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RESEARCH

Arrow of Time and Quantum Physics

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Abstract
Based on the hypothesis that the (non-reversible) arrow of time is intrinsic in any system, no matter how small, the consequences are discussed. Within the framework of local quantum physics it is shown how such a semi-group action of time can consistently be extended to that of the group of spacetime translations in Minkowski space. In presence of massless excitations, however, there arise ambiguities in the theoretical extensions of the time translations to the past. The corresponding loss of quantum information on states upon time is determined. Finally, it is explained how the description of operations in classical terms combined with constraints imposed by the arrow of time leads to a quantum theoretical framework. These results suggest that the arrow of time is fundamental in nature and not merely a consequence of statistical effects on which the Second Law is based.

Keywords Time as a semi-group · Loss of quantum information · Causal operations and quantum behavior

1 Introduction

The *arrow of time* is a subject of continuing discussions ever since this term was coined by Eddington [16]. In brief, this topic can be described as follows: the time parameter that enters into the fundamental equations of physics can be reversed, which in principle seems to allow physical systems to move backwards in time. On the other hand, there is overwhelming evidence that this does not happen in the real world. The standard resolution of this apparent clash between theory and reality is

This article is dedicated to Roberto Longo on the occasion of his 70th birthday.

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based on the argument that such time reversed processes are exceedingly unlikely (Second Law). Therefore, they were and will never be observed.

It is the aim of the present article to propose an alternative view on this topic. We will discuss the hypothesis that the direction of time is inherent in all systems, no matter how small. The evolution of time thus has to be described by a semigroup, there is no inverse. More concretely, consider an observer located at a particular moment at point o in Minkowski space $\mathscr{M}$. As time proceeds, such observers and all their offsprings will enter the forward lightcone $\mathscr{V}_o := o + V_+ \subset \mathscr{M}$, where V_+ is the semigroup of all positive timelike translations. Observers at o can neither re-enter the past lightcone $\mathscr{V}_{o\,-}$, where they came from, nor the spacelike complement of o; it is also not possible for them to send any instruments there. We denote this non-accessible region by $\mathscr{N}_o$.

Information obtained in the past is encoded in material bodies that accompany any observer. It is laid down in books or stored in other devices, not least in our brains. They contain information about past events, observations, experiments, the resulting data, and the theories developed on their basis. The amount of this kind of information grows steadily with time. Irrespective of their truth value, these informations can be treated as factual and described in classical terms (ordinary language). It is impossible to go backwards in time in order to verify their initial quantum features. Any setup for future experiments is described in such classical terms if they have been proven to work in the past. In experiments quantum effects can be explored, re-examined and confirmed. But with the registration of the results these become part of the past and therefore facts in the above sense.

Local quantum physics [21] allows one to make statistical predictions for future measuring results. It is even more important in the present context that, on the basis of theory, one can return to the past and develop meaningful scenarios of what has happened there and elsewhere. It turns out, however, that back-calculations in time do not allow unambiguous statements about the past in general. These facts suggest to explore in more detail the consequences of the hypothesis that time has no inverse. There appear the following questions:

- (I) Is the hypothesis of an intrinsic arrow of time compatible with the successful theoretical treatment of time as a group?
- (II) What are the uncertainties in the theoretical description of the past that arise from this hypothesis and how do they manifest themselves?
- (III) How big is the loss of information on the properties of states over time that arises from these uncertainties?
- (IV) Does the arrow of time enforce the quantum features of operations that are described by classical concepts?

In the present article we will provide answers to these questions. Our arguments are based on previous work and some measure for the information contained in states that was recently invented by Roberto Longo et al. [15]. We will be led to the conclusion that the hypothesis of a fundamental arrow of time is not only meaningful but also leads to the resolution of some theoretical puzzles.

Our article is organized as follows. In Sect. 2, we introduce our framework and recall from [13] how to identify ground states (vacua) by observables in a given lightcone $\mathscr{V}_o$, together with the semigroup V_+ of time translations. Any such state leads to a unitary representation of the semigroup. It can be extended to a representation of the group of spacetime translations on $\mathscr{M}$ that allows one to perform computations backwards in time. Section 3 contains a discussion of the ambiguities concerning the past which arise in these extensions and a proof that they are related to states of arbitrarily small mass. In Sect. 4 we use the measure of information introduced in [15] and show that the information which can be extracted from such states by observables in lightcones decreases monotonically in time. We then recall in Sect. 5 an approach to local quantum physics which proceeds from the concept of operations [11]. These operations are described in terms of classical physics and are subject to causal relations that incorporate the arrow of time. Without imposing any quantization rules from the outset, this approach gives rise to genuine quantum theories. It shows that the arrow of time entails the non-commutative structures of quantum physics. So this arrow may be regarded as a fundamental property, complementing its statistical explanations, based on the Second Law.

2 Time as a Semigroup and Its Extension to a Group

In this section we introduce our framework and explain the construction of a unitary representation of the semigroup V_+ which induces the action of time translations on the observables in a given lightcone. We then exhibit an extension of this representation to the group of all spacetime translations $\mathbb{R}^4$. It determines lightcone algebras all over Minkowski space and acts covariantly (geometrically) on them. This provides an affirmative answer to question (I).

In order to justify our input let us begin with a brief remark. At each moment, there exists a multitude of observers at points $o_1, \dots, o_n \in \mathscr{M}$ who follow their momentary arrow of time, depending on their velocity. The union of their forward light cones is contained in some bigger lightcone with apex o in their pasts. If they agree on this point, they can set their clocks to 0 there. For the present discussion it implies that it suffices to consider a single lightcone $\mathscr{V}_o := o + V_+$ and the semigroup of time translations $V_+ = \{\tau := (t, t\boldsymbol{v}) : t \geq 0,\ |\boldsymbol{v}| < 1\}$ acting on it.

Let $\mathscr{V}_o$ be given. We consider for all lightcones $\mathscr{V} \subset \mathscr{V}_o$ the subalgebras of observables $\mathfrak{A}(\mathscr{V})$ localized in $\mathscr{V}$. If $\mathscr{V}_1 \subset \mathscr{V}_2$ one has $\mathfrak{A}(\mathscr{V}_1) \subset \mathfrak{A}(\mathscr{V}_2)$. So the assignment $\mathscr{V} \mapsto \mathfrak{A}(\mathscr{V})$ defines an (isotonous) net of lightcone algebras. The local structure of this net (spacelike commutativity of oberservables) will be used later. We assume that these algebras are unital C*-algebras. The action of the time translations $\tau \in V_+$ on these algebras is induced by morphisms α_τ,

$$\alpha_\tau(\mathfrak{A}(\mathscr{V}_o)) = \mathfrak{A}(\mathscr{V}_{o+\tau}) \subset \mathfrak{A}(\mathscr{V}_o)\,, \quad \tau \in V_+\,. \tag{2.1}$$

Composing them yields $\alpha_{\tau_1}\alpha_{\tau_2} = \alpha_{\tau_1+\tau_2}$ for $\tau_1, \tau_2 \in V_+$.

Turning to the states, experience shows that ensembles with given properties can repeatedly be prepared. It suggests that there is some stationary background state which permits these operations, *e.g.* an equilibrium state or a ground state. We discuss here the latter scenario and use the following definition, where assumptions made in [13, Sect. 3] are slightly weakened.

Definition Let $\mathfrak{A}(\mathscr{V}_o)$ be given. A state ω_0 on this algebra is said to be a ground state (vacuum) for all inertial observers in $\mathscr{V}_o$ if it satisfies the following conditions.

(a) $\omega_0 \, \alpha_\tau = \omega_0$ for $\tau \in V_+$.
(b) $\tau \mapsto \omega_0(A^* \alpha_\tau(B))$ is continuous for $A, B \in \mathfrak{A}(\mathscr{V}_o)$.
(c) The functions $\tau \mapsto \omega_0(A^* \alpha_\tau(B))$ extend continuously to the complex domain $V_+ + i\, V_+$ and are analytic in its interior. Their modulus is bounded on this domain by $\sqrt{\omega_0(A^*A)\omega_0(B^*B)}$ for $A, B \in \mathfrak{A}(\mathscr{V}_o)$.

Remark These properties can in principle be tested by observers in $\mathscr{V}_o$.

Vacuum states on $\mathfrak{A}(\mathscr{V}_o)$ determine a continuous unitary representation of the semigroup V_+ in the corresponding GNS-representation. It extends to a representation of the spacetime translations $\mathbb{R}^4$ on the net of all lightcone algebras. We add here to results given in [13, Sect. 3].

Proposition 2.1 *Let $\mathfrak{A}(\mathscr{V}_o)$ be given, let ω_0 be a vacuum state on this algebra, and let $(\pi_0, \Omega_0, \mathscr{H}_0)$ be the corresponding GNS-representation.*

(i) *The vector Ω_0 is cyclic for each algebra $\pi_0(\mathfrak{A}_0(\mathscr{V}_{o+\tau}))$, $\tau \in V_+$.*
(ii) *There exists a continuous unitary representation $\tau \mapsto U_0(\tau)$ on $\mathscr{H}_0$ that implements the action of the semigroup V_+ on $\mathfrak{A}(\mathscr{V}_o)$,*

$$\mathrm{ad}\, U_0(\tau)(\pi_0(A)) = \pi_0(\alpha_\tau(A)) \,, \quad A \in \mathfrak{A}(\mathscr{V}_o) \,. \tag{2.2}$$

It leaves the representing vector invariant, $U_0(\tau)\Omega_0 = \Omega_0$, $\tau \in V_+$. This unitary representation is unique.
(iii) *The representation U_0 of V_+ can be extended to a continuous unitary representation U of the group $\mathbb{R}^4$ of spacetime translations on $\mathscr{M}$. Its adjoint action on the given algebra defines a net of lightcone algebras on $\mathscr{M}$ on which it acts covariantly (geometrically). Moreover, U satisfies the relativistic spectrum condition and leaves the vector Ω_0 invariant.*

Proof (i) According to property (c) of vacuum states, the vector-valued functions $\tau \mapsto \pi_0(\alpha_\tau(A))\Omega_0$ extend continuously to functions on the complex domain $V_+ + i\, V_+$ that are analytic in its interior, $A \in \mathfrak{A}(\mathscr{V}_0)$. Now if $\Psi \in \mathscr{H}_0$ is a vector in the orthogonal complement of $\pi_0(\alpha_\sigma(\mathfrak{A}(\mathscr{V}_o))\Omega_0$ for some $\sigma \in V_+$, it follows from the isotony of the net of lightcone algebras and the covariant action of the time translations that $(\Psi, \pi_0(\alpha_\tau(A))\Omega_0) = 0$ for all $\tau \in \sigma + V_+$. The edge-of-the-wedge theorem then implies $(\Psi, \pi_0(\alpha_\tau(A))\Omega_0) = 0$ for all $\tau \in V_+$ and hence $(\Psi, \pi_0(A)\Omega_0) = 0$,

$A \in \mathfrak{A}(\mathscr{V}_o)$. Whence $\Psi = 0$ since the GNS vector Ω_0 is cyclic for $\pi_0(\mathfrak{A}(\mathscr{V}_0))$, proving the first statement.

(ii) Making use of property (a) of vacuum states, one can consistently define isometries $U_0(\tau)$, $\tau \in V_+$, on $\mathscr{H}_0$, putting

$$U_0(\tau)\, \pi_0(A)\Omega_0 \, := \pi_0(\alpha_\tau(A))\Omega_0\,, \quad A \in \mathfrak{A}(\mathscr{V}_o)\,. \tag{2.3}$$

By the preceding step, these isometries have a dense range in $\mathscr{H}_0$ and hence are unitary. So they induce the endomorphic action of V_+ on the lightcone algebras and leave the vector Ω_0 invariant. This fixes them uniquely. Moreover, they are weakly continuous according to property (b) of vacuum states and satisfy

$$U_0(\tau_1)U_0(\tau_2) = U_0(\tau_1 + \tau_2) = U_0(\tau_2)U_0(\tau_1)\,, \quad \tau_1, \tau_2 \in V_+\,. \tag{2.4}$$

(iii) For the proof that U_0 can be extended to all spacetime translations, we make use of the fact that any $x \in \mathbb{R}^4$ can be presented as difference of elements of V_+. So let $x = \tau_1 - \tau_2 = \tau_3 - \tau_4$, hence $\tau_1 + \tau_4 = \tau_2 + \tau_3$. Making use of Eq. (2.4) it follows that $U_0(\tau_1)U_0(\tau_2)^{-1} = U_0(\tau_3)U_0(\tau_4)^{-1}$. So the operators

$$U(x) \, := U_0(\tau_1)U_0(\tau_2)^{-1}\,, \quad x = \tau_1 - \tau_2 \in \mathbb{R}^4\,, \tag{2.5}$$

are well defined, unitary, and they extend U_0. By a similar computation one finds that $U(x)U(y) = U(x + y)$ for $x, y \in \mathbb{R}^4$. We put now

$$\mathfrak{A}_0(\mathscr{V}_{o+x}) \, := \mathrm{ad}\, U(x)(\pi_0(\mathfrak{A}(\mathscr{V}_o)))\,, \quad x \in \mathbb{R}^4\,, \tag{2.6}$$

hence $\mathfrak{A}_0(\mathscr{V}_{o+\tau}) = \pi_0(\mathfrak{A}(\mathscr{V}_{o+\tau}))$, $\tau \in V_+$. The covariant action of U on these algebras follows from its definition. Since the translations act transitively on the lightcones in Minkowski space, one obtains all corresponding algebras. Next, let $y - x \in V_+$. There is some $\tau \in V_+$ such that $(x + \tau), (y + \tau) \in V_+$, hence

$$\begin{aligned} \mathfrak{A}_0(\mathscr{V}_{o+y}) &= \mathrm{ad}\, U(\tau)^{-1}\mathrm{ad}\, U(y+\tau)(\mathfrak{A}_0(\mathscr{V}_o)) = \mathrm{ad}\, U(\tau)^{-1}\pi_0(\mathfrak{A}(\mathscr{V}_{o+y+\tau})) \\ &\subset \mathrm{ad}\, U(\tau)^{-1}\pi_0(\mathfrak{A}(\mathscr{V}_{o+x+\tau})) = \mathrm{ad}\, U(\tau)^{-1}\mathrm{ad}\, U(x+\tau)(\mathfrak{A}_0(\mathscr{V}_o)) = \mathfrak{A}_0(\mathscr{V}_{o+x})\,. \end{aligned} \tag{2.7}$$

So the resulting family of lightcone algebras is isotonous, i.e. it constitutes a net.

To verify the continuity and spectral properties of U, let $A, B \in \mathfrak{A}(\mathscr{V}_o)$ and let $x + \tau \in V_+$ for suitable $\tau \in V_+$. Property (a) of vacuum states implies that

$$\langle \pi_0(A)\Omega_0, U(x)\pi_0(B)\Omega_0 \rangle = \langle \pi_0(\alpha_\tau(A))\Omega_0, \pi_0(\alpha_{\tau+x}(B))\Omega_0 \rangle\,. \tag{2.8}$$

Thus, by property (c), the function $x \mapsto \langle \pi_0(A)\Omega_0, U(x)\pi_0(B)\Omega_0 \rangle$ is continuous and can analytically be continued into the domain $\mathbb{R}^4 + i\, V_+$. Moreover, its modulus is bounded there by $\|\pi_0(A)\Omega_0\| \|\pi_0(B)\Omega_0\|$. It then follows from standard arguments in the theory of Laplace transforms that the spectrum of U is contained in the closed forward light cone $\overline{V}_+$. The invariance of Ω_0 under the action of U follows from Eqs. (2.5) and (2.3), completing the proof. □

This result shows that the hypothesis of a fundamental arrow of time leads, under meaningful assumptions, to the theoretical description of arbitrary spacetime translations, forming a group and acting covariantly on observables all over Minkowski space. So that hypothesis is compatible with the common assumptions made in theoretical physics. However, as we shall see in the next section, this theoretical extension of the semigroup of time to a group is in general not unique. Statements about the past then involve unavoidable uncertainties.

3 The Uncertain Past

We exhibit now possible ambiguities arising in the extension of the semigroup of time translations to a group, constructed in the preceding section, and relate them to specific properties of the energy-momentum spectrum. The results provide answers to question (II), raised above. In this analysis we make use of the net of local subalgebras contained in the given lightcone algebra, $\mathscr{O} \mapsto \mathfrak{A}(\mathscr{O}) \subset \mathfrak{A}(\mathscr{V}_o)$, whose elements are assumed to commute at spacelike distances. In representations induced by a vacuum state, this net extends to a local net $\mathscr{O} \mapsto \mathfrak{A}_0(\mathscr{O})$ on Minkowski space $\mathscr{M}$, obtained by the adjoint action of the unitary group U, cf. Eq. (2.7). Let $\mathfrak{R}$ be the von Neumann algebra on $\mathscr{H}_0$, which is generated by this net. By a result of Araki on vacuum representations, cf. [24, Sect. 2.4], its commutant coincides with the center of the algebra, i.e. $\mathfrak{R}' \subset \mathfrak{R}$, and it is pointwise invariant under the adjoint action of U. Thus, by central decomposition, we may assume that the multiples of Ω_0 are the only U-invariant vectors in $\mathscr{H}_0$ and that $\mathfrak{R}$ coincides with the algebra $\mathscr{B}(\mathscr{H}_0)$ of all bounded operators on $\mathscr{H}_0$. Moreover, the spectrum of U is a Lorentz invariant subset of the lightcone $\overline{V}_+$ in momentum space, cf. [4].

Whereas the theoretical extension of the algebra $\mathfrak{A}(\mathscr{V}_o)$ to all of Minkowski space, obtained in this manner, comprises maximal information, this extension is in general not unique. Whenever this happens, the spectrum of U fills the whole lightcone, i.e. there exist excitations of arbitrarily small mass. In the proof of this assertion we make use of the following lemma, where the weak closure of the lightcone algebra is denoted by $\mathfrak{R}(\mathscr{V}_o) := \pi_0(\mathfrak{A}(\mathscr{V}_o))^-$.

Lemma 3.1 *Let $Z \in \mathfrak{R}(\mathscr{V}_o)$ be the largest projection which annihilates the vacuum, $Z\,\Omega_0 = 0$. Then* $\mathrm{ad}\,U(x)(Z) = Z$ *for all* $x \in \mathbb{R}^4$.

Proof Let $x \in \mathbb{R}^4$. Because of the covariant action of U on the lightcone algebras, the operator $Z_x := \mathrm{ad}\,U(x)(Z)$ is the largest projection in $\mathfrak{R}(\mathscr{V}_{o+x})$ which annihilates Ω_0. If $(y - x) \in V_+$ it follows that $\mathfrak{R}(\mathscr{V}_{o+y}) \subset \mathfrak{R}(\mathscr{V}_{o+x})$ and consequently $Z_y \leq Z_x$.

We pick now a one-parameter family of time translations $t \mapsto \tau(t) = (t, t\boldsymbol{v})$ for fixed $\boldsymbol{v}$. By the preceding argument, the projections $Z_{\tau(t)} = \mathrm{ad}\,U(\tau(t))(Z)$, $t \in \mathbb{R}$, commute and generate, together with 1, an abelian von Neumann algebra. Because of the spectral properties of U, this implies by a theorem of Borchers that $Z_{\tau(t)} = Z$, $t \in \mathbb{R}$, for any admissible choice of $\boldsymbol{v}$, cf. [24, Theorem 2.4.3]. The statement then follows. □

The projection Z in the preceding proposition encodes information about the degree of ambiguity involved in the extension of time translations to the past. Depending on the underlying theory, there occur the following possibilities.

(a) $Z = (1 - P_{\Omega_0})$, where P_{Ω_0} is the one-dimensional projection onto the vacuum vector. Since $1 - Z$ is an element of $\mathfrak{R}(\mathscr{V}_0)$, this algebra coincides with $\mathscr{B}(\mathscr{H}_0)$, the unitaries $U \in \mathscr{B}(\mathscr{H}_0)$ are uniquely fixed, and all lightcone algebras coincide. This possibility occurs in theories where the spectrum of U has a gap between the point 0, corresponding to the vacuum, and the rest of the spectrum [23].

(b) $0 < Z < (1 - P_{\Omega_0})$. Then $\mathfrak{R}(\mathscr{V}_o)$ is a proper subalgebra of $\mathscr{B}(\mathscr{H}_0)$ and thus has a non-trivial commutant $\mathfrak{R}(\mathscr{V}_o)'$. Disregarding multiples of 1, the operators in $\mathfrak{R}(\mathscr{V}_o)'$ do not commute with the translations U; otherwise, Ω_0 would not be unique. Now let $W \in \mathfrak{R}(\mathscr{V}_o)'$ be unitary. Then $x \mapsto U_W(x) := \mathrm{ad}\, W(U(x))$ is another unitary representation of the translations $\mathbb{R}^4$ whose adjoint action on $\mathfrak{R}(\mathscr{V}_o)$ for time translations $\tau \in V_+$ coincides with those of the unique initial U_0. More generally, one can modify U by cocycles with values in $\mathfrak{R}(\mathscr{V}_o)'$. It shows that the past and spacelike complement $\mathscr{N}_o$ of the spacetime point o can be described in different ways without modifying any observations in the future $\mathscr{V}_o$.

(c) $Z = 0$. Then Ω_0 is cyclic and separating for $\mathfrak{R}(\mathscr{V}_o)$. By modular theory, its commutant $\mathfrak{R}(\mathscr{V}_o)'$ is anti-isomorphic to $\mathfrak{R}(\mathscr{V}_o)$. As in (b), U is not fixed, which has the same consequences. This special case occurs in asymptotically complete theories describing exclusively massless particles, cf. [10, Proposition 4.2].

Thus, apart from case (a), one is faced with theoretical uncertainties involved in back-calculations to the past. As was mentioned, such uncertainties appear in theories of massless particles. We show next that in cases (b) and (c) the spectrum of U never has a gap between the vacuum and the excited states. In the proof we rely on a fundamental result by Borchers on modular inclusions [5, 20].

Proposition 3.2 *If the extension U of the unitary time translations U_0 on $\mathfrak{A}_0(\mathscr{V}_o)$ is not unique, its spectrum consists of the closed cone $\overline{V}_+$. In particular, it contains contributions with arbitrarily small mass.*

Proof As was shown above, the extension U is not unique iff $(1 - Z) > P_{\Omega_0}$. By the definition of Z, any positive operator $A \in \mathfrak{R}(\mathscr{V}_o)$ that annihilates Ω_0 is dominated by a multiple of Z, i.e. $0 \leq A \leq \|A\|\, Z$. Hence Ω_0 is separating for the algebra $\mathfrak{S}(\mathscr{V}_o) := (1 - Z)\, \mathfrak{R}(\mathscr{V}_0)\, (1 - Z) \subset \mathfrak{R}(\mathscr{V}_o)$. It is also clear that $\mathfrak{S}(\mathscr{V}_o)\, \Omega_0$ is dense in $(1 - Z)\mathscr{H}_0$. Let Δ_0 be the modular operator on $(1 - Z)\mathscr{H}_0$ fixed by $(\mathfrak{S}(\mathscr{V}_o), \Omega_0)$. Since U commutes with Z, it leaves $(1 - Z)\mathscr{H}_0$ invariant and $\mathrm{ad}\, U(\tau)(\mathfrak{S}(\mathscr{V}_o)) \subset \mathfrak{S}(\mathscr{V}_o)$ for $\tau \in V_+$. So in view of the spectral properties of U one obtains the equality, cf. [5, 20],

$$\mathrm{ad}\, \Delta_0^{i\sigma}(U(x)) = U(e^{-2\pi\sigma} x), \quad \sigma \in \mathbb{R}\,, \ x \in \mathbb{R}^4\,. \tag{3.1}$$

Hence the spectrum of U on $(1 - Z)\mathscr{H}_0$ is invariant under scale transformations. Since $(1 - Z)\mathscr{H}_0$ contains, apart from multiples of Ω_0, only vectors which are not invariant under the action of U, the spectrum of U includes a ray. So the statement follows from the fact that the spectrum is Lorentz invariant. □

4 Loss of Quantum Information

As we have seen, the theoretical uncertainties about the past manifest themselves in specific spectral properties of the unitaries U on the subspace $(1-Z)\mathscr{H}_0$. We will now answer question (III) above and determine the corresponding losses of information about these states over time, disregarding the vacuum state Ω_0 which is stationary. Here we rely on definitions and results in [15].

For the analysis of the states in $(1-Z)\mathscr{H}_0$ it suffices to consider the subalgebras $\mathfrak{S}(\mathscr{V}_{o+\tau}) := (1-Z)\,\mathfrak{R}(\mathscr{V}_{o+\tau})\,(1-Z)$, $\tau \in V_+$. Putting $\mathscr{K}_0 := (1-Z-P_{\Omega_0})\mathscr{H}_0$, one proceeds to the net of closed, real linear subspaces

$$\mathscr{L}_\tau := \{(A-\omega_0(A)1)\Omega_0 : A = A^* \in \mathfrak{S}(\mathscr{V}_{o+\tau})\}^- \subset \mathscr{K}_0\,, \quad \tau \in V_+\,. \tag{4.1}$$

Since Ω_0 is cyclic and separating for $\mathfrak{S}(\mathscr{V}_{o+\tau})$, they are standard subspaces,

$$\mathscr{L}_\tau \cap i\,\mathscr{L}_\tau = \{0\}\,, \qquad (\mathscr{L}_\tau + i\,\mathscr{L}_\tau)^- = \mathscr{K}_0\,. \tag{4.2}$$

Let Δ_0 be the modular operator determined by the initial standard space $\mathscr{L}_0$. It coincides with the restriction of the modular operator fixed by $(\mathfrak{S}_0(\mathscr{V}_o), \Omega_0)$ to $\mathscr{K}_0$. Equation (3.1) implies that $\Delta_0^{-i\sigma}\mathscr{L}_\tau = \mathscr{L}_{e^{2\pi\sigma}\tau} \subset \mathscr{L}_\tau$ for $\sigma \geq 0$. Thus the inclusions $\mathscr{L}_\tau \subset \mathscr{L}_0$ are half-sided modular. Since $U(\sigma)\mathscr{L}_\tau = \mathscr{L}_{\tau+\sigma}$, $\sigma, \tau \in V_+$, it also follows from the spectral properties of U that $\mathscr{L}_\tau$ has no non-trivial element in common with its symplectic complement $\mathscr{L}_\tau{}'$,

$$\mathscr{L}_\tau \cap \mathscr{L}_\tau' = \{0\}\,, \quad \tau \in V_+\,. \tag{4.3}$$

We follow now the discussion in [15] and define the (real linear, unbounded) cutting projections

$$P_\tau : \mathscr{L}_\tau + \mathscr{L}_\tau' \to \mathscr{L}_\tau\,, \quad \tau \in V_+\,. \tag{4.4}$$

The modular operator determined by the standard subspace $\mathscr{L}_\tau$ is denoted by Δ_τ. Given any vector state $\Phi \in (1-Z)\mathscr{H}_0$, we proceed to $\Phi^\perp := (1-P_{\Omega_0})\,\Phi \in \mathscr{K}_0$ and put

$$I_\tau(\Phi) := \operatorname{Im}\langle \Phi^\perp, P_\tau\, i\, \ln \Delta_\tau\, \Phi^\perp \rangle\,, \quad \tau \in V_+\,. \tag{4.5}$$

This quantity is interpreted as information which can be extracted from Φ by measurements with observables in $\mathfrak{S}(\mathscr{V}_\tau)$, $\tau \in V_+$; the information contained in the stationary state Ω_0 is put equal to 0. As has been shown in [15], this interpretation is related to the concept of relative entropy between Φ and Ω_0, invented by Araki

[1]. The following result, established in [15], describes the information on Φ in the course of time.

Proposition 4.1 *Let* $\Phi \in (1-Z)\mathscr{H}_0$ *be a vector state.*

(a) $I_\tau(\Phi) \in [0,\infty]$ *and there is a dense set of vectors* Φ *for which this quantity is finite*, $\tau \in V_+$.
(b) $I_\tau(\Phi) \leq I_\sigma(\Phi)$ *if* $(\tau - \sigma) \in V_+$.
(c) *Let* $t \mapsto \tau(t) := (t, t\boldsymbol{v})$ *for fixed* $\boldsymbol{v}$, $|\boldsymbol{v}| < 1$. *If* $I_{\tau(t_0)}(\Phi)$ *is finite*, $t_0 \geq 0$, *then* $t \mapsto I_{\tau(t)}(\Phi)$ *is continuous for* $t \geq t_0$, *decreases monotonically, and is convex.*

The loss of information over time, described in this proposition, can be understood in simple terms in the presence of massless single particle states in $\mathscr{H}_0$, such as the photon. There then exist corresponding outgoing scattering states of massless particles in $\mathscr{H}_0$ and corresponding outgoing fields. Denoting by $\mathfrak{A}_0^{\text{out}}(\mathscr{V}_o)$ the algebra generated by outgoing fields that are created in $\mathscr{V}_o$ and, similarly, by $\mathfrak{A}_0^{\text{out}}(\mathscr{V}_{o\,-})$ the algebra generated by outgoing fields that were created in the past cone $\mathscr{V}_{o\,-}$, one has the inclusions [9]

$$\mathfrak{A}_0^{\text{out}}(\mathscr{V}_o) \subset \mathfrak{R}(\mathscr{V}_o) \subset \mathfrak{A}_0^{\text{out}}(\mathscr{V}_{o\,-})' \,. \tag{4.6}$$

Whereas the first inclusion holds also for outgoing fields of massive particles in $\mathscr{H}_0$, the second one is a consequence of locality and the fact that massless particles propagate with the speed of light. Thus the loss of information can be visualized in this case geometrically. It is a consequence of Huygens's principle according to which the outgoing massless particles in a state, which were created in the past cone $\mathscr{V}_{o\,-}$, will miss the future cone $\mathscr{V}_o$ and thus leave no observable effects there. It is noteworthy that by this mechanism the notorious infrared problems in Minkowski space, caused by infinite clouds of massless particles, disappear for observers in lightcones [13].

5 The Arrow of Time as Origin of Quantization

As we have seen, the hypothesis of an intrinsic arrow of time is compatible with the standard theoretical treatment of spacetime translations as a group. However, if one is located at some spacetime point o, there arise ambiguities about the action of these translations and the properties of the underlying physical system in the non-accessible past and spacelike complement $\mathscr{N}_o$. They are due to the presence of massless particles. In other words, one never has perfect control on the initial data of states which would be needed for an exact prediction of the results of future measurements in $\mathscr{V}_o$. The best one can hope for are statistical statements. They must be based on informations, collected in the past and typically formulated in classical terms, including quantum features that have been recorded. This leads us to the last question (IV), namely, whether the arrow of time entails the quantum properties of operations that lie ahead, but are described by classical concepts.

That this is a meaningful idea has been expounded in recent work [7, 11]. It is based on a set of operations which comply with a specific version of the causality principle: namely, the effects of an operation in a given space-time region become visible exclusively in its future. As first observed by Sorkin [25], this is a strong restriction on possible operations, and its relevance for the measurement process in relativistic quantum physics was recently analyzed in [6, 18, 19]. We determine here, with a simple example, a consistent choice of operations. These operations are characterized by concepts of classical field theory. They are conceived to describe perturbations which are caused by adding interaction terms to a given Lagrangian. As we shall see, the causal constraints, i.e. the arrow of time, imply that the operations generate a non-commutative group which, together with the dynamical constraints, leads to the formalism of quantum field theory.

We consider a scalar field which propagates in Minkowski space $\mathscr{M}$. Its classical configurations are real, smooth functions $x \mapsto \phi(x)$. The dynamics of the field is described in classical terms as well and given by relativistic Lagrangians. We treat here the simple case of a non-interacting field with Lagrangian density

$$x \mapsto L(x)[\phi] = (1/2)(\partial_\mu \phi(x) \partial^\mu \phi(x) - m^2 \phi(x)^2) \,. \tag{5.1}$$

Here ∂_μ is the partial derivative with regard to the μ-component of x and $m \geq 0$ is the mass of the field. The variations of the corresponding action are given for real, smooth functions with compact support, $\phi_0 \in \mathscr{D}(\mathbb{R}^4)$, by

$$\begin{aligned} \phi \mapsto \delta L(\phi_0)[\phi] &:= \int dx \, \big(L(x)[\phi + \phi_0] - L(x)[\phi] \big) \\ &= (1/2) \int dx \, \big(\partial_\mu \phi_0(x) \partial^\mu \phi_0(x) - m^2 \phi_0(x)^2 \big) \\ &\quad - \int dx \, \big(\Box \phi_0(x) + m^2 \phi_0(x) \big) \phi(x) \,, \end{aligned} \tag{5.2}$$

where $\Box$ is the d'Alembertian. It is a special functional on the fields of the specific form

$$\phi \mapsto F[\phi] = c + \int dx f(x) \phi(x) \,, \quad c \in \mathbb{R} \,, f \in \mathscr{D}(\mathbb{R}^4) \,. \tag{5.3}$$

These functionals are regarded as perturbations of the dynamics. They arise by adding to the Lagrangian (5.1) a c-number function $x \mapsto c(x)$, which integrates to c, and a term $x \mapsto f(x)\phi(x)$ which is linear in the field. We restrict our attention to pertubations of this simple form, cf. [11] for more general examples. The support of a functional F in Minkowski space is identified with the support of the underlying test function f. The constant functional $\phi \mapsto c[\phi] = c$ has empty support and can be assigned to any spacetime region. We note in conclusion that the functions $\phi_0 \in \mathscr{D}(\mathbb{R}^4)$, appearing in the variations of the action, induce shifts of the functionals. They are denoted by $\phi \mapsto F^{\phi_0}[\phi] := F[\phi + \phi_0]$.

After this outline of the classical input, we consider now operations which are labeled by the functionals F. The symbols $S_0(F)$ denote operations, determined by F, in presence of the unperturbed dynamics. They are conceived to describe perturbations which are caused by a local change of the dynamics through F. Similarly, the symbols $S_G(F)$ denote the same operations in presence of the dynamics changed by G. According to this interpretation, the products of these operations (their composition) are assumed to satisfy for functionals F, G, H the relation

$$S_H(G) S_{G+H}(F) = S_H(F+G), \quad S_H(0) = 1\,. \tag{5.4}$$

It follows that the operations have an inverse, $S_H(G)^{-1} = S_{H+G}(-G)$. Moreover, for any choice of F, G, one has the relation $S_G(F) = S_0(G)^{-1} S_0(F+G)$, known as Bogoliubov-formula [3].

The operation $S_G(F)$ is assumed to be localized in Minkowski space in the support region of F, irrespective of the choice of G. In order to express the causality properties of these operations, we must compare the supports of the underlying functionals. We write $G \succ F$ if G is later than F, i.e. there is some Cauchy surface such that $\operatorname{supp} G$ lies above and $\operatorname{supp} F$ beneath it. According to the causality condition, indicated above, the operation $S_{G+H}(F)$ does not depend on the choice of the functional H if it is later than F, i.e.

$$S_{G+H}(F) = S_G(F) \quad \text{if} \quad H \succ F\,. \tag{5.5}$$

The preceding relations suggest to consider the group $\mathscr{G}_0$ generated by the operations $S_0(F)$ for the chosen Lagrangian (5.1). It is characterized by the following three relations:

$$S_0(F) S_0(G) = S_0(F+G) \quad \text{if} \quad F \succ G\,. \tag{5.6}$$

In this factorization condition the arrow of time enters. Note that this product of operations is not commutative, the causal order of the functionals matters. In the given relation, the operations are performed one after the other, in accordance with the time direction. There are no *a priori* restrictions for the swapped product. As a matter of fact, its concrete form depends on the dynamics, as we will see. The relation implies that the constant functionals $\phi \mapsto c[\phi] = c$ determine elements $S(c)$ of the center of $\mathscr{G}_0$. They satisfy $S(c_1)S(c_2) = S(c_1 + c_2)$. By choice of a scale factor one fixes these central operations and postulates as second relation

$$S_0(c) = e^{ic}\, 1\,, \quad c \in \mathbb{R}\,. \tag{5.7}$$

We will see later that the chosen scale factor is related to Planck's constant.

The dynamics induced by the Lagrangian L imposes further conditions on the operations, put forward in [11]. They are given for functionals F and test functions $\phi_0 \in \mathscr{D}(\mathbb{R}^4)$ by the third relation

$$S_0(F) = S_0(F^{\phi_0} + \delta L(\phi_0))\,. \tag{5.8}$$

It emerged from an exponentiated version of the Schwinger-Dyson equation in perturbative algebraic quantum field theory [11]. In the absence of perturbations the operations do not affect the underlying field, i.e. $S_0(\delta L(\phi_0)) = 1$.

By standard arguments one can proceed from the group $\mathscr{G}_0$ of operations to a C*-algebra $\mathfrak{A}$. It is generated by a net of local subalgebras on Minkowski space which is determined by operations having support in the corresponding spacetime regions. Note that this construction depends on the choice of a global Lagrangian on $\mathscr{M}$. Thus, in accordance with the results in the preceding sections, it can be modified in the presence of massless excitations by perturbations in the past of any given light-cone $\mathscr{V}_o$. The algebra $\mathfrak{A}$ satisfies all axioms of local quantum physics, which were used in the preceding sections, cf. [11]. This feature is a consequence of the defining relations of $\mathscr{G}_o$, most prominently of the arrow of time that enters in the causal factorization condition.

In the example considered here, this assertion can be established by some straightforward algebraic computations [11]. One considers for arbitrary test functions $f \in \mathscr{D}(\mathbb{R}^4)$ the functionals

$$\phi \mapsto F_W(f)[\phi] := (1/2) \int dxdy\, f(x)\Delta_D(x-y)f(y) + \int dz\, f(z)\phi(z)\,, \tag{5.9}$$

where $\Delta_D := (1/2)(\Delta_R + \Delta_A)$ is the mean of the retarded and advanced solutions of the Klein-Gordon equation with mass m. Putting $W(f) := S(F_W(f))$, the following relations are obtained for arbitrary test functions $f_1, f_2, f_3 \in \mathscr{D}(\mathbb{R}^4)$:

$$\begin{aligned} &W(f_1)W(f_2) = e^{-(i/2)\int dxdy\, f_1(x)\Delta(x-y)f_2(y)}\, W(f_1 + f_2)\,, \\ &W((\Box + m^2)f_3) = 1\,, \end{aligned} \tag{5.10}$$

where $\Delta := (\Delta_R - \Delta_A)$ (Pauli-Jordan function). Thus the operators $W(f)$ are exponentials of a real, scalar, local quantum field of mass m that satisfies the Klein-Gordon equation and is integrated with test functions f (Weyl operators). The exponent of the phase factor in (5.10) reveals that the scale chosen in Eq. (5.7) amounts to putting Planck's constant equal to 1. Note that these equations are obtained without having imposed any quantization rules from the outset. They emerge from the constraints imposed by the arrow of time.

6 Summary

In the present article we have examined the consequences of the hypothesis that the arrow of time is a fundamental fact which enters in the evolution of all systems. There is no return to the past. We have clarified in a first step how the assumption that time forms a semigroup is related to the standard description of spacetime transformations, forming a group. That the latter description is consistent with the present input relies on the empirical fact that experiments can be repeated, i.e. one can prepare the same state many times. This suggests that there is some stationary background. We have discussed here the case of a vacuum state. As we have seen, its properties imply that the semigroup of time translations can be unitarily implemented in the corresponding GNS-representation. One can then proceed to a unitary group of all spacetime translations. It allows one to move theoretically backwards in time. Let us mention as an aside that similar results obtain if one proceeds from a thermal equilibrium state that satisfies the KMS condition.

Next, we studied the question whether the extension of the semigroup of time translations to the group of spacetime translations is unique. It turned out that in general there arise ambiguities. Namely, given a future lightcone, where the evolution of operations and measurements is described by a given semigroup of time translations, there can exist different extensions of this semigroup which describe different dynamics in the past. As a result, past properties of states can not be reconstructed with certainty in this case. We have also seen that if such ambiguities occur, there exist states describing excitations of arbitrarily small mass.

In order to clarify whether these states are responsible for the loss of control of past properties, we made use of a novel quantity, introduced in [15]. It measures the quantum information contained in a state, relative to the vacuum. We have shown that the information in the states of interest here decreases monotonically with time. Alternatively, one may speak of an increase of entropy. The underlying excitations escape continuously into the non-accessible part of Minkowski space. In case of massless particles in the vacuum sector, this is known to be a consequence of Huygens's principle; but there may well exist other entities with this property. These excitations cause dissipation and an inevitable loss of quantum information. What remains accessible over time are material systems. They can carry along information which may be expressed in classical terms (ordinary language). In order to exhibit their quantum features one needs to perform renewed operations, which will produce again the transient excitations.

These points were complemented in a final step by a survey on recent results in [11], where the arrow of time was shown to be a source of quantization. Given a system, its properties are described in classical terms, which may be thought of as being based on informations obtained in the past. One then considers localized operations which are described by functionals on the trajectories of the underlying classical system. They are interpreted as perturbations caused by local changes of the dynamics. There are two fundamental relations between these operations. The first one describes the net effect of successive operations. There the arrow of time enters. The second relation involves the dynamics in form of a Lagrangian. These relations determine a dynamical group. No quantization rules were assumed from the outset. Nevertheless these ingredients determine concrete algebras which fit into the framework of local quantum physics, cf. [7, 11]. It is worth mentioning that a similar approach works also in case of non-relativistic quantum mechanics [12].

In contrast to the standard approach to quantum physics, where the observables are in focus, the basic ingredients are here the operations on the underlying system. Such operations, described by unitary operators, can be used as a substitute for observables. As has been shown in [14], they allow to determine with arbitrary precision the expectation values of given basic observables (projections) in given subspaces of states. Moreover, after their action the states are elements of the corresponding spectral subspaces, there is no collapse of wave functions. For this reason, these operations were called primitive observables in [14].

The present results are surprisingly close to the view of Niels Bohr, who has argued that observations must be described in ordinary language supplemented with classical physical concepts [17, p. 124]. What Bohr did not know at his time is the fact that their quantum features can be traced to the arrow of time.

So, in summary, we come to the conclusion that the hypothesis of an intrinsic arrow of time, inherent in all systems, is meaningful. It needs no justification by the Second Law. As a matter of fact, it implies the increase of entropy (loss of information) over time, as we have seen in a simple example. The initial problem, that this hypothesis is in conflict with the efficient theoretical usage of the group of spacetime translations has a surprisingly simple solution. But this solution also reveals that the standard theoretical treatment is to some extent ambiguous.

There remain, however, several questions. Among them is the description of events that can be regarded as secured facts, cf. [21, VII. 3] and [2]. They provide a steadily increasing wealth of information. From our present point of view, these events have to be attributed to the past, where they were recorded. Since they are the basis for the design of future operations, it would be desirable to describe their features in appropriate mathematical terms.

Another problem is the discussion of the arrow of time in curved backgrounds. There the evolution of systems can be described by a principle of local covariance [8]. Because of the lack of stationary states it is, however, less clear how to reconstruct from data in lightcones a consistent picture of the past. Nevertheless, one may expect that similiar ambiguities about the past, as in Minkowski space, occur there as well. This would shed new light on the information paradox, raised by Hawking [22]. As we have shown in the present investigation, the paradigm that there is no loss of information in quantum physics, may be questioned. It seems therefore worthwhile to take a fresh look at the foundations of quantum physics, based on our hypothesis about the nature of time.

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