction

The increase of urbanisation to full fill the need of a fast-growing population is created the dearth of adequate land to the construction in social facilities and infrastructure development. The lack of availability of suitable ground has inevitably led to the embankment's construction on the soft soil, that was treated as unsuitable from past decades. This soft soil generally possesses low shear strength, shows excessive settlement and large lateral movement. To overcome these difficulties, different types of ground improvement methods have been developed and adopted. Application of ground improvement techniques can enhance soft soil properties and enable undesired lands to be treated and be used for construction (Fatahi et al. 2012). Surcharging or preloading, providing vertical drains, stage-wise embankment construction to allow for consolidation, provision

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of reinforcements, provision of columns and usage of admixtures in foundation soil are being adopted nowadays based on cost, time and in-situ specific conditions of foundation soil. Out of several available ground improvement techniques, the columnar inclusion technique is most adequate to support the embankment constructed on soft soil. Column-supported embankments have been used more frequently in recent decades for construction on soft soils (Huang et al. 2020). In the past, various kinds of materials like stone, sand, lime, soil–cement (SC), lime-cement (LC), cement-fly ash-gravel (CFG) have been proposed for the columns. These columns may be extended to hard layer or may be floating. The main purpose of these columns in the soft soil is to transfer the loads to deeper stratum to reduce the excess pore water pressure, ground level settlement and lateral movement in foundation soil with an increase in the stability of embankment. According to Zhang et al. (2016), depending on their composition, the columns can be categorised as granular, flexible and rigid piles. Granular piles include stone and sand columns, flexible piles include lime and soil–cement (SC), and rigid piles include cement fly-ash gravel (CFG) piles.

Nowadays, various methods have been employed all over the world, such as grouting, jet grouting and soil deep mixing for installation of stone columns (Aboshi et al. 1979; Greenwood and Kirsch 1983) and lime columns (Okumura and Terashi 1975; Kawasaki et al. 1981, 1984; Terashi and Tanaka 1981, 1983). The construction methods for these columns are vibro-replacement method (dry process and wet process), vibro-composer method, cased borehole method or rammed columns, based on their demonstrated usefulness for in-situ ground condition, the availability of material, equipment's and skilled labours in the region. The use of granular piles has attracted considerable attention (Ghorbani et al. 2021). Granular piles such as stone columns are usually partially positioned with replacement of soil by stones to improve soft foundation soil beneath the embankment for rapid embankment construction. It is one of the most common and convenient methods with numerous advantages including increased bearing capacity and consolidation, reduced post-construction settlement and lateral movement and improved slope stability and liquefaction control along with others (Basack et al. 2017). Flexible piles such as lime columns, commonly known as Chemico-piles, are also recommended for ground improvement. The lime columns accelerate consolidation, which also serve as vertical drains. Compared to stone columns, lime columns are stiffer to some extent and likely to behave linearly elastic during the loading (Poorooshasb and Meyerhof 1997). Both the stone and lime columns are cost-effective and most commonly used ground improvement techniques. Improving the performance of poor-quality soils by means of the incorporation of columnar inclusions, such as stone or lime columns, represents an appropriate reinforcement technique for ensuring the stability of geotechnical structures, such as embankments lying over soft clay layers (Jellali et al. 2011).

Effectiveness of both stone columns (Bergado and Lam 1987; Christoulas et al. 1997; Li et al. 2000; Ghazavi and Shahmandi 2008; Borges et al. 2009; Abusharar and Han 2011; Das and Deb 2017; Nasiri and Hajiazizi 2021; Zheng et al. 2020; Nayak et al. 2020) and lime columns (Poorooshasb and Meyerhof 1997; Hossain and Rao 2006; Chong et al. 2018) have been investigated extensively using laboratory experiments, analytical methods and numerical tools, particularly to address the reduction in the settlement and for accelerating consolidation process. The columns effectiveness for improving the safety factor of the embankment constructed over soft soils has also been studied. But all these studies are limited to assess the embankments safety factor, constructed over non-consolidating soil, or either at the completion of the embankment construction and/or at the completion of consolidation in foundation soil. Evaluation of stability of embankments constructed over the soft consolidating soil in terms of both settlement and safety factor, from completion of embankment construction till completion of consolidation in foundation soil, has not been paid sufficient attention. It is observed that, when excess pore water pressure in the soft consolidating foundation soil dissipates, the safety factor of the embankment constructed over it varies with time. Settlement and safety factor are two most important parameters used to measure the stability of embankments. Hence, in this study, the usefulness of stone and lime columns to increase both settlement and safety factor at different time intervals from the construction until completion of consolidation in foundation soil is studied. The length and spacing effect of columns on the stability of embankments are also studied. Furthermore, the effectiveness of columns is also assessed for different types of embankments constructed using different types of soils. Finally, the suitability of stone and lime composite column system on embankment stability is studied.

The computation of settlement and safety factor for embankment founded on soft consolidating soil is iterative, lengthy and time-consuming and hence requires computer tools. From the past few decades, numerical analysis is gaining more importance to solve complex geotechnical problems and it is becoming a standard tool. The most popular numerical method for analyzing the problems related to the columns is the finite element method. The actual problem of embankments constructed on soil improved with multiple columns is three-dimensional. However, because of the complexities in the three-dimensional column's arrangement, generally, engineers prefer to perform a two-dimensional analysis for the assessment of the embankment behaviour and stability, due to its simplicity and ease of understanding. Besides, a two-dimensional analysis always gives a conservative estimation of the safety factor in comparison to the three-dimensional analysis (Arvin et al. 2019). Individual stone columns in three-dimensional problem are converted into equivalent column walls for plain strain analysis. The matching procedure is based on the assumption that the equivalent properties of column walls are equal to those of individual columns and surrounding soil. The equivalent properties for column walls are determined based on the area-weighted average of the properties of the stone columns and the surrounding soft soils within each row of columns. In this study, an embankment constructed over the soft soil reinforced with the column is studied in 2D plane strain condition from finite element method.

2 Method of Analysis

In present analysis, the actual three-dimensional problem comprising of soil and multiple columns is transformed into an equivalent plane strain problem of unit length, and the finite element method is used for solving the coupled consolidation problem. A method proposed by Zienkiewicz (1977) is adopted to formulate equations for the coupled system. In this procedure, change in displacement u and change in pore pressure p from the time t to $t + \Delta t$ due to the variation in force f is written as,

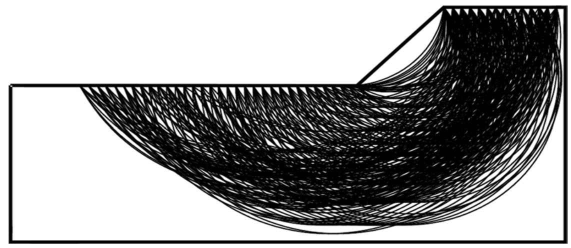


Fig. 1 Number of trial slip surfaces with different radii and centres generated for the study

$$\begin{bmatrix} k_s & L \\ 0 & H \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta p \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ L^T & 0 \end{bmatrix} \begin{Bmatrix} \dot{\Delta u} \\ \dot{\Delta p} \end{Bmatrix} = \begin{Bmatrix} \Delta f \\ 0 \end{Bmatrix} \quad (1)$$

where L, H and K_s are coupling, flow and stiffness matrices respectively. A computer programme using two-dimensional plain strain finite element method is developed, to model the embankment on soft consolidating soil with columns. In finite element modeling, both columns and soil are discretised from four noded quadrilateral elements having two translational degrees of freedom at each node. Also, four noded quadrilateral elements with one pressure degree of freedom at each node are also used to model fluid in columns and soil. In the present analysis, the lime columns are assumed as impervious material, and its permeability is assumed as similar to the permeability of the soil, whereas stone column is of pervious material. For modeling of embankment and foundation soil behaviour, the Mohr–coulomb model is used. The behavior of stone columns is also modeled using the Mohr–Coulomb model, whereas the behaviour of lime columns is modeled as linearly elastic. It may be noted that, due to the large value of cohesion, the lime column behaves elastically and the behaviour of lime columns has been modelled as linearly elastic in many of the studies reported in the literature (Poorooshasb and Meyerhof 1997; Abusharar et al. 2009; Hossain et al. 2006; Hossain and Rao 2006). At the location of columns, since the column elements are not continuous along the length of the embankment, the stiffness matrix is obtained separately for columns and for surrounding soil. The equivalent stiffness

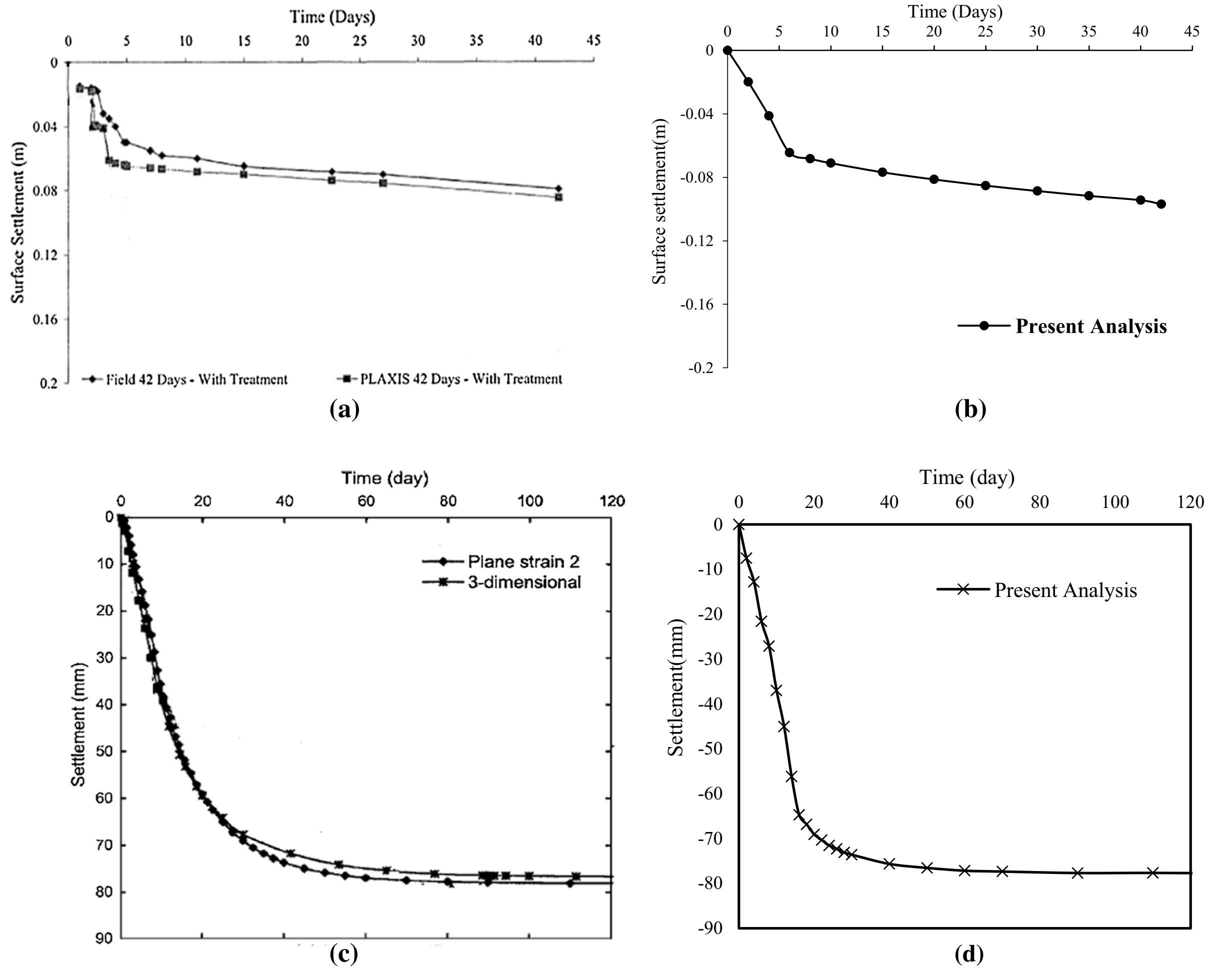


Fig. 2 **a** Settlement obtained at the ground surface below the centre of the embankment for the soil with lime columns reported by Hossain et al. (2006). **b** Settlement obtained at the ground surface below the centre of the embankment for the soil with lime columns from the proposed analysis. **c** Settlement obtained at the ground surface below the centre of the embankment for the soil with stone columns reported by Tan et al. (2008). **d** Settlement obtained at the ground surface below the centre of the embankment for the soil with stone columns from the proposed analysis

matrix for these elements is then obtained by adding the stiffness matrix of columns and stiffness matrix of soil, considering the areas contributed by columns and soil in unit length. The stiffness, flow and coupling matrices in Eq. (1) are computed and the resulting system of equations is solved from Newmark's method for obtaining the increments of displacements and pore pressure from time t to $t + \Delta t$.

2.1 Calculation of Safety Factor

A number of circular slip surfaces of varying radii and varying centers are generated at the time of consolidation of foundation soil. The surface with the lowest safety factor is taken into account for determination of safety factor. Safety factor is obtained from Eq. (2).

$$F.S. = \frac{\sum \tau_{f_i}..\Delta L_i}{\sum \tau_i.\Delta L_i} \tag{2}$$

where, τ_i, τ_{fi} and ΔL_i are shear stress of the soil, shear strength of the soil and the length for the i^{th} segment respectively. For certain segments, τ_{fi} and τ_i are obtained from Eq. (3),

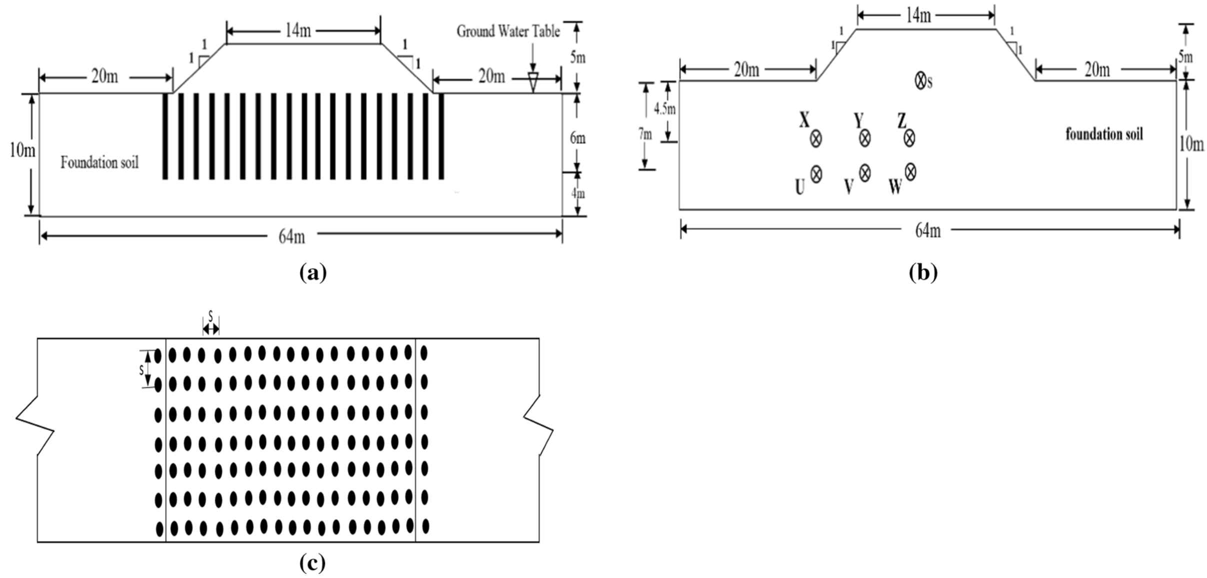


Fig. 3 **a** Embankment constructed on consolidating soil with columns considered for the study. **b** Figure showing the salient points considered for studying displacement and pore pressure. **C** Plan showing position of columns in foundation soil

$$\tau_{fi} = c' + \sigma'_{ni} \tan \phi'$$
$$\tau_i = 0.5\left(\sigma'_{yi} - \sigma'_{xi}\right) \sin 2\alpha_i + \tau'_{xyi} \cos 2\alpha_i$$
$$\sigma'_{ni} = 0.5\left(\sigma'_{yi} + \sigma'_{xi}\right) + 0.5\left(\sigma'_{yi} - \sigma'_{xi}\right) \cos 2\alpha_i - \tau'_{xyi} \sin 2\alpha_i \quad (3)$$

where, c' is effective cohesion, σ'_{ni} is effective normal stress acting on segment i and ϕ' is effective angle of internal friction, also σ'_{xi}, σ'_{yi} and τ'_{xyi} are the effective stresses on i^{th} segment. α_i is the angle of inclination with i^{th} segment with respect to horizontal.

To obtain the safety factor, at every interval of time throughout consolidation, the number of trial slip surfaces having different radius and centre are considered. The number of slip surfaces generated is shown in Fig. 1. The safety factor is computed for every trial surface generated by different radii and centres and at a particular time interval, safety factor is obtained by considering the surface with least safety factor. The equivalent shear strength parameters c' and ϕ' obtained using the column wall method proposed by Abusharar and Han (2011) are used to obtain the shear strength for column elements. The effective stresses at various points along slip surface are determined from the finite element analysis and these stresses are used to determine the safety factor during different time intervals.

2.2 Validation for Proposed Method of Analysis

The proposed method of analysis is validated by comparing the results obtained by the present analysis with the results reported in the literature. In order to validate the analysis when lime columns are used, the results are compared with the results reported by Hossian et al. (2006) and for the stone columns the results are compared with the results reported by Tan et al. (2008).

2.2.1 Validation of Proposed Analysis for the Consolidating Soil with Lime Columns

For comparison, the results reported by Hossain et al. (2006) for an embankment constructed on consolidating soil with a lime column is considered. Hossain et al. (2006) reported the settlement at various time intervals for an embankment with a height of 2 m and side slope of 1:1 constructed in four layers over consolidating soil and improved with lime columns. A plane strain finite element analysis was used to model both the soil and lime columns in the analysis. The length, diameter and c/c spacing of lime columns are 16 m, 0.4 m and 1.2 m respectively. The material properties of embankment soil, foundation soil and lime column are tabulated in Table 1. A similar problem with similar material properties is analysed using the proposed method of analysis. The columns are modelled using plane strain elements behaving as linearly elastic. Figure 2a shows settlement at ground surface below the centre of embankment when soil is improved with lime columns, as reported by Hossain et al. (2006) at different time intervals, till 42 days for field tests and obtained from two-dimensional finite element analysis. The settlement obtained at different time intervals till 42 days obtained from the present analysis is shown in Fig. 2b. From these figures it can be observed that the settlement obtained from the proposed analysis at various time intervals closely matches with the settlement reported by Hossain et al. (2006) for both field test data and two-dimensional finite element analysis.

2.2.2 Validation of Proposed Analysis for the Consolidating Soil with Stone Columns

In this section, the applicability of the proposed analysis for the embankment on consolidating soil with stone columns is evaluated. The settlement reported by Tan et al. (2008) for an embankment on consolidating soil with stone columns is considered for comparison. Tan et al. (2008) reported the settlement for an embankment of height 1.8 m, side slopes 2:1, constructed on soft clay soil improved with stone columns of 0.8 m diameter and 6 m long at 2.45 m c/c spacing. The material properties for embankment soil, foundation soil and stone columns reported by Tan et al. (2008) are shown in Table 2. In the proposed analysis, a similar problem with stone columns is also analysed. The settlements at ground surface below the center of embankment at various time intervals reported by Tan et al. (2008), from two-dimensional finite element analysis is shown in Fig. 2c. The settlement reported by Tan et al. (2008) for three-dimensional finite element analysis is also shown in Fig. 2c, and obtained from present analysis for a similar problem at different time intervals, when columns are represented using plane strain elements, considering nonlinear behavior is shown in Fig. 2d. Based on these figures, it appears that, the settlement determined from proposed analysis is quite similar to settlement of soil with stone columns as reported by Tan et al. (2008) for both the two-dimensional and three-dimensional finite element analysis.

3 Problem Considered for the Study

A typical profile of the embankment of 5 m height, crest width 14 m and 1:1 side slope considered for

Table 1 Properties of embankment soil and foundation soil (Hossain et al. 2006)

		E (kpa)	μ	γ (kN/m^3)	c' (kpa)	ϕ' (degree)	k_x (m/day)	k_y (m/day)
Embankment soil		8000	0.3	20	1	30	–	–
Foundation soil	Chemicolizer	20,000	0.3	22	200	23	0.009	0.0009
	Very soft clay	2100	0.3	16	4	23	0.005	0.005
	Soft clay	2300	0.3	17	2.5	23	0.0009	0.009
	Medium clay	2900	0.3	18	5	23	0.0007	0.0006
Lime columns		20,000						

Table 2 Properties of embankment soil and foundation soil (Tan et al. 2008)

Material	Unsaturated (kN/m^3)	Saturated (kN/m^3)	μ	E (kpa)	k_h (m/sec)	k_v (m/sec)	c' (kpa)	ϕ' (deg)
Embankment fill	18	20	0.3	15,000	1.16×10^{-5}	1.16×10^{-5}	3	33
Crust	17	18	0.3	15,000	3.47×10^{-7}	1.16×10^{-7}	3	28
Soft clay	15	15	0.3	1100	3.47×10^{-9}	1.16×10^{-9}	1	20
Stiff clay	18	20	0.3	40,000	3.47×10^{-9}	1.16×10^{-9}	3	30
Stone column	19	20	0.3	30,000	1.16×10^{-4}	1.16×10^{-4}	5	40

Table 3 Properties of embankment soil and foundation soil considerd for the study

	E (kN/m^2)	μ	γ (kN/m^3)	c' (kN/m^2)	ϕ' (degree)	k (m/day)	References
Embankment soil	20,000	0.4	18.7	29.3	36.5	–	Zhang et al. (2016)
Foundation soil	2300	0.3	17	2.5	23	9×10^{-4}	Abusharar et al. (2009)

Table 4 Properties of lime columns and stone columns considerd for the study

	E (kN/m^2)	μ	γ (kN/m^3)	c' (kN/m^2)	ϕ' (degree)	References
Lime column	20,000	0.25	20	200	0	Poorooshasb and Meyerhof (1997)
Stone column	40,000	0.3	17	0	38	Zhang et al. (2014)

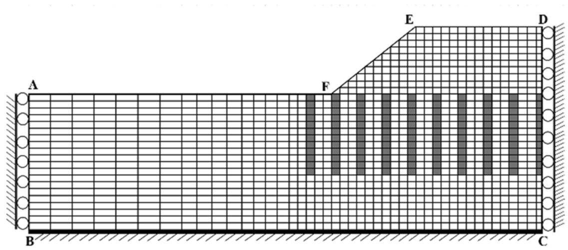


Fig. 4 Finite element discretisation of foundation soil, embankment soil and columns

studying the effectiveness of stone and lime columns is shown in Fig. 3a,b and c. The soft soil is of 10 m depth, over the hard stratum as shown in Fig. 3a. The ground water table is at the ground surface as shown in the Fig. 3a. The construction of the embankment is in five phases, each phase adding 1 m height over 2 days with another two days as a pause period after each phase of construction. The columns are of diameter 0.5 m, 6 m long at a c/c spacing of 1.5 m. The material properties for embankment soil and for foundation soil are shown in Table 3 and the columns properties are tabulated in Tables 4 and 5. The embankment and foundation soil properties are similar to the soils reported in the literature. Figure 3b also illustrates various points considered for studying the variation in excess pore water pressure and settlement. Figure 4 shows the finite element discretisation of problem. As shown in the figure, only half of the embankment is considered due to symmetry. The foundation soil and embankment soil are modeled as four noded plane strain quadrilateral elements with two translational degrees of freedom at each node, as shown in the figure. Three noded triangular elements with two translational degrees of freedom are also used to model the embankment near the side slope (Indraratna et al. 1994; Kim and Lee 1997; Chai and Miura 1999; Griffiths and lane 1999; Chai et al. 2001; Shen et al. 2005; Burman et al. 2015). The columns are also modeled as four noded plane strain quadrilateral elements. The extent of soil is set by trial and error, until increase in distance of soil from toe of the embankment, has no significant effect on the horizontal displacement and the horizontal displacements are prevented at 20 m away from the toe of the embankment along the boundary AB. Similarly, the horizontal displacements at all the nodes along the centreline of the embankment (along CD) are also prevented to represent the symmetry and along BC, displacements are prevented both horizontally and vertically to represent hard stratum. Since the embankment soil is dry and the ground water table is located at the ground surface, the excess pore water pressure is considered zero at the ground surface (along AF) and the embankment soil.

4 Results and Discussions

The effectiveness due to the provision of stone columns and lime columns to embankment stability when constructed over soft consolidating soil is studied. Since settlement and safety factor are the two major parameters generally used to measure stability of embankments, settlement and safety factor in different intervals of time, at the time of consolidation of foundation soil are considered for the study. In addition, the excess pore water pressure in foundation soil is also considered for the study, because both

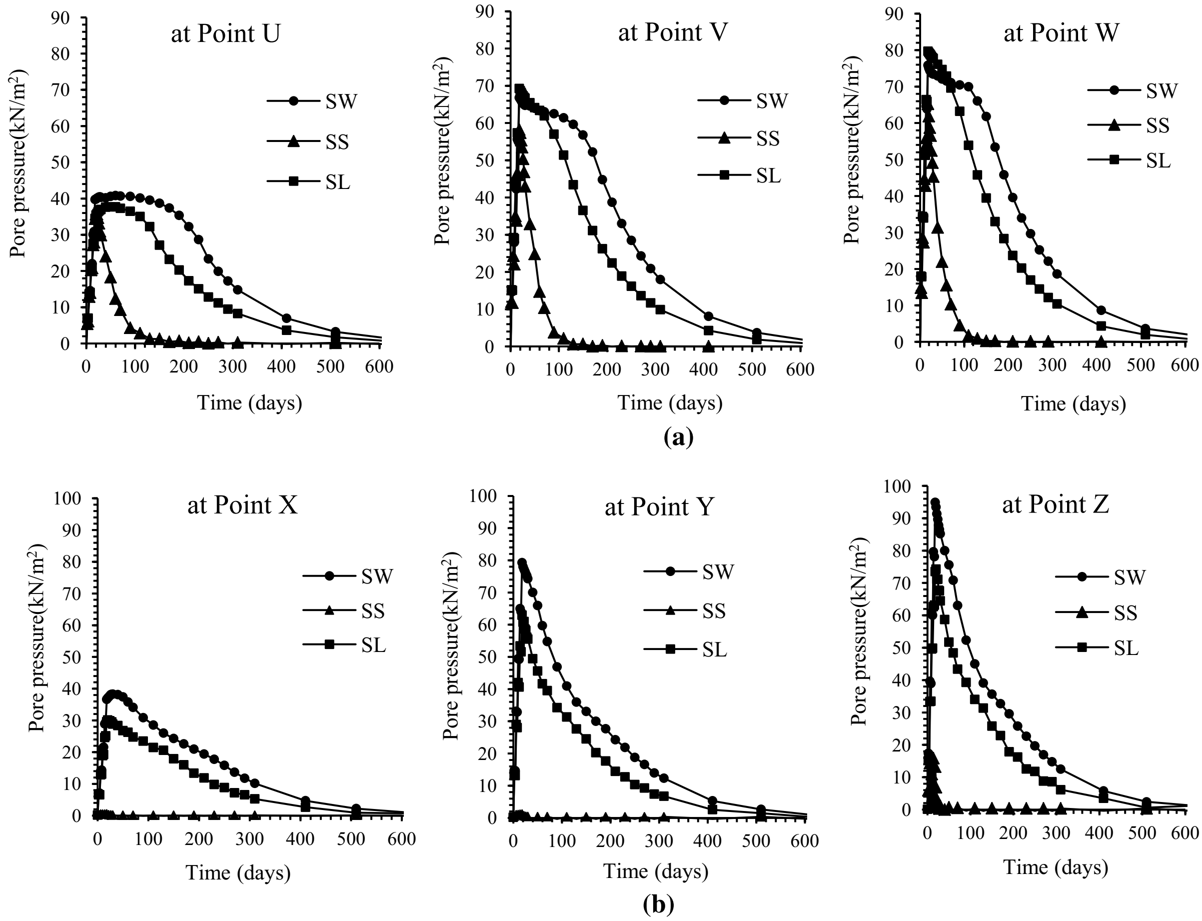


Fig. 5 **a** Variation of excess pore water pressure with time at points U, V and W for the soil with and without columns . **b** Variation of excess pore water pressure with time at points X, Y and Z for the soil with and without columns

the safety factor and settlement are influenced by rate of excess pore water pressure dissipation. The three study cases considered will be termed as,

Soil without columns (SW)
Soil with stone columns (SS)
Soil with lime columns (SL)

4.1 Effectiveness of Lime and Stone Columns on Excess Pore Water Pressure

The excess pore water pressure variation with time in foundation soil at points X, Y, Z and U, V, W are shown in Fig. 5a and b. As shown in Fig. 3b, all three points X, Y, Z are at a depth of 4.5 m from ground surface, but the point X is below the toe of the embankment, Z is below the center of the embankment and Y is in between the points X and Z. The points U, V and W are at a depth of 7 m from ground surface (just below the tip of columns) below the points X, Y and Z. In addition, the excess pore water pressure distribution in foundation soil at 200 days after the construction of embankment is also shown in Fig. 6 to study lime and stone columns effectiveness to accelerate consolidation process. In Fig. 5b, it is observed that, as expected, the excess pore water pressure at the end of embankment construction is largest at point Z, followed by point Y and then at point X for all the three cases of SW, SS and SL. Also, excess pore water pressures at points U, V, W are lesser than that of excess pore water pressures at points X, Y, Z when compared with Fig. 5a. This may be due to lesser stresses at points U, V, W than that of the points at X, Y, Z due to the construction of embankment. However, at all three points, the excess pore water pressure at the end of construction is largest for SW, closely followed by SL and is significantly lesser for the SS. In addition, it can also be observed from figures, the dissipation of excess pore water pressure is faster for SS compared to SW and SL. These observations clearly indicate that both the lime and stone columns are effective to reduce the excess pore water pressure, but, stone columns are more effective compared to lime columns, to reduce pore water pressure during construction and to accelerate the consolidation process after the construction of the embankment. Large permeability of stone columns prevents developing of excess pore water pressure generated at the time of construction and helps to accelerate consolidation after construction, whereas lesser permeability of lime columns prevents the rapid dissipation resulting in developing of pore pressure during construction and in addition, it also prevents the pore pressure to dissipate faster after construction. Although, the lesser excess pore water pressure for SL than that of SW is due to the larger stiffness of SL than that of SW. The larger stiffness of SL decreases the stress transferred to the soil and thus helps to dissipate the excess pore water pressure faster. The distribution of excess pore water pressure in foundation soil at 200 days after embankment construction shown in Fig. 6 also substantiates the better drainage performance of stone columns compared to SL and SW. As observed from this figure, the excess pore water pressure in all the points is lesser than 4 kN/m^2 for SS, whereas it is about 25 kN/m^2 and 50 kN/m^2 at some parts for SL and SW respectively.

4.2 Effectiveness of Stone and Lime Columns on Settlement of Foundation Soil

Figure 7 shows, settlement at different intervals of time from the beginning of embankment construction, till end of consolidation of foundation soil, at the ground surface beneath the center of the embankment (point S as indicated in Fig. 3b). The variation in post-construction settlement (settlement after the embankment construction in foundation soil) with time at

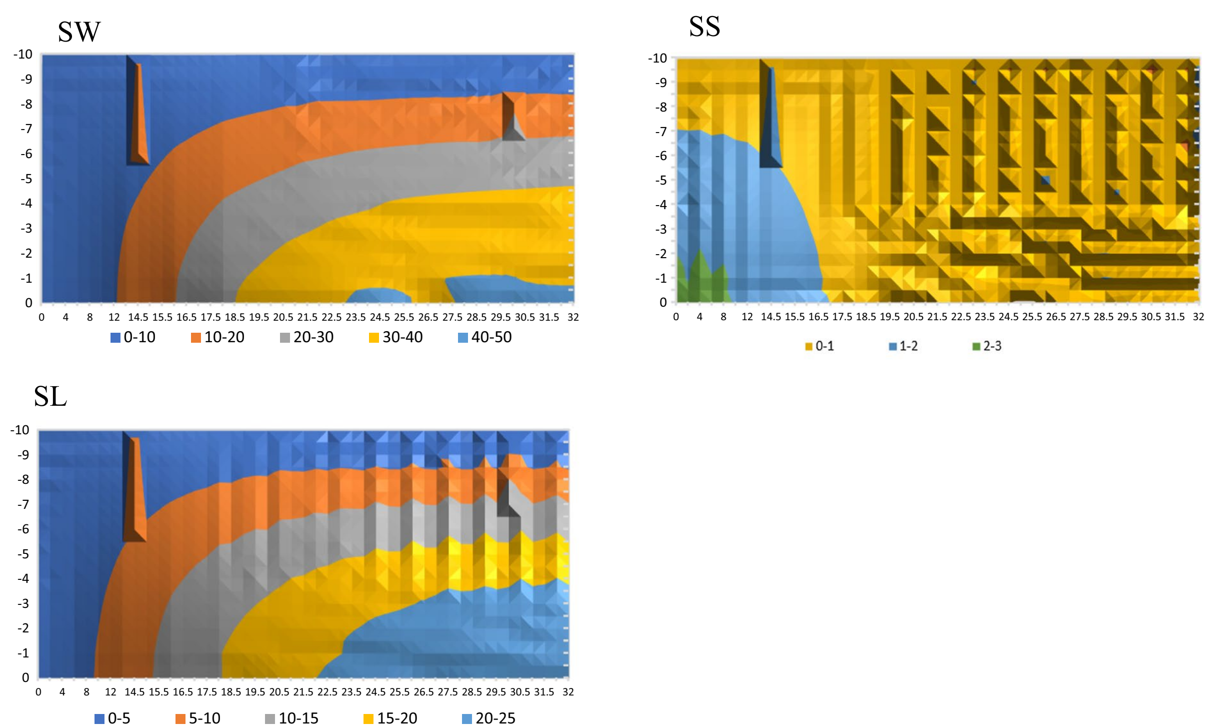
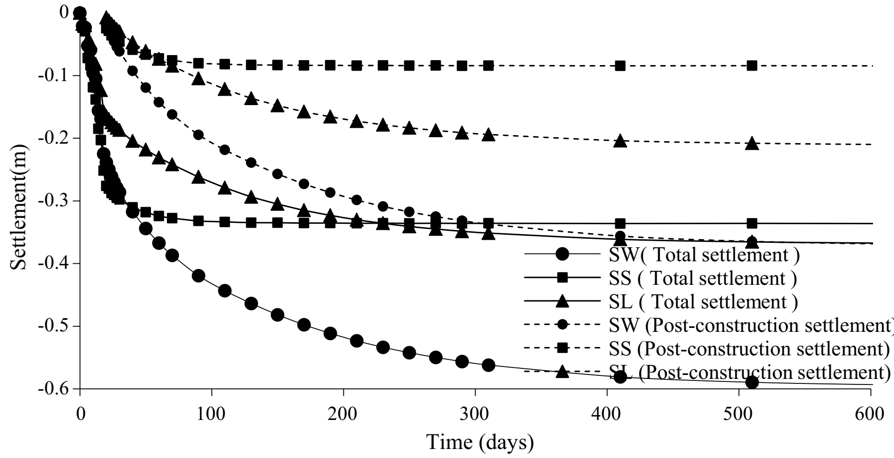


Fig. 6 Distribution of excess pore water pressure (kN/m^2) at 200 days for the soil with and without columns

Fig. 7 Variation of settlement and post construction settlement with time at point S for the soil with and without columns



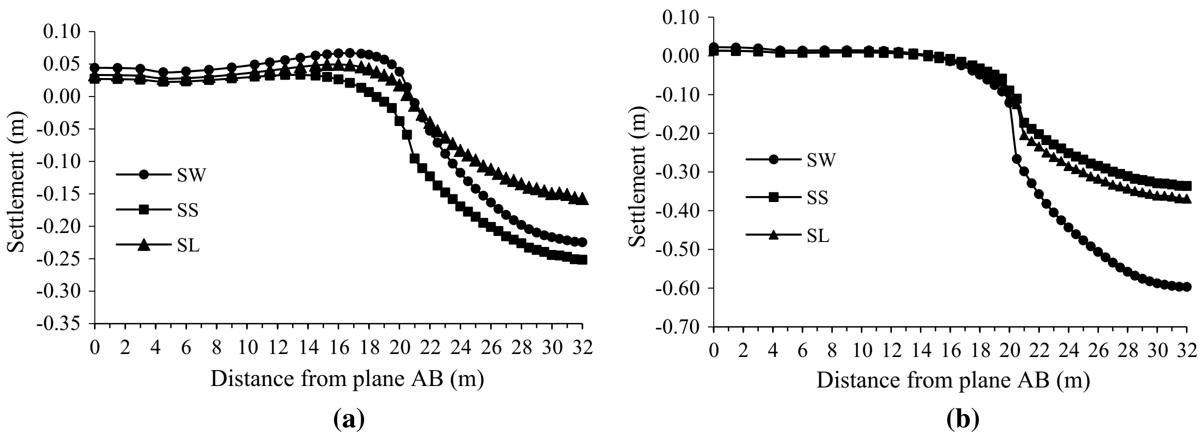


Fig. 8 **a** Settlement at the base of the embankment for the soil with and without columns at the end of construction. **b** Settlement at the base of the embankment for the soil with and without columns at the end of consolidation

point S is also plotted in Fig. 7. Figure 8a and b shows profile for ground surface settlement immediately as soon as the construction of embankment and at end of consolidation in foundation soil, respectively, and Fig. 9 shows the post-construction settlement profile at ground surface at the end of consolidation. It can be seen in Figs. 7 and 8 that, the settlement of soil beneath the base of the embankment for SS is larger than that of the SW at the end of construction and it does not vary much with time after the embankment construction, however the settlement for SL is lesser than that of the SW, at the end of construction, but it increases with time after the embankment construction. At the end of consolidation, the settlement for SS is marginally lesser than that of SL and settlements

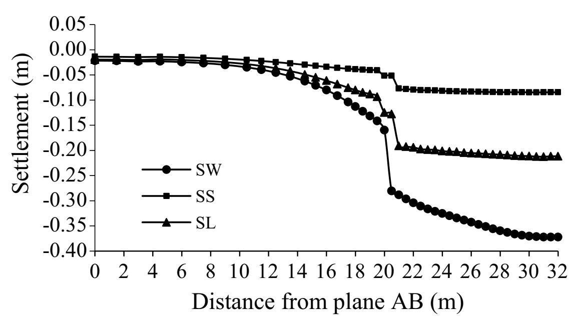


Fig. 9 Post construction settlement at the base of the embankment for the soil with and without columns at the end of consolidation

of both SS and SL are lesser than that of SW. Larger settlement of SS immediately at the end of construction and almost negligible settlement thereafter shows that most of the excess pore water pressure has dissipated during the embankment construction, when stone columns are provided. The better drainage performance of stone columns due to their large permeability helps to achieve final settlement (corresponding to settlement at the end of consolidation) immediately after construction of the embankment, whereas, SL requires a longer duration to achieve the final settlement due to its poor drainage performance. In addition, it may be noted that, although the stone columns are much stiffer than the lime columns (the modulus of elasticity of stone columns is 40,000 kN/m^2, whereas it is equal to 20,000 kN/m^2 for lime columns as shown in Table 4), the settlement of SS at the end of consolidation is only marginally lesser than that of SL. This implies that the stiffness of lime-soil composite system is more or less similar to the stiffness of the stone-soil composite system. This is mainly because the lime columns behave linearly elastic during loading and during consolidation of soil, whereas the behaviour of stone columns is non-linear. These observations indicate that both types of columns are effective to reduce the settlement, but stone columns may be more effective compared to lime columns, because final settlement at the end of consolidation can be attained faster in the case of SS and the post-construction settlement for SS is considerably lesser than that of SL as observed from Figs. 7 and 9.

4.3 Effectiveness of Lime and Stone Columns on Safety Factor

Variation of safety factor with time after embankment construction till the end of consolidation for the embankment on SW, SS and SL is shown in Fig. 10. This figure shows that, embankment's safety factor on SS is larger than that of embankment on SW at the end of construction and it does not vary much with time after embankment construction. The safety factor of embankment on SL is slightly lesser than the safety factor of embankment on SS at the end of construction, but it increases with time after the embankment construction and is substantially larger compared to the safety factor of embankment on SS at the end of consolidation. It is illustrated from Table 4, that lime columns shear strength is larger than that of stone columns due to comparatively higher value of cohesion, resulting in a larger safety factor for SL after construction of embankment. However, poor drainage performance of lime columns produces larger pore water pressure during construction and decelerates the dissipation of excess pore water pressure after construction, resulting in a longer duration to achieve the final safety factor. Although the angle of internal friction for stone columns is larger than that of lime columns, the shear strength of SS is yet comparatively lesser than that of SL. However, comparatively better drainage performance of stone columns contributes significantly to achieve the safety factor better than that of lime columns at the end of construction, since safety factor at the end of construction is 2.21 and 2.43 for the embankment with stone columns and lime columns respectively. These observations indicate that both the lime and stone columns are equally effective to improve the safety factor of embankment at the end of construction of embankment, but lime columns are observed to be more effective compared to stone columns to improve the safety factor after the construction of embankment. Stone columns also have an advantage over lime columns, because, safety factor corresponding to end of consolidation can be achieved immediately at the end of construction of embankment for SS, whereas it takes considerably longer duration for SL.

Thus, from the study of the effectiveness of stone and lime columns on the stability of embankment, it is interesting to observe that, stone columns are more effective compared to lime columns to reduce the post-construction settlement and to dissipate excess

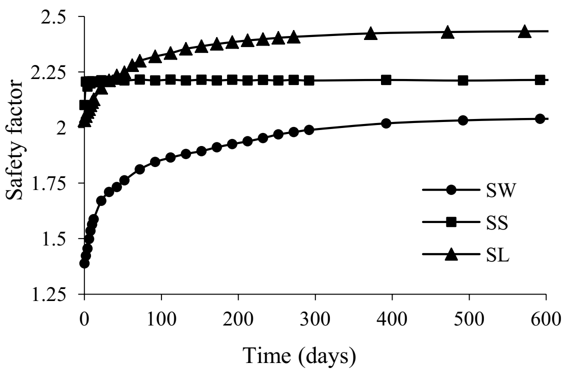


Fig. 10 Variation of safety factor with time for the embankment constructed on the soil with and without columns

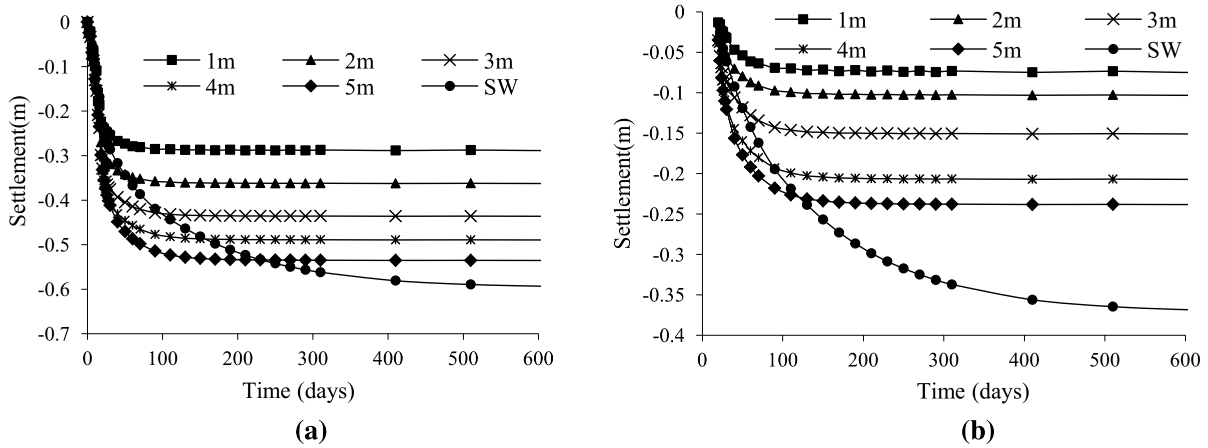


Fig. 11 **a** Variation of settlement with time at point S for foundation soil with stone columns at different spacing. **b** Variation of post construction settlement with time at point S for foundation soil with stone columns at different spacing

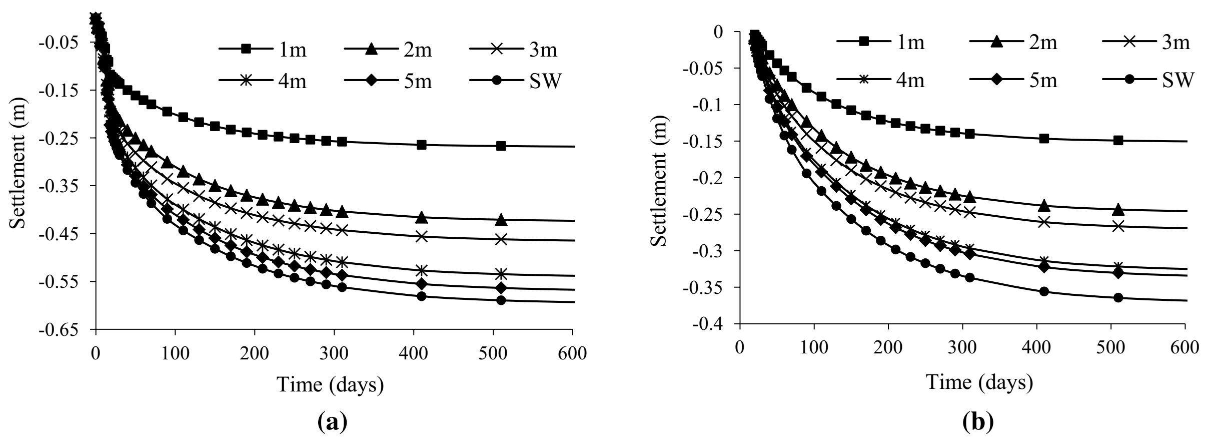


Fig. 12 **a** Variation of settlement with time at point S for foundation soil with lime columns at different spacing. **b** Variation of post construction settlement with time at point S for foundation soil with lime columns at different spacing

pore water pressure, whereas the lime columns are observed to be more effective than stone columns to improve safety factor after construction of embankment. Thus, the purpose for which the columns are required has a substantial effect on the choice of stone or lime columns.

4.4 Effect of Length and Spacing of Stone and Lime Columns on Stability of Embankment

In this section, the effect of length and spacing of stone and lime columns on settlement of foundation soil and safety factor of embankment is studied. In order to study effect of length of columns, the length of columns is varied from 1 to 10 m keeping the spacing of columns as 1.5 m, whereas the spacing of columns is varied from 1 to 5 m keeping the length of columns as 6 m, to study, effect of spacing of columns. Figures 11a and 12a shows the settlement variation with time at the point S (as indicated in Fig. 3b) and Figs. 11b and 12b shows , post construction settlement variation with time at point S (as indicated in Fig. 3b) for SS and SL respectively, at various spacing. The variation of safety factor of embankment with time for SS and SL at various spacing are shown in Figs. 13 and 14. As expected, the settlement increases and the safety factor decreases, at all the time intervals from end of construction of

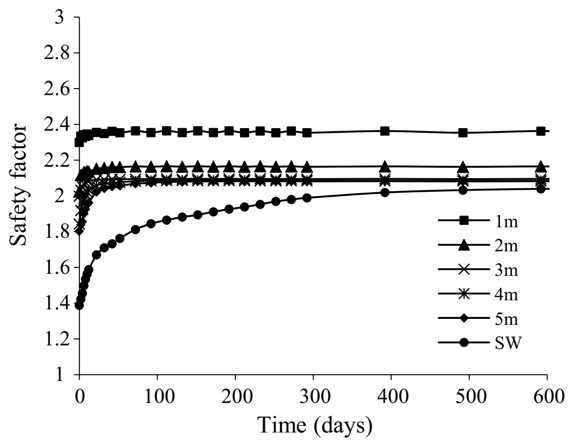


Fig. 13 Variation of safety factor with time for embankment constructed on soil with stone columns at different spacing

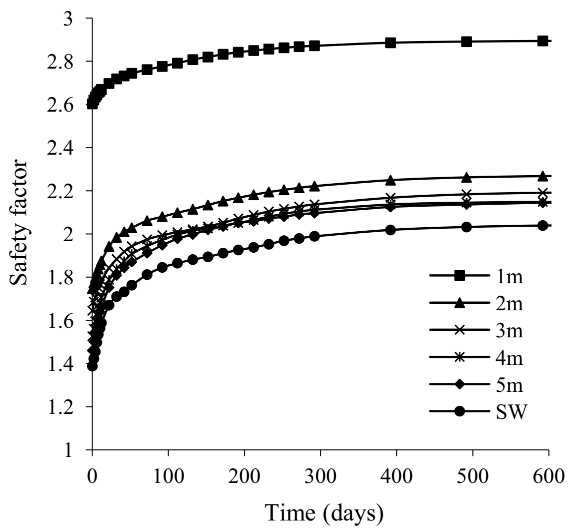


Fig. 14 Variation of safety factor with time for embankment constructed on soil with lime columns at different spacing

embankment till end of consolidation of foundation soil as spacing of columns increases. When spacing is increased from 1 to 5 m, the settlement increases from 0.28 m to 0.53 m for SS and from 0.28 m to 0.54 m for SL, and the post construction settlement increase from 0.07 m to 0.23 m for SS and from 0.149 m to 0.33 m for SL, whereas the safety factor at the end of consolidation decreases from 2.36 to 2.08 for SS and from 2.89 to 2.14 for SL. These observations indicate that the effect of spacing on settlement is almost similar for SS and SL, whereas, the effect of spacing on safety factor is larger for SL than that of SS. In addition, it can also be observed from the figure that, the effect of spacing on safety factor is larger when spacing is increased from 1 to 2 m and the effect decreases as the spacing increases for both SS and SL. Also, from Figs. 11a and 12a, it can be observed that the total settlement at 5 m spacing is more or less similar for the soil without columns. Whereas the post-construction settlement for the soil with stone columns for 5 m spacing is significantly lesser than that of the soil without stone columns. This shows that the stone columns are effective to reduce the post-construction settlement even at a spacing of 5 m. However, in the case of lime columns at spacing of 5 m, as observed from the Fig. 12a and b, both the total settlement and post construction settlement are more or less similar to the settlement of soil without columns. The other observation from these figures is that although the spacing has a considerable effect on both settlement and safety factor, the drainage performance of stone columns is not affected much due to spacing. For example, the time required to achieve final settlement and safety factor is about 20 days and 100 days when the spacing of columns is equal to 1 m and 5 m respectively, whereas it is more than 500 days for the soil without columns. i.e., at 5 m spacing of columns, although the settlement and safety factor of SS and SW are almost similar, the time required to attain these values is lesser for SS than that of SW. However, in the case of SL, the effect of spacing on drainage performance is more or less similar to that of the effect on settlement and safety factor. Larger stiffness of lime columns contributes to achieve better drainage performance of lime columns only at closer spacing, whereas the high permeability of stone columns contributes to achieve better drainage performance even at larger spacing.

The variation of settlement and post-construction settlement with time for various lengths of columns is shown in Figs. 15 and 16, and variation of safety factor with time for various lengths of columns is shown in Figs.17 and 18. Figures 15 and 16 show that both the settlement and post-construction settlement of foundation soil decreases at all time intervals as length of stone or lime columns increases and effect is more or less uniform with leng th of columns. This is mainly due to the decrease in the thickness of the soft compressible soil below the much stiffer composite soil, as the column length increases. Moreover, the effect of length of columns on the settlement of embankment is almost similar for SS and SL. The safety factor increases initially, as seen in Figs. 17 and 18, as the length of columns, increases till the column length is about 4 m to 6 m, whereas, when the length of columns is increased further, safety factor do not vary much with the increase in length of columns. These observations indicate that increasing length of the columns is effective only to achieve the improvement in the settlement, whereas, increasing the length beyond a certain limit may not be effective in increasing the embankment's safety factor. Extending the columns to hard layer is effective only in improving the settlement, and the floating columns are equally effective in increasing the safety factor.

4.5 Effectiveness of Lime and Stone Columns on Stability of Embankment Constructed Using Soils of Different Material Properties

The effectiveness of providing lime and stone columns on stability of embankment constructed using

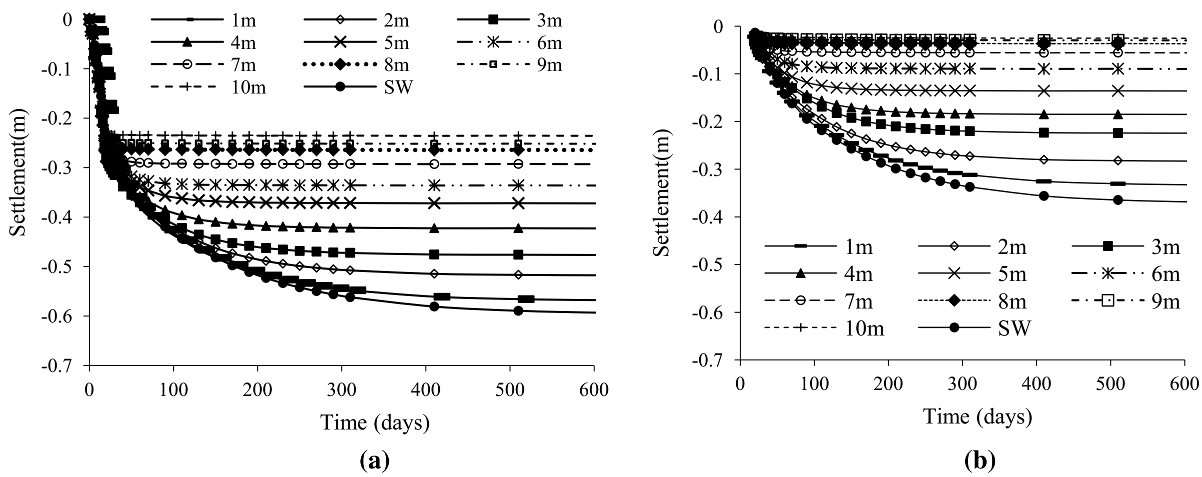


Fig. 15 **a** Variation of settlement with time at point S for the soil with stone columns of different length. **b** Variation of post construction settlement with time at point S for the soil with stone columns of different length

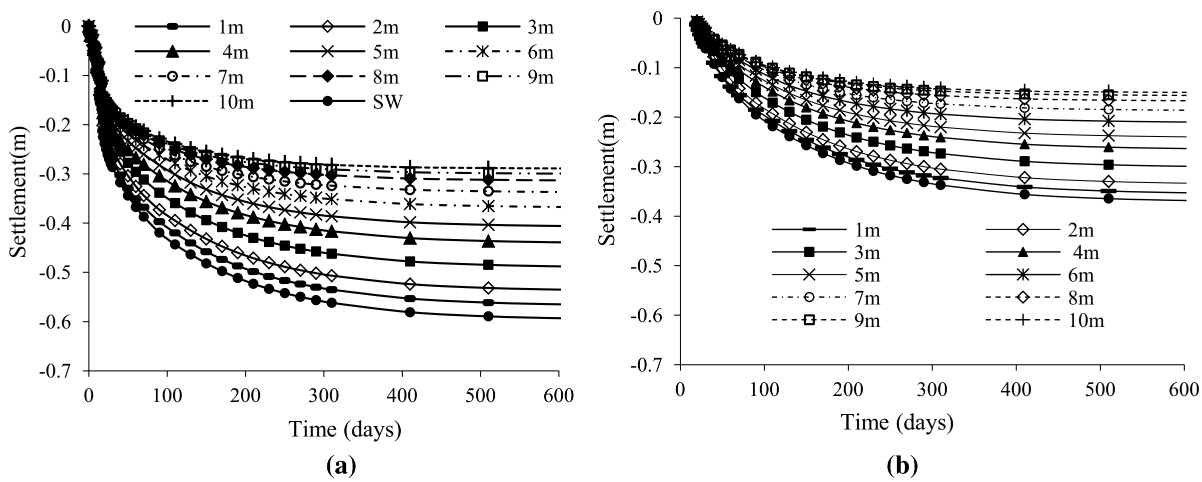


Fig.16 **a** Variation of settlement with time at point S for the soil with lime columns of different length. **b** Variation of post construction settlement with time at point S for the soil with lime columns of different length

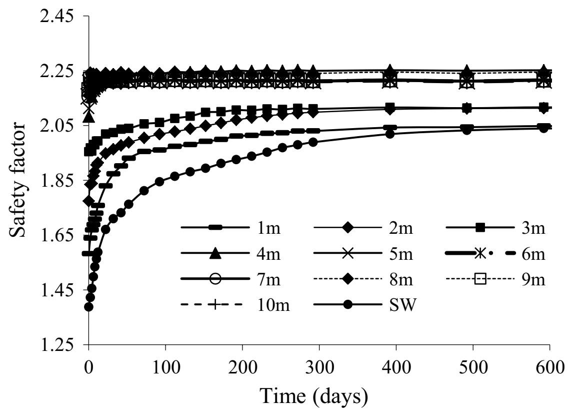


Fig. 17 Variation of safety factor with time for embankment constructed on soil with stone columns of different length

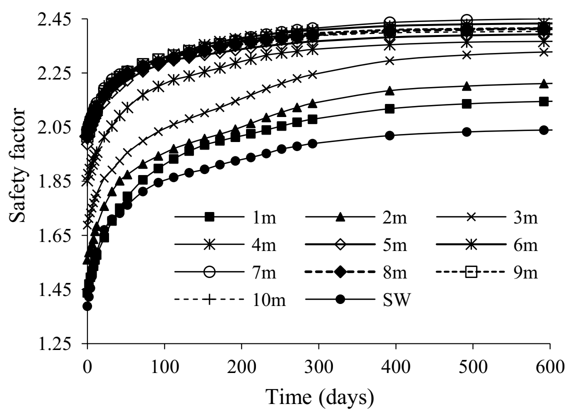


Fig. 18 Variation of safety factor with time for embankment constructed on soil with lime columns of different length

Table 5 Permiability of columns

	Lime column	Stone column
k (m/sec)	1.041×10^{-8}	3.009×10^{-6}

different types of soils is also studied. For this, embankments with similar geometry but constructed using three different types of soils are considered. The properties of three types of soil considered to construct the three types of embankments are tabulated as shown in Table 6. The material property of foundation soil and columns are similar for all three types of embankments and are already shown in Tables 4, 5 and 6. As observed from Table 6, $c^{'}$ and $\phi^{'}$ values are largest for soil type 1, followed by soil type 2 and are least for soil type 3. Also, the unit weights of embankment soil type 1 and type 2 are more or less similar, whereas for soil of embankment type 3 the unit weight is larger than that of soil type of embankment 1 and 2. The settlement variation in soil (at the point S) and safety factor of embankment with time for the three types of embankments are shown in Figs. 19 and 20 respectively. The critical slip surfaces at the end of consolidation are shown in Fig. 21. From Fig. 19, it can be observed that the settlement of foundation soil is slightly larger for embankment 3, followed by embankment 2 and then by embankment 1 for all three cases of SW, SS and SL. From Fig. 20, it can be observed that, due to provision of lime columns, the safety factor at end of consolidation improves from 2.04 to 2.43 for embankment 1, from 1.20 to 1.42 for embankment 2 and it does not improve for embankment 3, whereas, safety factor increases from 2.04 to 2.21 for embankment 1, from 1.20 to 1.25 for embankment 2 and it marginally decreases from 1.04 to 1.01 for embankment 3, when stone columns are provided. These observations indicate that the effect of providing lime or stone columns on a safety factor is largest for embankment 1, the effectiveness decreases for embankment 2 and the effect of providing lime or stone columns on a safety factor is negligible for embankment 3. These observations are important because the stone or lime columns are effective to improve the settlement irrespective of the type of soil used for the construction of the embankment and the effectiveness of providing columns on settlement do not vary much with the type of materials used for the construction of embankment, whereas, the type of soil used for construction

Table 6 Properties of embankment soils considerd for three types of embankments

	E (kN/m^2)	μ	γ (kN/m^3)	$c^{'}$ (kN/m^2)	$\phi^{'}$ (degree)	References
Embankment soil 1	20,000	0.4	18.7	29.3	36.5	Zhang et al. (2016)
Embankment soil 2	30,000	0.3	18	10	32	Zhang et al. (2014)
Embankment soil 3	8000	0.3	20	1	30	Abusharar et al. (2009)

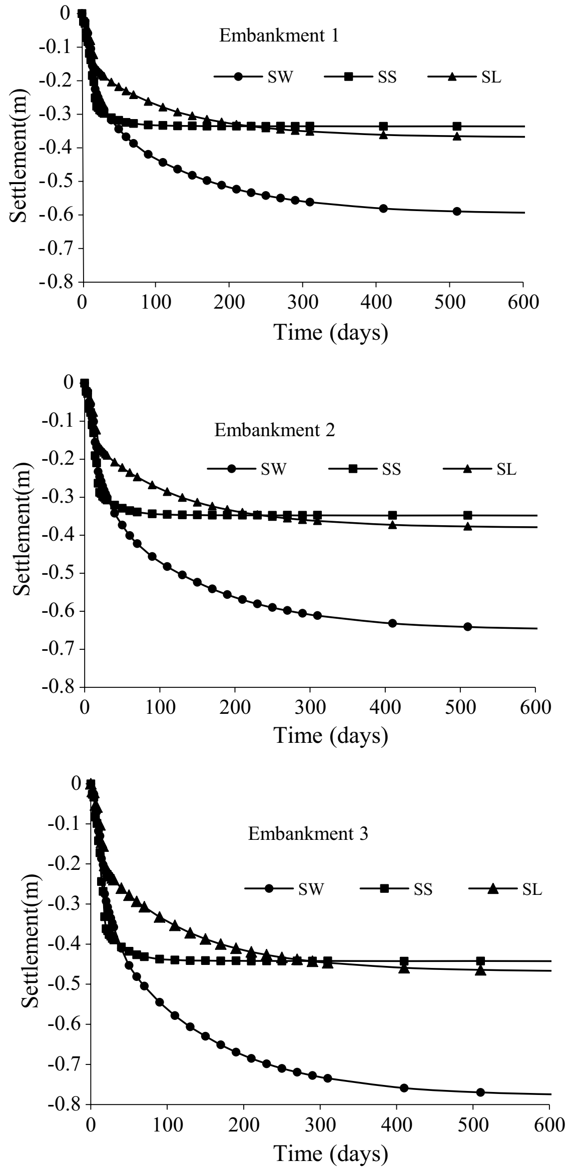


Fig. 19 Variation of settlement with time at point S for the soil with and without columns for various types of embankment

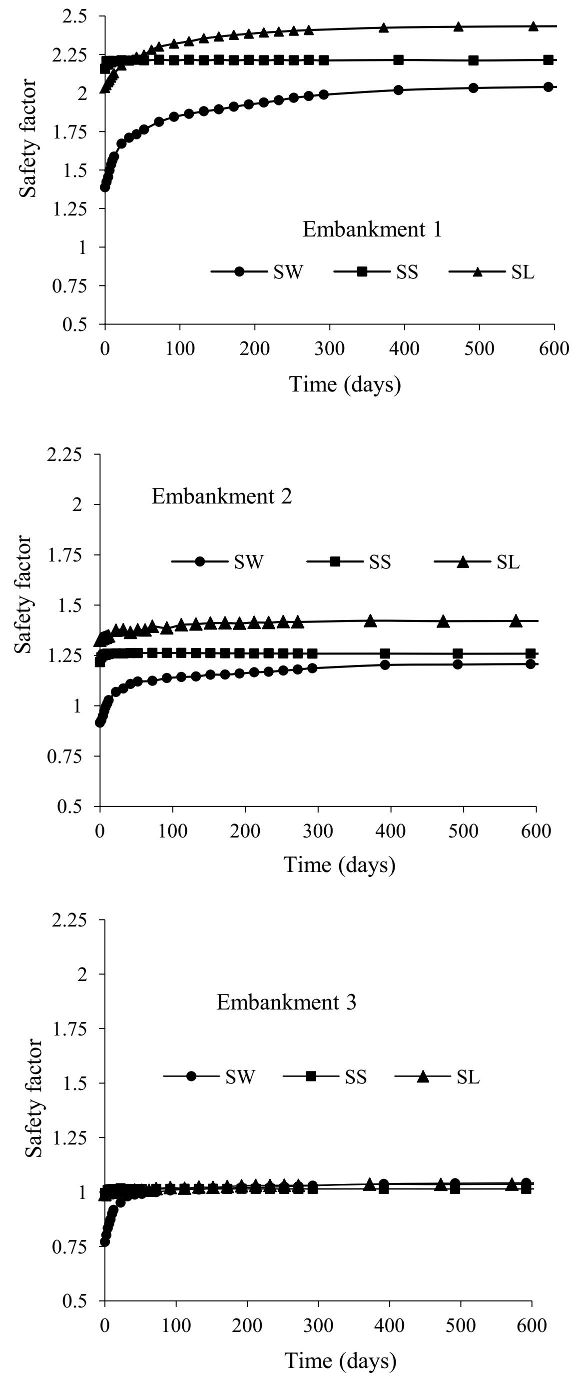


Fig. 20 Variation of safety factor with time for various types of embankment constructed on the soil with and without columns

of embankment has considerable influence on the effectiveness of columns on the safety factor. This is because for embankment 1, the shear strength of embankment soil is considerably larger than that of the foundation soil with or without columns, and hence, the major part of the critical slip surface passes through the foundation soil, as observed in Fig. 21. Provision of columns improves the shear strength of foundation soil resulting in improvement of a safety factor. In the case of embankment 2, due

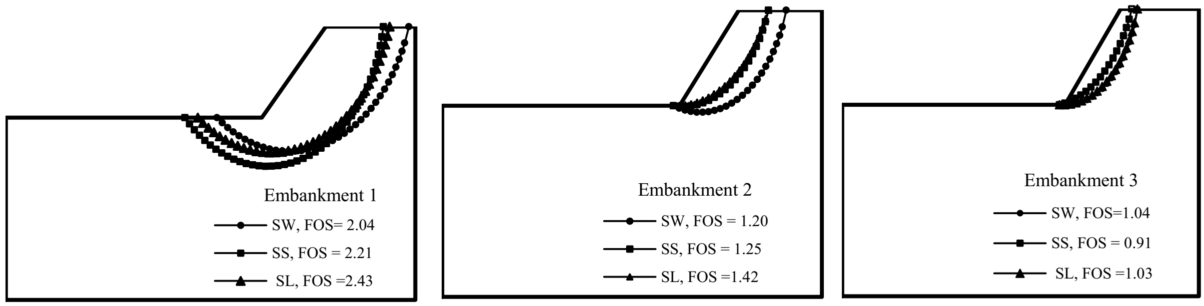


Fig. 21 Critical slip surfaces for various types of embankments constructed using different types of soil at the end of consolidation

to almost similar shear strengths of embankment and foundation soils, only a small portion of critical slip surface passes through foundation soil resulting in the lesser effect of columns on the safety factor, whereas, in the case of embankment 3, the shear strength of embankment soil is lesser than that of the foundation soil with or without columns, and hence, the critical slip surface passes only through embankment soil as shown in the figure for all the cases. Due to this, the columns provided on foundation soil do not have much effect on the safety factor for the embankment constructed using soil type 3. This may be the main reason for the columns not having any effect on the safety factor for the embankment constructed using soil type 3, although the unit weight of soil type 3 is larger than that of the other two types of soil. The marginal decrease in safety factor for embankment 3 when stone columns are provided, maybe, because of redistribution of stresses between the foundation soil and embankment soil, due to the stiffening of foundation soil when columns are provided in foundation soil.

Therefore, it may be said that the columns are effective to decrease the settlement irrespective in type of soil used for the construction of embankment, whereas effectiveness of columns on safety factor is influenced significantly by the type of soil used for construction of the embankment. The provision of columns on foundation soil may not be effective at all to improve the safety factor, depending on the types of embankments.

4.6 Effectiveness of Stone and Lime Composite System on Stability of Embankment

From above study, as already discussed, the stone columns are more effective in accelerating consolidation process, whereas lime columns are more effective to improve safety factor. Hence, in this section, effectiveness of combining lime and stone columns on the stability of embankment is studied for the problem shown in Fig. 3. The arrangement of stone and lime columns for composite system is as shown in Fig. 22a and b. In this arrangement stone and lime columns are placed alternatively at a spacing of 1.5 m as shown in the figure. It may be noted that since the cost of lime columns and stone column is more or less similar, the cost for arranging individual lime columns/ individual stone columns or combination of stone and lime columns in each alternate rows is similar for all the cases. The settlement of foundation soil at point *S* and the safety factor of embankment at various time intervals during the consolidation of foundation soil are shown in Figs. 23 and 24 respectively. From these figures, it can be observed that the settlement of composite system is almost similar to that of SS and safety factor of composite system is almost similar to that of SL. Moreover, compared to SL, the drainage performance of the composite system is improved significantly because, both the final settlement and final safety factor can be achieved much earlier with the composite system than that of SL. Thus, with the composite system the final settlement similar to that of SS and final safety factor similar to that of SL can be achieved at the time almost similar to that of SS, indicating the better performance of composite system than that of SS or SL.

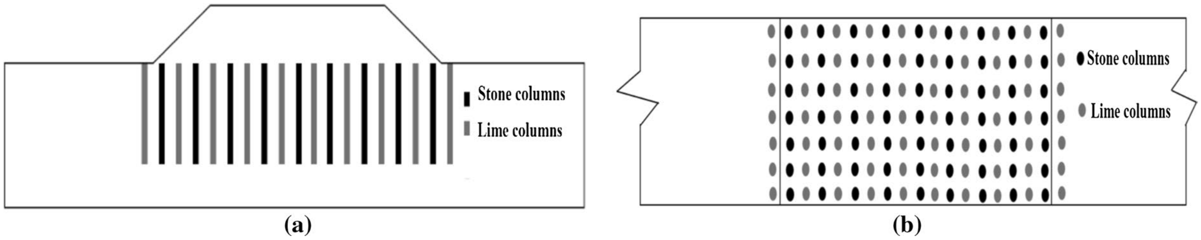


Fig. 22 **a** Elevation showing stone and lime column combinations considered for the study. **b** Plan showing stone and lime column combinations considered for the study

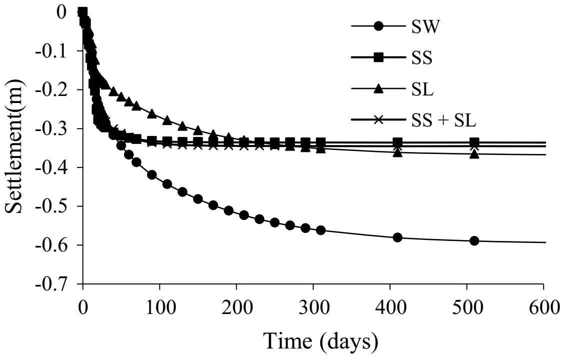


Fig. 23 Variation of settlement with time at point S for the soil with various combination of lime and stone columns

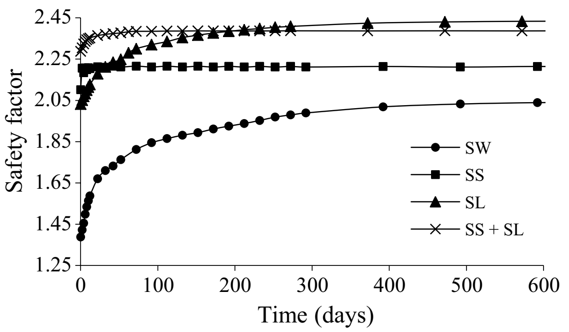


Fig. 24 Variation of safety factor with time for the embankment constructed on the soil with various combination of lime and stone columns

5 Summary and Conclusions

The effectiveness of stone and lime columns on the embankment stability when constructed on soft consolidating soil is investigated. The embankment stability is evaluated both in terms of settlement and safety factor at different time intervals during foundation soil consolidation. The effect of length of columns, spacing of columns and the properties of embankment soil on stability is also studied. In addition, the effect of stone and lime composite system on stability is investigated. In conclusion, it can be said that:

(1) The stone columns are more effective compared to lime columns to reduce the post-construction settlement and to accelerate the consolidation process, whereas the lime columns are more effective than stone columns to improve the safety factor after the construction of embankment, indicating that the use of stone or lime columns is strongly influenced by the purpose for which the columns are required.
(2) In the case of soil improved with stone columns, the settlement and safety factor are affected considerably with the spacing of columns, whereas the drainage performance is not affected much with the spacing of columns. However, in the case of lime columns, all the three parameters such as the settlement, safety factor and the drainage performance are affected due to the spacing of columns.
(3) The settlement of foundation soil decreases at all the time intervals as the length of stone or lime columns increases, whereas, the safety factor increases initially, as the length of columns increases up to a certain length of columns and when the length of columns is increased further, the safety factor does not vary much, indicating that when column length increases beyond certain length is effective only to achieve the improvement of settlement, whereas, it may not be effective to improve the safety factor of the embankment.

(4) The effectiveness of stone or lime columns on settlement is not influenced much by the type of soil used for the construction of embankment, whereas the type of embankment soil has considerable influence on the effectiveness of stone or lime columns on the safety factor. Use of stone or lime columns may not be effective at all to improve the safety factor when the shear strength of embankment soil is lesser than that of foundation soil.
(5) A composite system consisting of stone and lime columns placed alternatively performs better than providing either only stone columns or only lime columns, because, the safety factor of the composite system at the end of consolidation is similar to that of the system with only lime columns and the drainage performance of the composite system is more or less similar to that of the system provided only with stone columns.

Author Contributions Contribution in problem formulation, formal analysis, collection of data and introductory part of manuscript was written by the first author KGP, Contribution in designing computer program, write up of methodology and remaining part of manuscript was written by the second author AK. Manuscript was read by both authors and approved final manuscript.

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Data Availability Enquiries about data availability should be directed to the authors.

Declarations

Conflict of interest It is declared that the authors have no competing interests related to the manuscript.

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