An Extensional Mereology for Structured Entities

Ilaria Canavotto[1] · Alessandro Giordani[2]

Abstract

In this paper, we present an extensional system of mereology suitable to account for the intuitive distinction between heaplike and non-heaplike entities. Since the need to capture this distinction has been a key motivation for non-extensional mereologies, we first assess the main non-extensional systems advanced in the last years and highlight some mereological and metaphysical difficulties they involve. We then advance a novel program, according to which the distinction between heaplike and non-heaplike entities can be accounted for by bringing together the parthood relation characterized by classical extensional mereology and an Aristotelian extensional notion of potential parthood. Thus, while rejecting the thesis of mereological monism, our proposal is consistent with the thesis of mereological extensionalism. We show that within this framework it is possible to characterize the above-mentioned distinction, to define the relation of material constitution, and to capture three fundamental standpoints in metaphysics.

1 Introduction

In *Metaphysics Z 17* Aristotle introduces a primitive distinction between concrete heaplike composites, like a bunch of bricks or a pile of sand, and concrete non-heaplike composites, like a house or a clay pot. This distinction is based on the intuitive judgment that, although the entities in the two groups have parts, and so are composite, they differ both in the way in which they are unified and in the way in which

✉ Ilaria Canavotto
   i.canavotto@uva.nl

   Alessandro Giordani
   alessandro.giordani@unicatt.it

[1] Institute for Logic, Language and Computation, University of Amsterdam, Amsterdam, The Netherlands

[2] Department of Philosophy, Catholic University of Milan, Milan, Italy

they can be composed and decomposed. To illustrate, consider the entities depicted in Fig. 1.[1]

Hardly anyone would take it as problematic to classify what we see in Fig. 1a as a puzzle and what we see in Fig. 1b as a bunch of puzzle pieces. In addition, most of us would certainly be prepared to claim that both the puzzle and the bunch of pieces have parts, even if the pieces in the puzzle, unlike those in the bunch, are unified in an appropriate way, partly dependent on the form of the pieces.

We would also typically classify what we see in Fig. 2a as a complete puzzle and what we see in Fig. 2b as a puzzle lacking a piece.[2] But it would strike us as odd that a bunch of puzzle pieces can lack a piece: we would typically classify what we obtain by removing one of the pieces in a bunch as a new bunch rather than as an incomplete bunch. Hence, while a puzzle can be decomposed in such a way that it makes sense to say that it is no more a complete puzzle, a bunch of puzzle pieces is different in this respect.

Finally, in order to produce a puzzle, the pieces need to be assembled in a specific way, while no particular way of composition is required in order to produce a bunch of puzzle pieces. In fact, we tend to say that a puzzle has a certain structure and that it is in virtue of this that it has a unity allowing us to judge whether it is complete or not. So, unlike the bunch of pieces, the puzzle is what it is insofar as the pieces are arranged according to a specific structure. In light of this, we can also say that the bunch of pieces (in general, any heaplike entity) is a *mere sum*, while the puzzle (in general, any non-heaplike entity) is a *structured sum*.

Making the framework underlying these intuitions explicit, by assuming that there are both heaplike and non-heaplike entities, Aristotle's view on composition implies that

(i)   there exist composed entities;
(ii)   some composed entities are mere sums of their components;
(iii)   some composed entities are not mere sums of their components.

Claims (i)–(iii) represent a substantive standpoint in the metaphysics of composition. While claim (i) is the negation of *mereological nihilism*, the viewpoint according to which no composed entity exists (Rosen and Dorr 2007; Sider 2013), the recognition of different kinds of composites, as expressed by claims (ii) and (iii), is the negation of either *mereological extensionalism*, the viewpoint according to which no two composed entities can have the same proper parts (Lewis 1991; Varzi 2008), or *mereological monism*, the viewpoint according to which there is only one parthood relation and one corresponding way of composition (Lando 2017; Lewis 1991).

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[1] The following examples are introduced to highlight the pre-philosophical intuitions motivating the distinction between heaplike and non-heaplike entities, not to justify them. In discussing the examples, we thus take the perspective, say, of a kid looking at the pictures and answering questions about what she sees.

[2] In fact, if you ask a kid whether the puzzles in the figure are finished, you should not be surprised if she answers "yes, it is" in the first case, and "no, a piece is still missing" in the second case.
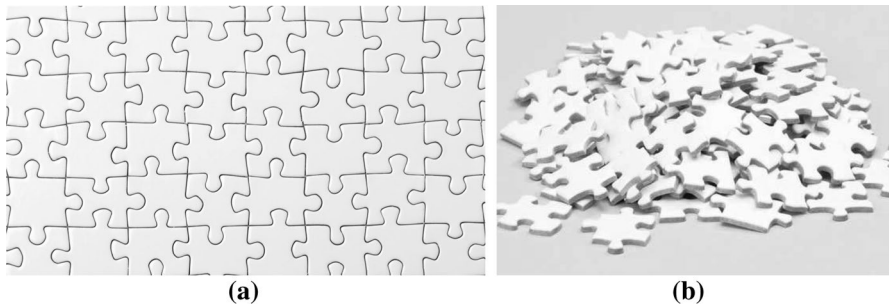
Fig. 1 **a** A puzzle. **b** A bunch of pieces

In the current literature, this view has led to the development of both non-extensional and non-monistic mereologies. On the one hand, some scholars maintain mereological monism, allowing for theories of parthood in which *either* different sums can result from composing the same proper parts in the same way—against mereological extensionalism (Cotnoir 2010; Cotnoir and Bacon 2012; Thomson 1983, 1998)—*or* different sums can result from composing the same proper material parts by adding a formal part to one of them—in accordance with mereological extensionalism (Koslicki 2008). On the other hand, other scholars renounce to both mereological monism and mereological extensionalism, proposing theories of parthood in which the same proper parts, composed either in the same or in different ways, can give rise to different sums (Armstrong 1997; Fine 1999, 2010; McDaniel 2009).

Interestingly, in the Aristotelian perspective, the distinction between kinds of sums is related to a distinction between parthood relations and corresponding ways of composition. This implies a negation of mereological monism but not necessarily of mereological extensionalism: according to Aristotle, while a collection of entities can, by itself, give rise to one and only one mere sum, it can give rise to one and only one structured sum only if it is unified in a substance by a formal *principle* (not to be intended, *pace* some Neo-Aristotelian philosophers including, e.g. Fine 1999 and Koslicki 2008, as a formal *part*[3]). The senses in which the entities in the given collection will be proper parts of the resulting sums will then differ accordingly.

By following the Aristotelian lead, the aim of the present paper is to develop an *extensional but pluralist* mereology for concrete entities, in which the distinction between mere and structured sums can be accounted for, at least in a first basic way. Although we will take this distinction as a given and not defend it further in what

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[3] See *Metaphysics* Z 17, where Aristotle argues that (i) the substance is more than the mere sum of its parts and that (ii) the form of a substance, which is that in virtue of which the substance is structured, cannot be a part of the substance. The idea is that, if we disaggregate a substance into its parts, we obtain the mere sum of the parts, but we destroy the substance. Hence, the substance cannot be more than the mere sum of its components in virtue of a formal element that is itself one of its parts, since by disaggregating a substance into its material *and* formal parts what we would obtain would still be the mere sum of the parts, but not the substance.
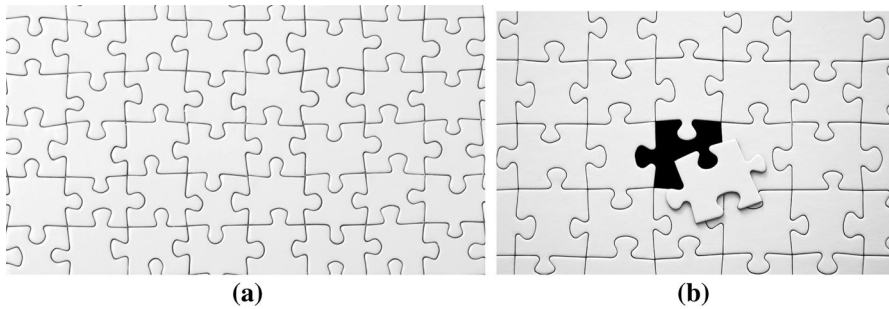
Fig. 2 a A complete puzzle. **b** An incomplete puzzle

follows, the formulation of a mereology capturing it can be seen as a preliminary defence against all potential arguments aiming at showing that the distinction cannot be modelled in a sensible way.[4] In particular, our system is intended to account for three seemingly conflicting pre-theoretic judgments about the composition and the existence of mere and structured sums. In terms of a concrete example, these can be summarized as follows.

1. *Unrestricted existence and uniqueness of heaplike entities*: when there are some puzzle pieces, e.g., on a table, we usually recognize one and only one bunch of puzzle pieces.
2. *Restricted existence and uniqueness of non-heaplike entities*: when there is a puzzle built on the table, we usually recognize one and only one puzzle made of those pieces.
3. *Apparent existence only of one non-heaplike entity*: when there is a puzzle on the table, there seems to be no bunch of puzzle pieces on the table.

The first two judgments support the assumption of extensionalism both for mere and structured sums. Despite sharing this feature, the two forms of composition differ in an important respect: as mentioned in the previous discussion of Fig. 2, the mere presence of some puzzle pieces on the table suffices to identify a bunch of pieces (heaplike composition is *unrestricted*[5]), whereas the pieces need to be arranged in a complete puzzle before we identify a puzzle (non-heaplike composition

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[4] This makes our approach methodologically close to Fine's development of a pluralistic theory of parts-wholes (Fine 2010). According to Fine, before assuming mereological monism, we should first consider whether it is possible to consistently account for different notions of parthood. In a similar spirit, by putting forward a coherent treatment of the distinction between heaplike and non-heaplike entities, we provide a preliminary rationale for not rejecting the distinction. The main difference between Fine's approach and our own is that, while Fine focuses on a "pure" theory of parts-wholes, leaving aside the question how the theory applies to familiar objects, we start from intuitions about these objects and develop a theory to account for them.

[5] We take the claim that heaplike composition is unrestricted as a pre-theoretic judgment because we think that, before reflection, a layperson would answer positively to questions like: "Are the Empire State Building, the Tour Eiffel, the Pyramid of Cheops, and the Colosseum a bunch of buildings?" or (assum-

is *restricted*). So far so good. Nevertheless, when we add the third judgment to the picture, we seemingly find ourselves in a predicament. On the one hand, *whenever* there are some entities, we are prepared to accept the existence of the bunch of them. On the other hand, *when* those entities constitute a structured sum, we seem to no longer recognize the corresponding bunch, even if the entities are still there. We assume that the three judgments reflect initial mereological intuitions that need to be taken into account and that, in a mereology implying the first two, there should be a way to explain why we intuitively accept the third. Our ambition is to propose a first system of mereology in which this can be done in a consistent way.

We will proceed as follows. In the next section we support (a weak version of) system *CEM* of Classical Extensional Mereology as our preferred extensional mereology for heaplike entities. In Sect. 3, we discuss monistic non-extensional systems of mereology and argue that they give rise to some difficulties in accounting for heaplike and non-heaplike entities.[6] In Sect. 4, we advance a new pluralist extensional mereology, for both mere and structured sums, based on a primitive distinction between mere and potential parts. We will see that, besides accounting for the basic intuitions listed above, our system can be used to provide a natural characterization of the relation of material constitution and to distinguish between three fundamental metaphysical standpoints on substances: atomism, monism, and pluralism. Finally, Sect. 5 concludes by pointing at some limitations of the present account and some interesting lines of research.

2 Extensional Mereologies for Heaplike Entities

In this section we support the idea that a mereology for heaplike entities should satisfy the principles of (a variant of) *CEM*. In doing this, we suggest that most of the standard concerns against these principles lose their grip when only heaplike entities are considered.

To get started, one of the crucial assumptions underlying the construction of a theory of parts and wholes is that the whole is greater than each of its parts.[7] In the case of heaps, this principle seems unproblematic: every heap is greater than each

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Footnote 5 (continued)

ing that the layperson accepts the existence of numbers, parts, and propositions) "Are the notebook on the desk at the office, the core of the sun, the number 5, and the first theorem of Euclid a bunch of entities?" We will come back to the principle of unrestricted composition in Sect. 2.

[6] We will consider neither pluralist non-extensional systems nor monistic extensional systems. As to the first kind of systems, we will show that rejecting both mereological extensionalism and mereological monism is not necessary to characterize the difference between mere and structured sums. As to monistic extensional systems, we will defer the discussion of the idea of introducing formal parts to ground the unity of structured sums to further work (but see footnote 3 for a statement of our position on that).

[7] This assumption coincides with the fifth common notion of Euclid's Elements and constitutes one of the standard examples of *propositions known in themselves* in medieval philosophy. For instance, in the *Summa Theologiae* I, Q2, a1, Aquinas comments on the notion of proposition known in itself as follows: "Those things are said to be self-evident [*per se nota*] which are known as soon as the terms are known [...]. Thus, when the nature of a whole and of a part is known, it is at once recognized that every whole is greater than its part [*omne totum maius est sua parte*]."

of the entities it is made of and than any sub-heaps made of some of these entities. Accordingly, the relation we initially introduce is a relation $\ll$ of proper parthood, characterized by the following principles:[8]

IR. (*Irreflexivity*) $\neg(x \ll x)$
TR. (*Transitivity*) $x \ll y \wedge y \ll z \rightarrow x \ll z$

We define the notions of composite entity, improper parthood, overlap, and binary fusion[9] in the following standard way.

Definition 1 (*Being composite*) $C(x) := \exists a(a \ll x)$.

Definition 2 (*Improper parthood*) $x \leq y := x \ll y \vee x = y$.

Definition 3 (*Overlap*) $x \circ y := \exists z(z \leq x \wedge z \leq y)$.

Definition 4 (*Fusion*) $Fu(s, x, y) := x \leq s \wedge y \leq s \wedge \forall a(a \leq s \rightarrow a \circ x \vee a \circ y)$.

According to the definitions, x is a *composite* just in case it has at least one proper part; x is an *improper part* of y just in case it is either a proper part of y or it is identical to y; x and y *overlap* just in case they share at least one improper part (when x and y do not overlap, we will say that they are *disjoint*); finally, s is a *fusion* of x and y just in case x and y are improper parts of s and every improper part of s overlaps either x or y.

As pointed out, for instance, by Hovda (2009), Pietruszczak (2005) and Varzi (2009), among the definitions of fusion currently available in mereology, Definition 4 is the most appropriate to account for the intuitive ideas that (i) a fusion includes the fused entities as its parts and that (ii) no proper part of a fusion is disjoint from the fused entities, so that, if we subtract the fused entities from their fusion, we are left without a remainder. These seem to be basic traits of any form of composition and especially of the one giving rise to heaps. To be sure, a heap of things is made of these things taken together, and so it must be at least as great as each of them. In addition, since it is made *only* of these things taken together, nothing remains of it if we remove all of them.

If this is correct, as we think it is, and if we accept that any finite collection of entities gives rise to a heap, then the following principle of finite fusion existence appears to properly characterize heaplike composition.

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[8] Irreflexivity and transitivity of parthood have been subject to criticisms (see Varzi 2016 for an overview). However, the problems raised against these principles typically concern notions of part, e.g. functional part, that are different from the one characterizing heaplike entities. To our knowledge, no one has ever called into question the principles that (i) no heap is greater than itself and that (ii) every heaplike-part of a sub-heap is a part of the heap.

[9] We decided to work with binary fusions to keep the presentation simple. The generalization to arbitrary fusions is not problematic and, as explained, e.g., by Hovda (2009), can be obtained in various ways.

FFE. *Finite Fusion Existence* $\exists sFu(s, x, y)$

However, two well-known objections have been raised, e.g., by Fine (1999), Simons (1987), van Inwagen (1987, 1994), against **FFE** and its stronger version, the principle of *Unrestricted Composition*, which states that any *arbitrary* collection of entities has a fusion. First, as a criterion for the existence of composed entities, **FFE** seems to be too loose, since basically any pair of entities, no matter how scattered they are, can form a composite in the introduced sense. Second, for any pair of entities we include in the inventory of the world, **FFE** commits us to also include a fusion of them, which is yet a further entity. It thus seems that **FFE** commits us to the existence of a huge number of entities we did not even consider before assuming the principle. How should we react to these arguments?[10]

We think that the two worries are legitimate if our aim is to lay down a mereology for non-heaplike entities, but not if we are concerned with a mereology for heaplike entities—as we are for now. Concerning the first argument, although it is clear that not any pair of entities can compose a structured sum, it is far less clear that the same applies to heaps: as we mentioned in the introduction, it seems uncontroversial that any two things form a heap. But of course, one may object that what is uncontroversial is that any finite collection of things *sharing some property*, and not just any finite collection of things, constitute a heap—just as any finite collection of things *sharing some property*, and not just any finite collection of things, can be counted. We agree. But since we are willing to accept that *being a thing* is a legitimate property to aggregate entities—just as it is to count things—in our view all things in any finite collection do share a property, and so they do, indeed, form a heap.

Concerning the second argument, the issue about the ontological costs of **FFE** relates to the complex problem of identifying suitable criteria of existence. In this respect, we just observe that the most uncontroversial criteria of existence, which include the possibility of identifying, distinguishing, and counting the entities whose existence is affirmed, seem to be satisfied by heaps: given a finite collection of things, we are typically prepared to identify a heap of these things; we are able to distinguish different heaps; and, finally, we are able to count heaps. We thus take the burden of presenting acceptable criteria of existence not satisfied by heaps to be on those who deny their existence.

In light of this, we assume the mereology consisting of the previous principles, i.e. **IR**, **TR**, and **FFE**, as the minimal mereology for heaplike composites. We will call this mereology *HM*. In our view, any system of mereology aiming at accounting for heaplike composites has to be at least as strong as *HM*, that is, it has to be able

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[10] Classic defences of the principle of Unrestricted Composition against the two objections are the argument from vagueness and the thesis of composition as identity, which can be found, for instance, in Lewis (1986, 1991). More recent arguments investigating the notion of ontological commitment and the explanatory benefits of the assumption of Unrestricted Composition have been proposed respectively by Varzi (2000) and Calosi (2015). Unlike these arguments, which aim at defending Unrestricted Composition in the context of mereological monism and, hence, as an appropriate principle for the only accepted form of composition, our defence aims at supporting the weaker **FFE** as an appropriate principle just for heaplike composition.

to introduce an irreflexive and transitive relation of proper parthood and to allow for the existence of a fusion, in the sense of Definition 4, of any pair of entities.

But, despite consisting of indispensable principles, *HM* is admittedly still too weak to capture the initial intuition that, when there are some puzzle pieces on a table, we count one *and only one* heap made of them. A candidate principle to account for this judgment is the following principle of extensionality, according to which two composites with the same proper parts are identical.

EXT.   (*Extensionality*) $C(x) \wedge \forall a(a \ll x \leftrightarrow a \ll y) \rightarrow x = y$.

Call a system of mereology *extensional* if and only if **EXT** is deducible in it, and *non-extensional* otherwise. Given our assumptions on heaplike composition, any extensional mereology aiming at modelling heaps has to be at least as strong as the system *EHM*, obtained by extending *HM* with **EXT**.

However, assuming **EXT** is still not sufficient to exclude that the same puzzle pieces give rise to two different heaps: Model 1 is a model of *EHM* in which s_1 and s_2 are two fusions of x and y (the proper parthood relation is represented by the transitive closure of the relation represented by arrows).
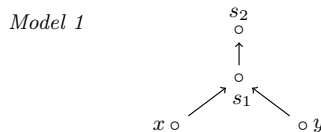
Model 1

What we need in order to exclude models like Model 1 is the following principle, stating that a unique fusion results from one and the same pair of entities.

FFU.   (*Finite Fusion Uniqueness*) $Fu(s_1, x, y) \wedge Fu(s_2, x, y) \rightarrow s_1 = s_2$.

Given a finite domain, the system *HM* + **FFU**, obtained by extending *HM* with **FFU**, is an extensional system of mereology equivalent to the system *CEM* of Classical Extensional Mereology.[11] Since we do not assume a finite domain, we will denote this weaker system *CEM**. Insofar as it allows us to account for our initial intuitions on heaplike composites, *CEM** is our preferred mereology for heaplike entities. Let us briefly review the main intuitions supporting three central *principles of decomposition* derivable in this system.[12]

First, it is natural to think that, whenever we subtract a proper part from a heaplike composite y, there should be at least another proper part of y we are left with.

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[11] *CEM*, in its full generality, is obtained by extending *HM* with the axiom of Unrestricted Composition rather than with **FFE**. In fact, this is the formulation of *CEM* famously assumed by Lewis (1991).

[12] For more details and proofs, we refer the interested reader to Hovda (2009), Simons (1987) and Varzi (2016).

This intuition is captured by the following principle of weak supplementation, which, since Simons (1987), has been taken by several philosophers, including Bennett (2013), Koslicki (2008), McDaniel (2009), Varzi (2008, 2009), as constitutive of the very notion of part.

**WSP.** (*Weak Supplementation Principle*) $x \ll y \rightarrow \exists a(a \ll y \wedge \neg(a \circ x))$.

Next, given two heaplike composites x and y, it is natural to think that, if all the proper parts of x are proper parts of y, then there is nothing of x that is not in y. Hence, x must be either a proper part of y or identical to y. This intuition is captured by the principle of proper parts.

PPP. (*Proper Parts Principle*) $C(x) \wedge \forall a(a \ll x \rightarrow a \ll y) \rightarrow x \leq y$.

Finally, if x is not a part of y, then there should be something of x that is not in y, and so there should be a part of x that does not overlap y. This idea is expressed by the following principle of strong supplementation.

SSP. (*Strong Supplementation Principle*) $C(x) \wedge \forall a(a \leq x \rightarrow a \circ y) \rightarrow x \leq y$.

We conclude this section with some comments on the interaction between these principles that will be useful later on. To start with, it is not difficult to see that, in mereologies not including composition principles, **WSP** allows for models, like Model 2, in which two different entities have the same proper parts.

Model 2
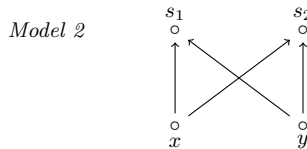
So, one would expect that the assumption of **WSP** has nothing to do with the assumption of extensionality. However, as shown in Varzi (2009), in the presence of **IR**, **TR**, and **FFE**, both **FFU** and **EXT** are deducible given **WSP**. That is, the systems *HM* + **WSP** and *CEM** are equivalent. What is more, as proved in Simons (1987), given **IR** and **TR**, **SSP** entails both **WSP** and **PPP**; in turn, **IR**, **TR**, and **PPP** entail **EXT**. This means that the system *HM* + **SSP**, obtained by extending *HM* with **SSP**, is equivalent to *HM* + **WSP**, and so to *CEM**. So, by extending *HM* with either **WSP** or **SSP**, we obtain mereologies equivalent to *CEM*and strictly stronger than the ones obtained by extending *HM* with **PPP** and **EXT**.[13] These results are summarized in the diagram in Fig. 3, where arrows represent inclusion.

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[13] That the inclusions are strict is easily seen. First, Model 1 is a model of *HM* plus **PPP** in which **WSP** fails, since there is no proper part of s_2 disjoint from s_1. Second, take Model 2 and modify it as follows: (i) delete the arrow from y to s_1, (ii) add an entity z, and (iii) draw arrows from s_1 to z and from s_2 to z. The resulting model satisfies all principles of *HM* plus **EXT**, but not **PPP**, since all proper parts of s_1 are proper parts of s_2 without s_1 being a proper part of s_2.

$$HM \longrightarrow EHM \longrightarrow HM+ \mathbf{PPP} \longrightarrow CEM^* \qquad \begin{array}{c} HM+ \mathbf{SSP} \\ \\ HM+\mathbf{WSP} \end{array}$$
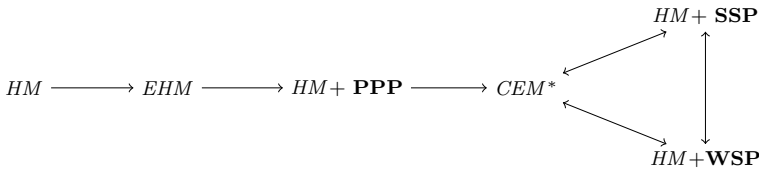
Fig. 3 Extensional mereologies based on *HM*

3 Non-extensional Mereologies for Non-heaplike Entities

In the previous section we suggested that *CEM** is the appropriate mereology for heaplike entities. Nevertheless, within this system, there is no room for both heaplike and non-heaplike composites: in the presence of **EXT** and **FFU** and with only one notion of part available, the same parts determine one and only one composite. If we want to differentiate structured sums from mere sums, we thus have to give up either mereological extensionalism, and so **EXT** or **FFU**, or mereological monism. We think that mereological extensionalism is an intuitive metaphysical thesis both for heaplike and non-heaplike entities, and hence that it is mereological monism that should go. But before developing this perspective, it is worth considering the possibility of accounting for non-heaplike entities by rejecting mereological extensionalism: after all, why should we develop a new system if existing ones are already satisfactory?

To assess non-extensional mereologies, we will keep the assumption that, in order to be appropriate for heaps, a system of mereology has to be at least as strong as *HM*. That is, it should allow for the introduction of a transitive and irreflexive relation of proper parthood and for the existence of a fusion of any pair of entities. As we will see, while some proposals do not satisfy this *desideratum*, the proposals that satisfy it still do not seem to be able to account, by themselves, for our intuitions about heaplike entities. In addition, and maybe more importantly, they give rise to a problematic proliferation of parthood relations.

3.1 Getting Started: Improper Parthood

A number of scholars (Cotnoir 2010, 2014; Cotnoir and Bacon 2012; Simons 1987; Thomson 1998) have suggested to account for non-heaplike entities by weakening *CEM*, so as to obtain non-extensional mereologies in which both mere and structured sums can be formed from the same proper parts. Mereologies of this sort have been developed on the basis of two key observations. First, given **IR** and **TR**, the relation $\leq$ of improper parthood has the following properties by definition:

REF. (*Reflexivity*) $x \leq x$
TR. (*Transitivity*) $x \leq y \wedge y \leq z \rightarrow x \leq z$
ANT. (*Antisymmetry*) $x \leq y \wedge y \leq x \rightarrow y = x$

Second, given **ANT**, **EXT** follows immediately from **PPP**. This suggests two main strategies to obtain a non-extensional mereology: one can either reject **PPP** (and hence **SSP**) or assume a non-antisymmetric relation of improper parthood as a primitive notion, and then re-formulate the proper parts and strong supplementation principles accordingly. The first strategy results in a mereology allowing for models like Model 2 above, where arrows represent the proper parthood relation, while the second strategy results in a mereology allowing for models like Model 3, where arrows represent a non-antisymmetric relation of improper parthood.
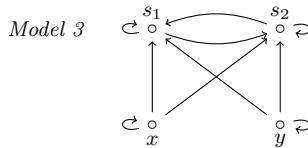
Model 3

So, suppose we have a puzzle with only two pieces, x and y. Then, by accepting Model 2, we can claim that the puzzle and the heap of puzzle pieces are distinct sums that share the same proper parts, but do not stand in any other mereological relation. By accepting Model 3, we can claim that the puzzle and the heap are distinct sums in a mutual relation of improper parthood. Clearly enough, since the two models are symmetric, they both represent the puzzle and the heap as mereologically indistinguishable, which is not what one would intuitively expect. But let us leave this aside for the moment. Our question is: can either one of the two strategies be successfully pursued in our framework?

Regarding the first strategy, the answer seems to be negative. Its proponents usually reject **PPP** by arguing that it is **WSP**, rather than **PPP**, that characterizes the concept of part (Simons 1987). However, as we have seen in Sect. 2, once the principles of *HM* are granted, it is not possible to reject **PPP** without also rejecting **WSP**. Hence, we cannot adopt this strategy without abandoning the very idea that **WSP**, together with any other of the supplementation principles we considered, is constitutive of the concept of part.[14]

Regarding the second strategy, the issue is more involved. Call a system of mereology *non-standard* if it includes a non-antisymmetric relation of improper parthood.[15] There are two main ways to obtain a non-standard system of mereology: first, we can assume a primitive relation of improper parthood that is reflexive, transitive, but not antisymmetric (Cotnoir 2010); second, we can assume a more generic relation of parthood that is only transitive, and then define improper parthood as we

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[14] Principles of supplementation weaker than those we introduced in the previous section have also been discussed. Yet, these principles seem to be too weak to characterize an intuitive notion of decomposition, and especially the one proper of heaps. We will not expand on this point here, but see Simons (1987, pp. 26–27) for a discussion of standard principles and Cotnoir (2013, pp. 836–837) for a discussion of a recent proposal advanced in Gilmore (Forthcoming).

[15] Non-standard systems of mereology were first suggested by Thomson (1983, 1998) and have been recently investigated from a formal point of view by Cotnoir (2010, 2014), Cotnoir and Bacon (2012), Parsons (2013, 2014) and Pietruszczak (2018). An overview of the debate can be found in Lando (2017, ch. 9).

did in Definition 2 (Cotnoir 2014; Cotnoir and Bacon 2012). To keep the discussion to a reasonable length (and since the two strategies are similar with respect to the points we wish to make), we will only focus on the first strategy.

3.2 Non-standard Mereology with Improper Parthood

Cotnoir (2010) proposes a non-standard system of mereology based on a reflexive and transitive parthood relation that we will denote with $<$. The notion of overlap is introduced by definition as follows.

Definition 5 (*Overlap'*) $x \circ' y := \exists a (a < x \land a < y)$.

A non-standard mereology, which we call *NS*, is then defined as shown in the next table.

NS (non-standard mereology based on $<$):		
(1) $\mathbf{R}^<$	(*reflexivity*)	$x < x$
(2) $\mathbf{TR}^<$	(*transitivity*)	$x < y \land y < z \rightarrow x < z$
(3) $\mathbf{SSP}^<$	(*supplementation*)	$\forall a (a < x \rightarrow a \circ' y) \rightarrow x < y$

In order to analyse the properties of *NS* and see whether it can be extended to a system at least as strong as *HM*, we need to introduce a relation of proper parthood. There are two candidate definitions.

Definition 6 (*Proper parthood*)

$$x \ll_1 y := x < y \land y \neq x;$$
$$x \ll_2 y := x < y \land \neg(y < x).$$

Although these definitions would be equivalent if $<$ were antisymmetric, in the absence of antisymmetry, $\ll_2$ turns out to be strictly stronger than $\ll_1$: on the one hand, $x \ll_2 y$ implies $x \ll_1 y$ by Leibniz's law of indiscernibility of identicals; on the other hand, the converse implication fails if $<$ is not antisymmetric. To see this, consider Model 3 above and take arrows to represent $<$. Then, both $s_1 \ll_1 s_2$ and $s_2 \ll_1 s_1$ hold, because s_1 and s_2 are different entities in the improper parthood relation; but, precisely because s_1 and s_2 are improper parts of each other, neither $s_1 \ll_2 s_2$ nor $s_2 \ll_2 s_1$ hold. So, while $\ll_1$ allows for loops, $\ll_2$ does not.

Which one of the two definitions captures a suitable proper parthood relation? In considering this issue, we rely on the following facts, which follow immediately from Definitions 5 and 6 by $\mathbf{R}^<$.

Fact 1 $x \ll_1 y \rightarrow x \circ' y$
Fact 2 $\exists a (a \ll_1 x) \land \forall a (a \ll_1 x \rightarrow a \ll_1 y) \rightarrow x \circ' y$

Note also that any proper part is also an improper part, and so every entity overlaps each of its proper parts.

3.2.1 Assessing the First Definition: $\ll_1$

To begin with, *NS* allows us to derive $\ll_1$-versions of the proper parts principle and of the principle of extensionality.

Proposition 1 *The following principle of proper parts is derivable in NS.*

PPP$^{\ll_1}$ $\quad \exists a(a \ll_1 x) \wedge \forall a(a \ll_1 x \rightarrow a \ll_1 y) \rightarrow x < y$

Proof Suppose that (i) $\exists a(a \ll_1 x)$ and (ii) $\forall a(a \ll_1 x \rightarrow a \ll_1 y)$. Consider an arbitrary b such that $b < x$. Then, either $b = x$ or $b \neq x$. In the first case, $b \circ' y$ by hypotheses (i) and (ii) and Fact 2. In the second case, $b \ll_1 x$ by Definition 6, so that $b \ll_1 y$ by hypothesis (ii). But then $b \circ' y$ by Fact 1. Hence, either case, $b \circ' y$. Since b was arbitrary, we can infer that $\forall b(b < x \rightarrow b \circ' y)$. By **SSP$^<$**, we can then conclude that $x < y$. $\qquad \square$

Proposition 2 *The following principle of extensionality is derivable in NS.*

EXT$^{\ll_1}$ $\quad \exists a(a \ll_1 x) \wedge \forall a(a \ll_1 x \leftrightarrow a \ll_1 y) \rightarrow x = y$

Proof Suppose that $\exists a(a \ll_1 x)$ and $\forall a(a \ll_1 x \leftrightarrow a \ll_1 y)$. Then, by **PPP$^{\ll_1}$**, $x < y$. If $x \neq y$, then $x \ll_1 y$ by Definition 6. But, since $\ll_1$ is irreflexive again by Definition 6, there is a $\ll_1$-part of y, namely x itself, that is not a $\ll_1$-part of x, against the hypothesis. So, it must be the case that $x = y$. $\qquad \square$

So, if the principle of extensionality is formulated as **EXT$^{\ll_1}$**, then *NS* turns out to be a *non-standard but extensional* system of mereology—even though, it is admittedly questionable whether, by allowing for models like Model 3, **EXT$^{\ll_1}$** captures the intuition at the basis of the principle of extensionality. The following propositions show that $\ll_1$ has further problematic properties.

Proposition 3 *The relation $\ll_1$ is not transitive.*

Proof Model 3 is a model of *NS* in which $s_1 \ll_1 s_2, s_2 \ll_1 s_1$, and yet $\neg(s_1 \ll_1 s_1)$. $\qquad \square$

Proposition 4 *The next principle **WSP$^{\ll_1}$** is not derivable in NS. In addition, antisymmetry of $<$ is derivable from **TR$^<$** and **WSP$^{\ll_1}$**.*

WSP$^{\ll_1}$ $\quad x \ll_1 y \rightarrow \exists a(a \ll_1 y \wedge \neg(a \circ' x))$

Proof Model 3 is a model of *NS* that violates **WSP$^{\ll_1}$**, since $s_1 \ll_1 s_2$ but there is no $a \ll_1 s_2$ that is disjoint from s_1. Turning to antisymmetry of $<$, assume **WSP$^{\ll_1}$** and suppose that $x < y$ and $x \neq y$. Then, $x \ll_1 y$ by Definition 6. Hence, by **WSP$^{\ll_1}$**, there is $a \ll_1 y$ such that $\neg(a \circ' x)$. But then, $a < y$ and $\neg(a < x)$, and so $\neg(y < x)$ by **TR$^<$**. $\qquad \square$

Proposition 3 implies that $\ll_1$ does not meet the minimal requirements for a proper parthood relation characterizing heaplike composition. Hence, by assuming $\ll_1$ as our proper parthood relation, we would lose the possibility of accounting for heaps. In addition, according to Proposition 4, $\mathbf{WSP}^{\ll_1}$ cannot be added to NS without making the relation $<$ antisymmetric and, thus, without ruling out the very possibility of distinguishing composites sharing all of their improper parts, like s_1 and s_2 in Model 3. Hence, in the context of NS, the assumption of $\ll_1$ would clash with the idea that $\mathbf{WSP}^{\ll_1}$ is constitutive of the concept of part. Be that as it may, since we are looking for a relation of proper parthood that is at least irreflexive and transitive, we will exclude $\ll_1$ as a candidate notion.

3.2.2 Assessing the Second Definition: $\ll_2$

As the next propositions show, $\ll_2$ has promising properties.

Proposition 5 *The relation $\ll_2$ is irreflexive and transitive.*

Proof Immediate by Definition 6 and transitivity of $<$.   □

Proposition 6 *The next principle of weak supplementation is derivable in NS.*

$\mathbf{WSP}^{\ll_2}$ $x \ll_2 y \to \exists a(a \ll_2 y \land \neg(a \circ' x))$.

Proof Suppose that $x < y$ and $y \not< x$. Then, by $\mathbf{SSP}^<$, there is $a < y$ such that $\neg(a \circ' x)$. Since $x < y$, if $y < a$, then $x < a$ by $\mathbf{TR}^<$. But then, since $<$ is reflexive, we would have $a \circ' x$, against the hypothesis. Hence, $\neg(y < a)$. We thus obtain that $a \ll_2 y$ and $\neg(a \circ' x)$, as desired.   □

Proposition 7 *The following principle of proper parts is derivable in NS.*

$\mathbf{PPP}^{\ll_2}$ $\exists a(a \ll_2 x) \land \forall a(a \ll_2 x \to a \ll_2 y) \to x < y$.

Proof Suppose that (a) $\exists a(a \ll_2 x)$ and (b) $\forall a(a \ll_2 x \to a \ll_2 y)$. Consider an arbitrary b such that $b < x$, and let us show that $b \circ' y$. By the law of excluded middle, either $b \ll_2 x$ or $\neg(b \ll_2 x)$. In the first case, $b \ll_2 y$ by hypothesis (b). Hence, $b < y$ by Definition 6 and so $b \circ' y$. In second case, $x < b$ by Definition 6, since $b < x$ by hypothesis. Consider an $a \ll_2 x$ (which exists by hypothesis (a)). By $\mathbf{TR}^<$, $a < b$ and $\neg(b < a)$, since otherwise $x < a$, against the hypothesis that $a \ll_2 x$. Hence, $a \ll_2 b$. But since $a \ll_2 x, a \ll_2 y$ by hypothesis (b), and so $b \circ' y$. Hence, either case, $b \circ' y$. Since b was arbitrary, we can infer that $\forall b(b < x \to b \circ' y)$. By $\mathbf{SSP}^<$, we can then conclude that $x < y$.   □

Proposition 8 *The following principle of extensionality is not derivable in NS.*

$\mathbf{EXT}^{\ll_2}$ $\exists a(a \ll_2 x) \land \forall a(a \ll_2 x \leftrightarrow a \ll_2 y) \to x = y$

Proof Model 3 is a model of NS in which $\mathbf{EXT}^{\ll_2}$ fails.   □

According to Propositions 6–8, by assuming $\ll_2$ as our notion of proper parthood, we obtain a *non-standard and non-extensional* system of mereology characterized by interesting decomposition principles, including **SSP**$^<$, **WSP**$^{\ll_2}$, and **PPP**$^{\ll_2}$. What is more, according to Proposition 5, $\ll_2$, unlike $\ll_1$, satisfies the minimal requirements we imposed on proper parthood, that is, irreflexivity and transitivity. In order to see whether *NS* can be made at least as strong as *HM* it thus remains to be shown that this system can allow for the existence of a fusion of any pair of entities. Following Cotnoir (2010) and Cotnoir and Bacon (2012), let us introduce the notion of fusion as follows.

Definition 7 (*Fusion'*) $Fu'(s, x, y) := x < s \wedge y < s \wedge \forall a(a < s \rightarrow a \circ' x \vee a \circ' y)$.

Definition 7 is a reformulation of Definition 4 by using a weaker notion of part. The next proposition highlights the main difference between the two notions.

Proposition 9 *The principle of uniqueness of fusion' is not deducible in the system NS + **FFE'**, obtained by extending NS with the next principle of finite fusion' existence.*

FFE' $\quad \exists s Fu'(s, x, y)$

Proof Model 3 is a model of *NS* + **FFE'** in which both $Fu'(s_1, x, y)$ and $Fu'(s_2, x, y)$ hold. $\square$

Hence, given Definition 7, **FFE'**, unlike **FFE**, does not entail uniqueness of fusions (nor **EXT**$^{\ll_2}$) in the presence of the weak supplementation principle **WSP**$^{\ll_2}$. So, if we assume $\ll_2$ as a notion of proper parthood, then *NS* + **FFE'** is a non-standard and non-extensional system of mereology that is at least as strong as *HM*. Does this system do justice to the intuitive notion of heaplike composition? Here we only observe that, if the friends of non-standard mereologies share our intuition that there is only one heap formed of the same puzzle pieces, then, *by itself*, system *NS* + **FFE'** is not enough: a separate account of what, among two entities sharing the same proper parts, makes one of them a heap and the other a non-heaplike entity is needed.

3.3 Problems for Non-extensional Mereologies

We have considered two main strategies to weaken *CEM** so as to obtain a non-extensional mereology. We have seen that, while the first, i.e. rejecting **PPP**, does not meet the initial requirement of allowing for at least the core principles of *HM*, the second, i.e. rejecting **ANT**, leads to a system that, *by itself*, cannot account for our basic intuitions about heaplike entities. In general, it seems that, by tweaking *CEM** in order to allow for structured sums, we make the system no longer suitable to account for mere sums. This supports the idea that a unified account of heaplike

and non-heaplike composition may not be possible. In any case, there is another independent difficulty affecting non-extensional mereologies.

Our worry is as follows. As we have already seen, given the standard transitive and irreflexive relation $\ll$ of proper parthood, it is possible to define a corresponding standard transitive, reflexive, and antisymmetric relation $\leq$ of improper parthood as in Definition 2. Conversely, given a primitive partial order (reflexive, transitive, antisymmetric) relation $\leq$, it is possible to define a corresponding strict partial order (irreflexive, transitive) relation $\ll$ by assuming either one of the two notions in definition 6. It can then be shown that the relations obtained by assuming either one of the two primitives are essentially the same. Specifically, let $\ll$ be a strict partial order and $\leq$ a partial order, and consider the relations in the next table.

Primitive:	$x \ll y$	$x \leq y$
Primary defined:	$x \leq' y := x \ll y \lor x = y$	$x \ll' y := x \leq y \land \neg(y \leq x)$
Secondary defined:	$x \ll'' y := x \leq' y \land \neg(y \leq' x)$	$x \leq'' y := x \ll' y \lor x = y$

The following facts can then be proved without difficulty:

(1)	$\leq'$ is a partial order	(3)	$\ll'$ is a strict partial order
(2)	$x \ll y$ just in case $x \ll'' y$	(4)	$x \leq y$ just in case $x \leq'' y$

Conditions (1)–(4) show that the two relations of standard proper parthood and standard improper parthood are interdefinable. This means that the two notions are *essentially the same*, in the sense that they encode the same amount of information. Parthood relations weaker than $\ll$ and $\leq$ do not behave in the same way: given a primitive preorder (reflexive and transitive) relation $<$ or a primitive transitive relation $\prec$, the duality disappears. As a consequence, by using standard definitions, a number of reflexive and irreflexive relations not reducible to one another become suddenly available, as shown in the following table.

Primitive reflexive and transitive relation $<$	
Defined irreflexive relations	Defined reflexive relations
$x \ll_1 y := x < y \land x \neq y$	
$x \ll_2 y := x < y \land \neg(y < x)$	$x \leq' y := x \ll_2 y \lor x = y$
Primitive transitive relation $\prec$	
Defined irreflexive relations	Defined reflexive relations
$x \ll'_1 y := x \prec y \land x \neq y$	$x \leq y := x \prec y \lor x = y$
$x \ll'_2 y := x \prec y \land \neg(y \prec x)$	$x \leq'' y := x \ll'_2 y \lor x = y$

So, the assumption of a primitive parthood relation weaker than a strict or partial order leads to a proliferation of reflexive and irreflexive relations. These relations are obtained by exploiting standard definitions mereologists normally use to introduce *parthood* relations, and so there is no *prima facie* reason why they should not

be parthood relations themselves.[16] But then, by assuming a weak primitive, we end up with a genuine proliferation of parthood relations.

Interestingly, dropping the proper parts principle **PPP** gives rise to a similar situation. As Simons (1987, pp. 112–116) observes, given a relation $\ll$ of proper parthood, two ways to define a relation of improper parthood come to mind. Besides using Definition 2, we could also use the following definition:

$$x \leq^* y := \begin{cases} \forall a(a \ll x \to a \ll y) & \text{if } C(x) \\ x \leq y & \text{if } \neg C(x) \end{cases}$$

It is immediate to check that, in the presence of **PPP**, $x \leq y$ holds just in case $x \leq^* y$ holds, so that $\leq$ and $\leq^*$ are the same relation. Still, without **PPP**, the implication from $x \leq^* y$ to $x \leq y$ fails, and hence $\leq$ and $\leq^*$ turn out to be different parthood relations. So, suppose that we relax our initial requirements concerning a mereology for heaps, and consider a system of mereology, let us call it *MM* following Varzi (2016), based on a primitive proper parthood relation $\ll$ and obtained by assuming **IR**, **TR**, **WSP**, and no composition principle. Let us use $x \leq_P y$ to abbreviate $\forall a(a \ll x \to a \ll y)$. Then, at least the following relations can be introduced in *MM* by definition and shown to be irreducible to one another.[17]

Primitive relation $\ll$ satisfying the principles of MM

Defined irreflexive relations	Defined reflexive relations
	$x \leq^* y := (C(x) \wedge x \leq_P y) \vee (\neg C(x) \wedge x \leq_P y)$
$x \ll_1^* y := x \leq^* y \wedge x \neq y$	$x \leq_1^* y := x \ll_1^* y \vee x = y$
$x \ll_2^* y := x \leq^* y \wedge \neg(y \leq^* x)$	$x \leq_2^* y := x \ll_2^* y \vee x = y$

This proliferation of parthood relations reveals that weakening *CEM** in one of the considered ways does not only imply a negation of mereological extensionalism but also ends up threatening the spirit (if not the letter) of mereological monism. This leads to a serious interpretative problem: our intuitions clearly support the classical parthood relation for heaps; weaken it and there will be a proliferation of parthood relations with no good reason to select any one among them. So, take a tiger and its tail, a house and one of its windows, a pile of sand and one of the grains in it as instances of possibly different parthood relations. Given either *NS* or *MM*, which one among the primitive and defined parthood relations is the most appropriate to describe each case? In light of the formal similarity of the available notions of part, there is no good reason to drop one of the supporting intuitions instead of the other. Note that the same problem affects any principle of mereology, since, as Sect. 3.2

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[16] Note that this does not mean that we should accept that all relations displayed in the table are "genuine" parthood relations. What it means is just that, *once a non-standard mereology is assumed*, there is no immediate justification to exclude that they are, given that the procedures used to introduce them are the same as the ones used to introduce what are typically taken to be legitimate notions of parthood.

[17] More relations than those displayed in the table can actually be introduced in *MM*. To give an example, by replacing $\ll$ with $\ll_1^*$ or $\ll_2^*$ in the definition used to introduce $\leq^*$, we obtain two further relations of improper parthood.

illustrates, the proliferation of parthood relations induces a proliferation of the principles of mereology, which can now be formulated in a number of different ways.

4 Extensional Mereologies for Heaplike and Non-heaplike Entities

In this section we develop a system of mereology enabling us to account for the difference between heaplike and non-heaplike entities without abandoning the thesis that *CEM** is the most appropriate mereology for heaps. To do this, we reject mereological monism and assume a primitive distinction between two kinds of parthood relation, namely *mere parthood* and *potential parthood*. This choice is based on two reasons. First, we want to maintain the idea that the identity of the components plus the way of composition are sufficient for determining the identity of a composite entity, without the need of introducing alien elements. Second, since the distinction between heaplike and non-heaplike entities is related to the way in which such entities are composed and since mereology is the general theory of parthood and composition, we want to maintain that this distinction is a mereological one.

The main idea we want to exploit is summarized in the following passage of *Metaphysics Z 13*:

> A substance cannot be composed of substances which are actually present in it: for things that are actually two are never actually one, even if things that are potentially two will be actually one (e.g. the whole line consists of two halves existing potentially, for actuality separates). Therefore, if the substance is one, it will not consist of substances present in it. [*Metaphysics Z 13*, 1039a3–1039a13, our translation.]

In this passage, Aristotle introduces the idea that substances have parts potentially. Even though this idea is partly explained by relying on a distinction between actual and potential existence, let us abstract away from this distinction and just extract the characterizing features of the relation of potential parthood. The main point is that *substances are structured sums constituted by parts, call them potential parts, that are not themselves substances*. In fact, the parts of a substance, like the two halves of a line, can be substances only if they are separated, i.e. only if they cease to be parts of *one* substance, thus becoming independent entities. The general idea is that potential parts are *unified* in a structured sum in virtue of the structure of the composite entity—the Aristotelian form—where unification results in an *identity-dependence* of the parts on the composite.[18] According to this view, while structured sums, in virtue of their unifying structures, have potential parts, mere sums can only have mere parts, precisely because they lack a unifying structure.

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[18] Although the notions of unification of a plurality of entities in a substance and of identity-dependence of a potential part on a structured sum are useful to grasp the relation of potential parthood, our aim here is not to analyse or further investigate these notions. Our intention is rather to develop a mereology based on a distinction between mere and potential parthood, where this distinction is assumed as primitive. For more discussion on the metaphysics of potential parthood, the reader is referred to Haslanger (1994) and Scaltsas (1994).

To illustrate, let us go back to the example of the puzzle. A complete puzzle can be described as a collection of puzzle pieces that are arranged according to a suitable structure and such that, in virtue of their arrangement, produce a complete picture. Now, suppose that the complete puzzle depicts a cat. When a puzzle piece is included in the complete puzzle, it is *reidentified* in the puzzle: it is no longer just a piece with some color on it, but it is a piece on which a part of the cat is depicted.[19] In our terminology, this makes the piece *identity-dependent* on the complete puzzle, even if the way in which the piece exists does not change. This identity-dependence of the piece on the puzzle is what makes the piece a potential rather than a mere part of the puzzle. This motivates us to adopt the following terminology:

- *x* is a *mere part*: *x* is an entity that exists in a mere sum;
- *x* is a *potential part*: *x* is an entity that exists in a structured sum;
- *x* is a *dependent entity*: there is a structured sum in which *x* is unified;
- *x* is an *independent entity*: there is no structured sum in which *x* is unified.

We will come back to the precise relations between these notions in Sect. 4.1. Before that, let us make explicit some crucial tenets of the mereological perspective that emerges from the ideas discussed so far.[20]

Tenet 1. Since it is impossible for an entity to have and lack a unifying structure, no entity can have both potential and mere parts. Hence, no entity can be both a potential and a mere part of the same sum.

So, the pieces in a bunch of puzzle pieces and the pieces in a complete puzzle are parts of the respective composites in two distinct ways: the former are not unified in the bunch, and so they are mere (but not potential) parts of it; the latter are unified in the puzzle, and so they are potential (but not mere) parts of it. Accordingly, the bunch of puzzle pieces only has mere parts, while the complete puzzle only has potential parts.

Tenet 2. Since the identity of an entity cannot depend on more than one principle, no entity can be a potential part of two composites at the same time.

So, two structured sums cannot have potential parts in common: one and the same puzzle piece can be unified in only one complete puzzle at a time.

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[19] Note that, in principle, one and the same puzzle piece can be included in different puzzles representing different things. Hence, it would not be correct to say that the piece can be identified as a piece depicting a part of the cat even when it is not in the puzzle, yet.

[20] Whether Aristotle actually agreed with all the principles we assume is a difficult question we will not try to answer: as we mentioned in footnote 18, although the view we assume is inspired by Aristotle, our aim is not to propose an analysis or a formalization of Aristotle's theory of potential parts, nor to advance a final such theory. The tenets characterizing our view are an initial attempt to understand the notion of potential parthood along the lines suggested by Aristotle (and consistently with Haslanger (1994) and Scaltsas (1994)) and the axioms that we will present in Sect. 4.1 are a corresponding attempt to make our understanding precise; we thus do not exclude the possibility of further developments.

Tenet 3. Since only one principle ultimately unifies the parts of a structured sum in an independent entity, a structured sum cannot have another structured sum among its potential parts.

This corresponds to the Aristotelian idea that a substance cannot be part of a substance. The idea is that a complete puzzle depicting, say, a cat next to a couch does not have a complete puzzle depicting a cat and a complete puzzle depicting a couch as its potential parts; otherwise, it would not be *one* complete puzzle but two puzzles juxtaposed.[21]

The rest of this section is divided into three parts. In the first, we propose a mereology based on a set of axioms that capture the previous ideas, and highlight some important consequences deriving from these axioms. In the second, we briefly show how the three intuitive judgments discussed in the introduction and the resulting predicament can be clarified. Finally, in the third part we explore the potential of our system as a general framework to model different metaphysical standpoints.

4.1 Mereology of Mere and Potential Parthood

The mereology of mere and potential parts *CAM*, for Classical Aristotelian Mereology, is based on two primitive proper parthood relations $\ll$ and $\ll_P$. Intuitively, $x \ll y$ means that x is a mere proper part of y, while $x \ll_P y$ means that x is a potential proper part of y. When $x \ll_P y$ holds, we will also say that x is unified in y. Given these two primitives, we define two notions of atomicity and composite entity (the notions of overlap and fusion are defined in terms of $\ll$ as in Definitions 3 and 4).

Definition 8 (*Being an atom*) An entity x can be an atom in either one of two ways:

1. atom with respect to mere parthood: $A_M(x) := \neg \exists a(a \ll x)$.
2. atom with respect to potential parthood: $A_P(x) := \neg \exists a(a \ll_P x)$.

Hence, an entity can be an atom either in the sense that it lacks mere proper parts or in the sense that it lacks potential proper parts. An entity that is an atom in both senses can be called an *absolute atom*.

Definition 9 (*Being composite*) An entity x can be a composite in either one of two ways:

1. heaplike, i.e. mere, composite: $C_M(x) := \neg A_M(x)$.
2. non-heaplike, i.e. structured, composite: $C_P(x) := \neg A_P(x)$.

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[21] We take Tenet 3 as a simplifying, methodological assumption rather than as an assumption characterizing non-heaplike composition as such. Even though it clashes with the pre-theoretic intuition that the parts of a substance can be hierarchically organized according to different levels of unity, it allows us to focus on the relation between heaplike and non-heaplike composition without, at the same time, diving into the problem of modeling hierarchical non-heaplike composition. We think that a full discussion of the latter problem requires a separate paper.

Hence, the relations of mere and potential proper parthood allow us to introduce a distinction between heaplike and non-heaplike composites. According to Definition 9, the former are entities with at least one mere proper part, while the latter are entities with at least one potential proper part. As we will see in a moment, the principles of *CAM* will ensure that heaplike composites have *only* mere proper parts, while non-heaplike composites have *only* potential proper parts.

The relation of potential parthood can be used to introduce a distinction between independent and dependent entities.

Definition 10 (*Being an independent/dependent entity*)

1. Independent entity: $\neg\exists a(x \ll_P a)$
2. Dependent entity: $\exists a(x \ll_P a)$

Hence, an entity is independent just in case it is not a potential proper part of any composite, in line with the Aristotelian idea that potential parts are *identity-dependent* on the composite of which they are parts. Indeed, a straightforward consequence of the previous definitions is that any non-heaplike entity is composed of dependent entities, i.e. the entities that are unified in it, and any dependent entity is a potential part of a non-heaplike entity, i.e. the structured sum unifying it.[22] We thus obtain an interesting classification of independent and dependent entities, based on the possibility of distinguishing atomic and composite entities with respect to mere parthood.

Entity	atomic: $A_M(x)$	composite: $C_M(x)$
independent	puzzle	heap of pieces
dependent	piece of the puzzle	part of the puzzle

In general:

Entity	atomic: $A_M(x)$	composite: $C_M(x)$
independent	substance	heap of substances
dependent	atomic material part	composite material part

Having introduced these notions, we can now turn to the principles of *CAM*. Since it is meant to represent the proper parthood relation characterizing heaps, we assume that $\ll$ obeys all the principles of *CEM**, that is, **IR**, **TR**, **FFE**, and **FFU**. Since **FFE** and **FFU** ensure that any pair of entities have a unique fusion, we introduce the functional symbol $+$ and use the expression $x + y$ to denote the fusion of x and y. The properties of $\ll_P$ and the interaction between $\ll$ and $\ll_P$ are then determined by four basic principles.

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[22] Note that this does not exclude the possibility that a dependent entity is also a *mere* part of some heaplike entity: in fact, as shown in the tables below, any heap of potential parts has dependent entities as its mere parts. We will come back to this later.

P1 $a \ll_P x \rightarrow A_P(a)$

According to **P1**, the potential proper parts of an entity cannot have potential proper parts. Equivalently, a structured sum cannot be a potential part of a structured sum; that is, *all structured sums are independent entities*. This is in line with *Tenet 3*, i.e. that a substance cannot be part of a substance.

P2 $x \ll_P a \wedge x \ll_P b \rightarrow a = b$

According to **P2**, two different composites cannot share potential proper parts. This principle is justified by the idea, expressed by *Tenet 2*, that an entity cannot be unified by more than one structured sum, since this would make it identity-dependent on two different structures.

P3 $x \ll_P a \wedge y \ll_P a \leftrightarrow x + y \ll_P a$

From left to right, principle **P3** states that the mere sum of any pair of potential proper parts of a composite is a potential proper part of that composite, so that *mere sums of potential proper parts are dependent entities*. In fact, taking together two entities that are unified in a composite can only result in something that is itself unified in that composite. So, the part of a complete puzzle that depicts a cat and the part that depicts a couch next to the cat taken together are still unified in the puzzle. From right to left, **P3** states that the only mere sums that can be potential proper parts of a composite are mere sums of potential proper parts of that composite. In fact, if one of the fused entities is not unified in a composite, then there is no way in which their fusion can be unified in it. So, there is no way in which the part of a complete puzzle that depicts a cat and a puzzle piece still in the box taken together can be unified in the puzzle.

P4 $A_M(x) \vee A_P(x)$

According to **P4**, no entity can be at the same time a heaplike and a non-heaplike entity. In fact, if x has mere proper parts, thus being a heaplike entity, then it is not an atom with respect to $\ll$, and so, according to **P4**, it must be an atom with respect to $\ll_P$. Similarly, if x has potential proper parts, thus being a non-heaplike entity, then it is not an atom with respect to $\ll_P$, and so, according to the principle, it must be an atom with respect to $\ll$. Hence, heaplike composites only have mere proper parts, while non-heaplike composites only have potential proper parts, in line with the idea expressed by *Tenet 1*.

The basic principles **P1** to **P4** have several key consequences.

Corollary 1 *The following properties of $\ll_P$ and $\ll$ are deducible in CAM.*

C1   (*Irreflexivity of $\ll_P$*) $\neg(x \ll_P x)$
C2   (*Transitivity of $\ll_P$*) $x \ll_P y \wedge y \ll_P z \rightarrow x \ll_P z$

C3 (*Combined Transitivity*) $x \ll y \wedge y \ll_P z \rightarrow x \ll_P z$

Proof **C1** follows immediately from **P1** and Definition 8. **C2** follows from the fact that the antecedent is always false by **P1**. Finally, for **C3**, suppose that $x \ll y$ and $y \ll_P z$. By Definition 4, we know that $y = x + y$, so that $x + y \ll_P z$. But then, $x \ll_P z$ by **P3**. $\square$

According to **C1** and **C2**, the relation of potential proper parthood satisfies the minimal requirements for being a proper parthood relation. According to **C3**, which is a reformulation of the right-to-left direction of **P3**,[23] a structured sum unifies all the mere proper parts of its potential proper parts, in line with the ideas that whatever is in a substance is unified in the substance.

Corollary 2 *The following principle of proper parts is deducible in CAM, where* $\leq_P$ *is standardly defined as in Definition 2*:

PPP$^{\ll_P}$ $\quad C_P(x) \wedge \forall a(a \ll_P x \rightarrow a \ll_P y) \rightarrow x \leq_P y$

Proof Immediate, since the antecedent is always false by **P2**. $\square$

Since, given Corollary 1, $\leq_P$ is antisymmetric by definition, **PPP$^{\ll_P}$** entails that potential proper parthood satisfies the principle of extensionality.

Corollary 3 *The following principle of extensionality is deducible in CAM:*

EXT$^{\ll_P}$ $\quad C_P(x) \wedge \forall a(a \ll_P x \leftrightarrow a \ll_P y) \rightarrow x = y$

Hence, *CAM* is an extensional system of mereology with respect to both the parthood relations on which it is based. By assuming *CAM* as our mereology for heaplike and non-heaplike entities we thus keep the thesis of mereological extensionalism, in the sense that no two composed entities of the same type can have the same proper parts.[24,25]

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[23] It is not difficult to prove that **C3** entails the right-to-left direction of **P3** and hence that the two implications are equivalent in *CAM*. In fact, suppose that **C3** holds and that $a + b \ll_P x$. Then, by Definition 4, there are four cases: (i) $a = a + b = b \ll_P x$; (ii) $a \ll a + b = b \ll_P x$; (iii) $b \ll a + b = a \ll_P x$; (iv) $a \ll a + b$ and $b \ll a + b$ and $\forall z(z \ll a \rightarrow z \circ a \vee z \circ b)$. It is immediate to check that **C3** ensures that $a \ll_P x$ and $b \ll_P x$ in all cases.

[24] Note that **P2** actually allows us to capture the idea that the same pair of entities cannot give rise to more than one structured sum. In this sense, principle **P2** is close to **FFU**: as **FFU** ensures that the fusion of a pair of entities, if it exists, is unique, **P2** ensures that the structured sum of a pair of entities, if it exists, is unique.

[25] *CAM* can be consistently extended with the assumption that $\ll_P$ satisfies the principles of weak and strong supplementation. We have decided not to include these principles because the only arguments we found to support them are based on a (we think) unjustified analogy between mere and potential

Fig. 4 **a** Proton: atomic poten-
tial parts. **b** Proton: atomic parts
and their mere sums

        (a)   (b)
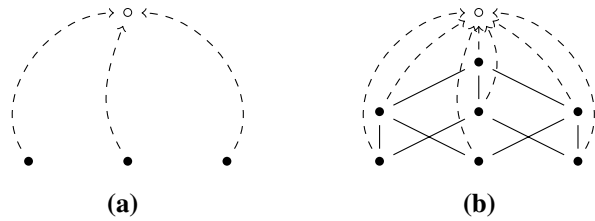
Before discussing further properties of *CAM*, let us illustrate the previous notions
by considering a (admittedly abstract) universe where entities have both mere and
potential parts.

Suppose that there is a unique proton and that a proton is constituted by two up
quarks and one down quark held together by the strong interaction. We construe the
proton as a structured sum composed by three atomic potential proper parts, as show
in Fig. 4, where dependent entities, i.e. quarks and their mere sums, are represented
by black circles, independent entities are represented by white circles, immediate
relations of mere parthood are represented by continuous lines,[26] and relations of
potential proper parthood are represented by dashed arrows pointing to the proton.
In order to have a complete representation of all the mere sums and all the relations
of mere parthood implied by *CAM*, we need to modify the diagram as shown in
Fig. 5a, where the dashed arrows from the mere sums of the atomic potential parts to
the proton are omitted.

We can now observe that the two primitive relations $\ll$ and $\ll_P$ should *not* be
thought of as specifications of a more general proper parthood relation holding
between *a* and *b* just in case either $a \ll b$ holds or $a \ll_P b$ holds. In fact, it can be
proved that the union of $\ll$ and $\ll_P$ is not transitive, and so it is not classifiable as a
relation of proper parthood. In order to see this, let us consider an evolution of the
previous world where the proton decays to a positron and a neutral pion, as show in
Fig. 6. This world contains two actual entities, pion *x* and positron *y*, such that *x* is a
structured sum, containing an up quark *u* and an up antiquark $\bar{u}$, and *y* is an absolute
atom. In this scenario, *x* is the only structured sum (its potential parts are repre-
sented by black circles). A complete representation of the generated *CAM*-universe
is provided in Fig. 5b , where the dashed arrows from the mere sums of the atomic
potential parts to the pion are again omitted.

It is not difficult to check that all the axioms on $\ll$ and $\ll_P$ are satisfied, so that
the model in Fig. 5b is indeed a model of *CAM*. If we consider *x* and its potential
proper part *u*, we note that there are three circles immediately above *x* along the

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Footnote 25 (continued)

parthood. Besides, since the principle of weak supplementation is put into question in some accounts
of cases of material coincidence resulting from mereological diminution, as in Deon and Theon (Sed-
ley, 1982) or Tib and Tibbles (Wiggins, 1968), and in some hylomorphic accounts of ordinary objects
(see Cotnoir 2018 for a recent discussion), we prefer to remain neutral as to its validity in *CAM*.

[26] The order is the standard order of Boolean graphs, i.e. bottom up. We say that *x* is an immediate mere
proper part of *y* just in case $x \ll y$ and there is no $a \ll y$ such that $x \ll a$.
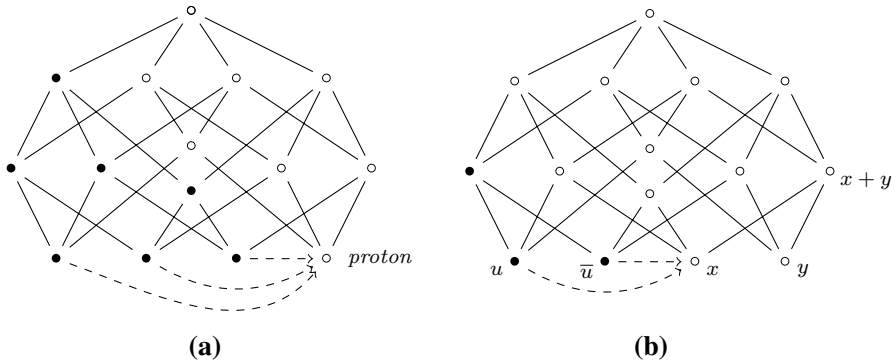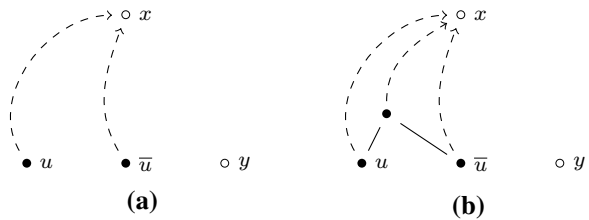
Fig. 5 **a** Proton: complete picture. **b** Decayed proton: complete picture

Fig. 6 **a** Decay: atomic potential parts. **b** Decay: atomic parts and their mere sums

relation of mere proper parthood and that only one of them, which is the mere sum of u and x, is also above u. Thus, even if $u \ll_P x$ and $x \ll x + y$, it is not the case that either $u \ll x + y$ or $u \ll_P x + y$. Hence, the union of our two primitive relations is not a transitive relation, and so cannot count as a relation of proper parthood.

So far so good. But what about the commitment to these two parthood relations *CAM* ask us to accept? Besides uncovering important properties of the potential proper parthood relation, corollaries 1–3 also play a central role in assessing the form of mereological pluralism to which we commit ourselves by endorsing *CAM*. Unlike the non-extensional mereologies we analysed in Sect. 3, our mereology of mere and potential parthood supports what we can call a *controlled mereological pluralism*: we can keep under control the number of different reflexive and irreflexive parthood relations that can be legitimately introduced in the system. In fact, since $\ll$ and $\ll_P$ are both strict partial orders that satisfy the principle of proper parts, the only parthood relations to which we genuinely commit ourselves by assuming *CAM* are no more than the two primitive relations $\ll$ and $\ll_P$. In addition, since they are introduced with the specific intention to capture precise intuitions about the mereological structure of heaplike and non-heaplike entities, these parthood relations do not give rise to an interpretative problem. To repeat the test we proposed in Sect. 3.3, consider a tiger and its tail, a house and one of its windows, a pile of sand and one of the grains in it. Our framework allows us to interpret the first two pairs of

entities as instantiating the relation of having a potential proper part, while the last pair of entities as instantiating the relation of having a mere proper part.[27]

4.2 Meeting the Desiderata

Let us now go back to the intuitive judgments concerning the existence of, and the connection between, heaplike and non-heaplike entities we considered in Sect. 1, and see how they can be handled in *CAM*.

1. *Existence and uniqueness of heaplike entities.*

The intuition underlying this condition is that any plurality of entities gives rise to one specific heap, so that, when we see some puzzle pieces on the table, we recognize the existence of one and only one heap made of those pieces. Since *CAM* includes *CEM**, this intuition is captured by principles **FFE** and **FFU**, which, as we saw in Sect. 2, ensure the existence of a unique mere sum of any plurality of entities. Hence, if $x_1, x_2, \ldots, x_n$ are some puzzle pieces on the table, $m = x_1 + x_2 + \cdots + x_n$ is the heap made of those pieces.

2. *Existence and uniqueness of non-heaplike entities.*

The intuition underlying this condition is that entities arranged according to a specific structure give rise to one substance, so that, when we see a puzzle on the table, we recognize one and only one puzzle made of the pieces constituting it. This intuition is captured by the fact that, within *CAM*, it is possible to define an appropriate notion of structured sum of potential parts. To do this, let us first define the notion of matter of an entity.

Definition 11 (*Matter of an entity*) $Matter(m, y) := \forall x (x \ll_P y \leftrightarrow x \leq m)$.

Hence, m is matter of y precisely when every potential proper part of y is either a mere proper part of m or coincides with m, i.e. when m is a potential proper part of y including all potential proper parts of y. As it is immediate to check, the matter of an entity, *if it exists*, is unique by the antisymmetry of $\leq$. The relation of structured sum can now be defined as the relation occurring between an entity and its matter: $Fu_{Str}(s, m) := Matter(m, s)$. The structured sum of m, *if it exists*, is unique by **P2**.

Going back to the puzzle we see on the table, let $x_1, x_2, \ldots, x_n$ be the pieces constituting it. Then, $x_1, x_2, \ldots, x_n$ are all and only the atomic potential proper parts of the puzzle. By **FFE** and **FFU**, $x_1, x_2, \ldots, x_n$ have a unique mere sum $m = x_1 + x_2 + \cdots + x_n$. We can now see that m is the matter of the puzzle and, hence, that $Fu_{Str}(puzzle, m)$. In fact, for the right-to-left direction of Definition 11, iterated applications of **P3** ensure that $m \ll_P puzzle$ and all mere proper parts of m are potential proper parts of the

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[27] Of course, there are cases in which intuitions might diverge, but this is not problematic for the point we want to make. We are not arguing that our system provides us with a criterion to distinguish heaplike and non-heaplike entities, but that, once we classify certain entities as heaps and others as structured sums, it is clear which one of our parthood relations can be used to represent the different cases.

puzzle. For the left-to-right direction, if $z \ll_P puzzle$ and $z \neq m$, then z must be either one of $x_1, x_2, \ldots, x_n$ or a mere sum of potential proper parts of the puzzle, ultimately made of some of $x_1, x_2, \ldots, x_n$. Since m is the fusion of $x_1, x_2, \ldots, x_n$, z must be a mere proper part of m in both cases. Hence, $m = x_1 + x_2 + \cdots + x_n$ is the unique matter of the puzzle and the puzzle is the unique structured sum made of it. That is, the puzzle is the unique structured sum made of $x_1, x_2, \ldots, x_n$, as it was desired.

3. *Apparent existence only of one non-heaplike entity.*

As the discussion of the first two intuitive judgments concerning the existence of heaplike and non-hepalike entities reveals, in the framework of *CAM*, whenever there is a puzzle built on the table, both the mere and the structured sum of the puzzle pieces are on the table, and so they both actually exist. But then, why are we prepared to recognize only the existence of the puzzle and not that of the bunch of puzzle pieces when a puzzle is on the table? This apparent predicament can be explained in the framework of *CAM* by relying on the relation between a structured sum and its matter. When a mere sum is unified in a structured sum, we prioritize the existence of the structured sum and ignore the existence of its matter, because, in general, we tend to prioritize the existence of independent entities (like the puzzle) over the existence of dependent entities (like the bunch of pieces constituting the puzzle). So, when there is a puzzle built on the table there *seems* to be no bunch of puzzle pieces on the table, because the puzzle is the only independent entity on the table.

Overall, the discussion of points 1–3 suggests that a natural characterization of material constitution (see Wasserman 2004) is available in *CAM*: given a structured sum consisting of finitely many atomic potential proper parts, the mere sum of its potential parts constitutes it. That is, material constitution is the relation between the matter of a structured sum and the structured sum. In line with Aristotle's view, *CAM* characterizes the matter as a dependent and non-structured entity, which is different from the substance, despite sharing with it all atomic components. Importantly, the difference is a mereological one: while the structured sum is an atom with respect to the relation of mere proper parthood (i.e. $A_{\ll}$(structured sum)), its matter is an atom with respect to the relation of potential proper parthood (i.e. $A_{\ll_P}$(matter)).

4.3 Modelling Metaphysical Systems

In this final part we show how the descriptive power of our mereology can be exploited in order to distinguish and characterize three metaphysical viewpoints on the structure of the world: atomism, substance monism, and substance pluralism.[28] In accordance with an extremely general account, we characterize the three positions as follows.

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[28] Atomism was first endorsed and developed by Democritus and it is the philosophical viewpoint shared by scholars close to the empiricist tradition. Substantial monism was first endorsed and developed by Parmenides and it is the philosophical viewpoint shared by scholars close to the idealist tradition. Substantial pluralism was first endorsed and developed by Aristotle and it is the philosophical viewpoint shared by scholars that tend to adopt a framework which is close to our basic intuitions on the world.

1. Atomism is the view that the entire world is composed of a number of fundamental entities that lack proper parts: everything that exists is a non-fundamental arrangement of atoms. Atoms are characterized exclusively by a certain number of intrinsic properties and every existent property derives from properties of the atoms.
2. Substantial monism is the view that the entire world, the universe, is a fundamental entity and that all its proper parts are abstract aspects of it: everything that exists is a way of being of this one entity. The universe is characterized by a certain structure and every existent property is a relational property connecting abstract parts of the whole.
3. Substantial pluralism is the view that the entire world is composed by a number of fundamental and structured entities, the substances: everything that exists is a way of being of these entities or a certain arrangement of them. Substances are characterized both by a certain structure and by a certain number of intrinsic properties.

Although these three viewpoints are not distinguishable in standard systems of extensional or non-extensional mereology, in *CAM* they can be characterized in terms of the distinctions between independent and dependent entities and between mere and structured sums, given the assumption that the universe only contains a finite number of entities.[29]

1. *Atomism is the view that there are no structured sums*.

Hence, according to atomism, there are *many* independent entities, but *none* of them is a structured sum. The following model represents an atomistic world constituted by four fundamental atomic entities. It is not difficult to see that the mereology we obtain by assuming that the relation of potential proper parthood is never instantiated is *CEM*. Hence, as expected, the atomistic version of *CAM* on a finite domain is precisely *CEM*. The mereological relations captured by *CEM* are thus the mereological relations seen from the viewpoint of an atomistic philosopher.

2. *Substantial monism is the view that there is only one structured sum*.


Atomistic Model

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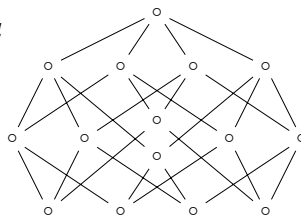
[29] If we drop the assumption that the universe is finite, then we might end up with a non-atomistic universe, and hence fail to capture atomism. If we generalize the atomistic model depicted below to a (non-atomistic) infinite model, we obtain the picture of a universe in which only independent unstructured entities exist. This corresponds to the elegant universe proposed by Calosi (2015) and Lewis (1991). Similarly, if we generalize the monistic model depicted below to a (non-atomistic) infinite model, we obtain the picture of the universe proposed by a priority monist (Schaffer 2010).

Hence, according to substantial monism, there is *only one* independent entity, and *this* entity is a structured sum. The following model represents a monistic world constituted by four non-fundamental atomic aspects (as before, black circles represent dependent entities and dashed lines represent the relation of potential proper parthood). It is not difficult to see that the mereology we obtain by assuming that the relation of mere proper parthood is instantiated only by dependent entities is again *CEM* on the domain constituted by these dependent entities (again, provided this domain is finite). Hence, the monistic version of *CAM* on the (finite) domain of potential entities is again precisely *CEM*. Thus, the mereological relations captured by *CEM* are also the mereological relations seen from the viewpoint of a monistic philosopher who wants to analyse the net of the relations of potential parthood between the dependent entities constituting the independent whole.

3. *Substantial pluralism is the view that there are independent structured and mere sums*.

Monistic Model
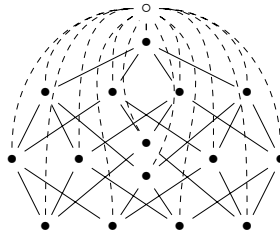
Hence, according to substantial pluralism, there are *many* independent entities, and *some* of them are structured sums. The pluralistic version of *CAM* on a finite domain is richer than *CEM*, even if it coincides with it provided that only mere sums are considered.

The main upshot of this final part is that the present mereological framework is sufficiently rich to allow us to characterize three significant metaphysical views on the structure of the world. In addition, we discovered that the extreme views, atomism and monism, are unified by the fact that in both cases the mereological structure of the world turns out to be properly described by the system of classical extensional mereology. This result is of interest: it tells us that *CEM* can be viewed as the mereology supported by specifying *CAM* in terms of assumptions that depend on precise metaphysical theses rather than on intuitions on the concept of parthood.

5 Conclusion

The aim of this paper was to introduce a system of mereology sufficiently strong to capture the intuitive distinction between heaplike and non-heaplike entities. Systems *CAM* of Classical Aristotelian Mereology seems to satisfy our request. *CAM* is interesting in at least three respects: (a) it is consistent with classical extensional

mereology when only heaplike entities are considered, as shown in Sect. 2; (b) it is extensional, thus avoiding the intuitive and mathematical costs of endorsing a non-extensional mereology, as highlighted in Sect. 3; (c) it is descriptively powerful, thus providing us with a framework where the distinction between mere and structured sums is consistently captured and our intuitive judgments on these kinds of sum are properly accounted for, as shown in Sect. 4. That said, we still think that *CAM* constitutes just a first step in the direction of developing an adequate mereology for heaplike and non-heaplike entities. Here, we briefly mention two lines of research that we think are among the most promising ones. Firstly, as Model 6 illustrates, the axioms of *CAM* do not allow for chains of potential proper parts: in our framework, potential proper parthood is a direct relation between absolute atoms and substances. We plan to explore the possibility of refining this relation so as to be able to account for the intuition that the parts of a substance can be hierarchically organized according to different levels of unity. Secondly, in order to do so, we plan to further elaborate our framework by making the notion of structured sum relative to an explicit principle of unity. In this way, we will be able to differentiate not only levels of unity within a substance but also kinds of structured sums, like organisms and non-organic wholes.

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