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ERRATUM

Erratum to: The application of Lie point symmetries to the resolution of certain problems in financial mathematics with a terminal condition

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Abstract In an analysis of the Lie point symmetries of the equation which allows for the inclusion of transaction costs into the Black–Scholes model (J Eng Math 82:67–75, 2013) one of the symmetries was omitted. We rectify that failing and demonstrate that the correctness of the subsequent analysis is not compromised.

Keywords Evolution partial differential equations · Financial mathematics · Lie symmetries

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The governing evolution partial differential equation for the Black–Scholes model when transaction costs are included is [1] (their Eq. 3.1)

$$0 = u_t + \frac{1}{2}\tilde{\sigma}^2 x^2 u_{xx} + b\sigma^2 x^3 u_{xx}^2 + r(xu_x - u) \tag{1}$$

subject to the terminal condition

$$u(T, x) = f(x), \tag{2}$$

where $f(x)$ is initially unspecified and is to be determined from the analysis.

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We analyse (1) for the possession of Lie point symmetries. We find that in general there are five Lie point symmetries, namely

$$\Gamma_1 = \partial_t, \quad \Gamma_2 = x\partial_u, \quad \Gamma_3 = e^{rt}\partial_u, \quad \Gamma_4 = x\partial_x + u\partial_u,$$
$$\Gamma_5 = t\partial_t + rtx\partial_x + \left\{\frac{\tilde{\sigma}^2 x}{8b\sigma^2}\left(2 + 2rt + \tilde{\sigma}^2 t - 2\log x\right) + (rt-1)u\right\}\partial_u. \quad (3)$$

To solve the boundary-value problem we apply the general symmetry, $\Gamma = \sum_{i=1}^{5}\alpha_i\Gamma_i$, where the $\alpha_i, i = 1, \ldots, 5$, are as yet arbitrary constants, to the terminal conditions (2). We obtain the two relations

$$\alpha_1 + \alpha_5 T = 0$$

and

$$\alpha_2 x + \alpha_3 e^{rT} + \alpha_5\frac{\tilde{\sigma}^2 x}{8b\sigma^2}\left(2 + 2rT + \tilde{\sigma}^2 T - 2\log x\right) + (\alpha_4 + \alpha_5(rT-1))f = (\alpha_4 + \alpha_5 rT)xf'.$$

Note that we have eliminated α_1 in favour of $-\alpha_5 T$. The second relation may be considered as a first-order equation for $f(x)$. If $\alpha_5 \neq 0$, its solution is, obtained using Mathematica,

$$f(x) = -\frac{\alpha_3 \mathrm{e}^{rT}}{\alpha_4 + \alpha_5(-1+rT)} + \frac{\left(8\alpha_2 b\sigma^2 + \tilde{\sigma}^2\left(2\alpha_4 + \alpha_5\left(2 + 4rT + \tilde{\sigma}^2 T\right)\right)\right)x}{8\alpha_5 b\sigma^2}$$
$$+ c_1 x^{1-\alpha_5/(\alpha_4+\alpha_5 rT)} - \frac{\tilde{\sigma}^2 x\log x}{4b\sigma^2}, \quad (4)$$

in which a constant term,

$$(\alpha_4 + \alpha_5 rT)^{1-\alpha_5/(\alpha_4+\alpha_5 rT)},$$

has been incorporated into the constant of integration to give c_1, and, if $\alpha_5 = 0$, we have

$$f(x) = c_1 x - \frac{\alpha_3}{\alpha_4}e^{rT} + \frac{\alpha_2}{\alpha_4}x\log x \quad (5)$$

where c_1 is the constant of integration. The form of $f(x)$ in (5) provides the solution (3.8) in [1], while the form in (4) will be examined elsewhere.

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Reference

1. O'Hara JG, Sophocleous C, Leach PGL (2013) The application of Lie point symmetries to the resolution of certain problems in financial mathematics with a terminal condition. J. Eng. Math. 82:67–75. doi:10.10007/s10665-012-9595-4