Foundations of Science
https://doi.org/10.1007/s10699-022-09868-8

On Why Quine's Ontological Relativity Requires Reconsideration

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Accepted: 11 August 2022

Abstract
We aim to show from a new perspective that Quine's ontological relativity, based largely on his so-called "proxy-function argument", falls short of being a rigorously coherent philosophical conception, as it exhibits significant formal defects. This new perspective enables exposing the shortcomings of Quine's position and suggests a possible reformulation of the original position. Moreover, we argue that his ontological relativity is inconsistent with the empirical data associated with some of our best physical theories, such as quantum mechanics. We refer to fundamental concepts of philosophy and the foundations of mathematics in order to clarify our critique of Quine's position concerning the relation between formalized theories and both what we can know about the real world and how we come to know it.

Keywords Quine's ontological relativity · Ontology of scientific theories · Proxy function · Ontological commitments · Empiricism

1 Introduction

Take a formalized scientific theory such as, for example, the arithmetic of natural numbers. It is intended that this theory should pertain to natural numbers, describing their properties and relations between them. Quine argues that such a theory will not have any distinctive ontology, so it is impossible to say what particular objects the theory is talking about. Now imagine particular objects, of whatever type you will, but equinumerous to, for instance, the natural numbers in the sense of von Neumann, Frege, Zermelo, or Peano arithmetic (PA). According to Quine, if we "replace" objects of one kind with objects of another using a special "proxy function", then the theories under consideration will have no formal means for perceiving and describing any difference between these objects. The ontology of a given theory is thus relative. This is, leaving (temporarily) aside many other relevant

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Published online: 02 November 2022

details, the intuitive import of Quine's important and influential set of views on the relations between formalized theories on the one hand, and both what they refer to in the real world and how they do so on the other. We aim to show that Quine's ontological relativity, based largely on the so-called "proxy-function argument", is not a strict and coherent philosophical conception, as it possesses significant formal flaws. In our analysis, we do not refer to an absolute determination of an ontology, which Quine considers impossible, but to its relative determination: for example, with reference to an appropriate background theory. Such a relative determination was explicitly postulated by Quine. Moreover, we shall argue from a new perspective that Quine's ontological relativity is inconsistent with the empirical data associated with some of our best physical theories, such as quantum mechanics. This new perspective, which uses the concept of substitution and substitutional models described below, enables exposing the shortcomings of Quine's position and suggests a possible reformulation of the original position. We will invoke fundamental concepts from philosophy and the foundations of mathematics in order to clarify our critique of Quine's position on the relation between formalized theories and both what we can know about the real world and how we come to know it. Our focus, however, will be on the ontology of science.

Quine's argument involves constructing a philosophical theory that is "holistically coherent" with empiricist assumptions: this sophisticated kind of coherence provides the main justification for his views. Thus, our first and basic task in these considerations will be to check *whether Quine's ontological relativity is really consistent with basic empirical and naturalistic suppositions and whether it is precise and consistent from a formal point of view*.

In "General Comments on Quine's Analytical Empiricism and Ontological Relativity" section below, we recall the main determinants of Quine's philosophical position insofar as these are relevant to our considerations. We cannot analyse here all of the theories and concepts relevant to Quine's position—and, in particular, we will not analyse the details of his theory of the indeterminacy of translation or its relationship to the inscrutability of reference. These matters have been extensively analysed elsewhere [cf., for instance, (Hylton, 2004; Kirk, 2004; Romanos, 1983), and Quine's works (1976a, 1977, 1981, 1983, 1986, 1992a, 1992b, 1995a, 1995b)]. The basic forms taken by translations in the context of formalized theories are ontological reductions and the use of a proxy function. In "Critical Remarks on the Proxy-Function Argument" section, we then consider those details of Quine's proxy-function argument that are fundamental to his account of ontological relativity. We point to formal deficiencies within this argument by referring to the concept of interpretation in Tarski's sense, and also by referring to the concept of intuitive models, and substitution operations within these models.[1] Quine's argument turns out to be imprecise, informal, and intuitively indeterminate, with serious negative consequences for his ontological relativity. In particular, we argue—contrary to Quine—that a certain ontology for extensional formalized theories is absolutely distinguished by its elementary simplicity and consistency with empirical data. In "Physics, Experiments and Ontological Relativity" section, we discuss various possible descriptions of such a minimal ontology and give one example of its exact formalization (Minimal Set Theory, MST). We present an example of the incompatibility of Quine's conceptualization with one of our best contemporary scientific theories, namely quantum mechanics, in the context of the discussion concerning

[1] See our more detailed description elsewhere (Król & Lubacz 2021), where we deploy substitutional models for the purpose of falsifying quantificational ontological pluralism.

the so-called "hidden parameters". It follows from this that ontological relativity cannot be applied *in an unrestricted way* to all scientific theories and that it can be empirically falsifiable in some contexts.

2 General Comments on Quine's Analytical Empiricism and Ontological Relativity

One can point to a number of elementary observations that *seem*, intuitively, to justify the plausibility of Quine's relativistic philosophy. Almost all expressions, including sentences of natural languages, can be used (even not necessarily by different users) in many different specific situations, at different times, etc., and yet retain their syntactic identity together with their referential capacities. "My name is John", "this is my mother", etc., do not designate any particular person or situation, do not refer to anything specific, and can be used by different people and across a variety of different contexts. However, they cannot be used in *any* type of situation. Similarly, we can describe a certain situation giving the same information, but in a very different manner and linguistic form. A situation does not designate uniquely its description, nor does a certain set of sentences (a "prototheory") designate the only possible mode of reference to objects.

According to Quine, it is quite the same with mature scientific theories and, in particular, with formalized mathematical theories. This carries consequences for the so-called "Duhem-Quine thesis" [cf. (Quine, 1998)], and for Quine's ontological relativity: the creator of a theory may have, in their head, their own individual ontology or privately intended model, but we do not have any extra-theoretical or extra-linguistic and objective access to reality itself, such as would be "the same" for all users of a given language or theory. In consequence, only the linguistic forms are the same for everyone, even though they do not refer to the same "things" [cf., for instance, (Quine, 1969a, 1976a, *passim*].

For the purposes of this discussion, we shall consider Quine's position in its entirety and examine its internal coherence in some detail using only the means permitted by his philosophy. To clarify the key elements of that philosophy that are relevant to the definition of ontological relativity we first need to consider his commitment to scientism. Quine treats all ontological issues as issues belonging to natural science. This conviction follows from the assumption that the only access to the ontology of the so-called "real world" is provided by the mathematical and natural sciences. The result of this *decision* is that Quine is obliged to deny the objective existence of the whole world given to us in *direct experience*, along with the very fact—and even the mere possibility—of the existence of such direct experience. If something is given directly, Quine considers it irrelevant, as he holds that it is only given in a purely subjective way, and is a subjective product of our neural apparatus [cf., for example, (Quine, 1969b, 1995a)]. Man's only objective epistemic ability consists in the ability to speak and use language.

What emerges from Quine's scientism is not so much an appreciation of the achievements of science as we actually know it, but an implicit demand contained in his philosophy for the reform of science by means of its formalization: that is, the "scientification" of all science and philosophy. Without this healing procedure, science has no precise ontological commitments and is as impervious to ontological analysis as any informal conversation. At most, certain fragments of knowledge develop to the point of attaining the status of science: e.g. certain formalized theories of physics, logic, pure mechanics, or mathematics. What follows from this is that existing scientific knowledge not formulated in a first-order theory will be incapable of having ontological referents, or will be an instance of defective empirical science; cf., e.g., (Quine, 1969a, 1976b).

Quine does not hold that every single sentence of formalized scientific theory must contain a direct empirical reference; science is rather a "web (or field) of beliefs" that comes into contact with empirical data only "at the edges"; cf. (Quine, 1969a, pp. 78–79). This is a manifestation of his holism—one that seems to follow in particular from his conviction that if physics uses some formalized first-order mathematical theory (or a fragment of it), then the mathematical theory itself need not have any direct empirical reference, and its empirical character will be revealed by its use in certain empirical and/or theoretical contexts.

In our view, Quine does not clearly distinguish between the *ontological* commitments and the *empirical* commitments of a theory: he identifies these commitments and treats the empirical reference of a theory as precisely expressible by his theory of quantificational criteria of ontological commitment. Moreover, he does not clearly distinguish the ontological commitments of pure (uninterpreted) mathematical theories from the ontological commitments of these theories as used in certain empirical or "impure" theories. Instead, he merely argues that any theory with some variables that has non-eliminable sentences of the form "there is an x such that…" has ontological commitments—even "pure" mathematical theories. This is what the possibility of their use in "more empirically committed" contexts further indicates. Putnam is more precise in this respect (cf. the Quine-Putnam indispensability argument).[2] To be sure, Quine figures in the name of this argument, but it is not entirely clear whether he agrees with Putnam throughout, or whether he simply recognizes that any pure mathematical theory can be used in some empirical and scientific context, and thus that even pure theories, meaning empirically uninterpreted ones, have an object of reference, indicated by the existential quantifier. However, judging by the examples Quine gives—e.g. concerning "pure" arithmetic of natural numbers [cf., for instance, (Quine, 1976b) at many points, such as p. 211]—he regards such theories as having the same ontological commitments as scientific theories. Therefore, all (quantifier-expressible) ontological commitments of formalized theories are equally valid, and in this sense, Quine votes for ontological monism—i.e. the idea that there is only one way of being. Thus, more particularly, he does not distinguish between different modes of existence, such as abstract and real: existence is univocal, as van Inwagen would say; cf. (2009, p. 482). If natural numbers in a given empirical (or impure) theory are needed in the same way that electrons are needed, then one must assume that the theory obliges us to accept the existence of both numbers and electrons. Whether something is needed (i.e. exists) is uniquely indicated by the *non-eliminable existence* of sentences of the form "there is an x such that x is a natural number", and "there is an x such that x is an electron".

Nevertheless, given that not all sentences of scientific theories have an empirical reference, we are apparently supposed just to assume that sentences of the form "there is an x such that …" will have such a reference in some theories. Moreover, Quine often speaks of the "empirical consequences of whole theories" [cf. (Quine, 1969b)], but does not indicate any specific linguistic forms that correspond to them, so they all seem to have the same form: namely, "there is an x such that …".[3] One could also consider the points of empirical contact with the world of a given theory to be provided by the predicates "of an empirical

[2] See (Putnam, 1971). Note that Putnam changed his views soon after this.

[3] For example, it may follow from a given theory that there will be a centre of equilibrium for some system, or that there will be some body possessing a temperature higher than the average one for the system, etc.

nature" used within it: e.g. by "x is a dog", or "x is white". However, these must still be used in the above context—i.e. with an existential quantifier. As we explain below, such predicates *alone* mean nothing "in reality", or from the extensional point of view preferred by Quine.[4]

According to Quine, only some regimented theories are ontologically useful: i.e. only certain theories have ontological commitments and thus, we suppose, also a precise empirical reference. In general, if "To be is to be the value of a variable", cf. (Quine, 1948, p. 32), then the only theories that can have ontological references will be those in which there are free variables bound by an existential quantifier. Therefore, these will be some first-order theories that have an infinite model. Theories with finite models of named objects have no *direct* ontological reference since any sentence using a quantifier—be it universal or existential—can be replaced by a finite alternation or conjunction of certain atomic sentences that do not contain bound variables. Quantifiers and variables are thus eliminable from such theories; cf. (Quine, 1968, p. 209). Such theories do not have ontological references that are self-defined by their own linguistic means, because the proper name of something cannot indicate something exactly (in a one-to-one way). Thus, according to Quine, such theories are not themselves ontologically committed to anything. We can, nevertheless, associate with a constant's denoting of concrete individuals a certain ontological interpretation ("decoration"), using some "background theory"; cf., e.g., (Quine, 1968, p. 209). Such a theory, however, will in principle not be unambiguously defined, and there are infinitely many possibilities for such an ontological interpretation: Many different individuals can be called "John", and this name alone does not indicate any particular "John".

A similar problem pertains to complete and decidable theories in which we know how to eliminate all quantifiers. Moreover, such theories may have a *mechanical* procedure for the assignment of truth values, and if that is so, then we are not interested in the values of variables and quantifiers (cf. *ibid*.). Only *some* theories with infinite models have ontological commitments. It seems to follow from Quine's assumptions (assuming that the majority of objects known to us from *direct informal* experience exist only in a finite number) that the only objects known to us that form infinite sets are certain abstract mathematical objects and that only theories that speak about abstract objects have an object reference—*ergo* only abstract objects do exist. But having an infinite model is not yet sufficient for a theory to have any object reference of an ontologically significant kind. In particular, theories that speak of an infinite number of certain objects, each of which has a particular individual name, are not ontologically committed. An example of such a theory is the Peano arithmetic of natural numbers (PA), where each of an infinite number of natural numbers has an individual name: 0, S0 (or 1), SS0 (or 2), etc. This theory is not decidable. For such a theory to have ontological commitments, its quantifiers must be employed in a *referential* manner, and so not as so-called *substitutional* quantifiers.[5] According to the latter interpretation, "$\exists x\ \phi(x)$" is true iff, for some singular term "*t*" of the language, "$\phi(t)$" is also true. Thus, "$\exists x\ \phi(x)$" is equivalent to "$\phi(t)$". In the case of an objectual (referential) interpretation, however, the formula "$\exists x\ \phi(x)$" is true iff there is *an object t* (not only a term/name in

[4] For example, Quine (1969a) denies the existence of a realm of meanings (cf. analyses of the "museum myth of meanings") while accepting the existence of extensions and designations of a concept. Cf. also (Quine, 1995a).

[5] This remark applies not only to theories that have names for each individual, but in general to all theories that aspire to have ontological commitments on the basis of their own structure, and not just through ontological interpretation at the level of a background theory.

the vocabulary of the theory) in the model of this theory which satisfies the *open* formula "$\phi(x)$". Therefore, in the objectual case, a given object in a model satisfying the formula "$\phi(x)$" need not have a name in the language of the theory. There may be more objects than names. For instance, not every real number has a name in the analysis of real numbers, because the set of real numbers is uncountable, and we only have a denumerable number of names. Ontologically, the difference between the two approaches to quantification is that, in the case of substitutional quantification, we can speak of certain non-existent objects which nevertheless have a name (e.g. Pegasus): so we can name them, and some sentences containing these names of non-existent objects are nevertheless true (for instance "Pegasus has wings"). In the case of objectual quantification, the object in question must be some individual in the model of the theory, even if it has no name in the theory. For example, we can then say that the general sentence "for every x, $\phi(x)$" entails the sentence "there exists such an x that $\phi(x)$", and even if we cannot point to such an x specifically (i.e. by name), we are obliged to assume its existence. Meanwhile, in the substitutional case, "variables are placeholders for words (only) of any syntactical category"; cf. (Quine, 1968, p. 209). However, even in the substitutional instance, as in the above case of a finite theory, one can assign it an ontological interpretation within the corresponding background theory—though only if, in such a theory, referential quantifiers are used.

With regard to the question of the ontological commitments of finitary and infinitary theories, it should be noted that Quine does not consider all possibilities. Although a theory can have an infinite model, it can also have finite models. It should also be noted that it does indeed follow from Quine's favorite (downward) Löwenheim-Skolem theorem that the existence of even just one infinite model of a given first-order theory will entail the existence of an infinite denumerable model, but the existence of a finite model will not. Similarly, it is not the case that if any first-order theory has a finite model, it cannot have an infinite model. Therefore, according to Quine's criteria, ontological commitments are both present and absent.

So let us assume that we have some first-order theory with referential quantifiers, and with a denumerable infinite model: e.g. PA. Of course—and independently of Quine's philosophical claims—this theory will have various non-isomorphic models, including a countable intended (standard) model. From a purely mathematical point of view, PA (or any other such theory) does not explicitly designate to which model its purely syntactically specified form refers. To take another example, the axioms of group theory apply just as well (i.e. correctly and efficiently) to a trivial one-element group as they do to a group with an infinite number of elements. The reader should note that we have here an example of a theory possessing both finite and infinite models.

3 Critical Remarks on the Proxy-Function Argument

Our considerations in "The Proxy Function and the Mutual Interpretablity of Formal Theories" section are confined to an analysis of Quine's definition of "proxy function" and the related argument for ontological relativity in the light of Tarski's notion of the interpretation of a theory formalized in another one. In "The Proxy Function Viewed in the Light of Substitutions and Substitutional Models" section we use the concepts of substitution and substitutional models introduced in this context to point to the formal flaws in Quine's account of ontological relativity.

3.1 The Proxy Function and the Mutual Interpretability of Formal Theories

Central to the definition and construction of Quinean ontological relativity is the notion of a proxy function. Usually, this is explained by giving an example concerning only one predicate, say P1, in a given theory T1 referring to certain objects from the set/class D1 (e.g. of elephants). Then, a second theory T2 is indicated, in which we can find a predicate R2 (corresponding to P1 in T1), and whose objects belong to the set/class D2 of objects of the second theory (e.g. ants). A proxy function F will be a function that assigns, usually mutually unambiguously, and in a one-to-one manner, objects from D1 to objects from D2, and to each predicate P1, P2, …, from T1, a corresponding predicate R1, R2, …, in T2, such that P1x is true in T1 iff F(P1)(Fx) is true in T2: i.e., more precisely, $T1 \vdash P(x)[\mathbf{a}/x] \Leftrightarrow T2 \vdash R(x)[F(\mathbf{a})/x]$. Obviously, if $\forall x.P1x \rightarrow P2x$ in T1, then $\forall x.F(P1)x \rightarrow F(P2)x \equiv \forall x.R1x \rightarrow R2x$ in T2. Thus, in fact, and from a purely syntactic point of view, we have two theories that have exactly the same number of corresponding sentences, predicates, relations, functions and constants (or else the first theory is embedded within the second one), but they are named differently (denoted by different "letters"), and in the second theory there is a domain (or, in the given model of T2, only a subdomain) D2 equinumerous to the basic set of elements of a model of T1. In terms of model theory, if we assume that the proxy function is one-to-one, then these theories will be identical. If the proxy function is not one-to-one, then T1 will be a subtheory of T2.

Quine's so-called "proxy-function argument" indicates that with the use of a proxy function we can freely change the ontology of a given theory; cf. (Quine, 1969a, 1976a). The ontology of T1 is determined by the full-blooded objects of D1—i.e. objects determined, for instance, in the relevant background theory—and it can be replaced *in an innocent way* (i.e. without changing the logical form and truth conditions of the given theory) by the ontology of different full-blooded objects from D2. T1, thanks to the proxy function, can start talking about objects from D2 and not, as before, from D1. Thus there is no absolute ontology, but only theories and an infinite number of possible ontologies. "Arithmetic is, in this sense, all there is to number: there is no saying absolutely what the numbers are; there is only arithmetic"; cf. (Quine, 1969a, p. 45).

It should be emphasized that the idea of ontology change using a proxy function is reliant on some extremely strong ontological and mathematical assumptions. Quine wants to show how a given theory, e.g. PA, which talks—in his view—about certain specified objects, can speak about certain other objects that are specified in some other theory (e.g. ZFC), and vice versa. Such new objects are not recognizable by the theory in question, so he postulates that this whole operation of object-changing should be described in some stronger (or at least equally strong) theory—a background theory. This means that the proxy function is based on the relationship between the models of these theories, rather than that obtaining directly between the theories themselves. He, therefore, assumes that these models are distinguished in some way. This distinction between them seems to be—at least at the beginning—some kind of extra-linguistic one, as (1) the theories in question cannot themselves distinguish between their models, and (2) a background theory cannot distinguish between its own models either. As will be shown below, it seems possible that the proxy-function argument rests on our assuming the absolute and extra-linguistic existence of certain objects.

At least in some cases, Quine (and most of his followers) treats both theories and background theories as if they were talking about "full-blooded" objects with certain characteristics, and as if the models of these theories were made up of "full-blooded" objects of a certain kind: e.g. numbers, electrons, stones or elephants. Yet he never *explicitly* constructs such a formal theory *of full-blooded objects*. Intuitively, the problem of the use of such "real" objects seems very simple and obvious. It also seems that some formalized physical theories say something *directly* about "atoms" and "electrons", at least in the opinion of most physicists.[6] If not, then a related Quinean background theory will be required: i.e. a theory in which it will be possible to explain, precisely and in formal terms, what "electrons" and "atoms" ontologically are. Note also that physicists usually use some mathematical objects such as natural numbers "as if" they were definite and independent from formal arithmetical theories and formal descriptions—e.g. as in QM without the formal use of PA. At the same time, it is well known that formal theories—especially first-order theories—can have different, non-isomorphic models. So it is surely worth asking what the significance of such a Platonist stance could be for physics itself.[7]

Nowhere did Quine give any regimented description of how background theories and proxy functions should be constructed. The only strict examples are the mathematical "reductions" he cites: e.g., pure and impure numbers (cf., for instance, (Quine, 1964, p. 209)), von Neumann's definitions of natural numbers, or Frege's, or Zermelo's or Gödel numbers (cf., for example, (Quine, 1964, p. 211; 1968, p. 197)). In fact, the procedures for constructing background theories and proxy functions are—in most cases—very complex, and it is difficult to furnish a single common method for constructing them. However, it is worth noting that the "mathematical reductions" Quine refers to are interpretations in Tarski's sense—albeit only in some situations. (We shall explicate this point in due course, at the end of the present section.) This lack of precision represents, we think, a significant flaw in Quine's argumentation. In order to substantiate this thesis, however, we will first need to define a few auxiliary terms.

Let us first briefly remind ourselves of the notion of the interpretability of a theory S1 in a theory S2. We will only give an intuitive description of this concept here. For this purpose, we must recall something we have described more precisely elsewhere (Król & Lubacz, 2021). There are different possible definitions of this concept: Tarski et al. (1968, pp. 20–22) defined it in a syntactical way (cf. also (Friedman, 2007, 2009, 2012)), but one can also do so using models—i.e. in a semantical manner. The details are rather well-known and the reader can find them in the works indicated above.

Thus, from an intuitive point of view, S1 is interpretable in S2 when one is able to extend S2 with definitions of all relations and functions present in S1, but making use only of formal means accessible in S2. When S1 is interpretable in Tarski's sense in S2, one can speak about objects of S1 in S2. However, from the formal point of view, S2 should be enriched with the corresponding translations of all functions and relations present in

[6] The same goes for the intended models of formalized arithmetic: i.e. it was intuitively obvious to everybody, as well as unproblematic, that these theories were devoted precisely to "the natural numbers". Then, after the discovery of non-standard models of arithmetic, it became clear just how many formal details had been previously unaccounted for, and how much of a simplification it had been to intuit the theory as applying only to the natural numbers.

[7] We characterize this as "Platonism" because physicists (usually) treat numbers as if they were singular objects existing independently and apart from formal theories or models. The elimination of this kind of Platonism in physics may well prove important—for example, in the context of recent applications of mathematical models to quantum gravity; cf. (Król, 2004,2005a, 2005b,2008). However, model-theoretic relativism is not the same as Quine's ontological relativism.

S1, but expressed only in the language of S2. Let us refer to such an enriched S2 as "S". For example, in three-dimensional Euclidean geometry, one can interpret two-dimensional plane geometry and one can describe all the properties of a plane surface known to planimetry. In the same way, the von Neumann definition, or the Quinean "reduction" of natural numbers to some sets in ZFC, is an interpretation of PA in ZFC in Tarski's sense.

Note that the S-theory described in the above definition is the only example of a precisely defined "background theory" that Quine speaks of on multiple occasions. S also allows for a precise descriptive account to be given of his proxy function. However, as explained below, the two descriptions, Tarski's and Quine's are not equivalent.

The second concept needed for a precise grasp of Quine's position turns out to be that of substitutional models and substitution operations. (See the next point in this section.) However, before we proceed to the definition of these, we must draw the reader's attention to the fact that in mathematics only some *pointwise models* are in use. In *pointwise models*, the objects of a formal theory are "points" with no other qualities and without any internal structure—or some sets of such "points". The possession of a property is modelled by the set-theoretic relations obtaining between these unqualified points: e.g. sentences of the form "x has the property R" come out as instances of the one-place relation R(x), properties obtaining between two points as R(x, y), etc. "To be an Elephant" means "to be an element of a set of Elephants", not "to be something with a trunk". Thus, in the case of *formalized* theories, we never meet directly any such non-point-like objects, either in an initial theory or in a "background theory". Take an example of Euclidean geometry, in either Hilbert-Ackermann's or Tarski's version; cf. (Hilbert, 1950; Tarski, 1959). We encounter only points, sets of points, sets of sets of points, etc. Models and languages in *extensional* theories, which Quine prefers, are simply some point-like algebras: i.e. atoms, the empty set, sets composed of the empty set, set of sets composed of the empty set, etc.—or, at any rate, something like that. We only decorate these "points" externally and informally with certain additional "real" (intensional) properties not directly described by the given *extensional* theory; for instance, one can imagine some extensional set as a sphere.

The point-like models are therefore distinguished by minimalist requirements imposed on properties of the objects in the model of some theory. Proxy function does not change the *minimal* ontology in any way. Proxy function shows how to move from one *intuitively* "decorated" theory to another. Strictly speaking, usually only the names are subject to change. The only real change in the ontology is that, for instance, "points" representing numbers in PA can be replaced by certain sets of points, e.g. in ZFC: i.e. there emerge some additional properties pertaining to them (i.e. points). For example, you may now ask whether the number 7 belongs to a set called "number 9" (something which, in PA, is meaningless). If, in PA, we replace the numbers in the intended model with a "denumerable ontology for the elephant", then it can be said that the number 7 has a tail and a trunk. Similarly, there is no such thing as formally defined and full-blooded "numbers" in PA, or time–space areas in set theory. This issue becomes clearer when one moves to its equivalent description in category theory, where one can describe the concept of a set without any concepts of the internal structure of sets: i.e. without the concepts of "element" and the "∈"-relation; cf. (Lawvere, 1964). Sets are points in a category and possess only external properties.

On the other hand, from a *pragmatic* point of view, the mathematics of mathematicians, logicians and scientists is not a pure, uninterpreted formal game. They usually work with *intuitive models* and objects that possess some extra-formal properties: i.e. not only unqualified extensional "points", but also "more real" entities *not formally described* by the theories in question. It is possible to formulate a thesis (or at least a hypothesis) to the effect that a mathematician always works in a world of ideas that are somehow concretized, and concepts somehow intuitively interpreted. More precisely, one may claim that a mathematician cannot work in a world completely free of and devoid of "intuitive surroundings". For instance, we usually use the "full-blooded" letters "x", "y", etc., as elements of the alphabet (or language) of a formal theory. Such "letters" have a definite "shape" which is not precisely and formally described, yet it remains a fact that such objects prove useful in practical thinking. These informal "additional properties" not described by any formula of a given language of a given theory (usually) turn out to be inessential in formal proofs, as we never explicitly use any such extra-formal properties in formal proofs. For example, even though there is no definition of "spatial separation" in, say, ZFC, one can imagine or think of a set as a collection of "dots" separated by some spatial distances. In certain applications of formal theories (cf. the next section) the import of such an "intensional decoration" is responsible for the emergence of some additional empirical properties.

In order to make such a "substitution" of real objects for mathematical ones possible, real objects must be endowed realistically, or even just intentionally, with certain *minimal ontological* properties: they must be treated as (or just apprehended, conceived, and taken by implication to be) *separate (or distinguishable) individuals* (even if only mentally), and as preserving their ontological identity throughout the substitution operation. This means that the possibility of carrying out the whole operation without a formal description (e.g., without an explicit background theory) of certain real objects points to their "absoluteness" (in the Quinean sense). Quine, however, being a relativist, himself denies this. Yet the data of *direct intellectual* experience, and the mere description of what actually happens, stand here in stark opposition to Quine's a priori analyses—as will be clear from the justification of this assertion given below.

3.2 The Proxy Function Viewed in the Light of Substitutions and Substitutional Models

Our claim will be that substitutional models can be viewed as providing a necessary formal clarification of Quine's theory. We shall briefly present the main intuitive ideas concerning substitutional models and substitutions,[8] before illustrating how one can make use of them in a formal description of Quine's proxy-function argument.

Consider transformations between formal theories *in mathematics* and substitutions of objects from one mathematical (or formalized) theory for ones defined in another. We refer to such substitutions as *intuitive mathematical models*. There are two basic kinds of intuitive model in mathematics: models consisting of points without any internal structure and models consisting of *qualified objects*. The key idea when it comes to describing intuitive models in formal terms is the notion of *substitution* and *substitutional models*.

Suppose we are presented with some formal theory T1, and that the elements of the model of such a theory are some "unqualified points", or sets of such points, etc. T1 might be, for instance, ZFC, ZFA, PA, Euclidean geometry, group theory, linear continua, etc. Suppose, next, that every such point (or only some of them) is replaced or *substituted* by an object defined formally and present in the model of a second theory T2. For example, every point on a line is substituted by a linear algebra, a group, a circle or a whole model of ZFC or PA, etc. In effect, every unqualified (or "minimally" qualified in T1) point becomes

[8] We offer a more detailed description elsewhere (Król & Lubacz, 2021).

an object that has some additional internal properties defined formally in T2 but "invisible" to the formal tools of T1. Such substitutions of formal theories are *examples* of intuitive models in mathematics. The effects of many substitutions can be easily comprehended intuitively. For example, the substitution of every point of a Euclidean planar surface by a straight line can yield a planar surface, a 3-dimensional space, or some other objects. Even Tarski's concept of *truth in a model* implies that such a definition of truth from a formal point of view is an intuitive "decoration", as the relations it captures formally concern only certain algebras and morphisms; cf. (Rasiowa & Sikorski, 1963, Chapter VI and VII, pp. 354–362).

There are many possible types of substitutions, together with the corresponding substitutional, "intuitive" models: e.g. local substitutions that may be finite, infinite, countable, singular, etc., as well as global homogeneous or non-homogeneous substitutions, self-substitutions, and many others. Substitutions can be iterated, re-iterated, etc. For instance, in "local substitution", only one "point" or some "points" will be replaced by objects taken from the other theory, whereas "global substitution" refers to substitution of such objects for every "point" in T1. "Homogeneous substitution" means substitution of the same kind of objects, meaning that every point is substituted by the same type of object, while "non-homogeneous" designates a substitution of different types of objects taken from a single theory or from several different theories. Obviously, such substitutions can, in principle, be homogeneous or non-homogeneous, locally or globally. Self-substitutions are replacements of the points in a given theory with objects defined in that theory. Substitutions also make it possible to replace, say, the empty set with the entire model of ZFC or an infinite class of objects; cf. (Król & Lubacz, 2021).

The informal conception of such substitutions seems quite clear and easy to comprehend. However, there is no one universal method for formally describing the resulting substitutional theories and corresponding substitutional models. With respect to one and the same kind of substitution, there result many different possible theories. Such a formal description of it may involve certain standard concepts from model theory and category theory. In the next section, we describe in more exact terms one example of such a possible formal description in category theory.

To show the complexity of the formal description of such substitutions, let us consider now an example of global homogeneous substitution. The elements of a model of some theory T1, in which we substitute several items or copies of objects described in some other theory T2, can be used as a class $\boldsymbol{I}$ of indexes over which one can place objects (or even whole models) of T2. Thus, to every global homogeneous substitution corresponds a bundle of sets (or models) over the base space $\boldsymbol{I}$, and the stalk space **A**, containing the objects which we substitute for every point from $\boldsymbol{I}$. Then, in the usual way, we can construct a category **Bn(I)** of bundles over $\boldsymbol{I}$, which is a comma category **Set**$\downarrow\boldsymbol{I}$, and thus **Bn(I)** is a topos. When we also take into account some other possible structural properties such as the algebraic or topological structure of the elements of **A**, the corresponding bundle structure will be a *spatial* topos, or some other structures like this. A more exact description will depend on the special, individual properties of the theories that participate in the substitution, the type of substitution, and some other unspecified strictly formal choices of possible variants. (Compare the example in the next section.)

Such constructions as **Bn(I)** show what a proxy function can be in strict terms. It turns out that a proxy function corresponds to a whole range of formally very different situations, and it is difficult to speak simply of "the proxy-function argument". The proxy function does not explicitly designate a theory, and a formal description of the ontology of a given theory, after a change in respect of that ontology. It only states which objects are replaced, and which new objects are used for this purpose. However, as a result of such substitution or replacement, we do not obtain a single possible theory. The latter is evident if we actually try to give a formal description of either a new theory concerning new objects or a background theory. In both cases the number of possibilities is enormous, and an accurate description of the situation enriches the new objects with some more properties than just those of the theory in which the new objects are defined.

In the wake of these intuitive explanations, we can apply the concept of substitution to Quine's ontology. Now it becomes apparent what is missing from the formal side within his description of a proxy function in the context of full-blooded things: the intuitive construction of a proxy function is in fact a kind of substitution or interpretation. The approach of Quine himself, as regards the initial step of substitution and only this initial step, is more strictly captured by the topos structure indicated above. Moreover, the construction of a structure of, say, **Bn(I)**, is at most an incomplete beginning of such a strict description and not its culmination (cf. also the example in the next section). For instance, the "reduction" of PA to ZFC is an interpretation in Tarski's sense. On the other hand, "in the opposite direction", i.e. when we replace the natural numbers ("points") in PA with sets from ZFC defined in the manner described by von Neumann, we have a substitution, since PA is interpretable in ZFC, but ZFC is not interpretable in PA. The formal description of PA, where instead of the initial "points" representing natural numbers in the model we insert von Neumann sets, differs from the initial theory of PA and is de facto a completely different theory. Thus, one and the same proxy function works completely differently in one direction than in the opposite one. Moreover, there is also a corresponding change in the background theory—contra Quine, who considers only one background theory for "both directions". He thus fails to see these issues. Strictly speaking, if one wants to describe his proxy function, one should rather speak of a certain relation between some models of ZFC and PA. The problem, however, is that these are not simply two models linked by some "proxy function", but two completely different background models. These distinctions are well illustrated by the description, considered by us elsewhere (Król & Lubacz, 2021), of the result of substituting the whole model of the "other copy" of ZFC for the empty set in the ZFC model. To reiterate: Quine does not register these matters at all and treats the substitution of ontologies as formally conservative: i.e. as changing nothing in the theories under consideration from a formal perspective. His description does not concern a formal situation, but it is related to the implicitly Platonic approach of taking certain (absolute) objects (e.g. apples) from one "basket" and putting them in another.

The next point is that, as a result of the construction of substitutional models, Quine's relativity starts to prove too drastic—even for Quine himself. In his view, one can replace dogs and mammals in the sentence "every dog is a mammal" not with any other objects x and y, but with those that remain in the same structural relationship—i.e. those that fulfil the form "every x is y" and are related in a one-to-one fashion to "dogs" and "mammals". Here, again, it is assumed that there are some formalized theories of dogs and cats and, additionally, their infinite number and countable equinumerosity is also taken for granted. The construction of substitutional models demonstrates that the given structure can be forcible imposed upon *any kind* of objects—even ones that are such that, "taken in themselves", they cannot fulfil a given sentence. This is because substitutions can be performed between any (even not necessarily different!) theories, providing one neglects all possible empirical consequences.

We now show how a given structure can also be imposed on objects of any kind within the framework of Quine's theory: i.e. in the relevant background theory, and even without explicitly invoking our method of substitutions. It will then turn out that three, not just two, theories operating within a single background theory actually need to be involved.

Let us examine in more detail how the construction of local formal theories of elephants (H1) and ants (H2) could proceed in Quine's philosophy. If we restrict ourselves to introducing only the predicate "Ex", i.e. "x is an Elephant" (in H1) and "Ax", i.e. "x is an Ant" (in H2), and the theories contain only mutually corresponding axioms, H1 and H2 differ only alphabetically (being de facto identical, as certain merely "linguistic variants"). Note that the two theories are mutually interpretable in Tarski's way: one within the other. As elephants differ from ants, we need some specific axioms present in one theory but absent from the other: e.g. "For every x, x is the largest land mammal and never flies" (in H1) and "For every x, x can be either a queen or a worker or a larva" (in H2). It seems, therefore, that such theories cannot have the same logical and linguistic structure, because they have to possess some different axioms, and that will we not find a proxy-equivalent in H2 for every predicate in H1 (or vice versa), making it impossible to directly replace the "elephant ontology" with the corresponding "ant ontology". H1 and H2 have different *structures*. So there are clear formal limitations to the applicability of Quine's method.

However, one can try to find a way out of this problem, since it turns out that we can always construct an equivalent elephant-theory within the realm of ants. We only need (informally, of course) to consider some model of H2, say M2, whose elements are "ants", and then construct some H2* theory of ants along the following lines:

1. We create a one-to-one proxy function, F, between the set of elephants (given in some model M1 of theory H1) and some subset of the set of ants given in M2. We may assume that both ants and elephants are countably infinitely many, or simply that both creatures are finitely many, but that there are more ants than elephants.
2. For each predicate (or relation) from H1, e.g. Ex ("x is an Elephant"), let us define a corresponding predicate (or relation) E*x in the theory H2* under construction, such that "for every x, Ex" in H1 iff "for every x, E*Fx". E*Fx need not be defined in H2. Obviously, the relation E operates on elements of M1 (on "elephants"), and E* applies to elements of M2 ("ants").

The resulting theory H2* already applies to ants and, according to Quine's theory, allows one to move from the ontology of elephants in H1 to an ontology of ants also in H1. The only problem is that "Ax" is defined additionally in H2, and in H2* it is basically impossible to distinguish A from E. The latter is what Quine was trying to get at, but one might ask whether it was really that way. It now seems that in order to perform a little more meaningfully the shift from elephant-ontology to ant-ontology (in H1), Quine's postulated background theory should include three theories: H1, H2 and H2*. However, we have the impression that this is not what Quine had in mind, because he speaks only of one background language or theory (cf., for instance, (Quine, 1968, p. 201)). Note: we have assumed intuitively here that our initial theories say something about full-blooded "elephants" and "ants", although they are in fact certain "punctual" theories, and no real properties are in fact expressed *in the theories*.

According to Quine, in order to talk about "elephants", we can use a certain formalized background theory. (However, he himself does not explicitly construct any such theory.) It seems possible *prima facie* to construct a consistent "tower" consisting of a background theory, a meta-theory, a meta-meta-theory, etc. There are, however, limitations facing the construction of such an infinite "tower", of which Quine, and also Tarski, apparently were not aware. As Król has proved for some classical theories, such as the theory of smooth 4-manifolds, if we assume the existence of such an actually infinite tower of *classical* meta-theories, the basic theory of manifolds will necessarily be inconsistent: otherwise, one must assume that from some *finite* stage of the stack construction non-classical intuitionistic meta-theories are going to have to be used; cf. (Król, 2005a, pp. 24–26). Thus, it will be necessary at some finite stage to use some *informal* classical elementary meta-environment; this contradicts Quine, because it implies not only that the building of the stack has to be stopped, but also that no "next-stage" *classical* background theory will be possible. The pragmatic necessity at some stage of an intuitive meta-environment indicates that an intuitively and instantly determined ontology exists before any formalization. Moreover, on any particular layer of a "tower" of theories, there will be a practically unlimited number of possible ways of formalizing a given meta-background theory.

Let us therefore once again recall what it is that extensional and formalized theories talk about, and what objects they refer to. All extensional models of such theories must be "point-wise models": i.e. they must talk about objects that either have no internal structure (like the empty set or atoms in ZFA), or, if they do have such a structure, are "higher-order point-like objects"—i.e. sets of points, sets of sets of points, etc. Thus, we additionally assume that we are dealing with well-founded theories, which Quine seems to prefer, along with the extensionality of such theories. This means that in every set there will be an $\in$-minimal element, say $\boldsymbol{a}$ that is such that there will no longer be an x such that x belongs to $\boldsymbol{a}$. In short, $\boldsymbol{a}$ is an atom: a point without internal structure.

As was already mentioned, Quine often talks about "reductions"—e.g. when defining natural numbers as certain sets, for instance in ZFC. The arithmetic of natural numbers PA is interpretable in terms of Tarski in ZFC, but not vice versa. As a result of this interpretation, the natural numbers in ZFC are certain higher-level objects in ZFC: i.e. they are sets of certain elements in ZFC, and so have some additional internal structure in ZFC. However, all sets in ZFC are constructed from an "atom"—the empty set that no longer contains any elements. The moral of this is that an initial theory, namely PA, cannot speak of certain objects that have a pointwise ontology of a higher-level and that giving these points an ontological interpretation in the second pointwise ontology is possible not through the proxy function of Quine, but as the result of an interpretation of such a theory, meaning an interpretation in Tarski's sense, in another theory. The interpretation in the Tarskian sense is a special kind of formal description of some cases of our more general concept of substitutions. However, if some theories are mutually interpretable in Tarski's sense, then they are model-theoretically and syntactically equivalent and *speak of the same objects* in a way that is precise to the point of isomorphism. If one wants to give them certain individual characteristics, of the kind that some objects have and others not (and vice versa—after all, we guess that elephants are different from ants, at least in our frame of reference), then one goes beyond formalism and does so in an intuitive and informal way. This situation is not explicitly excluded by Quine from his considerations, since in the case of a one-to-one proxy function we may be dealing with such a case: i.e. the *extensional* ontology of one theory cannot be distinguished from that of the other. Therefore, the proxy-function argument is informal and imprecise, and in order to explain the "reduction" of one ontology to another in a precise manner, it is necessary to use not a proxy function, but an interpretation in Tarski's sense, or—more generally—the corresponding substitutions.

It can also be seen from the above discussion that for certain types of extensional theories, such as PA, group theory and other algebraic structures, there is a certain ontology that is *absolutely distinguished*. This "distinction" comes from the fact that this ontology is a kind of "minimal ontology". In order to show precisely that a certain kind of points-ontology is mathematically distinguished—contrary to what Quine claims—we owe the reader some additional explanations, which we present below; an explicit axiomatization of such a theory will then be set out in the next section.

One further point should be clarified. Arithmetic can be considered a formal codification of "the art of counting something". As such, it could not unambiguously determine just one area of its reference, because this would mean that it is only suitable for counting objects from that area (for instance, PA would count only "the" natural numbers)—and that, for some unknown reason (and against everyday experience), its use for anything else was precluded. In this, we have to agree with Quine. However, theories such as PA explicitly define a certain minimal ontology: i.e. they show what properties the objects we count must all have in common.[9] Certainly, they need not be discrete in the sense of being realistically separate from each other: the mere possibility of a (countable) purely intellectual distinction is sufficient. The natural numbers referred to by, say, PA, will therefore be objects such as are equipped with a certain minimal set of ontological features making it possible to count them. "The" natural numbers are therefore objects—non-existent in the real world—possessing only those features that PA attributes to them, together with some minimal ontological "decorative" assumptions enabling counting. (The theory models can be standard or non-standard, but from the ontological standpoint they all contain identical abstract point-objects.) They are therefore ideal (or abstract) objects, stripped of any individuality they might acquire as a result of some interpretation or reduction or other, and exhibiting when it comes to their need to possess some form of individuality, some additional minimal intensional decoration. MST (cf. the next section) is only one example of the attempts at formalizing such an additional ontology that is possible. Natural numbers must simply be some individual things, or be at least *something* (in the sense of a minimal ontology), to be countable. And this is what the proxy-function argument shows. However, this "being something" can be defined in terms of a minimal ontology, which as abstractly non-individual and general cannot be the ontology of any real object. The proxy-function argument thus shows that natural numbers, as defined by and exclusively in PA, are not real (sensuously manifested) objects, but ideal or at least abstract objects. Every formal mathematical theory is non-naturalistically directed to something ideal.

4 Physics, Experiments and Ontological Relativity

In this section, we aim to offer a concise demonstration to the effect that, applied unrestrictedly, Quine's doctrine of ontological relativity can be considered inconsistent with some of our best physical theories—e.g. quantum mechanics (QM). Moreover, there are situations in which his naturalized and relativistic ontology cannot be used in conjunction with the proxy-function method (cf. "The Proxy Function and the Mutual Interpretablity of Formal Theories" section). We will, further, point to examples of substitutions (cf. "The Proxy Function Viewed in the Light of Substitutions and Substitutional Models" section) involving a change of ontology that consists in changing (or imparting) a certain internal structure pertaining to the objects of one theory by means of objects defined by another,

[9] Thus, natural numbers are objects possessing only minimal-ontological properties according to given formal arithmetic and are absolutely distinguished as such. When it comes to different number theories, "the natural numbers" are the most minimal objects. Therefore, PA-ontology will be "more minimal" than ZFC arithmetic-ontology.

where this generates some additional empirical effects. We will, for this purpose, make use of the conceptual apparatus and conclusions of the analyses carried out here (mainly in "Critical Remarks on the Proxy-Function Argument" section). In particular, we will deploy the conclusions pertaining to the minimal ontology of theories, which will allow us to offer, in the present section, a formalization of the theory by describing an example of such an ontology.

As was already indicated, Quine considers ontology change to be empirically harmless, and even to be an operation empirically indistinguishable in terms of replacing certain objects with others. These matters undoubtedly call for a more systematic clarification. Therefore, we first show that Quine's ontological relativity, in conjunction with empirical theories, makes it necessary to assume the existence of some additional "hidden parameters": i.e. properties actually and absolutely undetermined by the theory. We will then consider the problem of the "theory of hidden parameters (or variables)" in QM, and show that Quine's unlimited ontological relativity contradicts QM itself.

The aim of using a proxy function is to change the original (or initial) ontology of a given theory. The objects of the original and transformed ontologies must differ with respect to some properties, even if they are defined only locally in the corresponding background theory. These properties may either be empirically determined (by the results of certain experiments, or even only by some sense-data), or they may be non-empirical in nature. Some "mixture" of these sorts of properties is also possible. In doing so, it is irrelevant whether such objects exist in reality or not. Quine denies the existence of any absolute reality, so he denies the existence of a world of things in themselves, and of properties of objects that would be independent of both the observer and any particular theory being considered. At first glance, then, it would seem that his theory contradicts the assumption of hidden parameters; cf. below. Note, however, that he allows for the possibility of a relative determination of ontology—locally in a background theory, or through an appropriate ontological reduction. This is described formally in "The Proxy Function Viewed in the Light of Substitutions and Substitutional Models" section in terms of substitutions and substitutional models. Such operations, even though intended by Quine merely as formal procedures with nothing behind them in reality, generate certain empirical consequences when described within an empirical theory (which may also involve thought experiments/concepts). Therefore, it seems that in the case of Quine's ontological relativity, we should rather consider those properties that are non-empirical at least *according to the given theory*, since relativity is only possible if changing the ontology has no effect on the form and results of our empirical theories. If a change in ontology were to produce empirically measurable consequences, we would be able to observe such a change experimentally, and it would be possible to say objectively that the two ontologies are different *according to the given theory*. For this reason, ontological relativity assumes a form of realism and some sort of absolute ontology: i.e. it assumes that there are, at least "locally" (i.e. for an ontologist, or a person using "his/her" ontology), objects possessing certain "private" properties, or that properties attributed in a certain context to these objects are regarded as belonging to them "in themselves", in that they are not formally described by the given theory,[10] or defined by the latter unambiguously and in an experimentally distinguishable way. These properties, in the case of QM ontologies, are therefore typically called "hidden

[10] If we substitute elephants for ants, then the individual characteristics of ants and elephants cannot be defined in the given theory, otherwise, the whole substitution becomes impossible. However, some minimal set of ontological properties must be identically present in ants and in elephants.

parameters"; cf. (Genovese, 2007, p. 4). Let us emphasize again that it is irrelevant here whether such substitution and refinement in respect of the ontology is only of a local nature (e.g., for a given observer, and in his or her environment), or of a global sort (i.e. affecting all objects in the universe of a given theory). This is because in both cases there are specific empirical consequences that can be rationally investigated. An operation may be formally permissible, i.e. described in a consistent way within the *formal* framework of a given theory, and yet may lead to impermissible empirical consequences. We will therefore seek to show that the changes with regard to ontology described by Quine lead to consequences that (1) are not empirically neutral, and (2) are to the best of our present knowledge inconsistent with QM.

The term "hidden parameters", in physics generally and QM in particular, refers to any situation in which a given physical theory is not complete, at least in terms of its reference domain. An example is the theory of ideal gases, since we are convinced that a real gas, e.g. hydrogen, has some other properties, for instance chemical and optical ones, that are not determined by an ideal theory, the latter being *eo ipso* a certain "idealization" of reality. Theories with hidden parameters are therefore obviously incomplete, as they give only a partial (or "minimal") description of *objective* full-blooded reality. The problem of the completeness of QM was already apparent right at the outset of its development. Immediately after the probabilistic interpretation of the wave function by Max Born (1926), problems with the realistic interpretation of this theory appeared. They concerned the question of whether objects of the micro-world "in themselves" possess certain properties, determined independently of the measurements made. It was also connected with the question of the possibility of the world's having a deterministic character, in contrast to the openly indeterministic character of QM itself. Einstein's saying that "God does not play dice" is well-known, and he was absolutely convinced that QM does not describe the whole of reality, even where just the micro-world is concerned; cf. (Einstein, 2018, p. 403; Einstein et al., 1935). In order to salvage determinism, it was necessary to recognize that QM is a partial, incomplete theory of reality, and therefore that there are certain properties of objects that "accrue" to them independently of the theory and that are indescribable by QM. It was therefore a significant question, whether there are any parameters hidden from QM "in reality". Indeed, such a QM theory, possessing an underlying deterministic character, has been proposed. This is the theory commonly known as the "de Broglie-Bohm mechanics/theory" (Bohm, 1952). In this theory, the hidden parameters are the trajectories of the electrons or the origin of such trajectories. However, this theory is not local and has other disadvantages.[11] The year 1964, when Bell formulated his famous theorem together with Bell's (1964) inequalities, should be considered a breakthrough moment. Bell's theorem implied that it was possible to perform certain experiments on "entangled states", the results of which should satisfy Bell's inequality as long as a given theory contained hidden parameters. If the inequality was not satisfied (i.e. was violated), the given theory could not have hidden parameters. It is not possible within the scope of this text to discuss, even just in review, the issue of hidden variables in QM and the possibility of testing them empirically. (We refer the interested reader to the literature on this matter: e.g. (Genovese, 2007; Nordén, 2016); or the relevant information in The Stanford Encyclopedia of Philosophy.) There are more and more papers and experiments concerned with testing the empirical consequences of assuming the existence of hidden parameters in QM (cf. *ibid.*). The results

[11] "Local" means that the properties of a physical system are describable in terms of its local environment, and not "all of the world"; cf. (Nordén, 2016, p. 2).

do not definitively falsify the possibility of the existence of QM with hidden parameters, although in the overwhelming number of cases they do falsify such theories—a fact which is in line with the spirit of the Copenhagen interpretation of QM.

Our present discussion of Quine's ontological relativity indicates that, interpreted in the context of hidden variables, it has empirically testable consequences that undermine the commitment to relativism. The unlimited version of this relativity, to the effect that ontologically different objects can be freely interchanged/substituted with others—even, let us again emphasize, purely theoretically and/or as a thought experiment—is untenable. It is not enough to have at one's disposal classes of objects that, so long as they are equinumerous, are arbitrary. It is not empirically the same whether we are talking about electrons or elephants or other macroscopic objects. Objects cannot have arbitrary features—not even just ones hidden from a given theory, or that are merely "coextensive". An elephant, or even just the "space–time complement of an elephant", is different from an electron, and one should not expect a positive result from an experiment concerning, for example, the diffraction of elephants. It is also impossible even to just freely interchange micro-world objects, such as photons with protons.

Of course, Quine nowhere explicitly proposes such a thing as, for example, the replacement of a local ontology of elementary particles by a local ontology of elephants; but the possibility undoubtedly follows from what he says in many places. (Compare, for instance, his remarks to the effect that every universe of any infinite theory can be reduced to the Pythagorean universe of natural numbers (at several points in (Quine, 1964), e.g., pp. 211–12.) Such a *prima facie* absurd consequence also follows from his repeatedly mentioned possibility of replacing a given object with its spatio-temporal complement—i.e. the whole of the remaining world, or "cosmic complement"; see, e.g., (Quine, 1995a, pp. 71–73). If this were not the case, one would have to restrict his ontological relativity to the interior of certain kinds of objects: for example, mathematical objects, elementary particles, etc. But how can we, according to Quine, precisely and non-relatively distinguish such broader kinds unambiguously? Note also that in the formulation of the proxy-function argument there are no extensional conditions concerning, for example, the coextensivity of the replaced objects, and excluding certain kinds of replacements.

Nevertheless, one can at least pose the question of whether there is a group of objects that allow for such "ontological permutations" (as Quine assumes). It turns out that in order for such objects to be absolutely indistinguishable, they should not have any internal hidden structure, in the sense of a structure describable by certain hidden parameters. Let us also note that this applies not only to the theory after ontological re-interpretation but also to the initial theory interpreted ontologically in terms of some "background theory" or other. The reason is that if the empirical ontologist, in his/her ontological frame of reference, uses his/her specific objects, this specificity must be described by some hidden non-empirical properties, which may turn out to be falsifiable. Thus, from a mathematical point of view, permutations and ontological changes can only be made using "point ontologies", and this kind of ontology is—contrary to Quine—empirically discerned.

We shall begin our discussion of this matter by giving an example of a theory that formalizes the notion of a minimal point ontology since it follows from our discussion so far that minimal point ontologies are the only ontologies admissible by QM, and in this sense are empirically distinguishable. In the case of QM, Quine's ontological relativism is only possible with respect to the substitution of one point ontology for another. Intuitively, a theory of "point-wise ontology" can be constructed by first defining an infinite and denumerable class of atoms in the sense of ZFA: i.e. where an atom **a** will be an object for which it is true that "for every x, it is not true that x is an element of **a**". The only sets will be sets of atoms. There will be no sets of which any set of atoms is an element, and so on. One will be able to create a sum, difference, product, etc., of sets of atoms, but the result of this operation will also have to be a set of atoms. Thus, one will be unable to create, say, the Cartesian product of two sets of atoms, and there will be no power set of a given set, but one will be in a position to say that one set is "smaller" than another set, etc. We shall describe one example of such a MST theory more precisely in due course. Moreover, we shall only state some of the axioms involved explicitly, leaving others that exhibit an analogous form to the corresponding axioms of ZFC to be only mentioned by name.

The language of MST:

A two-sorted domain: **A**—a class of atoms; **B**—a class of sets of atoms.

Symbols:

(,), =, ∈, logical connectives and quantifiers, symbols for variables and constants:

constants denoting elements of **A**—a_A, a_A', a_A'', b_A, …; constants denoting elements of **B**—a_B, a_B', b_B, ….

variables referring to the elements of **A**—a_i, b_1, b_i, …; variables referring to the elements of **B**—X_i, X_1, Z_i ….

(no functions).

Relations: ∈, =,

Terms: variables and constants.

Atomic formulae:

1. $a_A = b_A$;
2. $a_B = b_B$;
3. $a_A \in b_B$ (an element of **A** ∈ an element of **B**);
4. $a_A \in X_i$ (an element of **A** ∈ a set from **B**);
5. $a_i \in a_B$ (variable denoting an element of **A** ∈ a set from **B**);
6. $a_i \in X_i$: (variable denoting an element of **A** ∈ variable denoting an element of **B**).

Well-formed formulae (WFF): formed as usual from the atomic formulae with the use of logical connectives and quantifiers. Usual definitions of "open formula", "free variable", etc.

Rule of inference: modus ponens.

Axioms: Logic: Logical axioms of first order predicate calculus with identity.

Comprehension: () $\forall X_j \ldots \forall Z_k\ \exists Y_i\ \forall a_i\ (a_i \in Y_i \leftrightarrow \varphi\ (X_j, \ldots, Z_k, a_i))$ (and Y_i is different from all $X_j, \ldots, Z_k$ as a new object).

Extensionality: $\forall X_i\ \forall Y_i\ \forall a_i\ (X_i = Y_i) \leftrightarrow (a_i \in X_i \leftrightarrow a_i \in Y_i)$.

With the use of Comprehension and Extensionality, it is possible to define and prove the existence of the universal set of points, **V**, the empty set, the existence of singletons, etc.

Infinity: $\forall Y_i\ \exists a_i.\ \sim(Y_i = V) \rightarrow \sim(a_i \in Y_i)$.

The intuitive content of MST will be simple: sets are formed from atoms ("points"), and are themselves different objects from them. This elementary system corresponds to the first two layers in type theories, or to a simple subsystem of ZFA (MST is interpretable in ZFA). Therefore, if these systems are consistent, MST will also be so.

MST "generates" point-like structures that are "proxies" for the sets either from such theories as PA or from other theories of algebraic structures in which we operate without any need to discern an internal structure of objects. To use such a theory as a "point ontology generator" one can use the proxy function as described in the previous section (cf. "The Proxy Function and the Mutual Interpretablity of Formal Theories" section). Then MST acts as our theory H2, and the constructed theory H2* will be a theory H1, but already based on a well-defined point ontology. Also, natural numbers can be interpreted thus in MST in many different ways. For instance, if we accept the existence of an infinite set of atoms, this will mean that there is a one-to-one correspondence between the natural numbers 0, S0, SS0,.., and atoms from the set. One can also use the concept of Tarskian interpretability (cf. "The Proxy Function and the Mutual Interpretablity of Formal Theories" section) to demonstrate that PA is interpretable in MST (with some additions). Obviously, MST is not the sole possible theory descriptive of a "point ontology". For instance, any well-ordered infinite set of points can also be used, or some other structures based on pre-order categories with some additional conditions.

Formally speaking, MST is a certain subsystem of second-order arithmetic. This remark is important, because of the possible applications in reverse mathematics. MST can be said to be distinguished as a theory, because it is the simplest instance of set theory enabling one to talk about sets of objects without internal structure. The theory is also ontologically distinguished by the fact that it can be considered to provide models for theories that are minimally ontologically interpreted. A point ontology of objects without any additional internal structure seems like the simplest possible one. From the above, it follows that MST can also be regarded as an empirically distinguished theory.

Let us now return to the question of other possible "minimal set theories". The above MST seems to be the simplest of them, but some so-called "concatenation theories" can also be used; cf. (Corcoran et al., 1974). There are also other examples of MST formulated in the language of category theory, and we also find there so-called "forgetful functors", which allow one to "forget" (or omit) the internal structure of some or all objects in a given theory. For example, very many quotient structures of certain algebraic structures have an initial structure—e.g. groups. Forgetful functors make it possible to omit such an internal or underlying structure of elements of a given quotient structure, and de facto reduce these to an underlying point structure; cf., also, (Król & Lubacz, 2021).

The possibility of unrestricted construction of proxy functions runs counter to the practice and intentions of scientists for whom, for example, QM does not say anything about the "quantum states of cows or donkeys", but only about states of certain micro-world objects. There are also many substitutions that are known in science and that generate certain additional empirical effects. Below, we give an example of such a substitution for points in the space–time continuum of certain Boolean algebras which generates such empirical effects in quantum gravity—although, according to Quine, it should only be a neutral form of "ontological relativity" and this, moreover, should be consistent with naturalistic and holistic empiricism.

Take the following as an example of global homogeneous substitution: every point of $\mathbf{R}^4$ (space–time) is replaced by an atomless Boolean algebra $\mathbf{B}_x$: $\boldsymbol{i}: \boldsymbol{R}^4 \rightarrow \{\boldsymbol{B}_x \mid x \in \boldsymbol{R}^4\}$, $\boldsymbol{i} \in \boldsymbol{B}_x$. $\mathbf{B}_x$ is a complete atomless Boolean algebra in the countable transitive model V of ZFC, by which the forcing in V can be given. J. Król has demonstrated that for a local observer, the space–time will be the 5-dimensional structure $\mathbf{R}^1$ **x** $\mathbf{R}^4$. Thus, map $\boldsymbol{i}$ can be considered the replacement of (0,1) Boolean algebra corresponding to every point **x** of $\mathbf{R}^4$ by $\mathbf{B}_x$. There is a correspondence between every point **x** of $\mathbf{R}^4$ and the geometric morphisms $\mathbf{SET} \rightarrow \mathbf{Sh}(\mathbf{R}^4)$, where the category of sheaves **Sh(0,1)** is isomorphic to the category of sets **SET**, and $\mathbf{Sh}(\mathbf{R}^4)$ is a topos-category of sheaves over $\mathbf{R}^4$. The reader can find more details of the construction of a topos corresponding to this substitution in the most general case in the work of Król (2008, pp. 1786–7).

There is also an analogy between the emergence of higher dimensions for an observer in 4-dimensional space–time described in the work just referred to and, for example, the emergence of higher dimensions in the case of a global substitution in a line ($\mathbf{R}^1$) of infinite numbers of lines or planes.

Another example concerns the so-called "model-theoretic approach" to quantum gravity [cf. (Król, 2004, 2005a, 2005b)], in which it is possible to describe certain quantum gravity effects—taking into account the fact that the formal mathematical theories needed in QM (for instance, a number theory, exotic $\boldsymbol{R}^4$s, etc.) have different models. The so-called "exotic $\boldsymbol{R}^4$"—i.e. an infinite number of topologically non-equivalent differential structures that exist only in four dimensions (space–time) and are not present in any other dimension—are responsible for the emergence of quantum gravity fields.

In describing hidden parameter theories, we have employed a formulation according to which we refer either to empirical theories that are fully formalized or to theories that are strictly yet only partially formalized, in the given context of interest. As applied to QM, this remark is important, because the theory may not be axiomatizable; cf. (Wightman, 1976; Tsilerson, 1994; Deutsch, 1985). Quine does not consider this kind of possibility. Moreover, his remarks point to a formalization that is obligatory and necessarily couched in a first-order language. So Quine's theory may *ex definitione* not apply to some of our best empirically tested theories. However, this raises the possibility of describing ontological relativity and proxy functions for fragments of non-axiomatizable empirical theories (i.e. only "locally" formalized theories), or for "locally" formalizable theories—in the event of their global non-axiomatizability.

Let us emphasize that our discussion has left unmentioned a whole range of options with respect to the consistency of Quine's theory with different kinds of approach to QM. For example, a problem worth considering is the consistency of Quinean ontological relativity with the so-called "superdeterministic" account of QM. This account assumes that there are no QM-independent choices in the world as to what quantity is currently measured, and that such a choice is causally determined [cf. (Brans, 1988; Hall, 2010)]; in effect, one may assume the existence of some hidden parameters. Yet even in this case, completely arbitrary substitutions cannot be performed, because unlimited reductions and substitutions of ontologies are empirically impossible.

Another empirical limitation of Quine's ontological relativity is that certain kinds of ontological changes, which we described in "The Proxy Function Viewed in the Light of Substitutions and Substitutional Models" section as local and global substitutions, may violate the rules for describing entangled states in QM. If we consider a system of two entangled particles with opposite spins, then swapping one particle (object) with the other will be consistent with QM, providing that we swap them synchronously so that their spins are still opposite. This is an example of the limitations of the application of a "fixed-point rule" to ontological changes. Thus, again, it turns out that unlimited ontological relativity is inconsistent with QM.

Note, moreover, that were it to be possible to replace one of the electrons in a system containing several electrons with another indistinguishable electron, then the Pauli exclusion principle would not apply, and atoms would not be able to exist—only degenerate objects would then be possible. This shows that a purely formalist interpretation of QM is untenable, as it proclaims that there are no objects, in that only certain structures can exist, and that names and variables are merely empty and indistinguishable "placeholders".

5 Concluding Remarks

Let us bring together the results of our reflections, and the conclusions we seek to draw from them. In the introduction, we put forward the basic intuitions that *seem* to indicate the plausibility of Quine's ontological relativity. In "General Comments on Quine's Analytical Empiricism and Ontological Relativity" section, we reconstructed in some detail the basic explicit and implicit grounding of his views. In this section, we also indicated, without going into details, some general doubts concerning issues related to ontological relativity that are associated with his scientism, holism, naturalism, formalism, and empiricism. In particular, we pointed out that Quine's theory does not apply to science at large, but rather only to its formalized versions. In "Critical Remarks on the Proxy-Function Argument" section, we subjected his proxy-function argument to close scrutiny, this being the backbone of his ontological relativity. We have shown that the argument, seemingly obvious, is informal and leads to ambiguous results. This conclusion is based on strictly defined concepts and formal constructions. In particular, we have shown that in formulating this argument in the part dealing with the so-called "ontological reductions", the notion of interpretation in Tarski's sense should be taken into account, since most of the examples given by Quine are de facto based on this notion (although Quine does not mention this).

We have also considered in some detail the question of the introduction of "full-blooded objects" into formalized theories. To this end, we reasoned that extensional formalized theories mainly use what we call "pointwise models", and that real objects, such as elephants, have additional and completely informal features grounded in an intuitive and non-formal sense of conceptual "decorum" not defined by formalization. In particular, we considered the question of how to introduce objects strictly and formally defined in other theories, in place of "points", into a given formalized and extensional theory: e.g. how, in Euclidean geometry, certain algebraic structures (groups, interiors of circles, etc.) can be introduced in place of, say, points of a plane. These issues have been addressed formally here using our proposed account of "substitutional models" and the "operation of substitutions".

From the considerations presented, it follows that it is not sufficient to say in general terms that objects of one theory are replaced by objects of another theory via a proxy function. This is because the initial theory, after proxy-substitution of other objects, can, and even must, be further specified and completed by additional axioms. That implies that the proxy-substitution changes significantly the initial formal properties of a given theory. After the proxy-function-substitution, the theory becomes a significantly different theory in comparison to the initial one. This situation was not appreciated by Quine, who treats the initial theory as formally identical to the theory after proxy-substitution—i.e. he assumes that the change of ontology is "formally innocent". For this reason, proxy-models provide an infinite number of counter-examples to Quine's ontologically *conservative* ontological relativity. What is more important, such new theories, when used in science, can have different empirical consequences. We have also explained why a one-to-one proxy function can work in an asymmetric way: replacing objects of theory T1 by objects of theory T2, and replacing objects of T2 by objects of T1, are not symmetrical. A symmetry requires that, in Tarski's sense, T1 be interpretable in T2 and T2 interpretable in T1.

In "Physics, Experiments and Ontological Relativity" section, we have sought to show that there are no empirically neutral transformations of ontology. Contrary to Quine's conviction, theories distinguish a certain type of "absolute" ontology (i.e. minimal ontology), because the "real" objects imposed on the structure of a model are not neutral. Obviously, as theories determine only minimal point-structures, all other properties must be pre-existent in a non-linguistic way. Quine *implicitly* assumes in his proxy-function argument an absolute reality. The language(s) and structures determining "nodes" and models are also absolute. For this reason, in our discussion related to pointwise models, we also argued that certain models of formalized theories are distinguished by the fact that they have a set of minimal ontological-formal requirements—ones that the theories impose on them so that they are "the objects about which the given theory speaks". We have additionally indicated how the proxy-function argument could be formulated more precisely, showing that more theories must be involved in its description than Quine has considered.

In that same section, moreover, we presented the empirical consequences of the formal refinements of Quine's ontological relativity and gave there an example of strict formalization of minimal ontology in the sense of set theory, i.e. Minimal Set Theory (MST). At this stage, we mainly referred to the theory of hidden parameters, well-known to physicists, by showing that Quine's ontological relativity based on the proxy-function assumes operation on objects having hidden parameters. In quantum mechanics, there are convincing empirical reasons for the impossibility of objects with hidden parameters. Since a change in ontology, if it is to be formally and empirically neutral, must involve the substitution of hidden parameters, and since certain theories cannot possess objects with hidden properties, Quine's ontological relativity is falsified by such theories. This falsification is related to empiricism since the impossibility of the existence of hidden parameters is indicated by the results of experiments. A point worth noting is that if hidden parameters cannot exist in QM, then this would indicate that QM-ontology must be some kind of minimal ontology in the sense introduced in this paper.

To repeat again: there are no formally and empirically neutral changes of ontology. Quinean ontological relativity has serious flaws, both from a formal point of view and in terms of its being inconsistent with some important empirical data. Let us, however, note that science and ontology, for more than two thousand years, have developed brilliantly without such restrictive clarifications, making massive use of direct and informal contact with the real world.

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