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LETTER TO THE EDITOR

Misconceptions on Effective Field Theories and Spontaneous Symmetry Breaking: Response to Ellis' Article

Thomas Luu[1,2] · **Ulf-G. Meißner**[1,2,3,4]

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Abstract
In an earlier paper Luu and Meißner (arXiv:1910.13770 [physics.hist-ph]) we discussed emergence from the context of effective field theories, particularly as related to the fields of particle and nuclear physics. We argued on the side of reductionism and weak emergence. George Ellis has critiqued our exposition in Ellis (arXiv:2004.13591 [physics.hist-ph]), and here we provide our response to his critiques. Many of his critiques are based on incorrect assumptions related to the formalism of effective field theories and we attempt to correct these issues here. We also comment on other statements made in his paper. Important to note is that our response is to his critiques made in archive versions arXiv:2004.13591v1-5 [physics.hist-ph]. That is, versions 1–5 of this archive post. Version 6 has similar content as versions 1–5, but versions 7–9 are seemingly a different paper altogether (even with a different title).

Keywords Effective field theories · Emergence · Strong interactions · Chiral pertubation theory

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✉ Thomas Luu
t.luu@fz-juelich.de

Ulf-G. Meißner
meissner@hiskp.uni-bonn.de

[1] Institute for Advance Simulation 4 (IAS-4 & IKP-3) and Jülich Center for Hadron Physics, Forschungszentrum Jülich, Jülich, Germany

[2] Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Rheinische Friedrich-Williams-Universität Bonn, Bonn, Germany

[3] Center for Science and Thought, Rheinische Friedrich-Williams-Universität Bonn, Bonn, Germany

[4] Tbilisi State University, 0186 Tbilisi, Georgia

1 Introduction

Emergent phenomena are fascinating quantities that when viewed from lower level (more fundamental) constituents are highly complex in nature. The question of whether such phenomena can be totally derived from their underlying constituents provides an intriguing debate not only from a philosophical point of view, but also from a scientific point of view that spans all areas of science. At issue, from our point of view, is the invocation of *strong* emergence, which is the notion that certain (or all) properties of some emergent phenomena cannot be deduced from its fundamental constituents. The weaker claim, naturally called *weak* emergence, states that all properties of an emergent phenomena can, *in principle*, be deducible from its fundamental constituents.[1] A more detailed definition of these cases is given in [3].

In our paper [1] we gave an overview of the formalism of Effective Field Theories (EFT) and its relation to emergent phenomena, providing examples prevalent in nuclear and particle physics. We won't repeat our arguments here, but only state that in our paper we rejected the concept of strong emergence and argued in favor of weak emergence and reductionism in this context. Ellis has since criticised our arguments in [2], and we take this opportunity to respond to his criticisms.

Ellis' criticisms have not changed our views on the matter. Furthermore, many of his criticisms are based on incorrect assumptions and an incomplete understanding of the EFT formalism. Thus our response here also serves as a means to clarify any misconceptions that might occur from an initial read of Ellis' critiques by anyone uninitiated in the concepts of EFTs.

Our paper is simply structured. In the next Sect. 2 we respond to Ellis' critiques, pointing out incorrect statements he makes related to EFTs that lead to wrong conclusions. We make some further comments in Sect. 3 that are not directly related to Ellis' statements related to EFTs, but we feel are important nonetheless in view of the debate on strong emergence. We conclude in Sect. 4.

2 Our Response

2.1 On the Connection Between an EFT and Its Lower-Level Fundamental Theory

Ellis states that EFTs "$\cdots$ are an approximation and are not strictly derived from a more fundamental theory." He goes on to question whether EFTs, because of this limitation, can be used to even defend reductionism. To make his point he pulls quotes from various sources (Ref. [4] in particular) to demonstrate that EFTs often are no better than some ad hoc model of the system in question.

This is Categorically Wrong Effective field theories, defined as we outlined in [1], are often derived from a fundamental theory (we simplify the discussion for the moment, as cases exist, where this fundamental theory is not (yet) known and

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[1] Though weak emergent phenomena can *in principle* be deduced from fundamental constituents, such a deduction may *in practice* not be possible.

even might not be needed). A prime example is non-relativistic Quantum Electrodynamics (NRQED), which is an effective field theory of bound states, i.e. emergent phenomena, in Quantum Electrodynamics (QED) [5]. Here the coefficients of the low-energy effective field theory can be directly matched to non-pertubative resummations (i.e. calculations) of the fundamental theory, which is QED. NRQED and QED share the same symmetries, but what differentiates NRQED from QED is the reshuffling of diagrammatic terms such that operators representing the bound state degrees of freedom are explicit in NRQED. Such a reshuffling of terms results in low-energy constants (LECs) with each associated operator in NRQED. To calculate emergent bound states with fundamental QED requires an infinite number of resummations. Not surprisingly, the number of operators in NRQED is also infinite, but with a defined separation of scales there is a systematic hierarchy in relevance of these operators. One then truncates the number of terms in the EFT to achieve a desired accuracy. The truncation of terms to achieve a desired accuracy can be viewed as an "approximation" of the fundamental theory, but it is *in principle* possible to perform calculations with the EFT to any desired accuracy. Note also that in the fundamental theory there are practical limitations in performing calculations beyond some accuracy, as best exemplified in the tenth order calculation of the electrons anomalous magnetic moment [6], but this is another story.

Coming back to nuclear and particle physics, we admit that an explicit derivation of chiral perturbation theory (χEFT), the low-energy EFT of Quantum Chromodynamics (QCD) which governs quarks and gluons, from QCD is difficult to do, partly because the degrees of freedom of the emergent phenomena (pions and nucleons) are vastly different from their fundamental constituents. However, we stress that it has been shown that the Greens functions of QCD are indeed exactly reproduced by the ones in chiral perturbation theory [7], which leaves us with the LECs, as different theories can in principle lead to the same operator-structure but are characterized by different LECs. In QCD, the LECs of the EFT are very difficult to extract formally, but can, for example, be determined from non-perturbative numerical methods. Equally valid is the determination of the coefficients from empirical data, which is what is commonly done. It is quite amazing that these days some of the LECs can be determined more precisely from lattice QCD calculations than from phenomenology, which is related to the fact that one the lattice we can vary the quark masses but not in Nature. Either way, the principles are the same: The operators in the EFT share the same symmetry as the fundamental theory, and the separation of scales dictates the rate of convergence of the terms and thus how many terms are required for a desired accuracy. The EFT *is derived* from the underlying theory, and *in principle* can calculate observables to arbitrary precision. Clearly, scale separation is an important ingredient in any EFT. This will be stressed at various points in what follows. Another important remark is that chiral perturbation theory is what is called a *non-decoupling EFT*, as the relevant degrees of freedom, the Goldstone bosons, are only generated through the spontaneous symmetry breaking as discussed below.

The notion that EFTs are glorified models with multiple fit parameters (LECs) has been a misconception since its first application in nuclear physics by Weinberg [8]. Another historical example is the abandonment of the Fermi theory of the weak interactions because it violates unitarity at about 80 GeV (at that time a dream for any accelerator physicist). Now we know that it is indeed an EFT, with the breakdown scale given by the mass of the heavy vector bosons, alas 80 GeV. As stated above, the LECs of the theory are not *ad hoc* (the same cannot be said of many models in other areas of physics!) but are directly connected to the underlying theory. Furthermore, the number of LECs per given order of calculation is predetermined, and because of its strict power counting rules, estimates of the theory's *error* due to omission of higher order terms can be made. The same cannot be said for any type of model. The survey by Hartmann [4] completely overlooks these facts and thus gives a very poor and inaccurate description of the efficacy of EFTs. For a survey with philosophical connections, the reader should consider [9]. Note also that EFTs are widely used in all field of physics, in atomic, cold atom, condensed matter, astrophysics, you name it. This shows that EFTs are of general interest as they capture the pertinent physics in a certain energy regime. This, however, does not mean that they are all disconnected, often the reduction in energy and thus resolution leads to a tower of EFTs that fulfill matching conditions in the course of the reduction in resolution. A nice example is flavor-diagonal CP violation, where one starts with a beyond the standard model theory, like e.g. supersymmetry, at scales way above the electroweak breaking and runs down through a series of EFTs to the chiral Lagrangian of pions and nucleons including CP-violating operators, see e.g. [10]. Note that while the scale of supersymmetry is about 1 TeV, the one of the chiral Lagrangian is well below 1 GeV, thus one bridges about 3 orders of magnitude by this succession of EFTs.

2.2 On the Applicability of EFTs to Other Areas of Science

Ellis asks if EFTs can be applied "… to quantum chemistry, where methods such as the Born–Oppenheimer approximation and Density functional Theory (DFT) have been used?"

We see no reason why EFTs cannot be applied here. Indeed, both of these methods are mean-field approximations which, from an EFT perspective, are the leading order term of some EFT [11, 12]. In this view, an EFT description goes *beyond* both DFT and Born–Oppenheimer approximations, since it naturally includes dynamics of quasi-particles (emergent phenomena) above the mean-field approximation. To be more specific, the Born–Oppenheimer approximation shares all the features of an EFT, the light (fast) modes are decoupled form the heavy (slow) ones, all symmetries pertinent to the interactions are included and the proper degrees of freedom are identified. What is simply missing is to set up a power counting in which to calculate the corrections. This has recently been achieved in hadron physics, where the Born–Oppenheimer approximation has been in use for quite some time, but has since been surpassed by EFTs. For example, it was heavily utilized in the context of the bag model [13] leading to various model studies like e.g. by one of the authors [14]. More recently, in the context of heavy quark physics, this was even cast in terms of an EFT [12], which clearly shows that even in quantum chemistry the formulation of an EFT embodying the Born–Oppenheimer approximation should be possible. We point out that in quantum chemistry powerful techniques exist to perform extremely precise calculations like the already mentioned DFT approaches or the coupled-cluster scheme, originally invented for nuclear physics, thus the need for setting up an EFT has been less urgent than in strong interaction physics. However, as DFT in chemistry now also enters the stage to accomodate strong electronic correlations ab initio, this will change in the future. We will return to the topic of the Born–Oppenheimer approximation when we discuss phonons in the next section.

Ellis goes on to ask about the applicability of EFTs to signal propagation in neurons, neural networks,[2] Darwinian evolution, and election results. We admit we cannot answer these questions because some of these systems fall well outside our purview of expertise. We say "some" because one of us, however, has embarked in research in modelling neuron dynamics [17], and in fact we are presently setting up a simulation laboratory at Forschungszentrum Jülich that deals with the application of numerical quantum field theory to complex systems in particle and nuclear physics, solid-state physics, and also biological systems like the brain. Nevertheless, for certain fields, such numerical methods are not yet available or only based on simple modelling, but we do not dismiss the possibility of applicability just because of our ignorance. Ellis, on the other hand, answers with a definite *NO!* because he claims these are strong emergent phenomena.

2.3 Spontaneous Symmetry Breaking and Topological Effects in EFTs

Arguably the most egregious error that Ellis makes, from our point of view, is the statement that EFTs cannot capture spontaneous symmetry breaking (SSB), or explicit broken symmetries in general, and therefore cannot describe the emergent phenomena (which he claims are strong emergent) that ensue from these reduced symmetries. He uses the solid-state example of the reduced point symmetry of a lattice that ultimately leads to the creation of phonons. Indeed, he claims most of condensed matter and solid-state physics is off-limits to EFTs, because much of their emergent phenomena is due to SSB or explicit symmetry breaking.

We point out, however, that spontaneous symmetry breaking, and its ensuing consequences, is not solely relegated to the fields of condensed matter and solid-state physics, despite being the origin of its development [18]. One of the most prominent examples of SSB comes from the non-zero vacuum expectation value (VEV) of the scalar condensate in the QCD vacuum. While the Hamilton operator of the theory has the chiral symmetry $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$, in the QCD vacuum this symmetry is "hidden", that is, the *global* symmetry of $\mathrm{SU}(N_f)_L \times \mathrm{SU}\,(N_f)_R$ is spontaneously broken to $\mathrm{SU}(N_f)_V$.[3] As Ellis correctly points out, this symmetry breaking leads to emergent phenomena, which are gapless long-distance (pseudo-)scalar modes in the theory with reduced symmetry. The number of gapless, or massless, states is given by the much celebrated Goldstone theorem [22], which states that for each generator

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[2] Already statistical field theoretical techniques are being applied to neural networks (see for example [15, 16]). Such theories are amenable to EFTs.

[3] The interested reader might consult e.g. [19–21] for a detailed description of this process and the meaning of these symmetries.

of the hidden symmetry that is broken there corresponds a long-distance massless (pseudo-)scalar particle. These states are more commonly known as Nambu-Goldstone bosons. In QCD these bosons are the three (nearly) massless pions (for $N_f = 2$) observed in Nature. The basis of chiral perturbation theory is exactly this theorem—it provides the EFT of light-quark QCD with its proper degrees of freedom (for the heavy quark sector, a different type of EFT comes into play, which we will not discuss here). The emergent phenomena of pions due to SSB (and coupled eventually to nucleons, which act as matter fields) are the explicit degrees of freedom in this theory. Thus SSB and its subsequent reduced symmetry are not only *captured* but exactly reproduced (as stated above) in this EFT, as well as any other unaffected symmetries.

The same happens when, during the formation of a crystal, whether natural or synthetic, the underlying global Lorentz and Galilean symmetries are spontaneously broken to some reduced point symmetry of the crystal lattice. There are some differences between relativistic and non-relativistic formulations of the Goldstone theorem,[4] but the central point remains the same: The broken generators of the underlying continuous symmetries result in the formation of (pseudo-) scalar Nambu-Goldstone bosons, in this case the so-called phonons. Furthermore, Goldstone's theorem, and the subsequent phonon EFTs based off this theorem, go on to describe the interactions *between* phonons [24, 25]. One can imagine adding electron degrees of freedom as matter fields in such an EFT, all the while ensuring that the electrons respect the relevant point symmetry of the lattice, analogous to what is done with nucleons in χEFT. The LECs corresponding to electron-phonon interactions in the EFT can be constrained empirically or, *in principle*, calculated directly from QED with appropriate boundary conditions. This EFT captures, at its lowest orders, the Born–Oppenheimer approximation for electrons. Once the LECs are determined, the phonons themselves can be integrated out of the theory, which results in an effective electron–electron *attractive* interaction [26] [5].

With regards to Ref. [26] written by Polchinski in 1992, Ellis quotes various passages from this reference that seemingly lend credence to his claim that EFTs cannot be applied to high-T_c superconductors. We point out that this article was written as a series of introductory lectures on EFTs and the renormalization group. The underlying premise of these lectures was that the electrons were approximated as a fermi liquid. Non-fermi liquid behavior, which most certainly underlies unconventional superconductors like high-T_c superconductors, is thus not captured by these EFTs. From an EFT point of view, the separation of scale between the Debye length and inverse fermi momentum becomes less clear as electrons become more strongly correlated in non-fermi liquids. This signifies that the degrees of freedom of the EFTs in Polchinski's lectures are, at the very least, incomplete. Indeed, Polchinski points this out, stating the possibility that other degrees of freedom like anyons, or spin fluctuations, are required in a successful EFT of high-T_c superconductors. Thus

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[4] See e.g. the lucid discussion in Ref. [23]

[5] The analog of this process in chiral perturbation theory χEFT, that is integrating out pions, results in pionless EFT, see e.g. [27].

Ellis' chosen quotes, when taken out of context, belie this point. Though there still does not exist a successful EFT that captures high-T_c superconductors, there has been progress in formulating EFTs for finite-density systems [28] with quasi-particle excitations [29, 30], like magnons [31], as additional degrees of freedom.

Ellis also states that topological effects, prevalent in low-dimensional condensed matter and solid-state systems such as topological insulators and superconductors, cannot be captured by EFTs because they cannot be derived from a "bottom-up" framework. Again this is incorrect. One need only consider the decay process $\pi^0 \rightarrow \gamma\gamma$ and understand how this process is described in an EFT. On classical grounds, the decay of the neutral pion to two photons is not allowed, but quantum fluctuations allow such a decay through what is called the axial anomaly [32].[6] This process is non-local and is captured in an EFT by the Wess–Zumino–Witten (WZW) effective action, first presented in a top-down manner by Wess and Zumino in [34] and later derived by Witten [35] in a *bottom-up* fashion [36]. This action is topological in nature. Its geometrical interpretation relies on the fact that the physics it describes is confined to the surface (where our world "resides") of a five-sphere S^5. The WZW formalism for anomaly physics is universal and thus not constrained to the realm of particle and nuclear physics. Indeed, its lower-dimensional analogs are utilized in the classification of topological insulators and superconductors [37] which Ellis claims is beyond the purview of EFTs. Its application naturally leads to the Chern-Simons effective Lagrangian [38] that accounts for the integer quantum hall effect [39].

Another area of interplay of spontaneous symmetry breaking, topology (here the one of the QCD vacuum) and EFT is axion physics. The QCD vacuum exhibits a non-trivial topology, with different sectors given in terms of an integer winding number. Instanton effects allow for transitions between these sectors, which are, however, exponentially suppressed [40]. Another ramification is the appearance of the so-called θ term in QCD, which leads to CP violation. However, the value of the θ parameter accompanying this term is smaller than 10^{-11} as deduced e.g. from the experimental upper limit on the neutron electric dipole moment [41]. This constitutes the so-called "strong CP problem". An elegant solution was proposed by Peccei and Quinn [42], who elevated this parameter to a dynamical variable related to a new U(1) symmetry, nowadays called $\mathrm{U}(1)_{\mathrm{PQ}}$. This symmetry is spontaneously broken with the appearance of a Goldstone-like particle, the axion, whose interactions with light and matter can be cast into an EFT. For a recent high-precision calculation of the axion-nucleon coupling, see [43] which also contains references to earlier work in this important area of beyond-the-standard-model (BSM) physics, that has become one of the major playgrounds for EFTs.

Thus, EFTs can indeed capture both SSB and topological effects, and therefore their subsequent emergent phenomena.

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[6] A satisfactory description of this anomaly, and its ramifications goes beyond the point of this article, but a nice overview is given in [33] for the interested reader.

3 Additional Comments

3.1 Spontaneous Symmetry Breaking at the Micro Versus the Macro Scale

Ellis differentiates SSB that occurs at the *micro* scale and SSB that occurs at the *macro* scale. He labels them SSB(m) and SSB(M), respectively. As examples, he mentions the Higgs mechanism as a source for SSB(m) while the SSB that leads to crystallization is an example of SSB(M). Presumably the fact that the crystal exhibits long-range order at the macro scale is the reason why it falls under SSB(M).

Ellis uses this distinction to strengthen his arguments for strong emergence. We question the correctness of it, however, and thus the basis for such a classification. The mechanism for any SSB, whether in QCD due to the scalar condensate or in cyrstallization due to spontaneous nucleation, originates at the micro, or local, scale, but the symmetries that are broken are *global*. This means that the ramifications of the broken symmetries extend into the macro scale. How do we know this? In the case of the crystal we can definitely observe the long-range order of the crystal's point symmetry, as Ellis correctly points out. In QCD, on the other hand, we observe pions *everywhere*. Another example is the already mentioned Higgs field, that is generated in SSB at the electroweak scale and penetrates the *whole universe*.

Therefore, there is no distinction between SSB(m) and SSB(M). There is only just one SSB. Ellis' ensuing arguments based off this distinction are thus non sequiturs.

3.2 We Are Making Progress

In our original paper we challenged proponents of strong emergence to make a scientific prediction based off strong emergence that could be tested. Our goal here was to apply Popper's *Fasifiability* criterion [44]. Ellis accepted our challenge and answered:

> Neither LM (Luu & Meißner) nor any of their nuclear physics or particle physics colleagues will be able to derive the experimentally tested properties of superconductivity or superfluidity in a strictly bottom up way. In particular, they will be unable to thus derive a successful theory of high-temperature superconductivity.

He argues that we will never be able to do this since these phenomena are strong emergent. Let us be the first to admit that we (Luu & Meißner) have not derived such a theory since the publication of his challenge, and even with the amount of hubris that we already have, we would never claim that we ourselves will ever do so. We stress, however, that our inability to derive such a theory does not provide confirmation of strong emergence.

But we will point out that some of our colleagues of similar "ilk" have made progress in deriving bottom-up theories in areas that Ellis claims belongs to strong emergence. For example, in [45] a relativistic formulation of the fractional Quantum Hall effect was derived. Another example is the *ab initio* calculation of the so-called

Hoyle state in ^{12}C, which is not only making life on Earth possible, but has also been a barrier for nuclear theory calculations until 2011 [46]. Such problems were considered intractable but are now soluble due to advances in high-performance computing and the ingenuity of our colleagues. Theoretical physics requires optimism rather than a "can't do" attitude.

4 Conclusions

Conclusions based off misconceptions of EFT have historically lead to erroneous physical claims and added confusion about its ability to explain emergent phenomena as well as its connection to underlying theories. Over time many misconceptions have diminished, but unfortunately some still persist and one must remain vigilant to correct them and the conclusions based off them. In this article we corrected the misconceptions of EFT that Ellis uses in his arguments for the case of strong emergence.

Admittedly, because our imaginations are still too limited (and may be bounded!), we rely heavily on Nature to tell us how to define our physical boundaries that lead to exotic emergent phenomena, especially in cases where the environment is synthetic, like superconductivity in Yttrium-Barium-Copper Oxide. A proponent of strong emergence would say we "cheated", we "peaked" at Nature to tell us what to do. But we make no excuses for this since physics is an *experimental* science after all!

A common argument that Ellis makes related to strong emergent phenomena is that such phenomena would never be realized from a bottom-up procedure because the environment which enables the phenomena does not occur naturally in Nature, rather it is synthetic. He refers to superfluidity and superconductivity in solid-state physics, which he adamantly claims are strong emergent. If that is the case, then how do we classify neutron superfluidity in the crust of neutron stars,[7] or quark-color superconductivity predicted to occur deep within dense compact stars [47, 48]?[8] The physical mechanisms that underly these phenomena are analogous to those in solid-state systems, but clearly here the environment is not synthetic. So are they strong emergent because they share the same physical mechanism, or are they weak emergent because we predicted the phenomena ourselves? If we insist that the solid-state examples are strong emergent, while the others are weak emergent, than doesn't that imply that the synthetic materials are special? And by extension, that we humans who created the materials are special? Such hubris is best avoided, even by us.

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[7] Here we differentiate neutron superfluidity that occurs in neutron star crusts where crust nuclei provide a lattice with phonon interactions, as opposed to the superfluidity that occurs deeper in the star.

[8] The same could be asked about superconductivity in naturally occurring mercury and lead.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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