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ORIGINAL RESEARCH

The Significance of Value Additivity

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Abstract
Whether value is "additive," that is, whether the value of a whole must equal the sum of the values of its parts, is widely thought to have significant implications in ethics. For example, additivity rules out "organic unities," and is presupposed by "contrast arguments." This paper reconsiders the significance of value additivity. The main thesis defended is that it is significant only for a certain class of "mereologies", roughly, those in which both wholes and parts are "complete", in the sense that they can exist independently. For example, value additivity is significant in the case of a mereology of material objects, but not in the case of a mereology of propositions.

1 Introduction

By saying that value is "additive" I shall mean that the value of a whole must equal the sum of the values of its parts. Whether value is additive in this sense has been regarded as important by many philosophers.

Moore (1903) famously warned against assuming additivity. He posited "organic unities" whose values are non-additive. Familiar examples abound: a football team of mediocre players who coordinate well together may defeat a team full of individually brilliant lone wolves; a collection of tasty ingredients may combine to form a disgusting dish; a knife paired with a fork may be very useful, while either utensil alone is of only limited utility; and so on.[1]

Additivity is also widely thought to be implicated in a pervasive, yet controversial, style of argument in moral philosophy. Kagan (1988) calls arguments in this style "contrast arguments". Perhaps the most famous is advanced by Rachels (1975) in his discussion of active and passive euthanasia. Some believe active euthanasia is

[1] On organic unities, see, e.g., Carlson (1997), Hurka (1998), Lemos (1998, 2015), Zimmerman (1999), Dancy (2003), Brown (2007), Fletcher (2010).

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morally worse than passive euthanasia because the former involves killing a person, whereas the latter involves only letting a person die. Rachels rejects this view. He describes another pair of cases which differ only in that one involves killing and the other letting die, but in which, intuitively, neither is morally worse than the other. In both cases, a man sets out to drown his innocent cousin. In one case he carries out his plan, killing his cousin. In the other case, he finds his cousin already drowning, and so instead simply stands by and lets him die. If the bare difference between killing and letting die makes no moral difference in this pair of cases, Rachels argues, then it cannot make a difference in cases of active and passive euthanasia either. According to Kagan, arguments like Rachels' covertly assume that value is additive.[2]

On the other hand, one might be sceptical about the significance of additivity. One might doubt whether there's any substantive difference between additive and non-additive assignments of values, and regard this instead as merely a matter of "bookkeeping". One source of scepticism may be that it seems possible to divide a set of whole objects into parts in different ways, some of which are compatible with additivity and others not. But there may seem to be no further fact about which is the "correct" division, so it is merely a matter of taste whether one opts for an additive division or a non-additive one. I shall, however, pursue a different line of scepticism. One might fell that, ultimately, only the values of wholes matter. But additivity might not constrain the values of wholes. It might be the case that, for any assignment of values to wholes, one can find an assignment of values to the parts such that these add up in the required way. In that case, additivity may appear to be a non-issue.

The main thesis defended in this paper is that whether additivity is significant depends on the sort of mereology involved. By a "mereology" I mean a collection of objects (of whatever kind) related to each other by a parthood relation, so some are parts of others. Parthood relations hold between entities of diverse kinds. Perhaps the most obvious are physical objects: tables, chairs, dogs, mountains, and so on. In this case, parthood is determined spatially (and perhaps also temporally). The parts of a physical object occupy subregions of the space occupied by this object. But we talk also of parthood among abstract objects, such as properties, propositions, and states of affairs. For example, Socrates' being rational may be a part of his being a person. Here parthood is determined by "modal" space, instead of physical space. The possibilities in which Socrates is a person are a subset of those in which he is rational. The "region" of modal space occupied by the former set of possibilities is a subregion of that occupied by the latter. Notice, however, that the order here is reversed, in comparison to physical objects. For abstract objects, it is the whole that occupies the subregion, whereas for physical objects, it is the part.

The unifying general principle governing parthood in all these cases is that you cannot have the whole without the part, but you can have the part without the whole. (By "part" here I mean *proper* part.) In the case of physical objects, a whole cannot *exist* without its parts also existing, but each part can exist without the whole. For example, a table cannot exist without a tabletop, but a tabletop can exist without a

[2] On contrast arguments, especially Rachels' "bathtub" example, see, e.g., Philips (1987), Kamm (1996), Asscher (2007), Purves (2011), Woollard (2012).

table. The same holds for propositions, except what matters here is not existence, but truth. A whole proposition cannot *be true* without its parts being true, but each part can be true without the whole being true.[3] Likewise, a whole property cannot *be instantiated* without its parts being instantiated, and a whole state of affairs cannot *obtain* without its parts obtaining.

We may evaluate both concrete and abstract objects. We may say, for example, that a dog is good, or a computer is bad. But we may also say that it is good that a person is happy, or bad that she is depressed. When evaluation and mereology are combined, the question of value additivity arises.[4] We may ask whether the value of a whole must be the sum of the values of its parts. I argue, however, that additivity is significant only in a certain sub-class of mereologies, namely, those in which the parts are, as I say, “complete”. Consider, for example, the property of *being red*. It is impossible for anything to be *merely* red, without being any specific shade of red. This property cannot, therefore, be instantiated except *as a part* of some other property—as a part of being scarlet, or crimson, or some other shade of red.[5] In this sense, being red is incomplete. On the other hand, physical parts are complete. The pedal is a part of the bicycle. But the pedal *could* exist “on its own”, without being the part of any bicycle. It could exist as a whole in its own right.[6] So it is complete.

I am unsure how closely this distinction corresponds to that between abstract and concrete objects. It does seem that all concrete parts are complete, and that many abstract parts (e.g., properties, propositions, and states of affairs) are incomplete. But there may also be complete abstract parts. Consider, for example, a song, which may be regarded as abstract.[7] A song may have something like temporal parts: the introduction, the verse, the chorus, and so on. It may also have parts corresponding to the different instruments used to perform the song: the vocals, the drums, the guitar, and so on. These parts seem to be complete. We can imagine a band performing merely the introduction and then stopping, or a lone singer performing only the vocal part unaccompanied. These parts of the song may be performed as if they were whole songs in their own right. So the parts of a song, though abstract, seem nonetheless complete.

[3] Sometimes propositions are said to have parts, or “constituents”, of another sort. For example, the proposition that Papalymo is a bachelor might be said to have the individual Papalymo and the property of being a bachelor as parts. This is not the sense of parts that is relevant here. The parts of propositions, in the relevant sense, are also propositions.

[4] Strictly speaking, for additivity to be meaningful, it is not sufficient merely that we be able to evaluate the objects in question. These evaluations must be of a certain kind. As explained below, we need to be able to measure value on a ratio scale.

[5] Notice this is in accord with our general principle above. It is impossible for a thing to be scarlet without being red, though it is possible for a thing to be red without being scarlet.

[6] Here I am assuming a mereology of what might be called “ordinary” physical objects, which includes only those objects that would be recognised in ordinary discourse. This excludes arbitrary mereological fusions such as (if it exists) the object composed of my dog’s tail, the Palace of Culture, and the rings of Saturn. I cannot offer a more precise definition of ordinary object. But I hope it will be accepted that, for example, a bicycle is one, and moreover that (in some circumstances) it is not a proper part of any other; so it is a whole object in this mereology. Likewise, a bicycle pedal, when detached from a bicycle, is a whole ordinary object. Furthermore, even when attached to a bicycle, we may imagine its being detached, so we can, as it were, consider it as if it were a whole.

[7] I do not mean a particular performance of a song, but rather whatever it is that all performances of the song have in common – the unitary thing that is being performed in all these instances.

My thesis is that additivity is significant only in mereologies where at least some non-whole objects are complete. My argument has two main premises. The first is that, when additivity is properly defined, it places no constraints—or at most, only meagre constraints—on our evaluation of whole objects. For any non-additive evaluation, we may define an additive counterpart which is in (almost) complete agreement regarding the values of wholes. The reason this fact has been overlooked by some authors, I suggest, is that they have been insufficiently careful in applying additivity to particular cases. A whole may be divided into parts in different ways, and some of its parts may overlap each other. When applying additivity, it is essential to ensure that the parts whose values are summed do not overlap, to avoid "double-counting." Yet many purported counterexamples to additivity covertly employ overlapping parts. This fact is obscured by a failure to clearly define the mereology involved. When this is clarified, it can be shown that the constraints imposed by additivity on the evaluation of wholes are actually very slight, perhaps even non-existent, and they do not conflict with many of the standard "counterexamples" to additivity. The second premise is that only the values of *complete* objects matter. Since an incomplete object has no independent existence, one's evaluation of such an object is practically inconsequential.

Together, these two premises support my thesis as follows. Consider a mereology in which only the wholes are complete. Then, for this mereology, it is only the values of wholes that matter. But additivity does not constrain the values of wholes. So additivity does not constrain anything that matters, and is therefore insignificant.

A final clarification is needed. The specific kind of value that interests me here is so-called "intrinsic value"—the value that a thing has "in itself", in virtue of its intrinsic properties, as opposed to its relations to other things. I will not venture a more precise characterisation of intrinsic value. I hope the basic notion is clear enough. You may be sceptical about the existence of intrinsic value. It should be emphasised, however, the issue here is *structural:* assuming that (some) objects do have intrinsic values, how might these values related to each other? Below, for neatness, I continue to suppress the term "intrinsic", speaking simply of "value".

2 Defining Additivity

I begin by formally defining additivity. It is important to do this carefully, because some purported counterexamples to additivity tacitly assume an inadequate definition. When defined properly, I argue, we see that these are not really counterexamples after all.

2.1 Preliminaries

I shall focus on a mereolgy of propositions, where these are defined in the familiar way as sets of possible worlds. Let U be the set of all possible worlds (think of "U" for "universe").[8] So a proposition is a subset of U.

[8] Strictly speaking, the collection of *all* possible worlds is likely too big to be a mere set. It is probably a proper class. I ignore this technicality here.

Propositions exclude possibilities. To believe, for example, that *snow is white* is to rule out those possibilities in which snow is not white. When one proposition X excludes all the possibilities excluded by another proposition Y (and perhaps more besides), we may say that Y is a *part* of X. Belief in X, in a sense, already includes belief in Y. So, for example, the proposition that *snow is white* is a part of the proposition that *snow is white and grass is green*; and the proposition that *snow is white or grass is green* is a part of the proposition that *grass is green*.[9] Notice, a proposition must be *less specific* than a proposition of which it is a (proper) part, in the sense that it must convey less information, or narrow the range of possibilities to a lesser extent. The parthood relation is thus the *inverse* of entailment: Y is a part of X if and only if X entails Y, or $X \subseteq Y$.[10]

I assume that we are interested in evaluating only a subset of propositions, which I shall call Ω. This is defined as follows. Let $\equiv$ be an equivalence relation on U, where $x \equiv y$ is interpreted as meaning that x and y differ only in ways that are evaluatively irrelevant. Then Ω contains all and only those propositions which are closed under $\equiv$.[11] So a proposition X is included in Ω if and only if X contains any world that is evaluatively equivalent to any world in X.[12] The idea is to make Ω sufficiently fine-grained to "carve" the space of possibilities at its "evaluative joints", but no more fine-grained than this.

It follows that Ω is a Boolean algebra. This means that Ω is closed under the operations of complement (if $X \in \Omega$ then $\overline{X} \in \Omega$), union (if $X, Y \in \Omega$ then $X \cup Y \in \Omega$), and intersection (if $X, Y \in \Omega$ then $X \cap Y \in \Omega$). These operations may be thought of as, respectively, negation, disjunction, and conjunction. A benefit of working with a Boolean algebra is that propositions divide into parts in a well-behaved way. For simplicity, I assume, furthermore, that Ω is finite (this avoids dealing with infinite sums), though admittedly this is not guaranteed by the construction given above.

The universal set U and the empty set $\emptyset$ must both be included in Ω. These represent, respectively, the "tautologous proposition" and the "contradictory proposition". Though it is formally convenient to include these (doing so makes some definitions simpler), these are propositions only in a degenerate sense. The "purpose"

[9] As the second example helps to emphasise, we are interested here in parthood between *propositions*, not between *sentences*. The sentence "snow is white" is a part of the sentence "snow is white or grass is green", yet the proposition expressed by the latter sentence is a part of proposition expressed by the former.

[10] This may seem to get things backwards. One normally thinks of the subset as a part of the superset, whereas here it is the other way around. This is partly an artefact of the way in which I have defined propositions. One could instead represent a proposition by the worlds it *excludes*, rather than those it *includes*. That is, on this alternative definition, a proposition is the set of all worlds at which it is false, not true. In this case, the parthood relation between propositions would align with the subset relation, as seems more intuitive. A drawback of this approach, however, is that conjunctions of propositions would then be unions, and disjunctions would be intersections, which, again, is the opposite of what one normally expects.

[11] X is closed under $\equiv$ if, for any $x, y \in U$, if $x \in X$, and $x \equiv y$, then $y \in X$.

[12] To put this another way, let E be the partition of U induced by $\equiv$. Then X is in Ω if and only if X is the union of some subset of E.

of a proposition is to draw a *division* between possible worlds. But neither U nor $\emptyset$ does this. Moreover, it seems doubtful that we are interested in the values of these propositions. I will thus sometimes refer to "non-degenerate" propositions, by which I mean all propositions (in Ω) other than U and $\emptyset$.

Two subclasses of propositions are special: "atoms" and "wholes". An *atom* is a (non-degenerate) proposition of which no other (non-degenerate) proposition is a part. A *whole* is a (non-degenerate) proposition that is a proper part of no other (non-degenerate) proposition. Formally, these are defined as follows.

1. X is *atomic* (in Ω) if and only if $\forall Y \in \Omega : X \subset Y \Leftrightarrow Y = U$.
2. X is *whole* (in Ω) if and only if $\forall Y \in \Omega : X \supset Y \Leftrightarrow Y = \emptyset$.

These notions are "duals": a proposition is atomic just in case its complement is whole. Every atom is the complement of exactly one whole, and vice versa. This implies that the number of atoms must equal the number of wholes. It will be useful to enumerate the atoms and wholes. Thus let the atoms be $A_1, A_2, \dots A_n$, and the wholes be $W_1, W_2, \dots W_n$, where $n = \log_2 |\Omega|$. I also adopt the convention that the same number is assigned to a proposition and its complement. So $A_i = \overline{W_i}$.

It should be emphasised that atoms and wholes are defined relative to Ω. A proposition that is atomic in Ω may have (non-degenerate) proper parts in U, but these will not be included in Ω. In light of the construction of Ω given above, a whole (in Ω) may be equivalently defined as a proposition that is *minimally* closed under $\equiv$.[13] So wholes are maximally discriminating, from an evaluative perspective, whereas atoms are, conversely, minimally discriminating.[14]

We are interested in the values of these propositions. I represent this by a "value function" v that maps each proposition in Ω to a real number representing its value. I assume that v represents value on a ratio scale. This means that the unit of the value scale is arbitrary, but not the zero. So we may meaningfully compare ratios of values. We may say, for example, that X is twice as good as Y. This is necessary to make sense of additivity. Equalities between sums of values are preserved

[13] X is minimally closed under $\equiv$ if (a) X is closed under $\equiv$, and (b) no (non-empty) proper subset of X is closed under $\equiv$.

[14] It may be worth noting that these definitions of atoms and wholes are the opposite of what one might expect when dealing with a Boolean algebra of sets. In the usual representation of a power set (i.e., a set containing all the subsets of some set) as a Boolean algebra, the atoms are the (non-empty) sets containing the fewest elements, these being the singletons. On my definitions, however, these are instead the wholes, and their complements are the atoms. This may seem backwards. Notice, however, that Boolean algebras are completely symmetrical. If a Boolean algebra is, so to speak, "turned upside down", then the result will also be a Boolean algebra. When thinking of sets in mereological terms, it is natural to identify parthood with the subset relation, so the subsets of a set are its parts. On this way of thinking, the atoms are the singletons. However, as explained above, when these sets represent propositions, parthood should instead by identified with the superset relation, so the atoms and wholes are inverted.

by changes in the unit, but not by changes in the zero.[15] I should concede that this is therefore a significant implication of additivity, since it may be a matter of controversy whether value can be represented on a ratio scale (or even, for that matter, a cardinal scale). However, as I shall argue, this is (almost) the full extent of its significant implications.

2.2 Additivity

I turn now to defining Additivity. It will be instructive to consider first the following inadequate definition:

Naive Additivity $v(X \cap Y) = v(X) + v(Y)$.

This requires simply that the value of the conjunction of two propositions is the sum of the values of these propositions.

Naive Additivity is certainly not insignificant. It is incompatible with some substantive moral views. Oddie (2001) offers the following Kant-inspired example. On Kant's view, whether it is better that a person is happy or unhappy depends on her moral character. If she is virtuous, it is better that she is happy. But if she is vicious (non-virtuous), it is better that she is unhappy. Happiness enhances value when combined with virtue, but diminishes value when combined vice.

Let G be the proposition that a certain individual is virtuous (or good), and H be the proposition that this individual is happy. Then Kant's view may be stated as follows:

Kant's View $v(G \cap H) > v(G \cap \overline{H})$ and $v(\overline{G} \cap \overline{H}) > v(\overline{G} \cap H)$.

Assuming Naive Additivity, one might press the following "contrast argument" against Kant's View. The difference between $G \cap H$ and $G \cap \overline{H}$ is that H is a part of the former whereas $\overline{H}$ is a part of the latter. So the difference in value between $G \cap H$ and $G \cap \overline{H}$ must be the same as that between H and $\overline{H}$, i.e., we must have $v(G \cap H) - v(G \cap \overline{H}) = v(H) - v(\overline{H})$. But the difference between $\overline{G} \cap H$ and $\overline{G} \cap \overline{H}$ is precisely the same: H is a part of the former whereas $\overline{H}$ is a part of the latter. So the difference in value between this second pair of propositions must be the same as that between the first, i.e., we must also have $v(\overline{G} \cap H) - v(\overline{G} \cap \overline{H}) = v(H) - v(\overline{H})$. Therefore, $v(G \cap H) > v(G \cap \overline{H})$ if and only if $v(\overline{G} \cap H) > v(\overline{G} \cap \overline{H})$, which plainly contradicts Kant's View.

However, Naive Additivity is absurd. As Oddie shows, it implies "Nihilism," the view that all propositions have the same value (Oddie 2001, 324).[16] This is easy to show. Suppose v satisfies Naive Additivity. Then $v(X \cap X) = v(X) + v(X)$. But, since $X \cap X = X$, this implies $v(X) = 2v(X)$, and hence that $v(X) = 0$. So Naive Additivity is equivalent to the statement that all propositions have zero value (which implies

[15] The reason is that multiplication distributes over addition, but addition does not distribute over addition, i.e., $x * (y + z) = (x * y) + (x * z)$, but unless $x = 0$, $x + (y + z) \neq (x + y) + (x + z)$.

[16] More precisely, what Oddie shows is that a condition he calls "separability" implies Nihilism. Oddie does not explicitly discuss Naive Additivity. But he does discuss "additivity," which he defines only informally, and he argues that this implies separability. For further discussion of separability see below.

that all have the same value). It is no surprise, then, that Naive Additivity is incompatible with many moral views, including Kant's View. (Oddie suggests a different solution to this problem, which I discuss below.)

This problem with Naive Additivity stems from is its failure to recognise *overlap* between propositions. When propositions share a common part, summing their values has the effect of counting this common part twice. The example above exhibits the most extreme double-counting. Obviously, X overlaps X. Thus, if we add the value of X to itself to determine the value of $X \cap X$, we get the result that the value of X is twice as great as itself. The common part of X and X (i.e., X itself) is counted twice! Double-counting also occurs, though less extremely, when applying Naive Additivity to the propositions in Kant's View. For example, G and H share $G \cup H$ as a common part. If v is naively additive then we have the following:[17]

$$\begin{aligned} v(G \cap H) &= v(G) + v(H) \\ &= v(G \cup H) + v(G \cup \overline{H}) + v(G \cup H) + v(\overline{G} \cup H) \\ &= 2v(G \cup H) + v(G \cup \overline{H}) + v(\overline{G} \cup H) \end{aligned}$$

The value of $G \cup H$, the common part, is added twice.

To avoid double-counting, additivity must be defined more carefully. First, we must define overlap. Propositions overlap, I have said, if they share a common part. But this is not quite right. One proposition, U, is a part of all propositions. We should not say, however, that all propositions therefore overlap, because they all share U as a common part. So amend the definition: propositions overlap if they share a *non-degenerate* part, which excludes U. This means that X and Y overlap if and only if $X \cup Y \neq U$.[18]

A more plausible version of additivity may then be defined as follows:

Additivity $v(X \cap Y) = v(X) + v(\overline{X} \cup Y)$.

$\overline{X} \cup Y$ is the "largest" (or most determinate) part of $X \cap Y$ that does not overlap X. Additivity therefore avoids double-counting and is consistent with the denial of Nihilism.[19] Additivity implies only that $v(X \cap X) = v(X) + v(\overline{X} \cup X)$, or equivalently that $v(X) = v(X) + v(U)$. It follows that $v(U) = 0$. But this is

[17] Note that $G = (G \cup H) \cap (G \cup \overline{H})$ and $H = (G \cup H) \cap (\overline{G} \cup H)$.

[18] Proof. Suppose that X and Y overlap. So, for some Z, $X \subseteq Z$, $Y \subseteq Z$, and $Z \neq U$. It follows that $X \cup Y \subseteq Z$, and therefore that $X \cup Y \neq U$. Now suppose that X and Y do not overlap. So, for all Z, if $X \subseteq Z$ and $Y \subseteq Z$, then $Z = U$. But $X \subseteq X \cup Y$ and $Y \subseteq X \cup Y$. So $X \cup Y = U$.
I noted earlier that our definitions of atoms and wholes are inverted, compared to the normal definitions of these concepts in the context of a Boolean algebra of sets. In a similar way, our definition of overlap is also inverted. Normally, we would say that the sets X and Y are non-overlapping, or *disjoint*, if and only if their intersection is empty, i.e., $X \cap Y = \emptyset$. But our definition is precisely the inverse of this: X and Y are non-overlapping if and only if $X \cup Y = U$. The reason, as before, is that parthood in our framework is identified with the superset relation, rather than the subset relation.

[19] One might think a more natural condition, which also avoids the problem of overlap, is the following:

$$v(X \cap Y) = v(X) + v(Y) - v(X \cup Y).$$

This corrects for over-counting by subtracting the value of the overlapping part ($X \cup Y$). This alternative condition is actually logically weaker than Additivity, as defined above. The alternative is equivalent to the following (see Appendix for proof):

unproblematic. Since U is a tautology, it is quite natural to say that it has no value. Moreover, Additivity is consistent with Kant's View. According to the contrast argument above, we must have $v(G \cap H) - v(G \cap \overline{H}) = v(H) - v(\overline{H})$. For an additive value function, however, this need not be so. Rather, according to Additivity, $v(G \cap H) - v(G \cap \overline{H}) = v(\overline{G} \cup H) - v(\overline{G} \cup \overline{H})$. Kagan argues that contrast arguments commit a fallacy, which he call the "Additive Fallacy" Kagan (1988). Perhaps a more apt name would be the "Naive Additive Fallacy."

2.3 An Aside on Separability

Additivity presupposes that values can be quantified: that we can say, not only that one proposition is better than another, but moreover *how much* better it is. More precisely, as noted above, Additivity requires that value can be measured on a ratio scale. One might wonder, however, how much of Additivity can be salvaged in a purely ordinal context. A natural answer to this question is given by a condition commonly known as "Separability". This condition says, roughly, that each part of a whole makes an independent contribution to the value of the whole; so the contribution of a part remains the same regardless of which other parts it is combined with.

In the framework adopted here, a "naive" version of Separability may be defined as follows.

Naive Separability $v(X \cap Y) \geq v(X \cap Z)$ if and only if $v(Y) \geq v(Z)$.

Naive Separability is implied by Naive Additivity. However, the former condition, unlike the latter, is well defined even in an ordinal context. If Naive Separability is satisfied by a value function v, then it also satisfied by any monotonically increasing transformation of v. (Notice also that Naive Separability is plainly inconsistent with Kant's View.)

Naive Separability is absurd for the very same reason that Naive Additivity is absurd: it ignores the possibility of overlap between propositions. From Naive Separability, if $v(X \cap Y) = v(X \cap (X \cap Y))$ then $v(Y) = v(X \cap Y)$. But this implies that $v(X \cap Y) = v(Y)$.[20] By parallel reasoning we can show that $v(X \cap Y) = v(X)$, and therefore that $v(X) = v(Y)$. In fact, Naive Separability is equivalent to Nihilism.

This problem can be avoided by revising Naive Separability in a way similar to our revision of Naive Additivity above.

Separability $v(X \cap Y) \geq v(X \cap Z)$ if and only if $v(\overline{X} \cup Y) \geq v(\overline{X} \cup Z)$.

Separability, unlike Naive Separability, does not constrain the ordinal ranking of whole propositions. It is, for example, compatible with Kant's View.

Footnote 19 (continued)

$$v(X \cap Y) = v(X) + v(\overline{X} \cup Y) - v(U).$$

So Additivity is equivalent to the conjunction of this alternative condition with the further condition that $v(U) = 0$. This further condition seems to me very plausible, and it is also a helpful simplification. This is why I prefer to use Additivity, as defined.

[20] This follows from the fact that $X \cap Y = X \cap (X \cap Y)$.

3 Additivity and the Values of Wholes

I turn now to the first premise of my argument, that Additivity does not significantly constrain the values of *whole* propositions. I show, first, that *any* non-additive value function can be transformed into an additive one without altering the values assigned to whole propositions. Thus, whatever your view of the values of wholes happens to be, you will always be able to find a compatible additive value function. I then consider the implications of Additivity in combination with a further condition that requires the values of a proposition and its complement to be zero-sum. I show that this combination of conditions does constrain the ratios of values, but does not constrain either their order or the ratios of intervals between them.

3.1 Reverse Engineering

Additivity may be regarded as "atomistic". A proposition may be recursively divided into ever smaller non-overlapping parts, leading eventually to a set of atomic propositions. By definition, atoms cannot overlap: since they have no proper parts, they cannot share any common proper parts. In cases where the parts are non-overlapping, Additivity coincides with Naive Additivity. Thus Additivity implies that the value of a whole equals the sum of the values of its atomic parts, as does Naive Additivity. More precisely, Additivity is equivalent to the following condition (see Appendix for proof).
Atomicity $v(X) = \sum_{i=1}^{n} a_i(X)v(A_i)$, where

$$a_i(X) = \begin{cases} 1 \text{ if } X \subseteq A_i \\ 0 \text{ if } X \nsubseteq A_i \end{cases}$$

This may suggest that additivity commits us to a "bottom-up" approach, assigning values first to the "smallest" (more precisely: least specific) propositions, and then summing these to determine the values of larger propositions. And this may seem a substantive commitment, since many may reject such a bottom-up approach. Why should we "privilege" atomic propositions in this way?

However, Additivity is in fact also compatible with a "top-down" approach. One may first assign values to wholes, and then "reverse engineer" an assignment of values to atoms such that these add up as required. To illustrate, let $n = 3$. So the whole propositions are W_1, W_2, W_3. Suppose the value function v assigns the desired values to these. We now want to find an atomistic (and hence additive) value function u that assigns the same values to wholes. Note that A_i is a part of W_j if and only if $i \neq j$. For example, the atomic parts of W_1 are A_2 and A_3. So u must satisfy the following:

$$v(W_1) = u(A_2) + u(A_3)$$
$$v(W_2) = u(A_1) + u(A_3)$$
$$v(W_3) = u(A_1) + u(A_2)$$

This is a system of linear equations. Since the number of equations equals the number of unknowns, we would expect a unique solution, and indeed this is so. The system may be written in matrix form as follows.

$$\begin{bmatrix} v(W_1) \\ v(W_2) \\ v(W_3) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} u(A_1) \\ u(A_2) \\ u(A_3) \end{bmatrix}$$

By inverting the 3×3 matrix, we obtain:

$$\begin{bmatrix} u(A_1) \\ u(A_2) \\ u(A_3) \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} v(W_1) \\ v(W_2) \\ v(W_3) \end{bmatrix}$$

This gives us the solution:

$$u(A_1) = \frac{1}{2}(v(W_2) + v(W_3) - v(W_1))$$
$$u(A_2) = \frac{1}{2}(v(W_1) + v(W_3) - v(W_2))$$
$$u(A_3) = \frac{1}{2}(v(W_1) + v(W_2) - v(W_3))$$

For example, since u is atomistic, we have:

$$\begin{aligned} u(W_1) &= u(A_2) + u(A_3) \\ &= \frac{1}{2}(v(W_1) + v(W_3) - v(W_2)) + \frac{1}{2}(v(W_1) + v(W_2) - v(W_3)) \\ &= v(W_1) \end{aligned}$$

Generalising for any n, the solution is:

$$u(A_i) = \frac{1}{n-1}\left((2-n)v(W_i) + \sum_{j \neq i} v(W_j) \right)$$

What this shows is that Additivity places absolutely no constraints on the values of wholes. Perhaps it will be objected, however, that reverse engineering is not in the true "spirit" of additivity. The view that value is additive may be regarded as not merely "extensional," but *explanatory*. It holds not only that the value of a whole must equal the sum of the values of its parts, but moreover that this *explains* the value of the whole. The values of parts have a sort of explanatory priority. So ultimately the fundamental values are those of atomic propositions; these explain all other values. From this perspective, reverse engineering the values of atoms from the values of wholes looks like "cheating." We need some independent justification for our assignment of values to atoms. It cannot be merely that this assignment, when combined with Additivity, yields the values of wholes that we want.

My response to this objection is simply that this explanatory view is not what I am discussing here. Certainly, this view seems quite significant, and also, I would say, very implausible. Atomic propositions seem the least qualified candidates for being the fundamental bearers of values. Atomic propositions are the least specific, contain the least information. But evaluation should proceed from a position of more information, not less. Even if this is the more common understanding of additivity, I believe it is worthwhile to discuss the merely extensional version. For one thing, it is often suggested that additivity is extensionally significant, because, for example, it is extensionally inconsistent with moral doctrines like Kant's View. It is therefore worthwhile to demonstrate that this is not so.

3.2 Zero-Sum

Additivity on its own is insignificant at the level of wholes, as we have just seen. When combined with other conditions, however, it may have more bite. Some believe, for example, that value must be "zero-sum": the values of a proposition and its complement (or negation) must sum to zero. If a proposition is good, then its complement must be bad, and the goodness of the former must equal the badness of the latter.[21] Formally, this is the following condition:

Zero-Sum Complements $v(X) + v(\overline{X}) = 0$.

Additivity and Zero-Sum Complements jointly imply the following:[22]

Zero-Sum Wholes $\sum_{i=1}^{n} W_i = 0$.

This says that the values of the whole propositions must sum to zero. So it is impossible for all wholes to be good, or for all to be bad. If one whole is good, then at least one must be bad, and vice versa. Notice, the latter condition solely concerns the values of wholes. Thus, if one *already* accepts Zero-Sum Complements, then further accepting Additivity *does* constrain one's evaluations of whole propositions. In this context, it might be argued, Additivity is significant at the level of wholes.

I have two responses to this argument. First, it should be emphasised that this is not a consequence of Additivity on its own. If it is only the values of whole

[21] Though this may seem intuitive, it is not uncontroversial. For criticisms, see e.g. Chisholm and Sosa (1966).

[22] Suppose v satisfies Additivity and Zero-Sum Complements. It follows from Additivity (which is equivalent to Atomicity) that:

$$\begin{aligned}\sum_i v(W_i) &= \sum_i \sum_{j \neq i} v(A_j)\\ &= (n-1)\sum_i v(A_i)\end{aligned}$$

But from Zero-Sum Complements, we have $v(A_i) = -v(W_i)$. So we have:

$$\sum_i v(W_i) = (1-n)\sum_i v(W_i).$$

Since $n \neq 1$, this implies Zero-Sum Wholes.

propositions that matter, as I argue below, then there is no harm in rejecting Zero-Sum Complements, since this condition concerns the values of non-wholes. One may then accept Additivity while also rejecting Zero-Sum Wholes.

Second, even if Zero-Sum Wholes does constrain the evaluation of wholes, it might not do so in a significant way. Any value function that violates Zero-Sum Wholes can be transformed into one that satisfies this condition merely by uniformly subtracting a constant, effectively shifting the zero point in the measurement scale. For any value function v, we may define a value function v^* by

$$v^*(X) = v(X) - \frac{1}{n}\sum_{i=1}^{n} v(W_i).$$

So v is transformed into v^* by uniformly subtracting the mean of the values of the wholes. We then have the following:

$$\begin{aligned}\sum_{i=1}^{n} v^*(W_i) &= \sum_{i=1}^{n}\left(v(W_i) - \frac{1}{n}\sum_{j=1}^{n} v(W_j)\right)\\ &= 0\end{aligned}$$

So v^* satisfies Zero-Sum Wholes, and, moreover, v^* is cardinally equivalent to v. These value functions represent both the same ordering of values and the same ratios of value intervals.

To illustrate, consider again the Kantian example. Suppose we rank the whole propositions in this example from best to worst in the following order: $G \cap H$, $G \cap \overline{H}$, $\overline{G} \cap \overline{H}$, $\overline{G} \cap H$.[23] Let v be any value function that represents this ordering of wholes. For example, v may assign the values shown in the following table.

Whole	Value	
	v	v^*
$G \cap H$	6	3
$G \cap \overline{H}$	4	1
$\overline{G} \cap \overline{H}$	2	− 1
$\overline{G} \cap H$	0	-3

Subtracting the mean of the values the wholes then transforms v into v^*, as shown in the table. Notice, the values of the wholes, under v^*, sum to zero.

To be clear, these two value functions are not equivalent. I assumed above that values are measurable on a ratio-scale, but v and v^* do not represent the same ratios of values. For example, v says that $G \cap H$ is one and a half times as good as $G \cap \overline{H}$, whereas v^* says it is three times as good. Nonetheless, one might still deny that these differences matter. That is, one might think that, though ratios of values exist, they are not significant. As an analogy, one might hold that in racing, all that matters is

[23] Oddie suggests this would be Kant's ranking (Oddie 2001, 320).

the *order* in which the competitors cross the finishing line. Whether the competitor in first place beats the one in second place by a large or a small margin makes no difference; the result is the same in either case. This would not be to say that more fine-grained measurements of the competitors' performances are impossible, only that they are unimportant. Likewise, even if one accepts there is some fact about the ratios of these values—and therefore some fact about whether Zero-Sum Wholes is true—one might nonetheless deny that this fact is important. I return to this issue below. In any case, as explained above, even if one believes that ratios of values are important, one may still reject Zero-Sum Wholes while accepting Additivity. To do this, one may reject Zero-Sum Complements instead.

4 The Values of Incomplete Objects

I turn now to the second premise of my argument, that the values of incomplete objects do no matter. I begin with the specific case of propositions, before considering incomplete objects more generally.

4.1 Propositions

To summarise the findings of the previous section, Additivity alone places no constraints on the values of wholes, and in conjuction with Zero-Sum Complements, it constrains only the ratios of these values. Whatever one's view happens to be regarding cardinal value comparisons between whole propositions, one may find a compatible additive value function. It follows, I claim, that Additivity does not constrain one's evaluation of propositions in any significant way. There are two parts to this claim: first, only the values of wholes matter; and second, with regard to the value of wholes, only cardinal comparisons matter. This view seems to me very intuitive, but it is hard to give a decisive argument for it. The following considerations may help to persuade those who are more sceptical.

Why might we care about the values of the propositions in the Kantian example? Perhaps we need to decide whether to award or withhold a benefit to a person. Receiving the benefit will make her happy, and otherwise she will be unhappy. If we accept Kant's View, and this person is virtuous, we will award the benefit because we believe that $G \cap H$ is better than $G \cap \overline{H}$. On the other hand, if she is vicious, we will withhold the benefit because we believe that $\overline{G} \cap H$ is worse than $\overline{G} \cap \overline{H}$. Or perhaps we are uncertain whether she is virtuous or vicious. In that case, what we choose to do may depend on whether we judge the value interval between $G \cap H$ and $G \cap \overline{H}$ to be greater or less than that between $\overline{G} \cap H$ and $\overline{G} \cap \overline{H}$. There are two mistakes we might make here: we might either make a virtuous person unhappy, or make a vicious person happy. Our choice as to how to act in this case of uncertainty may depend on which of these, if either, we take be the greater mistake. This suggests that, from a practical perspective, only cardinal value comparisons between whole propositions matter.

It might objected that, in the case of uncertainty, the propositions whose values should determine our choice are H and $\overline{H}$. These are the propositions which are under our control, in the sense that we can choose whether they are true or false. We cannot choose which of the whole propositions is true, because this depends on whether G or $\overline{G}$ is true, and we do not know which one it is. So this is a case in which the values of non-whole propositions, H and $\overline{H}$, are significant. However, as stated above, my argument is restricted to *intrinsic* values. It seems to me implausible that our choice should depend on the intrinsic values of H and $\overline{H}$ (if indeed these even have intrinsic values). Our choice should be sensitive to the probability of G. But the intrinsic value of a proposition cannot be sensitive to something so clearly *extrinsic* as the probability of another proposition.

One may think that the values of non-whole propositions have some significance beyond the practical concerns raised above. For example, some philosophers favour thinking of intrinsic value in terms of "fitting attitudes". Roughly, on this view (applied to propositions), a proposition is good to extent that it is fitting to hold some positive attitude towards it, such as desire or admiration, and a proposition bad to extent that it is fitting to hold some negative attitude towards it, such as aversion or disgust.[24] One might also think that it is important to determine what attitudes are fittingly held towards non-whole propositions, even if this has no impact on practical matters. I do not wish to discuss here the merits of this view. So instead I weaken my claim: non-whole propositions have no *practical* significance.

4.2 Complete Versus Incomplete Objects

The reason that the values of non-whole propositions seem irrelevant, I suggest, is that these propositions are, in the sense defined earlier, *incomplete*. A non-whole proposition cannot be true without some other proposition of which it is a part also being true. For example, the proposition that *either snow is white or grass is green* cannot be true on its own. If it is true, then either the proposition that *snow is white* or the proposition that *grass is green* must also be true. Now consider, for example, the proposition that *either you win the lottery or you are killed*. Do you want this to be true? That probably depends on which of the propositions of which it is a part would also be true. Would it be true because you win the lottery, or because you are killed (or both)? These more specific propositions are what really matter to you.

The proposition given above may seem a tendentious example, since it is the conjunction of a good and a bad proposition. What about the conjunction of two good (or two bad) propositions? Consider, for example, the proposition that *either you are very happy or you are moderately happy*. You might want this to be true even in circumstances where you do not know how it would be true. However, this can again be explained by your attitudes towards whole propositions. You prefer both the proposition that *you are very happy* and the proposition that *you are moderately*

[24] On this view, therefore, the zero point of value is significant, since this determines whether one should hold a positive or a negative attitude towards a proposition. As I have shown, however, Additivity alone does not constrain the zero point (it does so only in combination with Zero-Sum Complements.)

happy to the proposition that *you are neither very happy nor moderately happy*. And this is why you prefer the disjunction of the former two propositions to the latter.

Contrast this with a mereology of material objects, where both wholes and parts are complete. Think of a painting, for example. We can imagine dividing this into, say, four quarters, each of which is a proper part of the whole. But each of these parts could exist on its own. We could physically cut the painting into four pieces, and display each of these on the gallery wall as an artwork in itself. Even without physically cutting the painting, we could focus our attention on just one part and consider its value in isolation, as if it were a whole artwork. Consequently, in the case of material objects, it is not only the values of wholes that matter. If for example, the painter felt that, although the painting as a whole was bad, some particular part was good, then this may be a reason for her to crop the painting, leaving only the good part.[25]

So additivity in the case of such a mereology seems quite significant, and also quite implausible. Imagine a painting composed of two contrasting halves. We may think that this contrast makes the painting good. On an additive representation, any value in the whole painting must also be present in its parts. But this may lead to a distorted representation, since it may be that neither half considered in isolation exhibits any contrast. In the case of material objects, it seems that organic unities may be quite common.

4.3 Evaluative Inadequacy

One might doubt whether it makes sense even to assign values to incomplete objects in the first place. What is the value of an individual's being *either virtuous or unhappy*? One may feel that this has no determinate answer. It could be true in different ways: the individual could be virtuous and happy, or virtuous and unhappy, or vicious and unhappy. But these may have quite different values. Therefore, one might think, $G \cup \overline{H}$ is too heterogeneous to be assigned a determinate value. On the fitting attitudes approach (mentioned above), it seems doubtful that any particular attitude is uniquely fitting in the case of such an incomplete proposition.

Zimmerman (2001) defends a more restrictive view of value bearers. Though he explicitly discusses only "states of affairs," his argument seems largely applicable also to propositions. He argues that some states of affairs, which he calls "evaluatively inadequate," cannot properly be regarded as bearers of value (Zimmerman 2001, 142). In particular, he excludes "negative" and "disjunctive" states of affairs. What prevents these from having values, he argues, is that they are "highly indeterminate". They cannot have a determinate value, and therefore have no value at all (not even zero).

The issue here is not really about negation or disjunction. A proposition expressed by a negative sentence (i.e., one containing the word "not") may be more specific than one expressed by a non-negative sentence. For example, "The population of

[25] For this reason, we might think it is pointless to consider the values of *very small* parts of the painting, because there is no way to contemplate such a part as an artwork in its own right.

New Zealand is *not* greater than six million" is more specific than "The population of New Zealand is less than a billion". Likewise, a proposition expressed by a disjunctive sentence (i.e., one containing the word "or") may be more specific than one expressed by a non-disjunctive sentence. For example, "Caprica lives either in Edinburgh *or* Glasgow" is more specific than "Caprica lives in Scotland".

Nonetheless, we can distinguish between evaluatively adequate and inadequate propositions, in a way suggested by Zimmerman. A proposition might be true in different possible ways. If all the ways for it to be true are equivalent in value, then we may assign it a determinate value, and so it is evaluatively adequate; otherwise it is inadequate. Earlier, in my construction of Ω, I introduced an equivalence relation $\equiv$ on U, representing the relation of evaluative equivalence. We may now say that a proposition X is adequate if and only if all the worlds contained in X are evaluatively equivalent; i.e., for all $x, y \in X$, $x \equiv y$. Then, following Zimmerman, we may say that the value function v is defined, not for *all* proposition in Ω, but only for those that are evaluatively adequate.

What should we say about Additivity in this revised framework? It follows from our definitions that the evaluatively adequate (non-degenerate) propositions in Ω are all and only the whole propositions. (Trivially, $\emptyset$ is also evaluatively adequate.) This might suggest that Additivity is false. The value of a whole proposition cannot be the sum of the values of its atomic parts, because its atomic parts are evaluatively inadequate and therefore have no values. However, in the context of this revised framework, it seems fair to also refine our definition of Additivity: the value of a proposition must equal the sum of the values of its evaluatively adequate parts. This version of Additivity is trivially true, because a whole proposition has only one evaluatively adequate part, namely this proposition itself. So in this case my central thesis still stands. If Additivity is trivially true, then it is certainly insignificant.

5 Oddie's Solution

Before concluding, I want to return to Oddie. As noted above, he also recognises the absurd consequences of Naive Additivity. His solution is to propose a different modification (Oddie 2001, 326–328). In the framework adopted here, his proposal may be stated as follows.

First, Oddie introduces "factors". A factor may be defined as a subset of Ω that forms a non-trivial partition of U.[26] For example, $\{G, \overline{G}\}$ is a factor, and so is $\{G, \overline{G} \cap H, \overline{G} \cap \overline{H}\}$. The elements of a factor Oddie calls "features." Next Oddie defines a "basis" as a set of factors satisfying some further conditions. Let $\mathcal{B}$ be a set of factors. Then a "selector" on $\mathcal{B}$ is a function $s : \mathcal{B} \to \Omega$ such that $s(F) \in F$ for all $F \in \mathcal{B}$. So a selector selects one feature from each factor. Let $\cap s$ be the intersection (or conjunction) of the features selected by s, i.e., $\cap s = \bigcap_{F \in \mathcal{B}} s(F)$. Now a basis is a set of factors $\mathcal{B}$ that satisfies the following conditions.

[26] By "non-trivial" I mean to exclude only the singleton partition $\{U\}$.

1. For any whole W_i, there exists a selector s_i on $\mathcal{B}$ such that $W_i = \cap s_i$.
2. For any selector s on $\mathcal{B}$, $\cap s \neq \emptyset$.
3. $\mathcal{B}$ contains at least two factors.

The first condition requires that a basis is able to characterise every whole proposition. The second condition ensures that the factors are independent, in the sense that selecting any feature from one factor is compatible with selecting any feature from another, and so on. The third condition rules out the "degenerate" set of factors that includes only the single factor $\{W_1, W_2, \dots, W_n\}$.[27] In the Kantian example, one basis is $\{\{G, \overline{G}\}, \{H, \overline{H}\}\}$. Oddie also discusses another basis. Let $J = (G \cap H) \cup (\overline{G} \cup \overline{H})$. Thus J is the proposition that, as Oddie puts it, the individual receives her "just deserts": she is happy if and only if she deserves to be. Then another basis is $\{\{G, \overline{G}\}, \{J, \overline{J}\}\}$.

A value function is additive relative to a basis, we may say, if the value of any whole equals the sum of the values of its features, as determined by this basis. So a value function may be additive relative to one basis, but not relative to another. On Oddie's proposal, a value function is additive *simpliciter* if it is additive relative to at least one basis. This gives us the following condition.

Basic Additivity There exists a basis $\mathcal{B}$ such that, for any whole proposition W_i, $v(W_i) = \sum_{F\in\mathcal{B}} v(s_i(F))$, where s_i is the selector on $\mathcal{B}$ such that $W_i = \cap s_i$.

Clearly, there are "non-nihilistic" value functions that satisfy Basic Additivity. So this version of additivity also avoids the problems of Naive Additivity. As Oddie shows, though Kant's View rules out additivity relative to $\{\{G, \overline{G}\}, \{H, \overline{H}\}\}$, it is compatible with additivity relative to $\{\{G, \overline{G}\}, \{J, \overline{J}\}\}$. So Kant's View is also compatible with Basic Additivity.

However, Basic Additivity does significantly constrain the evaluation of whole propositions. Oddie shows that it is incompatible with a class of value functions he calls "absolutist." This is the class satisfying the following condition.

Absolutism $v(G \cap H) > v(G \cap \overline{H}) = v(\overline{G} \cap H) = v(\overline{G} \cap \overline{H})$.

Every basis must have the form $\{\{X, \overline{X}\}, \{Y, \overline{Y}\}\}$. Suppose the features of $G \cap H$ are X and Y. Then, from Absolutism, must have

$$v(X) + v(Y) > v(X) + v(\overline{Y}) = v(\overline{X}) + v(Y) = v(\overline{X}) + v(\overline{Y})$$

It is easy to see that this system of equations is inconsistent. So Absolutism is inconsistent with Basic Additivity. (On the other hand, Absolutism is of course consistent with Additivity, since the latter allows *any* ordering of wholes, as we've seen.)

It seems to me, however, that this significance is achieved only by defining additivity in an arbitrarily restrictive way. For one thing, whether Ω has any bases at all depends on its cardinality. It follows from the definition of a basis that there is a one-to-one correspondence between selectors and whole propositions: every whole is the

[27] Strictly speaking, these conditions define what Oddie calls an "admissible basis." His definition of a basis includes only the first condition. The second and third conditions he calls, respectively, "independence" and "non-degeneracy" (Oddie 2001, 328–329).

intersection of a selector, and every intersection of a selector is a whole. Thus the number of wholes must equal the number of selectors. Now, the number of selectors is $\Pi_{F\in\mathcal{B}}|F|$, where $|F| \geq 2$ for all $F \in \mathcal{B}$. It follows that this number cannot be prime. Therefore, if Ω contains exactly five wholes, for example, then it is impossible to define a basis for Ω. In this case, no value function can satisfy Basic Additivity. This seems an arbitrary restriction.

Here is another way in which Basic Additivity seems arbitrary. In the Kantian example, there are exactly three possible bases, as shown in the following tables.

	H	$\overline{H}$
G	W_1	W_2
$\overline{G}$	W_3	W_4

	J	$\overline{J}$
G	W_1	W_2
$\overline{G}$	W_4	W_3

	J	$\overline{J}$
H	W_1	W_3
$\overline{H}$	W_4	W_2

In these tables, the columns represent one factor, and the rows represent another. Each whole combines two features, one from each factor. The wholes are numbered to show how they are related to each other in each basis. Essentially, what differs between bases is which wholes are represented as being "similar", in the sense of sharing a common feature. There are six pairs of (distinct) wholes. In each basis, four of these pairs are similar, and two are not. The dissimilar pairs are those on the diagonals, e.g., (W_1, W_4) and (W_2, W_3) in the first basis. In general, if the two dissimilar pairs are (W_i, W_j) and (W_k, W_l), then the basis is as follows.

	$W_i \cup W_k$	$W_j \cup W_l$
$W_i \cup W_l$	W_i	W_l
$W_j \cup W_k$	W_k	W_j

Basic Additivity requires that the parts whose values are summed to give the values of the wholes must have this specific structure.

Again, however, this seems an arbitrary restriction. It seems to go beyond the basic idea that the value of whole must equal the sum of the values of its parts. Why must the parts have the specific structure of a basis? Oddie's rationale for imposing this restriction is to avoid Nihilism. Say that a pair of propositions (X, Y) is additive, relative to a value function v, if the value of their conjunction equals the sum of their values, i.e., $v(X \cap Y) = v(X) + v(Y)$. Now consider, for example, the propositions G, H, and J. The conjunction of any (distinct) pair of these propositions is the same (i.e., $G \cap H = G \cap J = H \cap J$). Therefore we should not say that all of these pairs are additive (as required by Naive Additivity). Oddie realises that this would lead to absurdity. His solution is to say that additivity requires only that *one* of these pairs is additive. Which pair is the additive one depends on the basis. For example, in the first basis above, it is (G, H). But we have already seen a better solution. Notice, every pair of these propositions overlaps. So additivity should not require that *any* of these pairs is additive. Rather, it should require that all and only *non-overlapping* pairs are additive, as in Additivity. This avoids absurdity without imposing the structure of a basis.

6 Conclusion

I have argued, first, that additivity, properly defined, does not constrain the values of wholes; and, second, that the values of incomplete objects do not matter. In a mereology where only the wholes are complete (e.g., a mereology of propositions), it follows that additivity does not significantly constrain anything that matters. If our interest is in the (intrinsic) values of propositions, properties, states of affairs, and the like, then additivity is a non-issue.

Appendix

Proposition 1 *A value function is additive if and only if it is atomistic.*

Proof To prove additivity implies atomicity, suppose v is additive. Let u be a value function such that

$$u(X) = \sum_{i=1}^{n} a_i(X)v(A_i) \tag{1}$$

Notice, u is atomistic. We need to show $v = u$.

Let X contain m atoms, i.e., $\sum_{i=1}^{n} a_i(X) = m$. Let f be an injective function from $\{1, \dots m\}$ into $\{1, \dots n\}$ such that $a_{f(i)}(X) = 1$ for all i. (So f renumbers the atomic parts of X from 1 to m.) For $i \in \{1, \dots m\}$, let $X_i = \bigcap_{j \leq i} A_{f(j)}$. Notice $X_m = X$. We prove that $v(X_m) = u(X_m)$ by induction.

First, since $X_1 = A_{f(1)}$ and $v(A_{f(1)}) = u(A_{f(1)})$, we have

$$v(X_1) = u(X_1) \tag{2}$$

Now consider X_{i+1}. Since $X_{i+1} = X_i \cap A_{f(i+1)}$ and $\overline{X_i} \cup A_{f(i+1)} = A_{f(i+1)}$, and v is additive, we have

$$\begin{aligned} v(X_{i+1}) &= v(X_i \cap A_{f(i+1)}) \\ &= v(X_i) + v(\overline{X_i} \cup A_{f(i+1)}) \\ &= v(X_i) + v(A_{f(i+1)}) \end{aligned} \tag{3}$$

Since $v(A_{f(i+1)}) = u(A_{f(i+1)})$ it follows from 3 that

$$v(X_i) = u(X_i) \implies v(X_{i+1}) = u(X_i + 1) \tag{4}$$

From Eqs. 2 and 4 we have $v(X_m) = u(X_m)$.

To prove atomicity implies additivity, suppose v is additive. We have $a_i(X \cap Y) = a_i(X) + a_i(\overline{X} \cup Y)$. Thus we obtain

$$\begin{aligned} v(X \cap Y) &= \sum_{i=1}^{n} a_i(X \cap Y) v(A_i) \\ &= \sum_{i=1}^{n} (a_i(X) + a_i(\overline{X} \cup Y)) v(A_i) \\ &= \sum_{i=1}^{n} a_i(X) v(A_i) + \sum_{i=1}^{n} a_i(\overline{X} \cup Y) v(A_i) \\ &= v(X) + v(\overline{X} \cup Y) \end{aligned}$$

□

Proposition 2 *For any value function v, there exists an additive value function u such that $u(W_i) = v(W_i)$ for all i.*

Proof Let v be any value function. Let u be an additive value function such that

$$u(A_i) = \sum_{j=1}^{n} a_{ij} v(W_j) \quad \text{where } a_{ij} = \begin{cases} (2-n)/(n-1) & \text{if } i = j \\ 1/(n-1) & \text{if } i \neq j \end{cases} \tag{5}$$

We will show that $v(W_i) = u(W_i)$ for all i.

Since u is additive, it is also atomistic (see above). So we have

$$u(W_i) = \sum_{k=1}^{n} b_{ij} u(A_j) \quad \text{where } b_{ij} = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } i \neq j \end{cases} \tag{6}$$

Let $c = ba$. So, for any i, j,

$$c_{ij} = \sum_{k=1}^{n} b_{ik} a_{kj} \tag{7}$$

Thus, combining Eqs 5, 6, and 7 gives us

$$u(W_i) = \sum_{j=1}^{n} c_{ij} v(W_j) \tag{8}$$

Now, for any i, j, k,

$$b_{ik} a_{kj} = \begin{cases} 0 & \text{if } i = k \\ (2-n)/(n-1) & \text{if } i \neq k, j = k \\ 1/(n-1) & \text{if } i \neq k, j \neq k \end{cases} \tag{9}$$

Thus, for any i, j,

$$c_{ij} = \frac{m_{ij}(2-n) + p_{ij}}{n-1} \quad \text{where } m_{ij} = |\{k : i \neq k, j = k\}| \quad p_{ij} = |\{k : i \neq k, j \neq k\}| \tag{10}$$

Now, if $i = j$, then $m_{ij} = 0$ and $p_{ij} = (n-1)$. So in this case we have

$$i = j \implies c_{ij} = \frac{n-1}{n-1} = 1 \tag{11}$$

But if $i \neq j$, then $m_{ij} = 1$ and $p_{ij} = (n-2)$. So in this case we have

$$i \neq j \implies c_{ij} = \frac{(2-n) + (n-2)}{n-1} = 0 \tag{12}$$

It follows that, for any i, j,

$$c_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \tag{13}$$

Therefore, from Eqs. 8 and 13 , we have

$$u(W_i) = v(W_i) \tag{14}$$

$\square$

Proposition 3 *The following statements are equivalent:*

(i) $v(X \cap Y) = v(X) + v(Y) - v(X \cup Y)$

(ii) $v(X \cap Y) = v(X) + v(\overline{X} \cup Y) - v(U)$

Proof We prove first that (i) implies (ii). We have:

$$Y = (\overline{X} \cup Y) \cap (X \cup Y) \tag{15}$$

$$U = (\overline{X} \cup Y) \cup (X \cup Y) \tag{16}$$

So from (i) we have:

$$v(X \cap Y) = v(X) + v(Y) - v(X \cup Y) \tag{17}$$

$$= v(X) + v((\overline{X} \cup Y) \cap (X \cup Y)) - v(X \cup Y) \tag{18}$$

$$= v(X) + v(\overline{X} \cup Y) + v(X \cup Y) - v(U) - v(X \cup Y) \tag{19}$$

$$= v(X) + v(\overline{X} \cup Y) - v(U) \tag{20}$$

Next we prove (ii) implies (i). From (ii) we have:

$$v(Y) = v((\overline{X} \cup Y) \cap (X \cup Y)) \tag{21}$$

$$= v(\overline{X} \cup Y) + v((X \cap \overline{Y}) \cup v(X \cup Y)) - v(U) \tag{22}$$

$$= v(\overline{X} \cup Y) + v(X \cup Y) - v(U) \tag{23}$$

By rearranging we obtain:

$$v(\overline{X} \cup Y) = v(Y) - v(X \cup Y) + v(U) \tag{24}$$

Thus (ii) implies:

$$v(X \cap Y) = v(X) + v(\overline{X} \cup Y) - v(U) \tag{25}$$

$$= v(X) + v(Y) - v(X \cup Y) + v(U) - v(U) \tag{26}$$

$$= v(X) + v(Y) - v(X \cup Y) \tag{27}$$

□

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