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TECHNICAL NOTE

An Optimization Algorithm for Exponential Curve Model of Single Pile Bearing Capacity

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Abstract The ultimate bearing capacity of single pile is very important to engineering safety, so correctly predicting its value becomes an important part of engineering safety. Based on the traditional exponential curve model, a parameter optimization algorithm of the exponential curve model of single pile bearing capacity, which combines the golden section method and the linear least square method, is proposed. In order to verify the reliability of the proposed optimization algorithm, the measured data of the building engineering in the literature were optimized and calculated. Through comparison, it is found that the optimization algorithm is closer to the measured value than the traditional exponential curve model algorithm, which can better guide the engineering practice, and verify the effectiveness and superiority of the proposed parameter optimization method.

Keywords Ultimate bearing capacity · Exponential curve model · Golden section search · Linear least squares

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1 Introduction

The bearing capacity of piles can be determined based on conventional semi-empirical equations. In this regard, the Meyerhof equation is still well-respected. In addition, there are some simple correlations between the bearing capacity of piles and in-situ tests (like cone penetration test, CPT or standard penetration test, SPT); however, some studies suggest that the aforementioned correlations overestimate the bearing capacity. Furthermore, some other studies recommend the use of dynamic equations, which are based on the pile and hammer properties, to estimate the bearing capacity of piles; however, as stated by Milad et al. (Milad et al. 2015), there are many assumption and simplification in estimating the bearing capacity using the aforementioned equations. (Ehsan Momeni et al. 2020; HAN Jiwei et al. 2020).

Reasonable and correct prediction of vertical ultimate bearing capacity of single pile is the most important safety premise in pile foundation engineering. The existing models are widely used because of the advantages of exponential curve model. In the use of exponential curve model, the reasonable solution of its unknown parameters is the key problem for prediction. The existing methods mainly use the traditional intelligent optimization algorithm or gradient optimization algorithm to complete this task (Peng et al. 2010). In engineering, we find that we can explore more optimized algorithm, so we propose an optimization algorithm combining golden section method and linear least square algorithm. (Julong et al. 2012; Xing et al. 2016).

2 Problem Description

In order to predict the ultimate bearing capacity, the exponential curve model is used in this paper (Deling 2003; Nehdi 2015).

$$Q = Q_u\left(1 - Ae^{-Bs}\right), \tag{1}$$

In it, S represents stable settlement (mm), Q is the test load corresponding to s (kN), Unknown parameters in the model are A,B and theoretical ultimate bearing capacity Q_u (kN) at s → ∞

It is assumed that the stable settlement of n group S_i (i = 1,2,… n) and its corresponding test load Q_i (i = 1,2,…n) are obtained by test pile test, then the unknown parameter can be obtained by optimizing that nonlinear least squares cost function (Jiang et al. 2016).

$$\min J(A, B, Q_u) = \sum_{i=1}^{n}\left(Q_i - Q_u\left(1 - Ae^{-Bs_i}\right)\right)^2 \tag{2}$$

3 Exponential Curve Fitting Method

3.1 Separable Least Squares Problem

Genetic algorithm may also be used Genetic algorithms can also be used. The nonlinear least squares cost function defined by (2) can be solved by standard nonlinear least squares optimization method, such as Gauss–Newton method or Marquardt method. Genetic algorithm, differential evolution and other intelligent optimization methods can also be used to solve this problem. However (Yan et al. 2015; Zhang et al. 2015), the above methods do not take into account the characteristics of the unknown parameters in the model (1). For the convenience of analysis, order:

$$A_1 = Q_u \times A, \tag{3}$$

Then the formula (1) and (2) are converted into:

$$Q = Q_u - A_1 e^{-Bs}, \tag{4}$$

$$\min J(A, B, Q_u) = \sum_{i=1}^{n}\left(Q_i - \left(Q_u - A_1 e^{-Bs_i}\right)\right)^2 \tag{5}$$

And then it's easy to know, If B is known quantity, The nonlinear least squares problem defined by (5) will be simplified to a linear least squares problem. In addition, if let:

$$N = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix}, \ldots M = \begin{bmatrix} 1 & e^{-Bs_1} \\ 1 & e^{-Bs_2} \\ \vdots & \vdots \\ 1 & e^{-Bs_n} \end{bmatrix}, \tag{6}$$

Another two unknown parameters in the formula (4):

$$\begin{bmatrix} Q_u \\ A_1 \end{bmatrix} = \left(M^T M\right)^{-1} M^T N \tag{7}$$

Through the analysis above, we can see that the nonlinear least squares problem defined by (5) is variable separable. For this kind of nonlinear least squares problem, the nonlinear parameters and linear parameters can be divided into two groups estimated separately. The nonlinear parameter B can be estimated by nonlinear optimization techniques. In this paper, golden section method as a commonly used method in univariate optimization problems is used. Q_u and A1 are linear parameters, them can be obtained through formula (7).

3.2 Golden Section Method

The main idea of the golden section method is to find the optimal solution by narrowing the area of the optimal solution step by step. In the process of optimization, only the value of the finite cost function needs to be calculated. To calculate the objective function value required by the golden section method (Arnau et al. 2012), let B take a particular value, First, the linear parameters Q_u and A1 corresponding to B is calculated by formula (7), the objective function J corresponding to B, Q_u and A1 is calculated by the formula (5).

For one dimension optimization problem:

$$\min f(x), \quad x \in [a, b]$$

Order constant θ = 0.618, the first step in the Golden Section, let $x_1 = a + (1 - \theta) \times (b - a)$, $x_2 = a + b - x_1$, Calculate $f(x_1)$ and $f(x_2)$.

The second step in the Golden Section is to compare the size of $f(x_1)$ and $f(x_2)$. If $f(x_1) \leq f(x_2)$, the interval of the optimal solution is reduced to [a, x_2], this situation order $b = x_2$, the interval of the optimal solution can still be recorded as [a, b], at the same time, let $x_2 = x_1$, $f(x_2) = f(x_1)$, $x_1 = a + (1 - \theta) \times (b - a)$, calculate the value of $f(x_1)$; If $f(x_1) = f(x_2)$, the interval of the optimal solution is reduced to $[x_1, b]$, at this point, $a = x_1$, the interval of the optimal solution can still be recorded as $[a, b]$, at the same time, let $x_2 = x_1$, $f(x_2) = f(x_1)$, $x_1 = a + \theta \times (b - a)$, then calculate $f(x_2)$.

The second step is repeated several times until the length of the interval $[a, b]$ is less than a predetermined constant ε, so that the optimal solution $x_b = (a + b)/2$ can be obtained.

After each iteration, the golden section reduces the length of the search interval to θ times that of the previous one (Xue et al. 2015; Sun et al. 2015).

3.3 Algorithm Flow

In summary, the golden section-least square procedure of estimating the unknown parameters a\b and qu in the exponential curve model defined by the formula (1) is as follows:

(1) Sets the constant ε required to end the golden section search, order constant $\theta = 0.618$; Sets the range of values of the parameter B $[\alpha, \beta]$.

(2) Let $B_1 = a + (1 - \theta) \times (\beta - \alpha)$, the corresponding objective function J_1 is calculated by the formula (5); Let $B_2 = a + \beta - B_1$, the corresponding objective function J_2 is calculated by the formula (5).

(3) Compare the size of J_1 and J_2. If $J_1 \leq J_2$, order $\beta = B_2$, $B_2 = B_1$, $J_2 = J_1$, $B_1 = a + (1 - \theta) \times (\beta - \alpha)$, J_1 is calculated by the formula (5); Otherwise, order $\alpha = B_1$, $B_1 = B_2$, $J_1 = J_2$, $B_2 = a + \theta \times (\beta - \alpha)$, J_2 is calculated by the formula (5).

(4) *If* $\beta - \alpha \geq \varepsilon$, return to the step (3); Otherwise, let the optimal solution $B^* = (\alpha + \beta)/2$, the linear parameter and corresponding to B^* are calculated by formula(3), (4), the objective function J* corresponding to, and can be calculated by formula (5). Let $A^* = A^*\ 1/Q^*$ u, to find out the optimal solution B^*, A^*, Q^* u and the optimal objective function J*.

4 Example of Practical Calculation

To verify the performance of the above algorithm, Taking into account the static load test data of Wuhan Power Market Building provided by reference (Deling 2003), Set the constant $\varepsilon = 1 \times 10^{-4}$ required by the golden section method, Set the range of parameter $B[0, 20]$ (Fig. 1).

According to the static load test data of Wuhan power market building provided by literature (Deling 2003), the predicted value is calculated by golden section least square method according to the algorithm flow of 3.3, and compared with the prediction data in literature (Deling 2003) and Literature. The detailed results are shown in Table 1.

The model fitting accuracy in Table 1 is defined as:

$$\psi = \sqrt{1 - \sum_{i=1}^{n} (Q_i - Q_i^m)^2 \Big/ \sum_{i=1}^{n} (Q_i - \overline{Q})^2}$$

where Q_i^m corresponds to Q_i, The load value calculated by the model is KN, $\overline{Q}$ is the average value of measured load Q_i (i = 1,2,… n) (Fig. 2 and Table 2).

Fig. 1 The picture of single pile

Table 1 The optimal solutions obtained by different methods and their comparison

Calculation results	Literature (Deling 2003)	Literature	This paper
A	0.9470	0.8600	0.8607
B	0.0650	0.0552	0.0568
Q_u	7000	7151	7070
ψ	0.9855	0.9932	0.9934
J	1.34×10^6	6.24×10^5	5.96×10^5

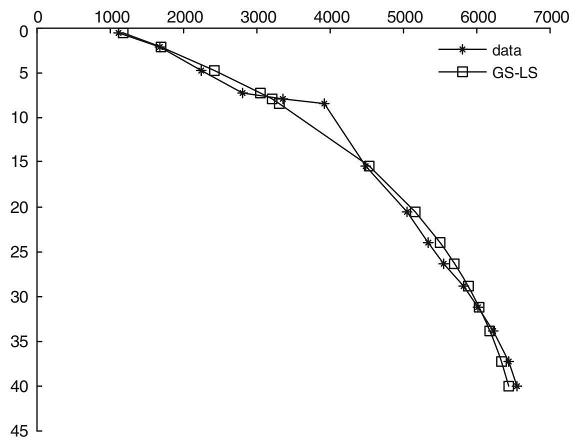

Fig. 2 A comparison figure between the golden section-the least squares solution and the measured values

5 Conclusion

According to the traditional exponential model curve fitting method, an exponential model curve fitting optimization algorithm is proposed, and its theoretical prediction of the ultimate load of a single pile is deduced. The calculation results for typical problems show that this method has the advantages of small amount of calculation and high model accuracy. From the data, the fitting precision of curve fitting optimization algorithm of exponential model is higher than that of traditional method. Through comparison, it can be seen that the exponential model curve fitting optimization algorithm is more safe and reliable to predict the limit load value, and can meet the need of high precision in prediction or design.

Table 2 Comparison table of different algorithm fitting results

Settle-Ments/mm	Load Q/kN			
	measured value	Literature (Deling 2003)	Literature	this paper
0.58	1123	616	1195	1182
2.20	1685	1254	1704	1699
4.75	2246	2132	2419	2423
7.31	2808	2878	3043	3052
8.02	3370	3064	3201	3211
8.51	3931	3187	3306	3317
15.43	4493	4569	4526	4536
20.56	5054	5258	5173	5177
23.98	5346	5605	5513	5511
26.33	5553	5803	5713	5706
28.85	5824	5984	5899	5888
31.24	6032	6130	6054	6038
33.91	6240	6269	6204	6183
37.35	6448	6415	6368	6340
40.12	6556	6511	6479	6447

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