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ORIGINAL RESEARCH

Janina Hosiasson-Lindenbaum on Analogical Reasoning: New Sources

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Abstract

Janina Hosiasson-Lindenbaum is a known figure in philosophy of probability of the 1930s. A previously unpublished manuscript fills in the blanks in the full picture of her work on inductive reasoning by analogy, until now only accessible through a single publication. In this paper, I present Hosiasson's work on analogical reasoning, bringing together her early publications that were never translated from Polish, and the recently discovered unpublished work. I then show how her late work relates to Rudolf Carnap's approach to "analogy by similarity" developed in the 1960s. Hosiasson turns out to be a predecessor of the line of research that models analogical influence as inductive relevance. A translation of Hosiasson's manuscript concludes the paper.

1 Hosiasson's Work in the 1940s

Janina Hosiasson-Lindenbaum[1] was a philosopher of induction and probability active from the 1920s until her untimely death during World War II. A member of the Lvov-Warsaw School, she is known primarily for her paper "On Confirmation" (1940), where she presented an axiomatic approach to the qualitative notion of degree of confirmation, as well as the first published Bayesian-like response to Hempel's paradox of confirmation (the raven paradox). In 1935, Hosiasson was the first woman whose work appeared in *Erkentnnis*. She published a significant number of articles on probability and related topics: on the logical form and the justification of inductive reasoning, the frequency theory of probability, and the psychology of inductive reasoning, among others. She also wrote popular science articles about logic and translated some of Bertrand Russell's books into Polish.

[1] She was also known under her married name Lindenbaum, and signed her works with various combinations of the two surnames. For simplicity, in the following I refer to her only as Janina Hosiasson.

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A student of Tadeusz Kotarbiński, she defended her PhD thesis on the justification of induction in 1926 at the University of Warsaw. She remained closely involved with the Lvov-Warsaw School, although she never held any official position at any philosophy department. She spent the academic year 1929/30 in Cambridge, where she became acquainted with Frank Ramsey's work on the subjective interpretation of probability, which was unpublished at the time. In her own related paper, likely written during that visit, she wrote that she had "previously thought independently on similar lines" as Ramsey (Hosiasson, 1931). This fact has prompted some historians to count her as one of the earliest adopters of the subjective interpretation of probability (Galavotti, 2008).

Hosiasson attended all of the Unity of Science congresses in the 1930s, except for the last one: in spite of having been invited and having prepared a paper, Hosiasson could not travel to Harvard in August 1939 and stayed in Poland. At the outbreak of the war in September 1939, she left Warsaw and in October 1939 settled as a refugee in Vilnius.[2] The city was at the time being taken over by still-independent Lithuania, which presented some opportunities for fleeing to the West. During her stay in Vilnius, apart from working to sustain herself, Hosiasson maintained an active research life, writing papers and giving talks. One of those papers is the manuscript translated here.

I have discovered this manuscript—to my knowledge, previously unknown and unpublished—in the form of a handwritten note, in the personal archive of Tadeusz Czeżowski.[3] Czeżowski was the head of the philosophy department at the Polish university in pre-war Vilnius. After the university was closed in December 1939, he organized a series of secret philosophy seminars, of which there were to be 143 until the end of the war. It was during one of those seminars, in May 1940, that Hosiasson presented her work on analogical reasoning seemingly for the first time. The talk she presented there was published in *Mind* in October 1941 (Hosiasson Lindenbaum, 1941); the published version is a translation of a Polish typescript which can be found in Czeżowski's archive and which the talk was based on.

It is clear that Hosiasson continued to work on that topic, building on the results of that first paper. This was interrupted in the summer of 1941, when the German-Soviet war broke out and the Nazis took over Vilnius. Hosiasson was arrested soon thereafter. She first appears, under the name Janina Pańska,[4] in the inmate register of the Lukiškės Prison on October 28th, 1941, as someone who had been moved over from a different location.[5] During her incarceration, she was taken for interrogation to the Gestapo headquarters[6] and on March 29, 1942, together with 28 other inmates, she was listed as "released" from prison and likely shot on the same day.[7]

[2] A few letters survive in which Hosiasson describes her flight from Warsaw. The one she sent to G. E. Moore was published by Szubka (2018).

[3] Currently at the archive of the Nicolaus Copernicus University in Toruń, Poland.

[4] There is no evidence that she actually married Antoni Pański, the man whose name she used when arrested. According to a (much later) account of their friend Anna Jędrychowska, Hosiasson was arrested while carrying a fake passport under the name of Pańska.

[5] Lithuanian Central State Archives (LCVA), R-730 ap. 2 b. 23.

[6] LCVA R-730 ap. 2 b. 85.

[7] LCVA R-730 ap. 2 b. 64.

Henryk Elzenberg, who lived in Vilnius at the time, mentioned in 1945 that while in prison, Hosiasson managed to write "a serious study on reasoning by analogy", which contained at least one example inspired by her prison life ((Elzenberg, 1990), p. 55). None of Hosiasson's publications to date match this description. However, the manuscript translated here, clearly written in her own handwriting and expanding on her previous work on analogical reasoning, does fit Elzenberg's recollection, up to the discussion of an example of analogical reasoning about prison life problems. We have therefore a good reason to believe that the manuscript translated here was written during Hosiasson's incarceration, in 1941 or 1942.

Another related item in the Czeżowski archive is a small handwritten note, in French, addressed to "C. G. Hempel, New York College" and signed by Hosiasson. It contains a number of corrections and additions meant to be added to the *Mind* paper before publication. The presence of the note in the archive, as well as the fact that none of those corrections are included in the published version, suggest that either it was never sent, or did not made it to Hempel or any of the editors of the journal on time (a fact not too surprising given a world war happening at the time). The note not only corrects some formulations of the original paper, but also contains an important extension of the analogy theory, which is why it will be included in the discussion below (but not transcribed in full, as most of its contents are repetitive in relation to the fist manuscript).[8]

Together with the translation of Hosiasson's manuscript, this paper provides a discussion of its contents and its context within Hosiasson's full body of work. As I show in Sect. 5, the importance of this manuscript goes far beyond a merely archival interest. Hosiasson's approach to analogical reasoning, based on considerations of statistical relevance of properties or propositions to one another, preceded the wave of similar work by Carnap and later inductive logicians by decades. The extent to which her approach foreshadowed Carnap's could not be so clearly seen from the published work, like the 1941 *Mind* paper, because at that point she was still entangled in the search for more structural, similarity-based, rules for analogy. In this later, unpublished work, she arrived at a more streamlined approach focused solely on inductive relevance.

In what follows, I sketch Hosiasson's work on inductive logic and analogical reasoning and comment on the new manuscript. In Sect. 2 I briefly introduce Hosiasson's general approach to modeling inductive reasoning and the search for its justification. In Sect. 3 I outline Hosiasson's published work on analogical reasoning which has been inaccessible to English speakers. Following that, Sect. 4 presents the contents of the new manuscripts, showing in particular how the published and unpublished materials are related. Finally, in Sect. 5 I show how the newly uncovered material relates to the later developments in the field, foreshadowing Carnap's

[8] Hempel wrote a short review of Hosiasson's 1941 analogy paper (Hempel, 1942), but there is no evidence that Hosiasson was able to react to it prior to publication.

theory of analogy by similarity of predicates. The translation of the manuscript follows these comments.[9]

2 Logical Analysis of Induction

Throughout her academic career, Janina Hosiasson worked exclusively on induction and probability. She participated in the debates on the foundations and the interpretation of probability, explored an axiomatic approach to confirmation, and conducted empirical investigations into the psychology of inductive reasoning. The manuscript translated here is a direct continuation of a line of work that starts as early as her 1926 doctoral dissertation on the justification of induction. Two papers, based on initial chapters of the dissertation, are the foundation of this line of work: "Definitions of Inductive Reasoning" (Hosiasson, 1928) and "On the Validity of Hypothetical Induction" (Hosiasson, 1934). As these papers were only published in Polish, I will discuss them here in a little more detail, to give the full background of Hosiasson's work on analogy.

In the first of these papers, Hosiasson aims to find out what the logical form of common inductive reasoning is. This intended formal definition of inductive reasoning is to be consistent with the established understanding of the term, which in turn is pulled from the opinions of experts on the topic: philosophers such as Bacon, Mill, Johnson, or Keynes. This focus on the use of the term is in line with the philosophical method of the Lvov-Warsaw School, of which Hosiasson was a member. Philosophical analysis was to always start with a clarification of the meaning of the terms involved. In this case, the clarification is sought with a particular goal in mind: that of specifying the conditions under which inductive reasoning is justified. This goal drives the final choice among potential logical forms that can be identified for inductive reasoning. Hosiasson goes for the most basic logical representation: only the logical relations between premises and the conclusion are relevant, and not their specific form. The standard inductive generalization, where a universal sentence is concluded from a sequence of particular instances of it, becomes only a special case of inductive reasoning, rather than its prototypical instance.

Hence, Hosiasson takes the most general form of inductive reasoning to be the following: when reasoning by induction, we raise our credence in a sentence, h, based on some (descriptions of) facts f_1, ..., f_k that are its consequences.[10] This is

[9] A note on the transcription: The original manuscript is handwritten in pencil. Some of the text is underlined, and parts of the formulas are underlined. Taking Hosiasson's published papers as an example, I decided to turn the various kinds of underlining into italics and bold font, when appropriate, keeping in line with the conventions in her earlier papers. The manuscript contains numerous footnote numbers, but no footnotes; I omitted the footnote numbers in the transcription. Some obvious typos were corrected. I collated some paragraphs to improve readability.

[10] Historically speaking, what Hosiasson was analyzing was inductive inference, not inductive reasoning. In the terminology accepted in the Warsaw School, reasoning and inferring both concern the mental activity of matching consequences to premises. In the case of inference, however, the truth of the premises is assumed (see (Kotarbiński, 1929), part 4). As can be seen in the manuscript here, Hosiasson recognized this distinction. However, in her other papers, she did not stress it at all and used the terms "reasoning" and "inference" mostly interchangeably, which is consistent with the modern usage adopted here.

a version of the inversion theory of inductive reasoning, as proposed by William Jevons: in inductive reasoning, we reason from the consequences of the hypothesis, to the hypothesis itself. It is likely that in choosing this particular logical form as the basic form of inductive reasoning, Hosiasson was influenced by Jan Łukasiewicz, who was the second reader of her thesis, and whose own 1903 dissertation was on the inversion theory of induction (Łukasiewicz, 1903a, b).

According to Hosiasson, what is additionally needed for reasoning to be inductive is that the inferential relation between the premises and the conclusion must be (more or less consciously) used in the reasoning: the credence in h is raised *because* the observed facts f_1, ..., f_k are consequences of h. This basic form of inductive reasoning Hosiasson calls the hypothetical induction. A variation on this form, in which on the basis of f_1, ..., f_k we raise the credence not in h, but in some further consequences of h, is the hypothetical subinduction—this form of reasoning will be reappearing in Hosiasson's work on analogy, as we will shortly see.

In Hosiasson's own terminology, the inductive hypothesis is also called a *reason* for its consequences: whenever f_1, ..., f_k are consequences of h, h is called a reason for f_1, ..., f_k (in other words, "being a reason of" is the inverse of "being a consequence of"). The relation "being a consequence of" is explicated in an inferential way: a sentence f_i follows from h whenever f_i can be derived from h in the deductive system that we are using at the given time: this includes both the rules of logic, and rules that are domain-specific (in the ideal case, the full language of science and its rules of inference).

With the definitions at hand, Hosiasson turns in the follow-up paper (1934) to the question of justification of inductive reasoning. In order to determine when it is appropriate to raise our credence in h, given some f_i's which are its consequences, some formal theory of credence, or rational credence, is needed. Hosiasson formalizes the notion of rational credence using axiomatically defined credibility functions p.[11] Every p is a function of two arguments: a sentence h and a set of sentences f_1, ..., f_k. The background knowledge s—the relevant facts known to the agent before learning the f_i's—is always taken into account. The axiomatization of the credibility functions was adapted from the probability axioms of Mazurkiewicz (1932) and used by Hosiasson in all her subsequent work, including the manuscript below, as well as the well-known "On Confirmation" (Hosiasson-Lindenbaum, 1940), where the same set of axioms is used to characterize confirmation functions. Note the lack of any other constraints that would further restrict the set of admissible, or rational,

[11] In her papers, Hosiasson used multiple notations for her credibility functions. She started with w, for Polish *wiarygodność* (credibility). Later on, she used c, and finally, in the draft translated here, she switched to p. This could simply be a consequence of the fact that the draft was not yet prepared for publication. It is also possible that at some point she decided to keep the c-notation reserved for confirmation functions, as in her 1940 paper "On Confirmation" (which, incidentally, is thought to have inspired Carnap's c-function notation in his inductive logic). For the sake of clarity, I decided to use the notation found in the manuscript translated here throughout my whole discussion. At times this leads to reporting Hosiasson's results in a different notation to the one she used in a given publication. A reader familiar with this subject matter, however, should have no problem to find the corresponding formulas in the original papers.

credibility functions—this is most definitely not any strong form of logical probability that assumes only one rational probability function.

Under what circumstances are we then justified in reasoning inductively, i.e., in raising our credence in the conclusions of our inductive arguments? Bear in mind that when Hosiasson asks whether inductive reasoning is justified, she is not asking the grand Humean question, but one much more specific, and spelled out within her formal framework. In the 1934 paper, she puts forward two such justification questions, referring to the two kinds of inductive reasoning: hypothetical induction and hypothetical subinduction. The two questions are, respectively:

1. If, when, and how strongly can we believe—or how much can we strengthen our belief—that a sentence is true, given data that is a consequence of that sentence? In a simple formal form: under what circumstances do we have $p(h, sf) > p(h, s)$, where f is a consequence of h, and s represents the background knowledge?
2. If, when and how strongly can we believe—or how much can we strengthen our belief—that some sentences are true, given data that is a consequence of the same sentence which those sentences also follow from? In a simple formal form: under what circumstances do we have $p(f_1, sf_2) > p(f_1, s)$, where f_1 and f_2 are consequences of the same h, and s represents the background knowledge?

The first of these justification questions has a straightforward positive answer under the assumptions that $p(f, s) < 1$ and $p(h, s) > 0$, where s is the background knowledge at the time of observing f. The first assumption formalizes the idea that f is not yet known—that it is a "new" piece of evidence. The second of these assumptions states that h is consistent with the background knowledge, i.e., not excluded by it. From these two assumptions it does follow in the probability calculus that $p(h, sf) > p(h, s)$, i.e., observing f raises the credibility of the hypothesis.

It is the second of these questions that is relevant for Hosiasson's later work on analogy. It concerns the kind of reasoning which at that point she called the hypothetical subinduction: we observe some facts and on that basis raise our expectation of other facts, which are consequences of the same hypothesis as the already observed ones. When are we justified in doing so? In parallel with the way Hosiasson approached the first justification question, we would expect the answer to be: this reasoning is justified whenever it is the case that $p(f_{k+1}, sf_1 \dots f_k) > p(f_{k+1}, s)$, under some suitable assumptions. However, this is not how she proceeds. What Hosiasson does prove is a theorem of the form: given some nontrivial conditions, we have $p(f_k, sf_1 \dots f_{k-1}) > p(f_{k-1}, sf_1 \dots f_{k-2})$ (see Theorem 3 and its corollary on pages 30–31 of Hosiasson (1934)). What is compared, then, is not the credibility of the k'th fact with and without having observed the first $k-1$ facts. Instead, it is the credibility of the k'th and $k-1$'th fact, relative to the observations accumulated up to those points, respectively.

While she concludes that the above theorem provides a reason to answer her second justification question in the positive, it does not seem right, given how the question was formulated. The theorem could, indeed, be of interest when it comes to the question of whether a sufficient number of observations can in the limit lead to the (subjective) certainty of the inductive hypothesis—which is a very different form of the justification question, and a stronger one at that.

But the above theorem does not imply any answer yet to the question of whether the prior credibility of a consequence of the inductive hypothesis is raised by observing any number of other consequences of that same hypothesis—which is what was needed for the hypothetical subinduction to be justified in the same way that the hypothetical induction was shown to be justified. While the second justification question remains without a positive answer in the early papers, it reappears in a new context later on: in the analysis of inductive reasoning by analogy. This is the line of Hosiasson's thought that culminates in the recovered manuscript.

3 Analogical Reasoning

Given that Hosiasson was a careful reader of Keynes—she often referred to "A Treatise on Probability" (Keynes, 1963) and generalized some of the theorems there—it is no surprise that she considered analogical and inductive reasoning to be closely related. Eventually, she turned to investigate analogical reasoning in its own right, as a special kind of inductive reasoning. But while her starting point was on the traditional side—focused on equating analogy to, basically, the degree of similarity between objects—she eventually developed a much more general approach, fully in line with her initial focus on a structural, logical approach to inductive reasoning.

The first step in that direction was the article "Induction et analogie: comparaison de leur fondement" (Hosiasson Lindenbaum, 1941). In it, she sets up her general strategy for studying the logic of analogical reasoning. In a manner similar to the earlier work, she starts by specifying two general forms of inductive reasoning: induction proper, and reasoning by analogy. The two types are the same as in her previous work. What she now calls reasoning by analogy matches the earlier description of hypothetical subinduction: raising the credence in some fact based on other facts that are consequences of the same hypothesis.

Assume that two facts, f_1 and f_2, are consequences of the same hypothesis h, given background knowledge s. In reasoning by analogy, says Hosiasson, we proceed from learning f_1 to raising our credence in f_2, on the basis of the two sentences sharing the common hypothesis h. In what sense is this kind of reasoning analogical? There is no explicit counting of the characteristics that f_1 and f_2 share; they are not even things, but sentences of any form. But in a very general sense, f_1 and f_2 can be thought of as two objects that share the property of being a consequence of h. Informally speaking, it is "through" h that the credibility of f_2 grows after observing f_1: observing f_1, by induction, gives us a reason to strengthen our belief in h, which, in turn, gives us a reason to strengthen our belief in f_2.

The probability axioms alone do not guarantee that this will hold. Hence, the inquiry into the justification of such reasoning by analogy means looking for the additional assumptions that have to be made in order for it to be rational to raise the credence in f_2 in the above setting. That is, we are looking for the necessary and sufficient conditions for the following inequality to hold:

$$\mathbf{A.}\, p(f_2, f_1 s) > p(f_2, s).$$

Once again, s is the sentence representing the background knowledge. Condition A does not hold universally, without any extra assumptions about the relationships between the credibility values of h, f_1, and f_2. It is Hosiasson's goal in all her subsequent work on analogy to find necessary and sufficient conditions for A, under the minimal assumptions of the probability axioms for p, and the same additional conditions that guaranteed that the basic inductive reasoning was justified (conditions (a) and (b) in the next section).

For the most part, the 1941 paper is an exploration of this question in some less general settings, i.e., assuming specific logical forms of the f's in the above formula A. Hosiasson tries to formally capture the intuition that analogical reasoning has something to do with objects sharing properties, with analogy the more justified, the more properties are shared. The final solution, however, is not attained in the paper (as it has also been unattainable for everyone working on the topic ever since, as the vast modern literature on analogical reasoning shows). This is probably why in the subsequent work Hosiasson dropped the more fine-grained formalizations of reasoning by analogy and returned to the analysis of what she considered to be its simplest form, given by A—where the logical form of the h, f_1, and f_2 is not spelled out any further.

4 The Manuscript

All of the above is what we knew until now about Hosiasson's approach to reasoning by analogy: that by 1940, after settling down on its simple logical representation, she explored various ways to expand her logical approach to inductive reasoning to include the "counting of similarities" view of analogy—but none of them led anywhere particularly interesting.

The manuscript translated here and the note that (most probably) preceded it, are the further steps of this development, steps that have been missing. The new material suggests that Hosiasson continued to work on this topic shortly after writing the *Mind* paper. The new results were eventually put together in a draft entitled "On Inference". As both the form and the title of the manuscript suggest, it was merely a sketch and not a finished paper. Yet, the presentation is clear and well-organized, and the argument is given from first principles, starting with the clarification of the concepts of reasoning and inference as well as the axioms for the credibility functions.

In "On Inference," the problem of the justification of analogical reasoning is once again stated using the formula A, and the manuscript is dedicated to the search for conditions under which A holds. Other than Hosiasson's usual probability axioms, there are two additional assumptions made (see Sect. 2):

$$\textbf{a)}\, p(h, s) > 0,$$
$$\textbf{b)}\, p(f_1, s) < 1.$$

She identifies two constraints on the credibility functions that are each necessary and sufficient for A to hold:

$$\textbf{Condition 1.}\quad \frac{p(\overline{f_2}, \overline{h} f_1 s)}{p(\overline{f_2}, \overline{h} s)} < \frac{p(\overline{h}, s)}{p(\overline{h}, f_1 s)},$$
$$\textbf{Condition 2.}\quad p(f_2, \overline{h} f_1 s) > \frac{p(f_1, s) \cdot p(f_2, s) - p(h, s)}{p(f_1, s) - p(h, s)}.$$

In the above formulas, the bars over sentential variables signify negation. Once again, h is the hypothesis, f_1 and f_2 are its consequences, and s is the background knowledge. From these two formulas we can easily derive another pair of conditions which are sufficient—but not necessary—for A. Reasoning by analogy from f_1 to f_2 is rationally permitted, therefore, if we can find a hypothesis h which implies the two facts, and which satisfies either of the following sufficient conditions:

$$\textbf{Sufficient Condition 1.}\quad p(f_2, \overline{h} f_1 s) \geq p(f_2, \overline{h} s),$$
$$\textbf{Sufficient Condition 2.}\quad p(h, s) > p(f_1, s) \cdot p(f_2, s).$$

Sufficient Condition 2 implies Condition 2, since the right-hand side of Condition 2 will have a negative value once Suffiecient Condition 2 is true. In the note to Hempel, Hosassion suggests its further generalization: for any sequence of facts f_1 ,...,f_2, and their common reason h, the condition sufficient for reasoning by analogy to be rational, becomes $p(h, s) > p(f_1, s) \cdot p(f_2, s) \cdot \ldots \cdot p(f_n, s)$. In that case the credibility of at least one of the f_i's would be raised upon having observed all of the others.

Sufficient Condition 1 is a good illustration of how deeply reasoning by analogy, as considered by Hosiasson, is related to induction in its most general form. In informal terms, putting aside the exact logical or probabilistic nature of the relation between hypotheses and their consequences, the interplay between analogical and inductive reasoning can be described as follows. Both f_1 and f_2 are consequences of a number of possible hypotheses. Some of these hypotheses are shared between them, like the h in the analogy formula A. Some of them are not shared and it can even be the case that some of the hypotheses supporting f_1 do not support f_2 at all. In probabilistic terms this would mean that such hypotheses make f_1 more probable, while making f_2 less probable; in the logical setting that Hosiasson uses, it would rather mean that those hypotheses imply the negation of f_2, or at least do not imply f_2.

Now, when we observe f_1, by simple inductive reasoning we can raise our credence in each of the hypotheses supporting f_1. The second fact, f_2, will be a consequence of some of those hypotheses, but not all—let us call those hypothesis that support f_1 but not f_2 the non-hypotheses of f_2. Whether the net result will be a higher credibility of f_2 (given f_1) will depend, so to speak, on whether such non-hypotheses of f_2 do not "cancel out" those hypotheses that f_1 and f_2 do share. It is definitely enough for that if all of the non-hypotheses of f_2 do not have a negative effect on f_2: they simply do not influence its probability at all—which is exactly what Sufficient Condition 1 ensures. In that case, the credibilty of f_2 is guaranteed to raise after f_1 is observed. However, calculating the exact ratios of influence of the different kinds of hypotheses that might be pointing out in different directions when it comes to the credibility of f_2, might be a very complicated task, hence only a very general clause like the Sufficient Condition 1 can be formulated.

This kind of reasoning is illustrated by the example at the end of the manuscript. Hosiasson describes the analogical reasoning that might be performed by someone who has spent the previous night in a prison cell and has experienced some rheumatic-like pain in her legs—that is the fact f_1. The second fact f_2 is the prediction that during the following night the leg pain will be present again. After having observed f_1, can we raise the credence in f_2?

One has to start by considering all possible hypotheses supporting f_1 or f_2—both the shared ones and the ones that the two facts do not have in common. Hosiasson assumes that in the scenario there are only two hypotheses supporting f_1: w which says that there is humidity in the prison cell, and w_1 which says that the previous day was rainy and the person in the example had to spend a lot of time queuing outside waiting for bread to be distributed. The first of these also supports f_2, which means that we have identified at least one shared hypothesis for f_1 and f_2.

For A to hold, then, it is enough for w to satisfy the Sufficient Condition 1. In order to determine this, we have to assume that w is not true, and check what influence observing f_1 will have on f_2 under this assumption. Because the only other hypothesis supporting f_1 is w_1, we have to check how the truth of w_1 affects the credibility of f_2. In the example this influence can go either way, depending on the additional background knowledge, such as whether the rain has been going on for some time (which lowers the probability that it will continue for yet another day), or whether the bread is distributed every day or not, which determines whether on the next day the person will also have to spend a lot of time queuing outside, etc. Once the character of this influence is established and it turns out that Sufficient Condition 1 does hold in this case, A will follow and the credence in f_2 can be rationally raised after having observed f_1.

5 Analogy as Probabilistic Relevance

Hosiasson was interested primarily in the logic of inductive reasoning, but this does not yet make her a supporter of the logical notion of probability, understood in the traditional sense. In her articles she took care to differentiate between rational credence—which is what she took herself to be modeling—and probability proper, the exact nature of which she was not clear about. Yet, given the influence that her paper on confirmation had on Carnap, and the general focus of her work, she could be, and in fact is, included in the inductive logic tradition broadly construed. This holds of her work on analogy as presented here as well, which was, moreover, pioneering in its generality.

Hosiasson and Carnap met multiple times in the 1930s, for the first time in November 1930 during Carnap's first visit to Warsaw, where he met the members of the Warsaw branch of the Lvov-Warsaw School. In his diary, Carnap mentions multiple conversations with Hosiasson, primarily about probablity and induction—a topic which he was not actively working on yet. These conversations continued in 1931 during Hosiasson's academic visits to Vienna in the spring and summer.[12] The two continued to meet during the subsequent Unity of Science congresses and related events, both official and social. Carnap praised Hosiasson's academic ability both in private[13] and as a reviewer.[14]

The most important point of comparison of Hosiasson's work translated here with what followed within the inductive logic tradition, is Carnap's late work on analogical reasoning. While Carnap made some earlier attempts to include analogical reasoning within his systems of inductive logic (see (Carnap, 1963)), it is the last of his proposals that is remembered best. In the last published account of his inductive logic project, "A Basic System of Inductive Logic," Carnap (1971, 1980) extended his single parameter continuum of confirmation functions (Carnap, 1952), into a much wider range of admissible, rational confirmation functions. A part of this extension was the introduction of the analogy parameter η, which quantifies how similar pairs of predicates are, which in turn influences how much they will raise each other's inductive probabilities in situations where not many observations have been made.

Carnap's formal setting is different than Hosiasson's: his confirmation functions are defined on first-order languages with unary predicates, as opposed to the propositional language of Hosiasson. Atomic sentences are of the form $P_j a_i$, ascribing the property P_j to the object named with the constant a_i. Let p be a confirmation function,[15] and a and b be, respectively, the first and the second objects observed in a particular investigation. The analogy parameter is defined on pairs of predicates: for each pair P_j, P_l, we look at how independent observing objects with these two properties is:

$$\textbf{Carnap's analogy parameter:}\quad \eta_{jl} = \frac{p(P_j a \cap P_l b)}{p(P_j a) \cdot p(P_l b)}.$$

(The set-theoretic notation comes from the fact that Carnap's probability at this point is taken to be applied to propositions, in contrast to Hosiasson's functions being applied to sentences.) When the η_{jl} value is greater than 1, the two predicates

[12] Rudolf Carnap Diaries, Series 5, Rudolf Carnap Papers, 1905-1970, ASP.1974.01, Archives of Scientific Philosophy, Archives and Special Collections, University of Pittsburgh Library System.

[13] Carnap to Neurath, June 9, 1933, item 029-11-15 in Rudolf Carnap Papers.

[14] Carnap's recommendation letter for Janina Hosiasson-Lindenbaum, September 9, 1940, in: Series 4, Box 3, Folder 147, American Council for Émigrés in the Professions Records, 1930-1974. M.E. Grenander Department of Special Collections and Archives, University Libraries, University at Albany, State University of New York.

[15] For simplicity, I do not distinguish here between conditional and unconditional probability functions. In the following, when p has only one argument, it is to be understood as probability conditional on the empty evidence, as in ((Carnap, 1971), p. 41).

are positively relevant for each other. The kind of inductive reasoning that uses this fact is called by Carnap the analogy by similarity of predicates: observing an object of one kind will raise the prior probability of observing the object of another kind—it is rational to raise the credence in the relevant propositions. This can be spelled out in the following statement:

$$\textbf{CA, Carnap's analogy:} \text{ if } \eta_{jl} > 1, \text{then } p(P_j a, P_l b) > p(P_j a).$$

Once we summarize Carnap's approach to analogical reasoning in this simple way, it becomes strikingly similar to the one of Hosiasson. Recall Hosiasson's Sufficient Condition 2, and the fact that it implies the analogy formula A: once we can find a common hypothesis h for two facts f_1, f_2, which satisfies Sufficient Condition 2, then $p(f_2,f_1 s) > p(f_2, s)$. Carnap's own analogy parameter supplies such a common hypothesis—simply the conjunction—for pairs of atomic propositions, leading to a version of A for atomic propositions involving pairs of predicates with sufficiently high η values. Hence, Carnap's analogy condition CA implies Hosiasson's analogy condition A, by providing a common hypothesis h: the conjunction of f_1 and f_2, which satisfies Sufficient Condition 2. Therefore, when two propositions are analogous to each other in Carnap's sense, they are also analogous in Hosiasson's sense.

There are two ways, however, in which Hosiasson's approach is more general than Carnap's. The first way concerns the kinds of sentences that are allowed to be used in analogical reasoning. Carnap considers only atomic sentences of his first-order languages. Hosiasson assumes no restrictions of this sort: the atomic propositions in her account can talk about any types of facts and relations whatsoever, as seen in her examples in the manuscript translated below. Moreover, Hosiasson did suggest generalizing Sufficient Condition 2 to include sequences, rather than pairs, of propositions (see previous section). Finally, her focus on any common hypothesis for the two facts under consideration, rather than just their conjunction, was not only more general, but also more in line with how analogical arguments are made in scientific practice. This greater generality holds also for the background knowledge. For Carnap, reasoning by analogy was permitted only when there was no other evidence available; after more observations are gathered, the analogy influence tapers down to nothing. Hosiasson, on the other hand, allows for any kind of extra information to be plugged into the background knowledge s in her conditions.

The second way in which Hosiasson's approach is more general is related to how the two of them relate to the idea of similarity as the backbone of analogy. Just like Hosiasson, Carnap was not particularly interested in measuring the degree of analogical relevance by counting the ways in which things (in this case: predicates) are similar. Instead, he chose a single, generalized relevance measure η which directly governs the inductive influence of one predicate over the other. Analogical reasoning is modeled as probabilistic relevance, based on a specific shared characteristic—having a common reason (Hosiasson), having a high η value (Carnap)—rather than a full list of similarities and dissimilarities.

There is a caveat to the above, however. It starts with how Carnap answers the question of how we are to know that two predicates are related in a way that leads to the η value greater than 1. He solves this by tying the η values to the distances between the predicates in an underlying conceptual space which provides a partial interpretation of the object language (see (Carnap, 1980), ch. 14 and 16). In short, when the two predicates P_j, P_l are sufficiently close to each other in the underlying value-space (attribute space for Carnap), the value of η_{jl} should be high, which in turn impacts the relevant rational confirmation values. This move does bring in considerations of similarity through the back door: the distances between predicates in the value space can be seen as direct correlates of the degree of similarity between them (this is also how such attribute spaces are constructed in the modern theories of conceptual spaces).

Hence, in the end, while Carnap does spell out analogical reasoning in terms of statistical dependence—as in the above condition CA—he does not give up considerations of similarity entirely. He simply moves them further down into the geometric underpinning of his partially interpreted formal languages. Hosiasson, on the other hand, abstracts from any measures of similarity altogether and focuses solely on the probabilistic relations. Hosiasson's approach to analogy, then, while significantly foreshadowing Carnap's version, retained a greater level of generality and a commitment to a probabilistic, rather than similarity-based, view of analogical reasoning. This is a very different view from the traditional ways in which analogical reasoning was represented, which was mostly by some form of similarity quantification between the source and target objects. Even within the early inductive logic literature, namely in Keynes's *Treatise*, analogical reasoning, while considered an important basis for inductive reasoning, was taken to be founded on the basic recognition of similarity between events of the same kind.

Carnap's discussion of the "analogy influence" appeared in print only in 1980, although he had worked on this approach at least from the early 1960s. The historical investigation into the development of the Basic System and the influences on it is still a work in progress. It is unclear at this point whether Carnap was influenced by Hosiasson on this topic in any way. Certainly, the only material he could have had access to, in the 1940s as well as in the 1960s, was Hosiasson's paper in *Mind*, where her analogy formula A is already present, but the Sufficient Condition 2 is not; the latter appears in the unpublished material. The common predecessor to both approaches could be Keynes, but his discussion of the role of analogy in inductive reasoning does not feature anything like Hosiasson's and Carnap's analogy conditions.

It was Carnap's approach to analogical reasoning as inductive, or probabilistic, relevance that spurred the subsequent line of research within the inductive logic tradition. As it turns out, this very approach has a history that is longer by at least two decades, and was started already in the early 1940s by Hosiasson—the point made even stronger by the manuscript translated here. Therefore, this draft and its origins are not only testament to Janina Hosiasson's strength of spirit and her devotion to philosophy, but also contribute an important missing link in the history of inductive logic.

6 Janina Hosiasson "On Inference. A Draft"

1. We perform an act of *inference* when, on the basis of some sentences (premises) which we consider as certain or take as assumptions, and on the basis of some reasoning, we attach to another sentence (the conclusion) a certain degree of belief (credence), or we change the degree of belief which we previously assigned to this sentence.

A necessary condition for the correctness of reasoning is that we assign degrees of belief to sentences as if we followed the following laws (and the laws that follow from them). [In the following] a, b, c are sentence name variables, $\overline{a}$ is the negation of a, $a \vee b$, ab are the logical sum and product of a and b, $p(a, b)$ is the degree of belief assigned to a given b ("under the assumption of") b. We will also call the latter the *degree of (subjective) certainty* of a given (based on) b.

We assume that $p(a, b)$, for the kinds of sentences we are interested in, is a number and we take $0 \leq p(a,b) \leq 1$.

Law 1. If a follows from b, then $p(a,b) = 1$

Law 2. If a excludes b, then $p(a \vee b, c) = p(a, c) + p(b, c)$

Law 3. $p(ab, c) = p(a, c) \cdot p(b, ac)$

Law 4. If $a \equiv b$, then $p(a, c) = p(b, c)$

An inference is correct if and only if it is based on correct reasoning.

The reasoning in which we move from some sentences to another sentence because it follows from them, is called *deductive reasoning*. As we can see from **Law 1**, in an inference which is based on assumptions that are certain, and which is based only on deductive reasoning, the conclusion should be a sentence which is also certain. This kind of inference we call *certain inference*.

[Here there shall be an insert about the conditions of higher and lower correctness of uncertain reasoning.]

Among uncertain inferences, ones that deserve special attention are: inductive inference and inference by analogy. In *inductive inference* we raise the degree of belief in the reason (conclusion) on the basis of its consequences (premises). In *inference by analogy* we raise the degree of belief in one consequence (conclusion) of some reason, based on some of its other consequences (premises).

2. The importance of reasons (hypotheses) for inference by analogy.

As I have shown elsewhere, not every inference by analogy is a correct one, i.e., not for all sentences (facts) f_1 and f_2 that have a common reason do we have

$$A.\ \ p(f_2, f_1 s) > p(f_2, s),$$

where s is the knowledge that was had before learning f_1.

This is because any two sentences have a common reason, for instance their logical product. Hence we would have at the same time $p(f_2, f_1 s) > p(f_2, s)$ and $p(\overline{f}_2, f_1 s) > p(\overline{f}_2, s)$ (because f_2 and f_1 have a common reason $f_1 f_2$, and $\overline{f}_2$ and f_1—a common reason $f_1 \overline{f}_2$). However, for these two inequalities to hold simultaneously would be inconsistent with **Laws 1-4**.

We shall state a couple of conditions which are necessary and sufficient for the inequality A to hold:

$$\textbf{Condition 1}\quad \frac{p(\overline{f_2}, \overline{h}f_1 s)}{p(\overline{f_2}, \overline{h}s)} < \frac{p(\overline{h}, s)}{p(\overline{h}, f_1 s)},$$

$$\textbf{Condition 2}\quad p(f_2, \overline{h}f_1 s) > \frac{p(f_1, s)\cdot p(f_2, s) - p(h, s)}{p(f_1, s) - p(h, s)},$$

where h is any reason of sentences f_1 and f_2 (any hypothesis, from which f_1 and f_2 follow).

Conditions 1 and **2** are also necessary, their equivalent does not mention h, and h is any reason. Therefore, if any reason of sentences f_1 and f_2 fulfills them, then any such reason does.

It is easy to show that if a) $p(h, s) > 0$, and b) $p(f_1, s) < 1$, then $p(h, f_1 s) > p(h, s)$, i.e., $p(\overline{h}, s) > p(\overline{h}, f_1 s)$. Therefore, we see from **Condition 1** that a sufficient condition for inference by analogy to be correct is (assuming a) and b)), among others, the condition:

$$p(\overline{f_2}, \overline{h}f_1 s) \le p(\overline{f_2}, \overline{h}s), \text{i.e., } p(f_2, \overline{h}f_1 s) \ge p(f_2, \overline{h}s).$$

Hence, if after learning f_1 we find a common reason h of f_1 and f_2 such that if h is taken to be false, then earning f_1 does not decrease the level of certainty of f_2—then we can (assuming *a)* and *b)*) infer by analogy, i.e., increase the degree of certainty of f_1, regardless of the degree of certainty of h.

This last inequality differs from **Conditions 1** and **2** in that not every reason of f_1 and f_2 will satisfy it, once one such reason does. E.g., the reason in the form of the logical product $f_1 f_2$ never satisfies this inequality under a) and b), because $p(f_2, \overline{h}f_1 s) = p(f_2, f_1\overline{f_2}s) = 0$. However, this reason can satisfy **Conditions 1** and **2**, and it does so for $\mathcal{A}$.

Let us take the following example:

We are sleeping for the first time in a prison cell, where throughout the night we have aches and pains in our legs. Call this fact f_1. Can we raise the degree of belief in the fact that the next night we will have the same aches and pains in the legs? Let us call this future fact f_2. We are therefore asking if A: $p(f_2, f_1 s) > p(f_2, s)$. We look for a possible cause (reason) of our nightly ailment. We have reasons to believe that the humidity level in the cell is high enough to bother us in this way, given the state of our rheumatism or arthritis. Let us call the existence of such humidity in the cell w. Hence, w is a reason for f_1 and f_2. Let us ask if the above sufficient condition for A holds, i.e., if $p(f_2, \overline{w}f_1 s) \ge p(f_2, \overline{w}s)$. Let us assume, then, that the humidity level in the cell (if there is any humidity at all) is not sufficient for f_1 and f_2, i.e., assume $\overline{w}$. Now our question comes down to whether there are any possible causes of f_1 such that, if we assume them, f_1 does not lower the degree of certainty of f_2. Suppose for instance that besides w the only possible cause of f_1 is (next to some humidity in the cell) rainy weather on the day preceding the night when we were in pain, and the fact that on that day we stood outside for a long time, waiting for bread.

In that case, f_1 can just as well decrease the level of certainty of f_2—when, e.g., the weather has been rainy for a few days already, so that each following rainy day makes it less likely that it will continue to be so; and if we go outside for longer periods of time only to get bread, and bread is distributed only every second day, etc. Hence, the above sufficient condition can fail to be satisfied by w. It does not mean, however, that some other condition sufficient for A cannot be satisfied; for instance the one listed below.

On the other hand, if it is the first day of rainy weather; if bread is given out every day or the next day we are supposed to sweep the streets, etc.—then f_1 does raise the level of certainty of f_2 under the assumption w. The sufficient condition given above is then satisfied and we can therefore raise our degree of certainty of the fact that the following night, just like the current one, we will be suffering from leg pain.

From **Condition 2** we get the following sufficient condition for A (assuming a) and b)):

$$p(h, s) > p(f_1, s) \cdot p(f_2, s).$$

That is because the denominator of the right-hand side of **Condition 2** is positive, given that h is a reason for f_1 (and a) and b) hold).

Hence, if after establishing f_1 we find a common reason h of f_1 and f_2 whose degree of certainty is higher than the product of degreef of certainty of f_1 and f_2 "in advance", then we can (assuming a) and b)) infer by analogy, i.e., raise the degree of certainty of f_2.

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Declarations

Disclosure The translated manuscript is now in the public domain, as its author died more than 70 years ago. Eleonora Czeżowska, who is the legal guardian of Tadeusz Czeżowski's papers, where the manuscript is deposited, has agreed to the publication.

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