Foundations of Physics (2022) 52:109
https://doi.org/10.1007/s10701-022-00622-5

Lorentz Transformation Under a Discrete Dynamical Time and Continuous Space

Roland Riek[1]

Received: 18 June 2020 / Accepted: 5 January 2022 / Published online: 6 October 2022

Abstract
The Lorentz transformation of space and time between two reference frames is one of the pillars of the special relativity theory. As a result of the Lorentz transformation, space and time are only relative and are entangled, while the Minkowski metric is Lorentz invariant. For this reason, the Lorentz transformation is one of the major obstructions in the development of physical theories with quantized space and time. Here is described the Lorentz transformation of a physical system with a discrete dynamical time and a continuous space that fulfills Lorentz invariance while approximating the Lorentz transformation at the time continuous limit and the Galilei transformation at the classical limit. Furthermore, the discreteness of time is not mixed with the continuous nature of space, making time distinct from space.

Keywords Discrete time · Quantum time · Special relativity

1 Introduction

With the advent of quantum mechanics and its discrete nature of system properties, discrete-time physics was also introduced [1–9]. However, the quantum nature of time and space has only recently gained thorough attention in investigations towards unified physical theories such as quantum loop gravity theory [10], causal set theories [11] and others [7, 9, 12–14] reviewed in part in [8]. One of the major obstacles therein appeared to be the Lorentz transformation [15]. In special relativity both time and space are only relative and are entangled since the value of the time variable in one reference frame depends on the values of the time and space variables in the other frame as described by the Lorentz transformation [15, 16]. With the geometrical approach of Minkowski and Poincaré, time and space are further merged into a

✉ Roland Riek
roland.riek@phys.chem.ethz.ch

[1] Laboratory of Physical Chemistry, ETH Zurich, Wolfgang-Pauli-Strasse 10, HCI F 225, CH 8093 Zurich, Switzerland

single entity, establishing a cornerstone of general relativity: spacetime [17–20] (but see [21]; please note, general relativity is not further considered here).

Lorentz invariance of the Minkowski metric [17, 18] and the Lorentz transformation *per se* as introduced by Einstein [15] thereby appear to request a continuous time and space and are violated by the discreteness of both time and space [8, 10] unless a probabilistic approach is introduced as in the causal set theory [11] or non-physical systems are considered [12]. It is here demonstrated that a physical theory with a discrete, non-random dynamical time and a continuous space is Lorentz covariant. Apart from its conceptual novelty, this approach paves the way for discrete-time physics in fundamental physics theories. Furthermore, it supports the notion of time as a series of events that are linked together by causality [22], yielding a time that is only relative in character (as we know it from special relativity) and highlights the difference between time and space, feeding into the debates between special relativity and the line element nature of spacetime by Minkowski [21, 23, 24]. After a short summary on special relativity theory (2.1), and the introduction of discrete dynamical time physics following the approach by Lee (2.2), the Lorentz transformation between two reference frames is derived for a system having a discrete dynamical time and a continuous space (2.3) followed by a discussion (3) and finished by a conclusion (4).

2 Theory

2.1 The Lorentz Transformation Within Special Relativity Theory

Here, special relativity theory (under a continuous time) is revisited [15, 16]. By studying the coordinates of a point particle in two different inertial reference frames S and S' (i.e. t, $\mathbf{x}$ and t', $\mathbf{x}'$ with t the time and the space vector $\mathbf{x} = (x_1, x_2, x_3)$ describing the coordinates of the particle within the three dimensional space) which move relative to each other with the velocity $\mathbf{v} = (v_1, 0, 0)$, the Minkowski metric given by

$$s^2 = (ct)^2 - \mathbf{x}^2 = (ct')^2 - \mathbf{x}'^2 = s'^2 \tag{1}$$

is preserved (please note, c is the speed of light and constant by definition). The Lorentz transformation describing the transformation of the system from the reference frame S to S' is defined by

$$x_1^{'} = \frac{x_1 - v_1 t}{\sqrt{1 - (v_1/c)^2}} \tag{2}$$

$$x_2^{'} = x_2$$
$$x_3^{'} = x_3$$

and

$$t' = \frac{t - (v_1/c^2)x_1}{\sqrt{1-(v_1/c)^2}} \ . \tag{3}$$

It is evident that time is relative with the Lorentz transformation and that time and space are entangled (eqs. (2) and (3)), forming together, according to Minkowski [17], a single entity (ct, x_1, x_2, x_3) called spacetime, noting that the x_0 component is not time but rather the product between the velocity of light and time [21]. In special relativity theory, causality is preserved, however, time and space are relative and no longer absolute. That is, an observer sitting in the S or the S' reference frame will not monitor the time-dependent location and velocity of the particle under observation on an absolute scale. If the observer looks at the particle from the S reference frame, at a given time point t he observes the particle at position x, while from the reference system S', he observes the particle at time point t' and position x' related to t and x from S by the Lorentz transformation. Considering the mixing/entanglement of time and space by the Lorentz transformation between two frames of reference, it is not obvious that time can be discrete and space continuous.

2.2 Discrete Time Physics

In the dynamical discrete-time non-relativistic approach by Lee [9] time is a discrete dynamical variable of tensor nature [14]

$$\hat{t}_n = \begin{bmatrix} t_{1,n} & 0 & 0 \\ 0 & t_{2,n} & 0 \\ 0 & 0 & t_{3,n} \end{bmatrix}.$$

The continuous space function $\mathbf{x}(t)$ of a system (such as a point particle) is thereby replaced by a sequence of discrete values

$$\mathbf{x}_n = \left(\mathrm{x}_{1,\mathrm{n}}, \mathrm{x}_{2,\mathrm{n}}, \mathrm{x}_{3,\mathrm{n}}\right) = \left(x_1\left(t_{1,n}\right), x_2\left(t_{2,n}\right), x_3\left(t_{3,n}\right)\right)$$

and

$$\mathbf{v}_n = (v_1(t_{1,n}), v_2(t_{2,n}), v_3(t_{3,n}))$$

yielding:

$$(\mathbf{x}_0, \mathbf{v_0}, \hat{t}_0),\ (\mathbf{x}_1, \mathbf{v}_1,\ \hat{t}_1), \ldots\ldots\ldots,\ (\mathbf{x}_n, \mathbf{v}_n,\ \hat{t}), \ldots\ldots,\ (\mathbf{x}_{N+1}, \mathbf{v}_{N+1},\ \hat{t}_{N+1}) \tag{4}$$

with $(\mathbf{x}_0, \mathbf{v}_0,\ \hat{t}_0)$ the initial and $(\mathbf{x}_{N+1}, \mathbf{v}_{N+1},\ \hat{t}_{N+1})$ the final "time position". In this description $\mathbf{x}_n$ and $\mathbf{v}_n$ are still continuous, while $\hat{t}_n$ is of discrete character (and n is only changing if there is something happening [14, 18]). Under these circumstances $\Delta\hat{t}_n = \hat{t}_n - \hat{t}_{n-1}$, which is dynamic in nature if a time reversible/symmetric description is invoked that can approximate the continuous time theories [9]. Or alternatively $\Delta\hat{t}_n = \Delta\hat{t}$ is constant if a time irreversible description is permitted/requested [14].

There are several possible definitions for the velocity within a discrete-time physical theory. Here, the definition introduced in reference [14] is used

$$v_{i,n} := \frac{x_{i,n} - x_{i,n-1}}{\Delta t_{i,n}} \text{ with } i = 1, 2, 3 \tag{5}$$

defining the velocity $\mathbf{v}_n$ at time n from information at time point n (i.e. $\mathbf{x}_n$) and the past time point $n-1$ (i.e. $\mathbf{x}_{n-1}$), which is a logical definition regarding causality since only present and past information is used to decipher the present velocity.

This approach must be distinguished from discrete formalisms in dynamic simulations (including special and general relativity) in which discrete algorithms are used, as exemplified here with the leapfrog integration method [25] whereby

$$\mathbf{v}_{n+\frac{1}{2}} = \mathbf{v}_{n-\frac{1}{2}} + \frac{\mathrm{d}^2\mathbf{x}_\mathrm{n}}{\mathrm{dt}^2}$$

and

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \mathbf{v}_{n+\frac{1}{2}} \Delta t \ .$$

The leapfrog algorithm is time-reversible and symplectic in nature and thus conserves the energy of a dynamical system. However, velocity and position are calculated at different time points and the acceleration is defined by the second derivative of the position vector, highlighting its limited applicability to simulations. Other algorithms have similar properties.

The introduction of discrete-time physics (eqs. (4) and (5)) may not only describe experimental physics (which is always of discrete time nature due to finite energy sources) more precisely than its continuous counterpart, but has the profound effect of imparting an arrow of time and time-irreversibility at the microscopic level (before ensemble and statistical averaging as done in statistical thermodynamics) [14]. As such it also allows the introduction of causality which must be a unidirectional entity because a cause is antecedent of its effect [22]. While the reader is invited to read relevant literature on discrete time physics if interested [9, 14, 22] the two following points of relevance are recapitulated. First, discrete time naturally defines casual chains of events. For this a classical description of the Hamilton function $H(\mathbf{q}, \mathbf{p})$ with $\mathbf{q} = (q_1, q_2, q_3)$ and $\mathbf{p} = (p_1, p_2, p_3)$ for the generalized phase space canonical position and momentum, respectively, and the acting potential $V(\mathbf{q})$ is introduced (in cartesian coordinates the Hamilton function is given as $H(\mathbf{q}, \mathbf{p}) = \frac{1}{2\mathrm{m}}\mathbf{p}^2 + \mathrm{V}(\mathbf{q})$). Using the Hamilton equation $\dot{p}_i = -\frac{\partial H}{\partial \mathbf{q_i}}$, Newton's second law is obtained

$$-\frac{\partial V}{\partial q_i} = lim_{\Delta t \to 0} \frac{p_i(t_i + \Delta t_i) - p_i(t_i)}{\Delta t_i} \tag{6}$$

which in the discrete-time approach (i.e. without the lim) and by redistributing the variables in the equation becomes

$$p_{i,n+1} = p_i(t_i + \Delta t_i) = -\frac{\partial V}{\partial q_i}\Delta t_i + p_i(t) = -\frac{\partial V}{\partial q_{i,n}}\Delta t_{i,n} + p_{i,n} \tag{7}$$

thereby connecting the cause described by the force (expressed through its components $-\frac{\partial V}{\partial q_{i,n}}$) that acts on $\mathbf{p}_n$ with the effect described by $\mathbf{p}_{n+1}$. Hence, the sequence of discrete values of eq. (4) can thus be interpreted as causal hopping from event n to event $n+1$ and if nothing is happening (i.e. if there is no force acting) no new event occurs. Second, a discrete-time approach yields a time-irreversible physics in the presence of a changing force. This is described by eq. (8) in which a step forward $n \to n+1$ is followed by a step backward in "time":

$$p_{i,n+2} = -\frac{\partial V}{\partial q_{i,n+1}}\Delta t_{i,n+1} + p_{i,n+1} = -\frac{\partial V}{\partial q_{i,n+1}}\Delta t_{i,n+1} - \frac{\partial V}{\partial q_{i,n}}\Delta t_{i,n} + p_{i,n} \tag{8}$$

To request time reversibility $p_{i,n+2} = p_{i,n}$ (note that the step backward gives a negative time) must hold after the second step, which in presence of a changing force (i.e. $\frac{\partial V}{\partial q_{i,n}} \neq \frac{\partial V}{\partial q_{i,n+1}}$) is only obtained if

$$\frac{\partial V}{\partial q_{i,n}}\Delta t_{i,n} = \frac{\partial V}{\partial q_{i,n+1}}\Delta t_{i,n+1} \tag{9}$$

this being possible with the dynamic nature of time (for consequences thereof, we refer to references [14, 22]). In other words, with the introduction of a discrete time, an arrow of time at the microscopic physics level is introduced, thereby yielding one of the principle properties of time, namely directionality. Once its discrete nature is introduced, time can be replaced by a series of events n. The discrete nature of time was thereby introduced as a means to get an arrow of time and causality and not because of its potential quantum nature requested from quantum mechanics, although in principle it could be exploited as such.

2.3 The Lorentz Transformation Under a Discrete Time and a Continuous Space

The discrete-time physics introduced in Sect. 2.2 is now extended to the special relativity theory. In particular the Lorentz transformation under a discrete time and a continuous space between the reference frames S $(t_n, \mathbf{x_n})$ and S' $(t'_n, \mathbf{x_n}')$ with S' moving with a constant velocity $\mathbf{v_n} = (v_{1,n}, 0, 0)$ in respect to S (without loosing generality) is now investigated as in the standard description above (Sect 2.1). The corresponding Lorentz transformation for a single time step is given by

$$x'_{1,n} = \frac{x_{1,n} - v_{1,n}\Delta t_{1,n}}{\sqrt{1-(v_{1,n}/c)^2}}, x'_{2,n} = x_{2,n}, x'_{3,n} = x_{3,n} \tag{10}$$

and

$$\Delta t'_{1,n} = \frac{\Delta t_{1,n} - (v_{1,n}/c^2)x_{1,n}}{\sqrt{1-(v_{1,n}/c)^2}}$$

with $v_{1,n}(t_n) = \frac{x_{1,n}-x_{1,n-1}}{\Delta t_{1,n}}$ and $c := \frac{x^c_{1,n}-x^c_{1,n-1}}{\Delta t_{1,n}}$ denoting $\mathbf{x_n}^c$, the wavefront coordinate vector of light at time point n.

In the classical limit in absence of a velocity (i.e. $\mathbf{v_n} = (0,0,0)$), the Galilei transformation is obtained with $x'_{i,n} = x_{i,n}$ with $i = 1...3$ and $\Delta t'_n = \Delta t_n$.

In the limit $\Delta t_{1,n} \to 0$, $\lim_{\Delta t_{1,n}\to 0} \Delta t_{1,n} = \mathrm{d}t$, $\lim_{\Delta t_{1,n}\to 0} \mathbf{x}_n = \mathbf{x}$, $\lim_{\Delta t_{1,n}\to 0} v_{1,n} = v_1$ and the standard Lorentz transformation is obtained with

$$x'_1 = \frac{x_1 - v_1 dt}{\sqrt{1-(v_1/c)^2}}$$
$$dt' = \frac{dt - (v_1/c^2)x_1}{\sqrt{1-(v_1/c)^2}}$$

It is evident, that neither $x'_{1,n}$ nor $x_{1,n}$ are required to be of granular nature because

$$\frac{v_{1,n}}{c} = \frac{\frac{x_{1,n}-x_{1,n-1}}{\Delta t_{1,n}}}{\frac{x^c_{1,n}-x^c_{1,n-1}}{\Delta t_{1,n}}} = \frac{x_{1,n}-x_{1,n-1}}{x^c_{1,n}-x^c_{1,n-1}}$$

is independent of the time step size as is

$$x'_{1,n} = \frac{x_{1,n} - v_{1,n}\Delta t_{1,n}}{\sqrt{1-(v_{1,n}/c)^2}} = \begin{cases} x_{1,n-1}/\sqrt{1-(v_{1,n}/c)^2} & for\, v_{1,n} > 0 \\ x_{1,n} & for\, v_{1,n} = 0. \end{cases} \tag{11}$$

Thus, space is (allowed to be) of continuous nature while time is discrete, distinguishing the space from time as mandated from the arrow of time in thermodynamics [14, 18]. In other words, the discrete nature of time and the continuous nature of space do not mix in the Lorentz transformation of eq. (10).

Furthermore, the discrete-time time vector given by

$$(c\Delta t_{1,n}, x_{1,n}, x_{2,n}, x_{3,n}) = (x^c_{1,n} - x^c_{1,n-1}, x_{1,n}, x_{2,n}, x_{3,n}) \tag{12}$$

is of continuous nature.

The discrete-time Minkowski metric reads

$$s^2_n = (c\Delta t_{1,n})^2 - \mathbf{x_n}^2 = (x^c_{1,n} - x^c_{1,n-1})^2 - \mathbf{x_n}^2. \tag{13}$$

By replacing $\Delta t'_{1,n}$ and $\mathbf{x}'_n$ from eq. (10) into s'^2_n from eq. (13) it is evident that the discrete-time Minkowski metric is Lorentz invariant:

$$(c\Delta t'_{1,n})^2 - \mathbf{x_n'}^2 = (c\Delta t_{1,n})^2 - \mathbf{x_n}^2. \tag{14}$$

Of course, the time step size differs between the two reference frames. Starting from eq. (10)

$$\Delta t'_{1,n} = \frac{\Delta t_{1,n} - (v_{1,n}/c^2)x_{1,n}}{\sqrt{1-(v_{1,n}/c)^2}} = \frac{\Delta t_{1,n} - \left(\frac{\frac{x_{1,n}-x_{1,n-1}}{\Delta t_{1,n}}}{\left[\frac{x^c_{1,n}-x^c_{1,n-1}}{\Delta t_{1,n}}\right]^2}\right)x_{i,n}}{\sqrt{1-\left(\frac{x_{1,n}-x_{1,n-1}}{x^c_{1,n}-x^c_{1,n-1}}\right)^2}} = \frac{\Delta t_{1,n}\left[1-\left(\frac{x_{1,n}-x_{1,n-1}}{\left[x^c_{1,n}-x^c_{1,n-1}\right]^2}\right)x_{i,n}\right]}{\sqrt{1-\left(\frac{x_{1,n}-x_{1,n-1}}{x^c_{1,n}-x^c_{1,n-1}}\right)^2}} \tag{15}$$

which corresponds to the time dilation in the standard description, while time is still of a discrete nature. In other words, although the time step size is relative, the time step number n is absolute. Hence, causality is guaranteed with n always being the n'th causal event regardless of the reference frame (i.e. $n' = n$). This finding is in line with the proposition that discrete time is the metric of causality as suggested in referene [22]. Furthermore, in contrast to the time step number, the time step size is not fundamental and thus the irrelevancy of the time step as well as the time dilation in the standard special relativity theory becomes obvious.

In summary, with the presented axiom of a dynamic discrete time, a Lorentz transformation is defined that separates the discrete nature of time from the continuous nature of space and preserves the Lorentz invariance of the Minkowski metric.

3 Discussion

3.1 Discrete-Time Physics

The rather simple derivation presented shows that dynamic discrete-time physics enables Lorentz invariance. This is an important finding because discrete-time physics approaches are usually challenged and criticized by the lack of Lorentz covariance and the lack of Lorentz invariant entities, limiting their use not only to the classical limit but questioning their fundamental character per se. With the findings presented, the discrete nature of time can thus be revisited. The concept of a discrete time is inspired by various physical considerations. Starting with thermodynamics, with the definition of time as the variation of the macroscopic entropy of irreversible processes, time becomes the footprint of irreversibility and as such is of granular nature [26]. Alternatively, the introduction of a microscopic entropy and an arrow of time at the microscopic scale is only possible if time is discrete [9, 14, 22] (see also section 2.2). A theory that treats time as discrete and space continuous (see also section 3.3 which is on discrete time and discrete space) distinguishes time from space and is consistent with our every experience, in which objects move forward and backward in space but not in time, but is not consistent with the standard physics, including special relativity. In other words, the introduction of the discreteness of time allows it to be distinguished from from space because it yields thereby, most importantly, time-irreversibility of a dynamic system and correspondingly an arrow of time with all its consequences including entropy [14]. Furthermore, the Dirac equation which describes the free relativistic electron is more sound in presence of a discrete time (termed "chronon") than its continuous analog because it lacks runaway solutions and contains the field of the particle itself [3, 4] and similarly, a discrete time is introduced to study the exact equation of motion for a moving (accelerated) charged particle [27]. In addition, the evolution of the universe at the Newtonian limit of general relativity using discrete time physics exposes the inflation of the universe and the cosmic constant as artifacts of a time continuous description [28]. Furthermore, others state that irrational numbers are not physical due to the information content therein, thereby demanding discreteness of reality [29]. Finally, discrete time physics will likely play a central role in the development of unified physical theories and does so already in quantum loop gravity theory [10], causal set theories [11] and others [7, 9, 12–14].

3.2 Absolute Causality, Relative Time

The introduction of a discrete time allows for a straightforward definition of causality, relating an antecedent cause with its resulting event due to the existence of an arrow of time [22]. As described in the example given above with the discrete-time analog of Newton's second law (eq. (7)), a causal relationship with the momentum at event $n + 1$ by the acting potential and the momentum at point n is straightforward. Importantly, causality is observer independent since $n = n'$ as shown by eq. (14). Hence, causality along with its metric, the discrete time number n, is absolute and evidently Lorentz invariant as also shown by Zeeman [30]. In contrast, time in terms of time length (usually described by a continuous time) is relative. These statements get to the heart of the fundamental difference between the absoluteness of causality and the causal chain and the relativeness of time. This differs from the classical philosophical view point of Aristoteles, Kant and others (but not Reichenbach) where either causality and time can be interchanged or time is more fundamental than causality [31, 32]. Within this context it is interesting to note that causality is usually addressed in special relativity only at the level of argumentation [31].

Furthermore, Zeeman showed that if causality is required to be invariant, the Lorentz group is obtained [29]. The Lorentz transformation thereby provides for a kinematic calculation of a process from different observers/reference frames and relates their observations to each other as demonstrated by Einstein in 1905 [15] and now shown above also for the case of discrete time (eqs. (11), (14), and (15)). The discrete extension shows in addition that while discrete time and continuous space are connected they cannot entail a single line element (i.e. a spacetime vector) since time stays discrete and space continuous. This is in principle already obvious considering the difference in units of time (second) and space (meter) [21]. Only if time is replaced by *ct* it can be merged with space into the four dimensional spacetime vector introduced by Minkowski and Poincaré [17–19] because unlike *t*, *ct* is a distance of continuous nature. The spacetime vector along with the Minkowski metric thereby allows for the determination of relativeness between two chains of causality that are not correlated with each other (or the past connection is unknown) via the distance that light travels between the two, thereby giving rise to a causal interconnection between them. We hope that this discussion may also help to resolve the dispute on the relationship between special relativity and spacetime geometry and the physical nature of the spacetime [21, 23, 24].

3.3 Discrete Time, Discrete Space

Towards the unification of quantum mechanics with the general relativity theory and field theories [8, 20] a granular nature of both time and space has been invoked and studied, including quantum loop gravity [10, 33, 34] and causal set theories [11]. However, unless a probabilistic argument is introduced such as in the causal set theory [11] or the existence of a physical time is neglected or different times for different purposes are introduced [35] the concurrent discreteness of both time and space appears to interfere with the Lorentz transformation and the Lorentz invariance of the spacetime vector metric of Minkowski [8] but as demonstrated here, these barriers do not exist for a discrete time and a continuous space.

Next, it is elaborated on whether, in addition to a discrete dynamical time, a discrete space could be introduced. Towards answering this question it is stated that both time and space are continuous in quantum mechanics. Furthermore, it is noted that time in quantum mechanics is the classical time reflected by the lack of a time operator. The time-dependent Schrodinger equation is thus of semi-classical nature with a time therein that comes from an adjacent classical clock [36, 37]. The time evolution of the wave function described by the time-dependent Schrodinger equation demonstrates thereby the time evolution of the classical outcome (after the measurement). Hence, the introduced discrete nature of time holds also for quantum mechanics. The request of a granular nature of space is owed to the Heisenberg uncertainty principle between momentum and space (determinants) [38], which can be associated to the accuracy of a measurement of a system because of the observer interference to the system by the measurement. It thus must not be *per se* a fundamental property. If however a fundamental length (such as the Planck length) is demanded, then by definition space has an absolute character (and is by definition Lorentz invariant) which is in contradiction to theories of relativity for which space is relative. The remaining possibility of a discrete space is thus a dynamic discrete space in analogy to the dynamic discrete time introduced above. The realization of a discrete dynamical space might be possible because as shown above time and space do not mix. But it is not straightforward and appears to require additional conditions/assumptions. For example, the important ratio of the velocity of the system studied and the velocity of light at time point n given by the ratio of the distance traveled by the system versus light $\frac{v_{1,n}}{c} = \frac{x_{1,n} - x_{1,n-1}}{x^c_{1,n} - x^c_{1,n-1}}$ may only be approximatively defined. Moreover and more fundamental, while the space steps are relative, an unknown entity of unknown origin (which in the case of time is causality) may be absolute. Furthermore, with the request of a reversible space (unlike time) the same space can again and again be explored and revisited with experiments approximating its infinitesimal nature (please note, that these experiments are however not possible in the spacetime manifold description). In short, a discrete dynamical symmetrical space that is not mixed with the discreetness of time, but is entangled with time through the Lorentz transformation, can be introduced in principle but appears not to be necessary and may eventually open up other issues.

4 Conclusion

The finding that a discrete-time/continuous-space physics obeys the Lorentz transformation opens an avenue for a (dynamic) discrete time description in fundamental physics theories. Moreover, it demonstrates the difference between space and time, challenging the line element character of the spacetime moiety along with the Minkowski metric. Furthermore, it gives causality a prominent role as a fundamental entity and gives further insights into the relationship between time and causality and on the nature of time [39–42]. The reader is invited to explore these findings further.

Acknowledgements We would like to thank the ETH for unrestrained financial support and Dr. Peter Güntert, Dr. Alexander Sobol, Dr. Thomas Elze, Dr. Jason Greenwald, and Dr. Yaron Hadad for helpful discussions.

Funding Open access funding provided by Swiss Federal Institute of Technology Zurich.

Data Availability The scientific work is of theoretical nature and does not include any data.

References

1. Thomson, J.J.: The intermittence of electric force. Proc. Roy. Soc. of Edinburgh **46**, 90 (1925)
2. Levi, R.: Theorie de l'action universelle et discontinue. J de Physique et le Radium **8**, 182–198 (1927)
3. Caldirola, P.: A new model of the classical electron. Supplemento al Nuovo Cimento **10**, 1747–1804 (1927)

4. Farias, R.A.H., Recami, R.: Introduction of a quantum of time ("chronon") and its consequences for quantum mechanics, arXiv:quant-ph/9706059 (2007)
5. Yang, C.N.: On quantized space-time. Phys. Rev. **72**, 874 (1947)
6. Snyder, H.: Quantized spacetime. Phys. Rev. **71**, 38–41 (1947)
7. Heisenberg, W.: The self-energy of the electron. In: Miller, A. (ed.) Early quantum electrodynamics, pp. 121–128. Cambridge University Press, Cambridge (1930)
8. Hagar, A.: Discrete or continuous. Cambridge University Press (2014)
9. Lee, T.D.: Can time be a discrete dynamical variable? Phys. Lett. **122B**, 217–220 (1983)
10. Rovelli, C.: Space and time in loop quantum gravity, arXiv1802.02382v1 (2018)
11. Surya, S: The causal set approach to quantum gravity. Living Rev. Relat 22, s41114-019-0023-1 (2019)
12. Schild, A.: Discrete space-time and integral lorentz transformations. Can. J. Math. **1**, 29–47 (1949)
13. Elze, H.-T.: Discrete mechanics, "time machines" and hybrid systems, arXiv:1310.2862 (2013)
14. Riek, R.: A derivation of a microscopic entropy and time irreversibility from the discreteness of time. Entropy **16**, 3149 (2014)
15. Einstein, A.: Zur Elektrodynamik bewegter Körper. Ann. Phys. **17**, 891–921 (1905)
16. Greiner, W., Rafelski, J.: Spezielle Relativitätstheorie. Verlag Harri Deutsch, Frankfurt a. M., Germany (1993)
17. Minkowski, H.: Die Grundgleichungen für die electromagnetischen Vorgänge in bewegten Körpern. Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen, 53–111, (1908)
18. Einstein, A., Korentz, H.A., Weyl, H., Minkowski, H.: The principle of relativity. Translated by W. Perrett and G.B. Jeffrey with notes by AA. Sommerfeild,. New York, Dover (1956)
19. Poincaré, H.: Sur la dynamique de l' électron. Rendiconti del Circolo matematico di Palermo **21**, 129–176 (1906)
20. Einstein, A.: Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin. part 1: **142**(1917)
21. Cosgrove, J.K.: Relativity without spacetime, Palgrave Macmillan (Cham) (2018)
22. Riek, R.: Entropy derived from causality. Entropy **22**, 647 (2020)
23. Brown, H.R.: Physical relativity: space-time structure from a dynamical perspective. Oxford University Press (2005)
24. Balashov, Y., Janssen, M.: Presentism and relativity. Br. J. Philosophy Sci. **54**, 327–46 (2003)
25. Hockney, R.W., Eastwood, J.W.: Computer simulation using particles. McGraw-Hill, New-York (1981)
26. Lucia, U., Grisolia, G.: Time: a constructal viewpoint & its consequences. Sci. Adv. **9**, 10454 (2019)
27. Sheffer, Y., Hadad, Y., Lynch, M.H., Kaminer, I.: Towards precision measurements of radiation reaction. arXiv:1812.10188
28. Riek, R.: On the time continuous evolution of the universe if time is discrete and irreversible in nature. J. Phys. Conf. Ser. **1275**, 012064 (2019)
29. Gisin, N.: Real numbers are the hidden variables of classical mechanics. Quant. Stud.: Mathem. Found. **7**, 197–201 (2020)
30. Zeeman, E.C.: Causality implies the lorentz group. J. Math. Phys. **5**, 490 (1964). https://doi.org/10.1063/1.1704140
31. Bunge, M.: Causality and modern science; Dover classics of science and mathematics; 3rd rev. ed.; Dover Publications: New York, (1979); ISBN 978-0-486-23728-2
32. Gadamer, H.G.: Hermeneutik: Wahrheit und Methode, Tübingen, (1960). ISBN-13: 978-3161502118
33. Rovelli, C.: Zakopane lecture on loop gravity **12011**, arXiv:11-2.3660 (2011)
34. Ross, G.: Grand unified theories. Westview Press ISBN 978-0-8053-6968-7 (1984)
35. Rovelli, C.: Neither presentism nor eternalism. Found. Phys. **49**, 1325–1335 (2019)
36. Briggs, J.S.: Equivalent emergence of time dependence in classical and quantum mechanics. Phys. Rev. A **91**, 052119 (2015)
37. Schild, A.: Time in quantum mechanics: a fresh look at the continuity equation. Phys. Rev. A **98**, 052113 (2018)
38. Heisenberg, W.: Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. Zeitschrift für Physik (in German) **43**(3–4), 172–198 (1927)
39. Smolin, L.: Time Reborn. Houghton Micclin Harcourt, Boston,USA (2013)
40. Wharton, W.R.: Understanding time and causality is the key to understanding quantum mechanics arXiv:quant-ph/0310131 (2003)

41. Barbour, J.: The end of time. Oxford University Press, New York (1999)
42. Reichenbach, H.: The direction of time. University of California press, Berkeley (1971)