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Global natural projections

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Abstract

The paper contributes to the debate on natural interest rates and potential growth rates. We build a model-based projection of the world's most significant economies/ areas to improve understanding of their change over the long run and the factors behind their decline. We use a general equilibrium overlapping generation model to understand the simultaneous role of demographics, technology, and globalization. The novelty of the model lies in the way it constructs a human capital index based on UN population projections and an estimated increasing returns production function for major economies worldwide. We find that the decline in interest rates is well explained through labor market dynamics and the increasing obsolescence of capital goods. We also find that a reduced share of labor income has caused movement in the opposite direction, leading to an increase in natural interest rates, which runs counter to the empirical evidence. Moreover, the dynamics of economic integration predict an endogenous adjustment of global imbalances over the long run, with an increasing weight of the Chinese economy and, consequently, a phase of weakness in United States growth between 2030 and 2040. The model is also used to perform shock scenario analysis. We find that demographic decline can adversely affect the growth dynamics for European countries, while a change in the dynamics of globalization can have serious consequences, especially for the United States, with significant benefits for European countries and China.

Keywords Forecasting and prediction methods · Simulation methods · Demographic trends · Macroeconomic effects · Forecasts

JEL Classification C53 · C68 · J11

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1 Introduction

The importance of measuring the natural rate of interest and potential growth rate is crucial for both monetary and fiscal policy. This is particularly so when shocks of a large magnitude impact the economy, for instance the COVID-19 pandemic and the currently abnormally high inflation, which may be due to excessive expansionary monetary policies at the global level.

Recognition of the aforementioned quantities is fundamental for the design of economic policies. There is, however, no structural modeling framework in the literature that explains them at a global level. The task is left to empirical estimation strategies, which refer only to certain areas, such as the advanced countries. This paper addresses this gap. We used a general equilibrium model to calculate the *natural* interest and potential gross domestic output growth rates and project them at a *global* level into a possibly distant future. This was conducted using a micro-founded description of economic sectors (i.e., households, firms, and government, and the evolution of real and financial markets-in particular, private and public savings) in a multi-country context that includes advanced and emerging economies.

The potential gross domestic product (GDP) and the real natural interest rate (NRI) in the long term cannot be estimated unless the main components of the economy are taken into account, such as the saving and investment behavior of agents in the economy, the economy's institutional characteristics, and the interconnections of these components at the international level that affect productivity growth. Moreover, the projection of potential output is becoming increasingly crucial in estimating the long-term consequences of macroeconomic policies, such as those related to climate change policies or to defining the long-term public debt path. For those reasons, as in Holston et al. (2017), and Williams et al. (2020), we determine interest rates and growth simultaneously but working with a calibrated model rather than using econometric measurement. We add specific processes to the model like capital depreciation, labor share dynamics, and demographic changes so as to explicitly specify the trends in preferences and technologies, and we measure their contribution to the interest rates and and growth in GDP.

We build a multi-country overlapping generations (OLG) model that includes the United States, Germany, Italy, and France as advanced countries, and China, India, and Africa as emerging countries/regions.[1] We simulate the model over a long period in history (1960–2019) and the future (2020–2100) to calibrate the model parameters. We assume some exogenous trends that would affect the natural rate projection. For calibrating exogenous factors, we use an index of human capital based on Barro and Lee (2015) education attainment data, UN population projections (United Nations 2019), the capital stock (Berlemann and Wesselhöft

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[1] We enlarge the model presented in Catalano and Pezzolla (2016) adding non-European Union (EU) countries to provide a projection of the global economy. To save effort, we keep Italy, Germany, and France as a nucleus of countries representative of the EU, instead of aggregating in one larger area. In future work, we will integrate the model structure to take into account EU countries or EA areas as a whole.

2014), and, for historical matching, the GDP level and the debt-to-GDP ratio (see calibration, Sect. 4).

We use the model to estimate the effects of the interaction of trends that would be reflected as a contribution to the dynamics of natural rates and potential growth. The analysis is thus not specific to a single economy/area but considers emerging and advanced countries as being of equal importance. Moreover, our specification of capital markets in the OLG model allows for multi-country effects to be gauged via a capital market allocation mechanism that, over time, is increasingly open to considering a process of increasing globalization. In this kind of model, this is a novel feature that allows measurement of the effect of the heterogeneity of saving behavior among areas and also links to the long-run globalization process, that is, the increasing role of emerging countries in supplying and demanding real investment financing.

Our main contribution is to determine the weights of the exogenous components of the model in explaining the potential growth rate and the natural rate of interest. The long-run projection confirms the decreasing path of GDP and interest rates for all countries, with interest rates becoming steadily negative for the advanced economies. This would allow the debt-to-GDP levels to be stabilized in the long-run. A corollary of our analysis is to provide an estimate for natural rates both for China and for emerging countries such as India.

Understanding the determinants can thus assist in the design of long-run structural economic policies able to maximize growth and help monetary policy in its long-run operational functions. We find that, in addition to aging, the increasing obsolescence of capital goods has led to a reduction in natural interest rates in Europe and the United States; meanwhile, a reduced share of labor income has caused movement in the opposite direction, leading to an increase in natural interest rates, in contrast with the available empirical evidence. While, in the past, the human capital contribution has played an important role in terms of the potential growth of advanced countries, its contribution in the future would vanish due to weak demographic evolution. The opposite holds for the emerging countries where the main factor sustaining potential growth would be the pace of accumulation of human capital, which in the future would quickly emerge as a substitute for the importance of labor supply.

In addition, using calibration strategy and demographic projections of this kind, we can project the dynamics of public debt and public finance that are consistent with long-term growth and interest rate paths. Nevertheless, for many countries, we find a debt-to-GDP ratio that grows in size due to the aging process in the advanced countries. This projection could be useful for advising policymakers of the possible evolution of the "equilibrium levels of public debt," given the natural evolution of the economy.

To sum up, our approach allows us to obtain model-based projections which represent *non-filtered* estimations of the leading natural variables for the major advanced and emerging economies. In particular, we obtain natural long-run projections for GDP, interest rates, savings, wages, technological trends, net foreign asset position, and public debt.

The paper aims to show the methodology and the modeling strategy used and not to provide an update on the determination of natural levels. We have carried out the analysis until 2019 without including recent macroeconomic developments related to the COVID-19 crisis, which will be analyzed in a separate paper. The model is designed to allow further developments that we leave for future research, such as greater disaggregation of these areas/countries and an extension to other economies.

The paper is organized as follows. Section 2 discusses the literature related to natural rates and long-run drivers of economic growth. Section 3 describes the model and Sect. 4 shows how we calibrate the most relevant parameters and exogenous variables. Results are presented in Sects. 5 and 6 concludes. "Appendix 1" discusses stylized trends and empirical facts.

2 Related literature and empirical facts

The paper aims to provide a systematic and simultaneous estimate of natural interest rates and potential growth. We discuss the methodologies found in the literature for the two indicators and the explanatory power of exogenous trends, such as demographics and technology. Potential production and natural interest rates are not observable but are estimated on the basis of models and assumptions. The determination of the potential growth and natural interest rates has been pursued jointly or in isolation, depending on the model used. In the following sections, we briefly discuss the relevant literature.

2.1 Natural interest rate in the literature

The natural rate of interest changes over time due to shifts in aggregate supply and demand. Holston et al. (2017) and Laubach and Williams (2003) consider a multivariate model that jointly estimates the natural rate and the potential growth rate, taking into account the comovements in inflation, output, and interest rates. Figure 1
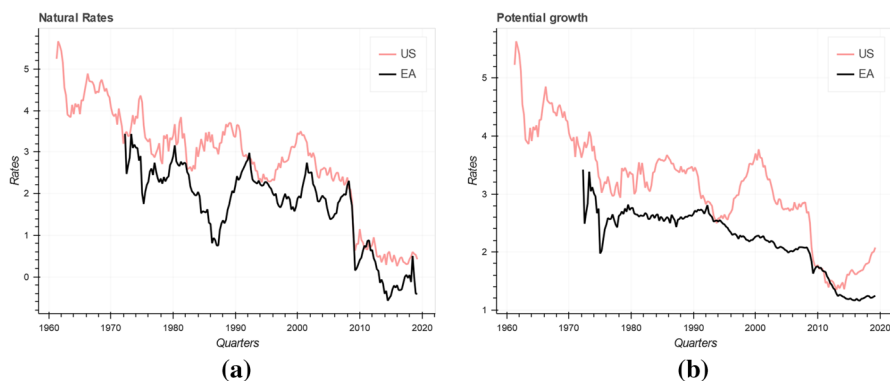
Fig. 1 Natural interest rate and potential GDP growth rate in the USA and the euro area (EA) *Source*: Holston et al. (2017)

shows the decreasing trends in natural rates in the Euro area (EA) and the United States (USA) since 1970. We will refer to these indexes to match our model results to their estimated natural interest rate levels. For example, estimates using the Laubach and Williams (2003) model indicate that the natural rate in the United States fell close to zero during the economic crisis and remained at that level throughout 2016 and afterwards. Explanations for this decline include shifts in demographics, a slowdown in trend productivity growth, and global factors affecting the real interest rates of the United States and three other advanced economies (i.e., Canada, the United Kingdom, and the euro area). Laubach and Williams (2003) find that significant declines in trend GDP growth and natural rates of interest have occurred over the past 25 years in all four economies. As we will see in the following, all such dynamics are partially confirmed by our estimates. These country-by-country estimates are found to display a substantial amount of comovement over time, suggesting an essential role for global analysis (i.e., multi-country modeling, as in our paper) in shaping trend growth and the natural rates of interest.

The European Central Bank (ECB) report on interest rate estimates (Brand et al. 2018) presents the various contributions in the literature. Structural approaches, similar to ours, have been developed to capture demographic effects on natural and potential growth (see Auerbach et al. 1987; Bielecki et al. 2018 and Papetti 2019).

In Papetti (2019), the OLG model is approximated with an aggregate representation and embodies perfect foresight transition with exogenous demographic change for a selection of Euro area countries (EA12) as a unique driver. Bielecki et al. (2018) calibrate a New Keynesian OLG with investment adjustment costs, using age cohorts 20-99, and modeling the entire life-cycle at an annual frequency. Both studies mainly attribute the secular decline in the natural rate of interest since the 1980s to aging, with an estimated impact of around one percentage point in Bielecki et al. (2018), and around 0.8 percentage points in Papetti (2019). Looking to the future, demographic developments will further lower the natural interest rate by another 0.5 percentage points by 2030, according to both studies. These results are in line with previous estimates in the literature. Bielecki et al. (2018), and Papetti (2019) conclude that the demographic situation had a positive influence on potential growth rates until 2000. In the 21st century, however, the influence turned negative and is expected to become even more so by 2030.

In a rich, multi-country OLG model, Krueger and Ludwig (2007) estimate a decrease in worldwide interest rate from 2005 to 2080 by around 0.86 percentage points. Carvalho et al. (2016) estimate that world interest rates declined by 1.5 percentage points between 1990 and 2014, while Kara and von Thadden (2016) project the natural rate of interest in the euro area to decrease by 0.9 percentage points between 2008 and 2030.

Although there is consensus in the literature regarding the overall effect of aging on the natural rate of interest, the relative roles of the three specific channels–life expectancy, lower labor supply, and the lower marginal product of capital-are still being debated. Carvalho et al. (2016) find that the channel of rising life expectancy is almost uniquely responsible for declining interest rates. However, they might be overestimating this channel, as they rely on a model with only two age groups and a constant mortality risk of the retired. In contrast, by using a multi-cohort OLG

model for the USA (as a closed economy), Gagnon et al. (2016) find that the channels of lower labor supply and the lower marginal product of capital are more prominent.

Papetti (2019) identifies an important role of the channel of aging-induced change in the savers-dissavers composition in depressing interest rates. Eggertsson et al. (2017) estimate that in the USA over the 1970–2015 period, the influence of changes in mortality and fertility were of equal strength. Neri and Gerali (2018) find that risk premium shocks–a shortcut for changes in agents' preference for safe assets–were the main driver behind the decline in interest rates in the euro area. In the United States, shocks to the efficiency of investment, which may capture the functioning of the financial sector, and shocks to the risk premium played a significant role.

Besides aging, rising income inequality could also be an important factor in lowering equilibrium real interest rates (see, e.g., Rachel and Smith (2017)). There is a striking coincidence in the dynamics of the non-growth component of the US interest rate (in Holston et al. (2017)) and income inequality in the USA. A secular increase in income inequality could depress the natural rate of interest if rich households save part of an increase in their permanent income, as found by Dynan et al. (2004) based on US microdata. Rannenberg (2019) finds that the increase in income inequality can be attributed to an increase in wage inequality. The increase in inequality can reduce the US interest rate by more than three percentage points.

Most importantly, natural rates also have a global dimension (e.g.Borio et al. 2017; Gourinchas and Rey 2019). Pescatori and Turunen (2016) have illustrated the increasing role of excess global savings in depressing the global interest rate. Global factors (i.e., excess global savings proxied by current account surpluses in emerging markets and an increase in the equity premium-as estimated by Duarte and Rosa 2015) after the global financial crisis) appear to play a prominent role in this. Using a panel of 17 advanced economies, Fiorentini et al. (2018) captured the contribution of demographic and other global macroeconomic developments of the late 19th century to the evolution of real effective rates. The effect of demographic developments is estimated to have been significant.

2.2　Potential output growth rate in the literature

Potential output determination can be seen from different perspectives (Álvarez and Gómez-Loscos 2018). From a purely statistical perspective, it can be seen as the trend or smooth component of the actual output series. From a theoretical point of view, it is often seen as characterizing the economy's sustainable aggregate supply capabilities (i.e., as consistent with stable inflation). Alternatively, potential output could be defined as the level of output attainable when full use is made of all the factors of production. Finally, the natural output can be associated with flexible prices (Kiley 2013). The European Commission, the IMF, and the Organisation for Economic Co-operation and Development (OECD) estimate potential output via a

production function approach-as a function of trend capital, labor, and total factor productivity.[2]

In general, potential output is computed on the basis of a production function with constant returns to scale. Schäfer and Steger (2014) found strong evidence, both direct and indirect, in favor of (aggregate) increasing returns to scale (IRS). For instance, Schmitt-Grohe (1997) reviews the empirical literature on IRS, reporting that the degree of IRS (at the level of industries) ranges from 1.03 to 1.4. Graham and Temple (2006) argue that models with IRS can explain a substantial part of international income disparity. Depending on the strength of the positive externality in the manufacturing sector, these authors attribute between 18% and 50% to the presence of IRS. Consequently, a careful discussion of the question raised above should take into account IRS and factor mobility. Baldwin and Forslid (2000) extended the well-known core-periphery model of Krugman (1991) by introducing endogenous growth à la (Romer 1990) and found evidence on the uneven process of economic integration. Prasad et al. (2006) showed the paradox of capital flowing from poor to rich countries. This perverse pattern of flows has been particularly striking since the beginning of the year 2000 decade, showing that, over the period 1970-2004, as well as over subperiods, the net amount of foreign capital flowing to relatively high-growth developing countries has been smaller than that flowing to the medium- and low-growth groups.

3 The model

The model is a multi-country OLG general equilibrium system that includes long-term demographic and human capital projections for different countries to evaluate growth, natural rates, and asset evolution in real terms. In particular, the model includes seven countries, namely advanced economies, such as the United States and three European countries- France, Germany, and Italy- as well as emerging and developing economies such as those of China, India, and Africa. These regions are characterized by very different aging and technological processes that affect the formation of labor and capital inputs through the saving behaviors of different agents and the interest rate levels. The approach, based on overlapping generations, makes it possible to analyze the determinants of capital accumulation in a general equilibrium growth model (i.e., the extent to which cohorts save, consume, work, and thus affect capital accumulation). At time t, each cohort is defined by the vector of states $x = (t - s, i, j)$, where $t - s$ denotes the age of each cohort, i the education level with $i = LS, HS, TS$, and j=Germany, Italy, France, China, India, Africa, USA. The core sectors in the model are households, firms, and government for each country j.

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[2] All three international institutions compute the cyclically adjusted balance (CAB) as the product of the output gap and a budget coefficient known as a sensitivity or semi-elasticity parameter. As a result, the cyclical budget balance varies by the output gap (i.e., the structural budget balance reproduces the behavior of the real balance when the economy is at its full potential).

The economy is populated by individuals divided into 101 age cohorts, with ages ranging from zero to a maximum of 100 years, split into three education levels (primary, secondary, and tertiary). We feed the model with United Nations long-term population projections (United Nations 2019) as in Börsch-Supan et al. (2006).

For ease of notation, we suppress index j and, whenever necessary, refer to index i to account for the education level. This implies that cohorts are differently populated, based on the age distribution resulting from the UN population projections, which provide a number of people for each age group, from 0 to 100+. Demography is thus taken as exogenous. In each country j and period t the size of population of age $t - s$, with s the birth year, is denoted by $P_{t-s,j}$. The dynamics of each cohort are defined by

$$P_{t+1-s,j} = P_{t-s,j}(q_{t-s} + x_{t-s}), \tag{1}$$

where q_{t-s} and x_{t-s}, are respectively, the survival and migration rate. We define m_{t-s} as the conditional survival rate at age $t - s$ given by $m_{t-s} = \prod_0^s q_{t-s}$.

Moreover, we calculate the share of population $g_{t,i}$ with education level i using data from Barro and Lee (2015). In each country, therefore, j and period t, the size of population of age $t - s$ is given by $P_{t-s,j,i} = P_{t-s,j}g_{t,i}$.[3] We approximate human capital quality as a measure of schooling duration (see Sect. 4), and consider three different education levels: primary and lower secondary *LS* (which implies that agents complete 8 years of study), upper secondary *HS* (completion of 13 years of study), and tertiary level *TS* (completion of 18 years of study).

This type of model allows us to introduce quite detailed modeling of fiscal policy. Indeed, Ricardian equivalence does not hold, which means that public finance decisions affect private decisions (i.e., savings, consumption, investment, and capital accumulation). In the baseline setup, the government draws resources from the active labor force via income taxes, consumption taxation, and interest rate income. Revenues from taxes on net wealth are used to finance investment in alternative scenarios (see Sect. 3.3).

The assumption of perfect foresight underlying the overlapping generation approach allows us to incorporate the anticipatory effects of fiscal policies into the expectations of government and households, affecting expected income and wealth today, consumer choices, and thus investment and growth. In this approach, the model is standard and makes it possible to study the effect of policies to be implemented in the future, so that it can be anticipated by firms and households.[4]

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[3] To avoid computational complexity, we neglect endogenous human capital formation and build an exogenous measure of labor productivity based on education levels.

[4] This perspective is shared by Semmler et al. (2018) but implemented in more detail through regime change techniques that allow for management of finite-horizon behavior and the change in the structure of the economy incurred after the policy implementation.

3.1 Households

Each cohort is represented by one household, which maximizes the discounted lifetime utility by choosing consumption and leisure over the life cycle from entry to the labor market (at earliest age 15) to death (age 101).

The households' life-cycle stream utility is given by:

$$U = \sum_{t=s}^{s+T} q_{t-s} \frac{u\big[c_{t-s}, (e_t - l_{t-s})\big]^{1-1/\xi}}{1 - \frac{1}{\xi}} \frac{1}{(1+\rho)^{t-s}},$$ (2)

where T is longevity (101 years for all agents), ρ denotes the discount rate which is cohort invariant, and ξ defines the intertemporal elasticity of substitution. q_{t-s} is the survival rate at age $t-s$. c denotes consumption goods, and l is the individual labor supply. Labor supply l is measured in efficiency units relative to the time endowment e. We assume that e grows at the human capital growth rate $\dot{h}$, i.e. ($e_{t+1} = e_t(1+\dot{h})$) as in Börsch-Supan et al. (2006). We assume that human capital stabilizes in the far long run. These conditions overcome inconsistencies about "growth compatible preferences".[5] Households maximize utility in Eq. (2) subject to:

$$\begin{cases} l = 0 & if \quad s \le T_i \\ 0 \le l \le e & if \quad T_i < s < T_r \\ l = 0 & if \quad s \ge T_r. \end{cases}$$ (3)

Equation (3) presents consumption and labour supply constraints during the life-cycle. The first constraint is relative to education lifetime, when labor supply is $l = 0$. The second constraint refers to the working-age period, and the third refers to the period of retirement. T_i denotes the age of individuals at the end of each phase of schooling/education which is 14 for the education level $i = LS$, 19 for $i = HS$, and 24 for $i = TS$. T_r indicates the retirement age (i.e., the years of contribution needed to obtain a pension).

Utility–of the constant elasticity substitution type-at time t for a particular working cohort aged $t-s$ is given by:

$$u(c_{t-s}, e_{t-s} - l_{t-s}) = \left[c_{t-s}^{(1-\frac{1}{\epsilon})} + \alpha \left(e_{t-s} - l_{t-s}\right)^{(1-\frac{1}{\epsilon})} \right]^{\frac{1}{1-\frac{1}{\epsilon}}}$$ (4)

where ϵ denotes the substitutability of consumption and leisure and α the intensity of preference for leisure relative to consumption.

The dynamic budget constraint is:

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[5] See (Börsch-Supan et al. 2006) for a discussion.

$$a_{t+1-s} = \begin{cases} \frac{1}{q_{t-s}}\left(1 + r_t - \tau_{a,t}\right)a_{t-s} + \bar{w}_{t-s}h_{t-s}l_{t-s} - (1 + \tau_{c,t})c_{t-s}, & \text{if } s < T_r \\[2ex] \frac{1}{q_{t-s}}\left(1 + r_t - \tau_{a,t}\right)a_{t-s} + pens_{t-s} - (1 + \tau_{c,t})c_{t-s}, & \text{if } s \geq T_r \end{cases}$$

where a_{t-s} denotes the wealth in time t of the cohort born in the period s and r_t and $pens_{t-s}$ are, respectively, the interest rate and the average pension at time t for the cohort aged $t - s$. The interest income tax rate is denoted by $\tau_{r,t}$ and $\bar{\tau}_{r,t} = (1 - \tau_{r,t})$. $\tau_{a,t}$ is the exogenous tax rate imposed by government on household wealth to finance public investment and $\tau_{c,t}$ is the rate of value-added tax (VAT). The term $\bar{w}_{t-s}h_{t-s}l_{t-s}$ represents the post-tax labor income, with $\bar{w}_{t-s} = (1 - \tau_{l,t})w_{t-s}$ and $\tau_{l,t}$ denoting the exogenous tax contribution rate. The human capital h_{t-s} is a specific education index, as discussed in Sect. 4.

The optimal labor/leisure choice gives the following first order condition:

$$e_{t-s} - l_{t-s} = \left(\frac{[w_{t-s}h_{t-s} + \mu_{t-s}](1 - \tau_{l,t})}{\alpha(1 + \tau_{c,t})} \right)^{-\epsilon} c_{t-s}, \tag{5}$$

where $[w_{t-s}h_{t-s} + \mu_{t-s}](1 - \tau_{l,t})$ denotes the effective cost of leisure and μ_{t-s} the shadow wage rate. If the optimal quantity of leisure is lower than the total time endowment, $\mu_{t-s} = 0$. If $e_{t-s} \leq l_{t-s}$, the shadow wage rate $\mu_{t-s} \geq 0$ (i.e., it reduces leisure demand to the time endowment).

The Euler equation for the intertemporal consumption choice is:

$$\frac{c_{t+1-s}}{c_{t-s}} = \left[\frac{1 + \tau_{c,t}}{1 + \tau_{c,t+1}} \frac{1 + r_{t+1} - \tau_{a,t}}{1 + \rho} \right]^{\xi} \left[\frac{1 + \alpha\epsilon(v_{t+1-s})^{1-\epsilon}}{1 + \alpha\epsilon(v_{t-s})^{1-\epsilon}} \right]^{\frac{\epsilon-\xi}{1-\epsilon}}, \tag{6}$$

where $v_{t-s} = [w_{t-s}h_{t-s} + \mu_{t-s}](1 - \tau_{l,t})$. When individuals retire $l = 0$ and leisure is equal to e. Moreover, we assume $\alpha = 0$ when households retire, meaning that households take care only of consumption goods, as leisure is at the maximum level. In this case, the Euler equation can be simplified as follows

$$\frac{c_{t+1-s}}{c_{t-s}} = \left[\frac{1 + \tau_{c,t}}{1 + \tau_{c,t+1}} \frac{1 + r_{t+1}) - \tau_{a,t}}{1 + \rho} \right]^{\xi}. \tag{7}$$

3.2 Firms

In each country j, the production sector is characterized by a segmented population of firms operating in the goods market. Each firm in region j uses a Cobb-Douglas technology which combines the capital stock from other region $i \neq j$, $K_{j,i,t}$ with the effective labor input $L_{j,t}$:

$$y_{j,t} = TFP_{j,t}K_{j,i,t}^{\beta}L_{j,t}^{1-\beta} \tag{8}$$

where $0 \leq \beta \leq 1$ is the capital share, $TFP_{j,t}$ the endogenous total factor productivity, and $L_{j,t} = H_{j,t}N_{j,t}$, with N_t denotes the aggregate hours worked and $H_{j,t}$ the human capital level. We assume that firm are homogeneous in total factor productivity and labor endowment.

Aggregate capital stock in country j is therefore the integral of individual capital firm endowment $K_{j,t}(s)$, located to a normalized population s in the interval $s \in [0, 1]$:

$$K_{j,t} = \sum_i \int^{\theta_i} K_{j,t}(s)ds = \sum_i \theta_i K_{j,t} = \sum_i K_{j,i,t} \tag{9}$$

where θ_i is the fraction of the population of firms in country i. The endogenous growth process is modeled by linking local aggregate available physical capital per worker, and human capital à la (Romer 1990) as follows:

$$TFP_{j,t} = \left(\frac{K_{j,t}}{N_{j,t}}\right)^g H_{j,t}^z, \tag{10}$$

where g and z denote the contribution of the production factors to TFP_t. In particular, g measures the capital-per-worker contribution in technology creation and z is the contribution of human capital.[6]

According to Eq. (10), productivity improvements are a result of the interaction between the capital-to-labor ratio (i.e., the number of machines per unit of labor) and human capital endowment (i.e., labor quality).

Labor is internationally fixed, while domestic capital stock could be used to provide a certain amount of output in region j, y_j, using the domestic labor supply and human capital stock. Profits are given by:

$$\pi_{j,t} = \sum_i y_{i,j,t} - (r_{j,t} + \delta_{j,t})K_{j,t} - w_{j,t}L_{j,t} \tag{11}$$

where δ_j denotes the depreciation rate on the capital used in region j. Firms set their optimal capital stock demand for $K_{j,i,t}$ and labor demand $L_{j,t}$. The first order conditions are:

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[6] Growth models, which include population projections, usually acknowledge the role of human capital and increasing return return to scale of returns to scale. Indeed, the economy's growth rate is proportional to the total amount of knowledge/ideas in the economy. An increase in the size of the population, other things equal, raises the average aggregate human capital and therefore leads to an increase in the per capita income. The inclusion of non-rivalrous input, such as human capital/ideas, linked to population dynamics, in the production function, leads to increasing returns to scale (Romer 1990). Given the following production function $Y_t = H_t^{z+1-\beta} K_t^{g+\beta} N_t^{1-\beta-g}$, there are constant returns to scale in capital K_t and labour N_t, and increasing returns to human capital, where the degree of increasing returns is measured by $z + 1 - \beta > 0$, with $z > 0$ and $0 \leq \beta \leq 1$. In particular, we estimate z, β and g (see Sect. 4 for details). For many countries the sum of exponents is positive. Therefore, human capital, H_t, in our model is exogenous but not constant over time, as it depends on exogenous population projections and education levels, which are also exogenous.

$$r_{j,t} = \sum_{i=1}^{n} \theta_{i,j,t} \left(TFP_{i,j,t} \beta_i f'_{K,i,j,t} - \delta_j \right),$$

$$w_t = TFP_t (1 - \beta) f'_L, \tag{12}$$

where f'_K and f'_L are the marginal productivity of capital and labor, respectively, and $\theta_{i,j,t}$ denote bilateral degree of openness. Equation (12) implies that firms do not take the positive externalities of physical and human capital into account when maximizing their profits.

3.3 Government

In each country j, the public sector consists of three areas, namely the social security and education departments and public spending in goods and services. The government raises funds through public debt and taxes paid by workers at the exogenous labor income tax rate τ_l, VAT rate τ_c, capital tax rate τ_k, and tax rate on wealth τ_a. The government uses revenues to finance education expenditure, sg, and current pension and social transfers to retired people, $\zeta pens$, where $\zeta_t = \sum_i \sum_{s=T_r+1}^{T} P_{t-s,i}$ denotes the retired population and $pens_t$ is calibrated to match pension expenditure to GDP data (see Sect. 4). Government issues new debt to finance the deficit:

$$\Delta B_t = r_t B_t - \tau_{l,t} w_t L_t - \tau_{c,t} C_t - \tau_{k,t} K_t - \tau_{a,t} A_t + sg_t + \zeta_t pens_t + G_t, \tag{13}$$

where $\tau_{a,t} A_t$ are wealth taxation revenues. We assume that $G_t = \gamma Y_t$ is public spending (public consumption and investment) as a fraction of GDP. $r_t B_t$ denotes the interest repayment on public debt and $\Delta B_t = B_{t+1} - B_t$ denotes public debt change. sg_t is public spending on education. To account for the burden of the school system on public finances in time t, for each level of education i, we allow the school department to bear a cost proportional to the number of students $\bar{P}$ enrolled in the school. In each country j, the total amount of public expenditure on education sg_t will be equal to the weighted sum of people that, at time t, are studying to achieve the schooling level $i = LS, HS, TS$:

$$sg_t = \sum_{i \in I} \sum_{s=0}^{T_i} sc_{t-s,i} \bar{P}_{t-s,i}$$

where $sc_{t-s,i}$ is the exogenous schooling expenditure per scholar born in year s with schooling level i (see Sect. 4). $\bar{P}_{t-s,i}$ is the number of people aged $t - s$ with education level i.

Finally, Eq. (13) adjusts labor tax rate τ_l to maintain the desired debt-to-GDP ratio deb_t, as discussed in the calibration section.

3.4 Aggregation

In each country j, the total labor supply $L_{j,t}$ is equal to

$$L_{j,t} = \sum_{i \in I} \sum_{s=s_{0,i}}^{T_r} l_{t-s,i,j} h_{t-s,i,j} P_{t-s,i,j}. \tag{14}$$

The aggregate amount of labor input is equal to:

$$N_{j,t} = \sum_{i \in I} \sum_{s=s_{0,i}}^{T_r} l_{t-s,i,j} P_{t-s,i,j}, \tag{15}$$

where $s_{0,i}$ is the year in which the cohort aged $t - s$ becomes employed; $P_{t-s,i,j}$ is the population aged $t - s$ in year t; and T_r denotes the contribution years required in year t to obtain a pension.

Aggregate wealth is given by:

$$A_{j,t} = \sum_{i \in I} \sum_{s=s_0}^{T} a_{t-s,i,j} P_{t-s,i,j}. \tag{16}$$

In each country j, assets change over the simulation period according to

$$F_{j,t} = A_{j,t} - K_{j,t} - B_{j,t}, \tag{17}$$

where $F_{j,t}$ denotes the amount of foreign assets. In the closed economy framework, the rate of return on capital in one country j is equal to the marginal productivity of capital in that country (equation 12). Moreover, any country's foreign assets are equal to zero $F_{j,t} = 0$, and the capital stock is equal simply to aggregate wealth $K_{j,t} = A_{j,t}$.

When the economy is open, and capital is internationally mobile, the rate of return on capital is equalized across countries $r_{j,t} = r_t$, such that the sum of all foreign assets across all countries equals zero, that is, $\sum_{j=1}^{J} F_{j,t} = 0$, with J denoting the number of countries in the model economy.

Capital markets in the model are not fully integrated, reflecting ongoing globalization or regional integration processes. Consequently, in line with empirical evidence, the model incorporates a gradual increase in the bilateral degree of openness $\gamma_{j,k,t}$, which allows for a progressive convergence of interest rates between countries j and k (see Sect. 4). However, different demographic trends in different countries imply diverging interest rate levels $r_{j,t}$, which are higher for countries with younger populations than for those economies that have already completed or are in the process of completing their demographic transition. This implies that countries most affected by aging (such as Italy and Germany in Europe) will initially be capital exporters, while countries (such as the emerging economies) with younger populations will be capital importers because the higher labor supply will increase demand for investment goods, given the need for more capital. As the population progressively ages, however, the savings rate decline and capital flows from faster-aging to younger countries decrease. These demographic trends, together with the gradual process of integration into the financial and goods markets, will lead to a progressive convergence of interest rates across countries, which will adjust endogenously to ensure that the sum of all countries' net positions is zero at any given time t.

4 Computation and calibration

In this section, we describe the computation and calibration strategy. We solve the model on a yearly basis until 2281, for a 470-year time range, starting from an initial steady state in 1812. We assume a phasing-in and a phasing-out period of 101 years at the beginning and the end of the simulation period. It is assumed that the levels of demography and human capital are stable. This makes the final steady state endogenous to the determination of the transition in the central simulation periods, including between phasing-in and phasing-out periods. The model is a system of large-scale non-linear equations ($101 \times 3 \times 3 \times 7$) due to inclusion of age (101), education levels (3), and the number of countries/areas (7). We use such a granular age structure to match the natural interest rates and GDP potential growth as much as possible. In this way, our estimations are of a higher precision, as the demographic pattern helps to explain our target variables. The calibration of the model is obtained using data for between 1970 and 2018 and is allowed to produce projections from 2019 to 2100. We use a dynamic calibration strategy (1970–2018) to calibrate the values of the GDP levels to the actual data for each economy using the exogenous total factor productivity (TFP) as an instrument to reach the level of real GDP trends in the data. Similarly, we use pension expenditure and labor tax rate data to calibrate the pension benefit and the historical value of debt-to-GDP, respectively. Moreover, to match interest rate data, we introduce changes in the discount rate time preference. In the following, we briefly describe the exogenous trends, and the parameters are calibrated in the model (for details, see "Appendix 3.2"). Finally, we estimate the production function to detect the degree of increasing returns in technology. We use a panel estimation technique to pool countries with specific elasticity parameters in order to estimate country-specific contributions to GDP growth (see "Appendix 3.2"). Based on the data, several exogenous trends are assumed (Table 1): demographics, human capital, technical obsolescence (via the depreciation rate), labor and capital share, and goods market integration (trade and capital flows). The latter allow us to consider the degree of integration between areas that is not captured by price determination (i.e., the institutional and behavioral factors that affect the degree of integration between economies). We refer the interested reader to "Appendix 3".

Table 1 Exogenous and semi-endogenous trends

Variable/parameter	Model	Equation	Source	Frequency
Population	P_t	14	ONU	Year
Human capital	H_t	10	Barro-Lee	5-year (interpolated)
Capital market	θ	12	Trade (OECD)	Year
Semi-endogenous				
Exogenous TFP	TFP_{ex}	8	Own elaborations	Year
Depreciation rate	δ_t	12	Own elaborations	Year
Capital share	β_t	8	Own elaborations	Year
Debt-to-GDP	β_t	8	IMF	Year

(a) Human capital index, 2000-2100

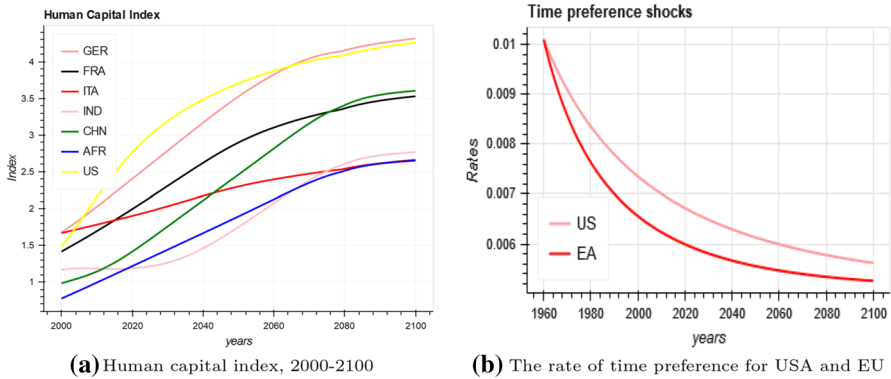**(b)** The rate of time preference for USA and EU
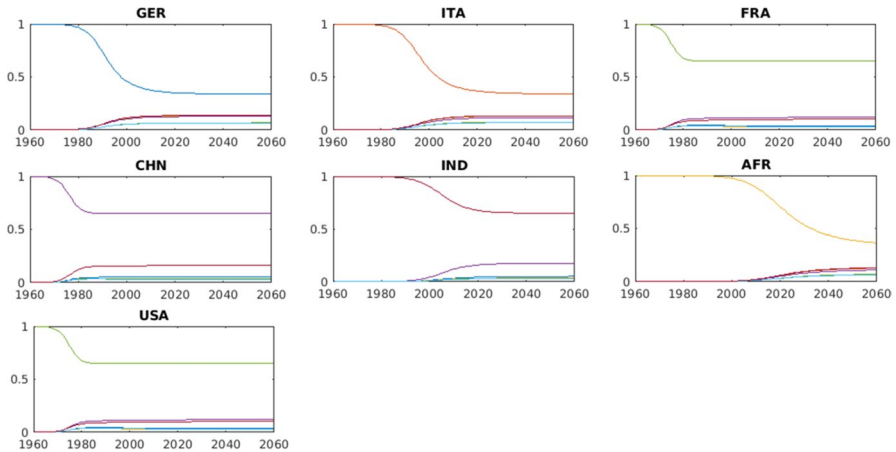
Fig. 2 Calibration results

Fig. 3 Dynamics of integration coefficient matrices. The line starting from 1 is domestic share; the rest represents the foreign share of the capital market. *Source*: our calculations based on WITS data (World Bank)

As shown in Fig. 2a, human capital is projected forward in line with the demographic projections. As can be seen in the figure, the initial level of the index reflects the productivity rankings between countries. Among the advanced countries, Germany and the USA have the highest human capital index, while Italy lags and shows the lowest initial level. Among the emerging countries, the dynamic of China stands out, as in the long term it would converge toward the levels of the advanced countries. Figure 2b shows the dynamic results for the rate of time preference, which is strictly related to demographic dynamics and is used to calibrate the capital-to-GDP ratio. For the euro area and the USA, the rate of time preference drops over the simulation period, while the other areas are assumed to remain at the same starting level, equal to 0.011 in 1960 (Ludwig et al. 2012). Thus, the model calibration suggests a dynamic adjustment of intertemporal preferences due to an increase in the rate of patience compatible with the aging process that characterizes the advanced economies.

Figure 3 shows the dynamics of the openness degree. Each country starts from perfect closeness (level 1), reflecting a closed economy in capital movements. At the same time, the coefficients of the matrix for the openness toward each of the other countries *j* starts increasing and converging to the level represented in Fig. 18. Emerging economies such as those of India and Africa open gradually with a significant delay compared to other advanced countries, simulating a process of capital market maturation that begins later with China and the other advanced countries.
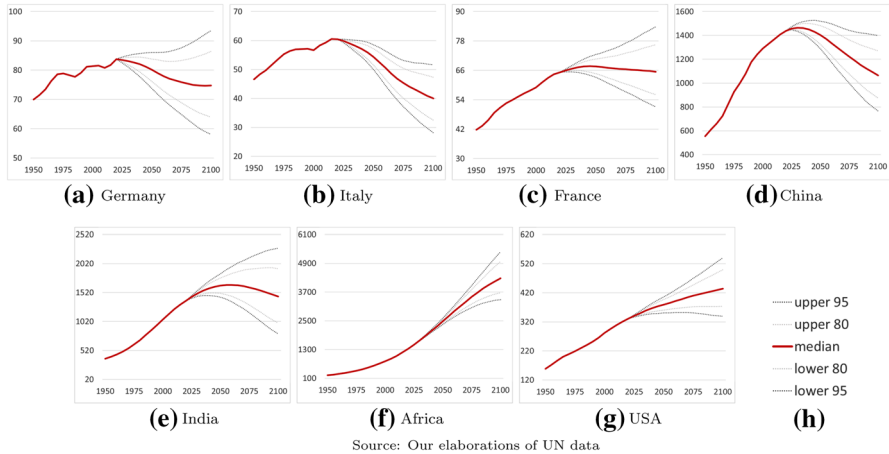

Source: Our elaborations of UN data

Fig. 4 Population dynamics and confidence intervals (1950–2100). *Source*: Our elaborations of UN data

Table 2 Model parameters

		Ger	Ita	Fra	Chn	Ind	Afr	Usa
ξ	Intertemporal elasticity of substitution	0.5	0.5	0.5	0.5	0.5	0.5	0.5
η	Labor–consumption elasticity of substitution	0.95	0.95	0.95	0.95	0.95	0.95	0.95
ρ	Pure time impatience rate	0.006	0.006	0.006	0.011	0.011	0.011	0.006
α	Weight of leisure utility	0.32	0.27	0.33	0.3	0.3	0.3	0.3
β	Share of capital in production	0.421	0.445	0.410	0.530	0.647	0.597	0.436
δ	Depreciation rate of capital	0.107	0.106	0.107	0.089	0.086	0.084	0.107
g	Capital spillover in TFP	0.030	0.001	0.011	0.041	0.007	0.082	0.048
z	Human capital spillover in TFP	0.307	0.016	0.138	0.476	0.088	0.816	0.014
τ	Labor tax rate and social contributions	0.50	0.48	0.24	0.15	0.29	0.23	0.30
τ_k	Corporate tax rate	0.30	0.24	0.31	0.25	0.30	0.28	0.27
τ_c	VAT rate	0.19	0.22	0.20	0.13	0.18	0.15	0.05
τ_a	Wealth tax	0.05	0.05	0.05	0.05	0.05	0.05	0.05
cs	Schooling expenditure on GDP	0.051	0.043	0.057	0.042	0.046	0.02	0.049
γ	Goods, services spending	0.195	0.188	0.234	0.145	0.111	0.145	0.140
debt	Debt-to-GDP ratio	0.597	1.317	0.985	0.504	0.697	0.530	1.066

In Fig. 4 we can appreciate the historical and projected evolution of the UN population data. Among advanced economies, the USA is the only country that would show a substantial growth in the projected period (to 2100). Among emerging countries, China will see a decrease in its population and, in the worst case (5° percentile) the population will be half that of the current level in the 2100s. The second-largest country is India, and its population will be the largest one in around 2025. India's population is then projected to decrease. Finally, it is expected that the populations of African countries will grow very fast, continuing Africa's historical trajectory and doubling its 20th century level.

To calibrate the model parameters (presented in Table 2) we assume standard parameterization common in the literature or we calibrate using targets built to match the data. For each country, we set the intra-temporal elasticity of substitution ϵ to 1 to avoid trends in the labor-to-consumption ratio, as in Auerbach et al. (1987), and the inter-temporal elasticity of substitution, ξ, to 0.5. To take into account heterogeneous labor market participation rates, we set the weights on leisure utility for Germany, Italy, and France to 0.32, 0.27, and 0.33, using the strategy employed in Bucciol et al. (2017) to match the fraction of one-third of the time devoted to labor.

The capital depreciation rate is set according to IMF data for total investment and capital stock in constant 2005 international dollars. We obtain a depreciation rate δ that is increasing for all countries over the period 1960–2013 (Fig. 5).
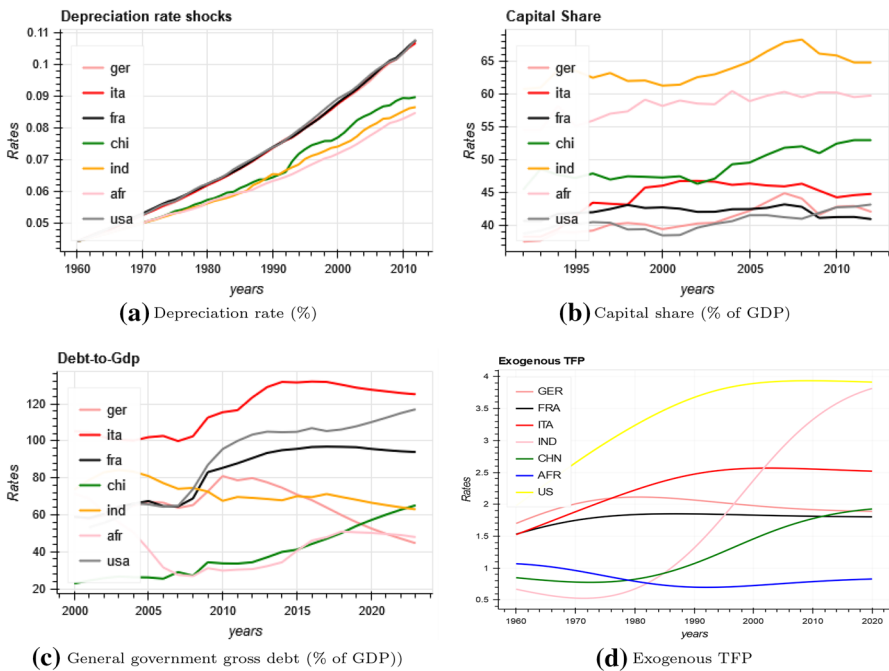

(a) Depreciation rate (%)

(b) Capital share (% of GDP)

(c) General government gross debt (% of GDP))

(d) Exogenous TFP

Fig. 5 Parameters trends

5 Results

5.1 The baseline

This section discusses the model results, focusing in particular on the projections of natural rates. We also describe the dynamics of government debt and net foreign positions, which should be treated with some caution. Many calibration processes are performed during the observed data period (1960–2018). The model is solved by projecting the processes as discussed in Sect. 4, (i.e., keeping the observed or calibrated historical values of the parameters constant or by assuming an exogenous scenario). In the following, we begin the discussion of our results with a description of the interest rates; for this we compare the dynamics resulting from the marginal effects of adding one hypothesis at a time until we obtain the baseline scenario. The other dimensions (growth and financial variables) are discussed by directly considering the final effects of the complete set of assumptions underlying our baseline.

5.1.1 Natural interest rates

In this section, we assess the effect of our calibration on the natural rate for all the economies in the model. Under the assumption of homogeneous total factor productivity (TFP) across countries, natural interest rates over the simulation period do not fall as much as shown in Holston et al. (2017), either for the Euro area (EA) countries or for the USA (Fig. 6a, b). This implies that exogenous demographic and human capital trends alone are not sufficient to explain the interest rate evolution observed in the data, even if the model predicts a decreasing trend. Other factors, such as differences in technological trends across countries, can play an important role. When we introduce heterogeneous TFP across countries, and the capital depreciation rate is calibrated on the data to account for technical obsolescence, the natural rates are lower than those of Holston et al. (2017). We also consider the effect of income distribution by calibrating the labor share as in the data and introduce a dynamic runtime calibration for time preference which is used for the saving and consumption decision (ρ). The simulation shows an increase in natural interest rates which is close to those estimated by Holston et al. (2017), especially for the USA. This implies that the reduction in the labor share interacts with, and counterbalances, the effects of technological innovation and aging on interest rates, the opposite of what is commonly found in empirical evidence. The interaction with demographic factors could explain this counterfactual result as the rise and fall in the natural interest rate for several advanced economics; this can be explained by the temporary demographic effect of the young baby boomers which alters the capital–labor ratio found in Fiorentini et al. (2018). This is an important corollary of our technological specification that provides increasing returns to scale and therefore shows an aggregate outcome that is not considered in the empirical literature: a world with increasing returns to scale leads to an incentive for institutional factors to reduce labor share, as the larger the capital share, the higher the interest rates and capital returns. This result suggests that there is room for improvement in further
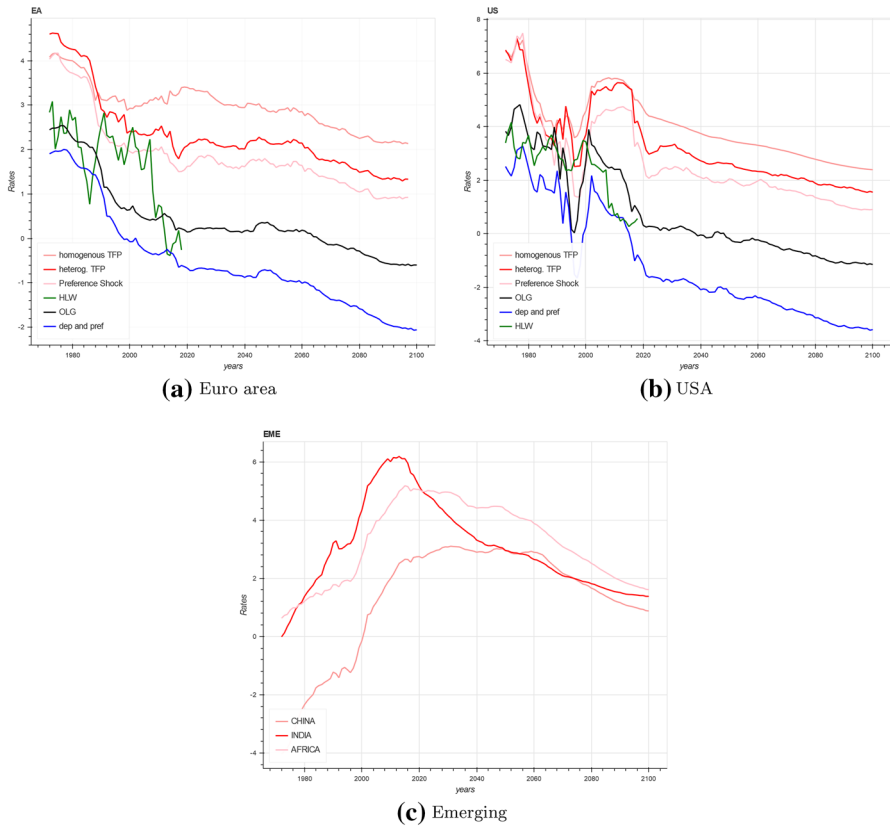
(a) Euro area

(b) USA

(c) Emerging

Fig. 6 Natural real interest rate

empirical investigations into the causal relationship between variables in a general equilibrium framework. Conversely, changes in the discount rate would result in an increase in savings and a reduction in the interest rate. Finally, emerging countries show a reversed dynamic of natural rates compared to advanced economies, with natural rates increasing before 2020. Thereafter, natural rates decrease but remain higher than in advanced countries (Fig. 6c).

5.1.2 Potential GDP growth

Using all the calibration tools described in Sect. 4, the model can replicate the observed average GDP growth rate in history providing a good starting point for the projections. In Figs. 7 and 8, both advanced and emerging countries show a decline in prospective growth. For all countries, the period 2008-2010 represents a watershed, as the potential growth rate decreases everywhere, and for some countries significantly. For European countries, it is mainly the contribution of the labor force to growth that is shrinking and becoming progressively negative.
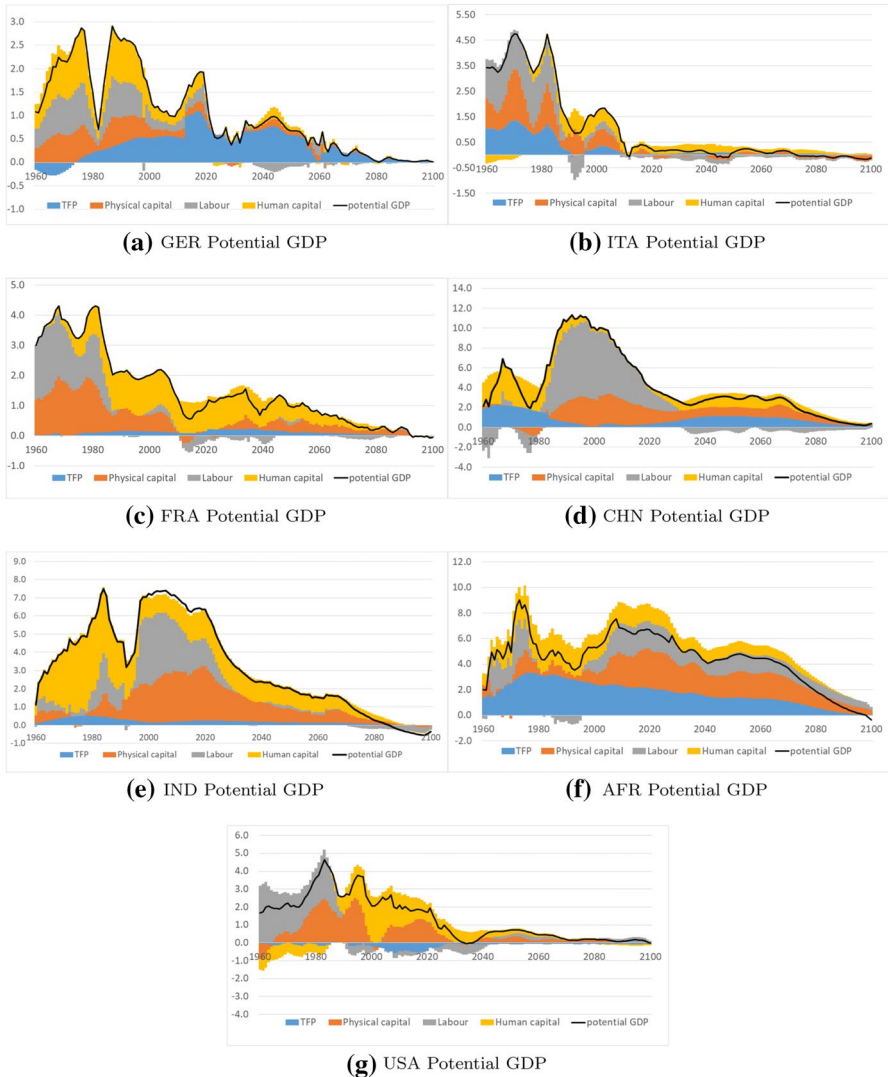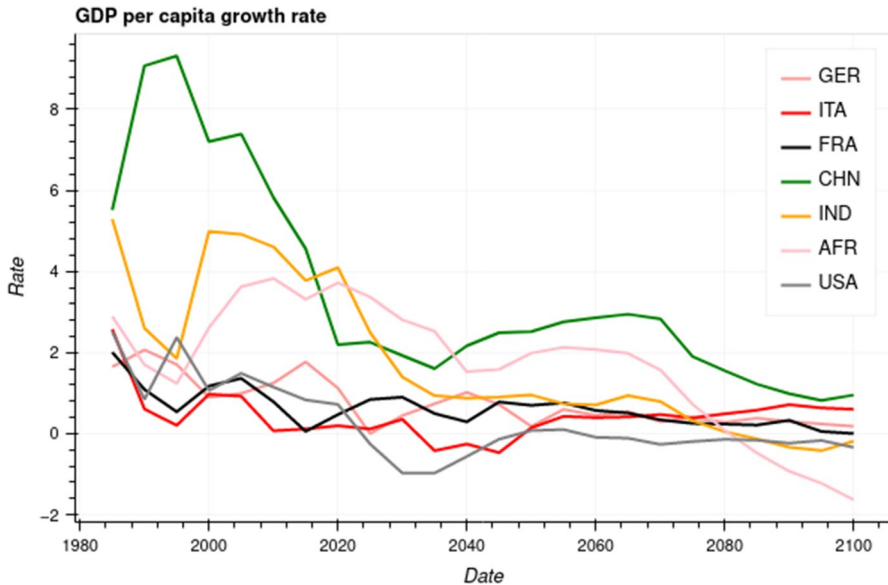
(a) GER Potential GDP

(b) ITA Potential GDP

(c) FRA Potential GDP

(d) CHN Potential GDP

(e) IND Potential GDP

(f) AFR Potential GDP

(g) USA Potential GDP

Fig. 7 Potential GDP growth rate

It should be noted that the model foresees a zero-average potential growth rate for Italy that becomes systematically negative after 2045. For France, the demographic dividend plays a positive role in growth, and is also reflected in a human capital that contributes to maintaining an average growth of 1.5% until 2040, for Germany, the situation is slightly different, with a growth rate remaining stable at around 1% on average until 2040 and then dropping to zero. Despite a negative demographic dividend, the factor that helps sustain growth is the endogenous total factor productivity, together with human capital and the performance of the economic system in spreading and generating innovation. For Germany, the increase in potential growth

Fig. 8 Potential GDP growth rate per capita

due to the total factor productivity just after the Great Recession, 2007–2009, and the 2012–2013 public debt crisis are worth noting. Indeed, the pattern of potential GDP growth has been variously affected in the European countries, signaling a stronger resilience for Germany than other countries in the wake of the two crises, as an increase in TFP signals an exogenous factor that is not determined by the simple endowment of production factors.

As far as emerging countries are concerned, China shows the prospect of a real slowdown with a largely negative demographic dividend. The contribution of human capital cannot offset the demographic effect in the long term. This new pattern of capital movements due to the strict interconnection between demographic trends (changing time preference propensity to consume), global value chain evolution, and technological trends would lead to a slowdown in potential growth for the Chinese and US economies around 2040. In fact, by 2040, the USA will reach zero potential growth. It will then slowly converge toward a growth of just above zero. For India, the model indicates a relatively rapid transition from a high growth rate of about 7% until 2020 to about 2% around 2050. Note that before 2100 the growth rate will be negative. Finally, it should be noted that the growth rate of the African continent will be between 6% and 8% at least until 2060; it should also be noted that, in recent years, several institutional and governance factors have limited growth in Africa to well below its potential. Our potential growth estimation allows the output gap realization to be observed. In Italy and France contingent macroeconomic factors have kept GDP below the potential levels resulting in a wider output gap than is shown by OECD metrics. This means that potential growth has accelerated in the last few years, leaving space for effective growth and reducing the risk of sudden

corrections. In this perspective, China's situation is much more severe, as its GDP has been much further above its potential level and for a prolonged time, thus signaling potentially higher risks of macroeconomic adjustments.

To show the effects in terms of GDP per capita, we can see in Fig. 8 that the growth pace for China will be substantially higher for the next decade. Moreover, Africa would, strictly speaking, follow the Chinese growth process except for at the end of the century due to a higher increase in population growth.

5.2 Fiscal and external financial development

Given the projection of the real natural rates, interest and potential growth, and the exogenous factors, we can infer the dynamics of two important variables to be monitored, namely, public and foreign debt, as we discuss in the following.

5.2.1 Public debt

As mentioned in Sect. 4, our objective is to calibrate fiscal policy, using the labor tax rate as an instrument, and thereby obtain a structural deficit dynamics consistent with the data and with the trend in public debt. Nevertheless, we validate the calibration process, observing that the initial (2020–2024) path of public debt is consistent with official short- and medium-term IMF forecasts for many countries. Figure 9 shows the dynamics of fiscal variables for all countries.

With regard to long-term projections, beginning with Germany, the model suggests a path of public debt that decreases until 2022, at which point the deficit becomes positive and public debt begins to grow, reaching a level of 90%. The decrease in potential growth toward zero and the expansion of the deficit due to increasing pension expenditure are partially related to aging. As we have seen, labor supply in Germany contributes negatively in the long run, leading public debt to
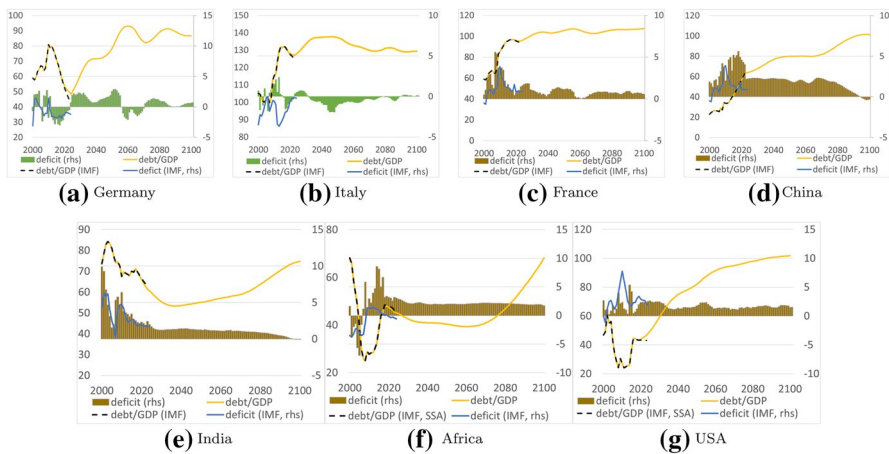

Fig. 9 Public debt and structural deficit

reverse its inertia and to reach 90% of GDP. In France, on the other hand, due to growth, a positive deficit and low-interest rates allow long-term stabilization of public debt at current levels to a level 100% of GDP. In Italy, the current public debt inertia would be followed by a slight recovery in growth that remains almost stable in the simulation period, allowing a stabilization of debt at the end of the period, thanks mainly to the large cumulative surplus balance. The low interest rate in this case allows the economy to remain balanced at a perpetual negative growth regime. In China, public debt is expected to increase over the simulation period, while in India and Africa, growth is expected first to decrease and then to increase over the long term and then to decline. For both India and Africa, the real natural interest rate becomes higher than potential growth, and is mainly due to the rapid decrease in the contribution of labor capital to growth. The modification of the specialization model, from labor-intensive to technology- and capital-intensive, together with the completion of the demographic transition, leads such regions to reverse the inertia of the public debt-to-GDP ratio at the end of the projected century. Finally, for the USA, the growth path of the debt-to-GDP ratio remains stable, mainly thanks to very low real interest rates, despite the structural persistence of the deficit that would lead the debt-to-GDP to reach a steady 100% ratio.

5.2.2 International variables–Net foreign asset position

The Great Recession produced a widescale reversal of the trend in savings behavior in many countries, which is particularly evident in China and the USA (Fig. 10). The calibration procedure helps the model capture the current account intensity and dynamics for different economies. The debate focused on the prospects for a further reversal or a return to the pre-crisis situation. Eichengreen (2014) argues that the USA and China will never return to pre-crisis patterns of growth and spending. Lane and Milesi-Ferretti (2014) argue that adjustments in deficit economies often take place through expenditure reduction; that is certainly the case for the 2006–2013 period. The net foreign asset positions are here considered at their natural level. By calibrating technological and demographic trends in detail, the model can generate
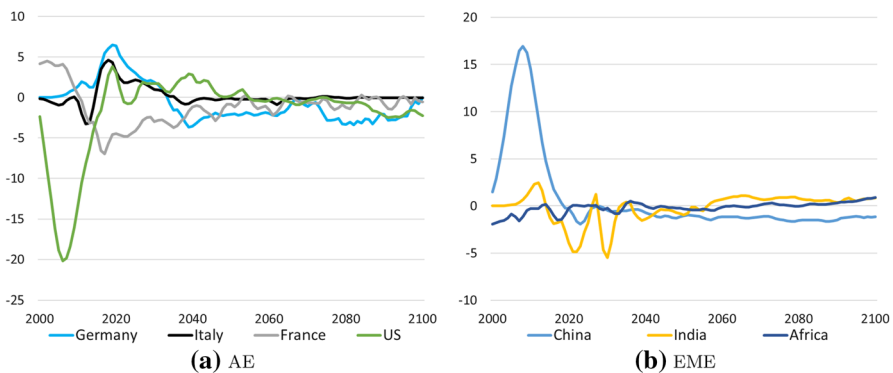
Fig. 10 Current account (% of GDP)

the necessary path of factor specialization among countries. Demographic and technological factors affect the capital-to-labor ratio, an aggregate measure of technological specialization. As shown in Fig. 10, the model provides a current account trajectory that faithfully reflects the historic dynamics, with a large USA deficit and a simultaneous Chinese surplus which suggests a deep reversal for the projected long run. This trajectory suggests that the emerging economies would make a leap forward by increasing the technological trend and deepening their demographic transition, moving the economy from a net creditor to a net debtor position–reflecting the US model during the projection period of the simulation. In the European countries, the trend toward the accumulation of trade surpluses will be important, but in the very long term, Germany will also become a net importer of goods and capital. Finally, the rest of the emerging countries show a stable trend in their net foreign asset position in the long run. The model suggests that this trend is a natural progressive evolution induced by technological and demographic trends. If distortion factors such as fiscal policies or exchange rate levels were to intervene temporarily (or even for long periods) this could divert actual current account levels. It is worth stressing that our model does not capture these aspects, as it provides current account dynamics consistent with potential growth levels. Any deviations could be caught with shocks that simulate policy changes able to change the current account dynamics in both the short and long term.

5.3 Shocks

In this section, we show the behaviour of the model when some exogenous trends change, that is, the population, the pension expenditure, and the global integration pattern (degree of openness). The goal is to assess the long-run effects of structural policies in a general equilibrium model.

5.3.1 Population

The first experiment is to assess the effects of different shocks on population dynamics. We perform the shock using the UN stochastic population projections and in particular using two levels of population: the worst-case scenario (5th percentile, l95) and the best-case scenario (95th percentile, u95) using (see Fig. 4 in the "Appendix 1"). In the Table 3, we show the response of GDP growth rates, natural interest rates, and the debt-to-GDP ratio, respectively, for all countries. Higher demographic growth prospects (scenario u95) lead to higher growth rates, translating into lower debt-to-GDP than the baseline for all countries, except for Italy, where debt-to-GDP in 2070 is higher than the baseline due to higher interest rates and lower growth compared with other European countries. Subsequently, the debt-to-GDP ratio stabilizes in response to a reduction in the interest rate.

In the long-run, debt-to-GDP in China and Africa is lower than the baseline, even in the presence of less positive demographic prospects (lower bound of the 95% confidence interval). This seems to be the result of interest rates that are steadily decreasing over the entire simulation period and growth rates that remain positive overall, albeit decreasing. Unlike the other emerging countries, India has more negative growth rates and lower long-term interest rates, leading to a higher debt-to-GDP

Table 3 Population shock—GDP growth rate %

Country	Scenario	Growth rate				Interest rate				Debt-to-GDP			
		2020	2040	2070	2100	2020	2040	2070	2100	2020	2040	2070	2100
Germany	u95	2.1	1.9	1.1	0.2					46.0	67.0	81.7	91.5
	Baseline	1.5	0.8	0.1	0.0					53.8	77.0	92.6	101.0
	l95	1.4	− 0.2	− 1.3	− 0.1					45.7	90.2	122.1	134.1
Italy	u95	0.9	0.9	0.7	0.2	− 0.1	0.4	0.5	− 0.3	106.0	125.0	135.6	135.0
	Baseline	0.3	0.2	0.2	− 0.1	0.1	0.2	− 0.1	− 0.6	127.9	137.4	130.0	129.4
	l95	0.3	− 0.3	− 1.0	− 0.1	0.1	− 0.2	− 0.9	− 1.2	106.0	137.4	167.7	154.3
France	u95	1.7	1.8	1.3	0.2					75.1	90.7	96.6	103.4
	Baseline	1.0	0.8	0.4	− 0.1					95.8	103.3	102.8	107.2
	l95	1.3	0.6	− 0.2	− 0.1					74.2	101.6	118.9	128.6
China	u95	4.6	3.1	3.2	0.9	2.2	2.5	2.8	1.1	43.0	16.9	25.5	35.7
	Baseline	4.1	2.7	2.7	0.6	2.7	2.9	2.2	0.9	56.7	76.7	81.6	101.1
	l95	4.5	2.1	1.6	0.4	3.7	3.0	1.4	0.3	39.0	19.0	37.7	47.7
India	u95	6.5	3.6	1.6	0.0	5.0	3.6	2.6	1.5	63.0	68.5	45.8	58.2
	Baseline	6.3	2.4	1.3	− 0.4	5.1	3.3	2.1	1.4	66.0	77.0	58.8	74.7
	l95	6.4	0.9	− 0.5	− 0.9	5.2	3.0	1.2	0.5	62.1	76.3	71.6	86.3
Africa	u95	9.0	6.6	6.6	1.0	4.9	4.9	5.0	2.9	42.4	24.0	14.3	17.8
	Baseline	8.7	5.2	4.7	0.6	5.0	4.4	3.0	1.6	47.4	40.9	40.9	68.1
	l95	8.6	3.9	2.6	− 0.0	5.2	3.9	1.5	0.5	41.1	28.6	30.7	49.2
USA	u95	2.5	1.0	0.8	0.1	0.1	0.0	− 0.1	− 1.0	26.7	67.0	83.6	91.3
	Baseline	1.7	0.4	0.2	− 0.0	0.3	− 0.0	− 0.6	− 1.1	43.9	74.8	94.9	101.8
	l95	2.8	0.1	− 0.4	− 0.2	0.3	− 0.2	− 0.9	− 1.4	21.3	70.6	92.1	99.2

ratio than the baseline in the period 2070–2100. Similarly, the USA is the only advanced country to benefit from a reduction in its debt-to-GDP ratio in the scenario of lower population growth (l95).

5.3.2 Debt

In Table 4 we show the effects of a change in pension expenditure to GDP on the main interest variables. The shocks are designed so that the pension expenditure-to-GDP ratio will gradually double (pens exp +) and halve (pens exp -), respectively, compared to the projection provided by the OECD for all countries until 2060, which constitutes the baseline scenario. From 2060 to 2100, we assume that expenditure remains stable at the level reached in 2060 (India is assumed to reach 1% of GDP in 2100). As expected, an increase in expenditure leads to an increase in public debt but also to an increase in growth potential in the short term. From 2050 onwards, the growth rate falls. Conversely, pension reforms that have negative short-term effects on real GDP (i.e., a reduction in the pension expenditure-to-GDP ratio) are broadly offset by the benefits of lower real interest rates in the long run.

5.3.3 Protectionism

We assess the effects of increased protectionism in all countries, comparing the case in which protectionist measures are introduced drastically and immediately ("sudden" scenario) with the case where, for the same magnitude of shock, they are implemented gradually ("fast" scenario). We move the parameter η, as shown in Fig. 18. In both cases, the effect of increased protectionism overall is to reduce the growth rates of emerging country GDP, while European countries seem to benefit not only from increased protectionism, especially France and Italy, but also in terms of debt-to-GDP reduction. The USA shows a fluctuating trend in GDP, with medium-term gains but long-term losses. Table 5 shows the interest rate response, with advanced economies recording an overall increase in interest rates compared to the baseline, and emerging markets a decrease, especially in the long run.

6 Conclusions

Consistent estimates of the main natural quantities are particularly important for economic policy (output gap, monetary policy, debt stabilization) and long− term prospects (e.g., climate change). To highlight the effect of structural trends on the natural rate and potential growth rate of GDP in the long term, we use a general equilibrium approach in a global context. The OLG model allows consideration of the effects of structural trends, such as demographic and human capital trends; to this is added, through calibration exercises, the contribution of increasing returns to scale of the technological progress, technical obsolescence, and international integration in the goods and capital markets. By including these factors, the model can produce projections of natural rates (i.e., potential GDP and interest rates) that are close to estimates in the literature. We also provide projections of government

Table 4 Pension spending shock

Country	Scenario	Growth rate				Interest rate				Debt-to-GDP			
		2020	2040	2070	2100	2020	2040	2070	2100	2020	2040	2070	2100
Germany	Pens exp +	1.5	0.9	0.0	– 0.1	– 0.0	0.1	– 0.1	– 0.6	54.7	82.4	99.1	107.4
	Baseline	1.5	0.8	0.1	0.0	0.1	0.2	– 0.1	– 0.6	53.8	77.0	92.6	101.0
	Pens exp –	1.5	0.9	– 0.1	0.0	0.1	0.2	– 0.1	– 0.6	53.1	72.3	89.9	97.5
Italy	Pens exp +	0.3	0.2	0.0	0.0					128.4	155.8	150.7	146.4
	Baseline	0.3	0.2	0.2	– 0.1					127.9	137.4	130.0	129.4
	Pens exp –	0.2	0.2	– 0.1	0.1					126.9	128.5	122.6	119.6
France	Pens exp +	1.0	1.3	0.3	0.0					96.5	117.9	128.8	131.6
	Baseline	1.0	0.8	0.4	– 0.1					95.8	103.3	102.8	107.2
	Pens exp –	1.0	1.2	0.4	– 0.1					94.6	93.6	90.9	95.6
China	Pens exp +	4.2	2.6	2.6	0.6	2.2	2.7	2.0	0.7	61.0	115.0	119.1	141.7
	Baseline	4.1	2.7	2.7	0.6	2.7	2.9	2.2	0.9	56.7	76.7	81.6	101.1
	Pens exp –	4.2	2.6	2.8	0.6	3.1	3.1	2.3	1.0	53.2	49.0	56.3	71.1
India	Pens exp +	6.3	2.3	1.2	– 0.4	5.2	3.4	2.2	1.3	65.6	77.2	59.1	74.7
	Baseline	6.3	2.4	1.3	– 0.4	5.1	3.3	2.1	1.4	66.0	77.0	58.8	74.7
	Pens exp –	6.2	2.3	1.2	– 0.4	5.2	3.3	2.1	1.4	65.7	77.0	59.1	74.7
Africa	Pens exp +	8.7	5.3	4.9	0.8	5.0	4.5	3.0	1.4	47.1	46.0	48.5	84.2
	Baseline	8.7	5.2	4.7	0.6	5.0	4.4	3.0	1.6	47.4	40.9	40.9	68.1
	Pens exp –	8.6	5.2	4.6	0.6	5.0	4.4	3.2	1.7	47.1	37.1	35.4	56.5
USA	Pens exp +	1.5	0.2	0.2	0.1	0.1	– 0.1	– 0.4	– 1.3	45.7	77.9	98.2	105.7
	Baseline	1.7	0.4	0.2	0.0	0.3	– 0.0	– 0.6	– 1.1	43.9	74.8	94.9	101.8
	Pens exp –	1.5	0.4	0.4	– 0.1	0.3	– 0.0	– 0.5	– 1.2	42.6	73.5	92.1	100.3

Table 5 Protectionism shock

Country	Scenario	Growth rate				Interest rate				Debt-to-GDP			
		2020	2040	2070	2100	2020	2040	2070	2100	2020	2040	2070	2100
Germany	Fast protect.	2.1	0.9	0.0	−0.1	−0.2	0.2	0.1	−0.3	51.4	76.3	92.3	102.0
	Baseline	1.5	0.8	0.1	0.0	0.1	0.2	−0.1	−0.6	53.8	77.0	92.6	101.0
	Slow protect.	1.9	0.8	−0.1	0.0	−0.5	0.2	0.1	−0.3	52.1	77.5	93.5	100.7
Italy	Fast protect.	0.6	0.4	0.2	−0.1					125.1	133.3	130.6	130.4
	Baseline	0.3	0.2	0.2	−0.1					127.9	137.4	130.0	129.4
	Slow protect.	0.6	0.1	0.2	0.0					125.4	134.9	130.3	128.6
France	Fast protect.	1.3	1.3	0.6	0.1					93.2	99.7	102.7	106.7
	Baseline	1.0	0.8	0.4	−0.1					95.8	103.3	102.8	107.2
	Slow protect.	1.7	1.0	0.5	−0.0					91.0	99.4	103.1	106.8
China	Fast protect.	4.4	2.6	2.6	0.6	2.5	3.0	2.1	0.7	55.4	77.0	82.1	103.5
	Baseline	4.1	2.7	2.7	0.6	2.7	2.9	2.2	0.9	56.7	76.7	81.6	101.1
	Slow protect.	4.1	2.6	2.9	0.6	3.3	3.1	2.0	0.7	56.1	76.4	81.3	103.2
India	Fast protect.	5.9	2.3	1.3	−0.4	5.8	3.2	2.0	1.3	67.1	77.3	59.6	75.1
	Baseline	6.3	2.4	1.3	−0.4	5.1	3.3	2.1	1.4	66.0	77.0	58.8	74.7
	Slow protect.	6.1	2.2	1.2	−0.4	5.2	3.3	2.1	1.3	66.1	77.3	60.0	75.0
Africa	Fast protect.	8.2	5.1	4.7	0.6	5.3	4.5	2.9	1.2	48.9	41.1	41.1	71.4
	Baseline	8.7	5.2	4.7	0.6	5.0	4.4	3.0	1.6	47.4	40.9	40.9	68.1
	Slow protect.	8.7	5.2	4.7	0.6	4.5	4.5	2.9	1.0	46.7	40.6	41.2	71.2
USA	Fast protect.	0.7	0.9	0.3	−0.1	0.6	0.4	−0.1	−0.7	49.5	74.0	93.4	100.4
	Baseline	1.7	0.4	0.2	0.0	0.3	−0.0	−0.6	−1.1	43.9	74.8	94.9	101.8
	Slow protect.	1.2	0.7	0.3	0.0	0.5	0.6	−0.1	−0.7	43.7	74.9	93.0	100.4

deficits and global imbalances over the long run, although the projections provide only qualitative assessments and do not attempt to be exhaustive. What emerges from the results is that all countries show substantial stability in their deficit/surplus trends, with the exception of Germany, where we expect a reversal with respect to structural public deficits in the coming decades. Regarding global imbalances, countries such as the United States and China will produce a turnaround in global imbalances by reversing the sign of current account flows over a long period of time; this is due to changes in the aggregate savings rate. In contrast, European countries are expected to experience a medium-term persistence of surpluses.

Our model's global and general equilibrium analysis could provide a useful tool for policy analysis and the determination of natural rates (interest and potential growth). In a further paper, we will use the model to determine the effect of the COVID-19 pandemic on natural growth and interest rate. The pandemic is a multiple-channel shock affecting, for example, the average discount rate by a reduction in life expectancy. Such excess mortality increases the impatience of the households and reduces their saving behavior; this, in turn, increases the natural interest rate and decreases capital accumulation, which negatively affects the pace of potential growth. A second important channel, even if it is reduced in size, could be an interruption in human capital growth due to a loss in the quality and quantity of school attainment for a large ratio of the world's young population. In future research, all these factors will be evaluated using the model developed in the current work.

Appendix 1: Exogenous trends and empirical evidence

According to the United Nations (2019) Population Prospects, the world's population continues to increase, but growth rates vary greatly across regions.

The global population is projected to grow from 7.7 billion in 2019 to 8.5 billion in 2030 (10% increase), and further to 9.7 billion in 2050 (26%) and to 10.9 billion in 2100 (42%). The population of sub-Saharan Africa is projected to double by 2050 (99%). Other regions will see varying rates of increase between 2019 and 2050: Oceania, excluding Australia/New Zealand, (56%), Northern Africa and Western Asia (46%), Australia/New Zealand (28%), Central and Southern Asia (25%), Latin America and the Caribbean (18%), Eastern and South-Eastern Asia (3%), and Europe and Northern America (2%).

In most of sub-Saharan Africa and in parts of Asia, Latin America, and the Caribbean, recent reductions in fertility have led the working-age population (25–64 years) to grow faster than other age groups, creating an opportunity for accelerated economic growth. To benefit from this "demographic dividend," governments should invest in education and health, especially of young people, and create conditions for sustained economic growth (United Nations 2019). Life expectancy at birth for the world, which increased from 64.2 years in 1990 to 72.6 years in 2019, is expected to increase further to 77.1 years in 2050. While considerable progress has been made in closing the longevity differential between countries, there are still wide gaps in life expectancy. In 2019 life expectancy at birth in the least developed countries lagged 7.4 years behind the global average, due largely to persistently high

Fig. 11 2020 population structure, % *Source*: Our elaborations based on UN data
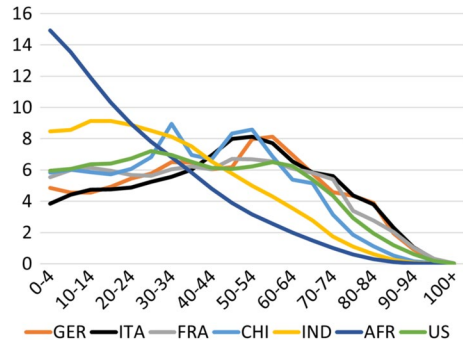

child and maternal mortality, violence, conflict, and the continuing impact of the HIV epidemic. The world's population is growing older, with persons over age 65 being the fastest-growing age group by 2050, and one in six people in the world being over 65 (16%), compared to one in 11 in 2019 (9%).
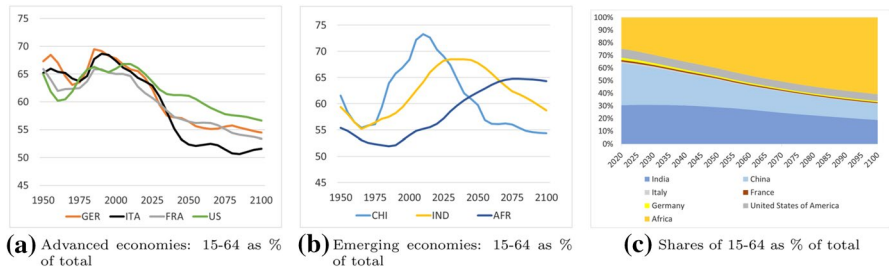
The United Nations (2019) indicates that an increasing number of countries are experiencing a population reduction. Since 2010, 27 countries or geographical areas have experienced a population reduction of 1% or more. This is caused by low fertility levels and, in some places, high emigration rates. Between 2019 and 2050, populations are expected to decrease by 1% or more in 55 countries or areas. Between 2010 and 2020, Europe and Northern America, Northern Africa and Western Asia, and Australia/New Zealand will be net receivers of international migrants, while other regions will be net senders. Fourteen countries or areas will see more than one million migrants, while ten countries will see a net outflow of more than one million migrants.

Figure 11 shows the profound difference in current cohort density distributions. Among advanced countries, France, and the USA show a constant population structure between 2020 and 2060. The decrease in Italian and German young cohorts will have a huge impact on the economy in the long run. Africa will keep its "triangular" shape in the future due to its eruptive population dynamics. The share of young cohorts for Indian and African countries is double that of advanced economies.

Figure 12 shows the evolution of underlying demographic rates. It is worth noting that among advanced countries, Germany and Italy closed their demographic transition in the period 1975-1990. The UN projections show a reversed regime in these countries, where the crude mortality rate is higher than the crude birth rate. The same applies to France, China, and India, but much later. Except for Africa and the USA, all countries will, in the long run, have a total fertility rate lower than the reproducibility threshold (2.0). For India, the reversal of the demographic transition will manifest itself around 2050. Africa will not close the demographic transition (until 2100) and will persistently show a net emigration flow for the entire UN projection period. Finally, the net migration rate in the USA will overtake the total fertility rate close to the reproducibility threshold. The high net migration rate in the presence of equilibrated endogenous

(a) Germany **(b)** Italy **(c)** France **(d)** China

(e) India **(f)** Africa **(g)** USA **(h)**
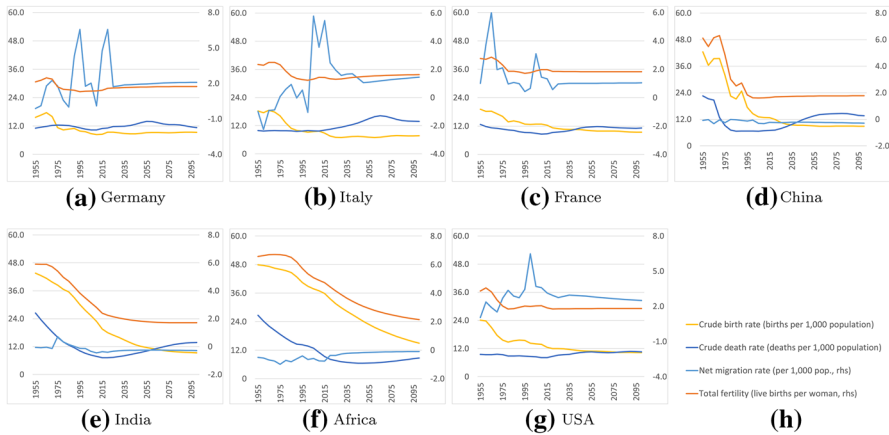
Fig. 12 Migration, fertility and mortality data: 1950-2100 *Source*: Our elaborations on UN data

(a) Advanced economies: 15-64 as % of total **(b)** Emerging economies: 15-64 as % of total **(c)** Shares of 15-64 as % of total

Fig. 13 Working age population dynamics, 1950–2100 *Source*: Our elaborations on UN data

demographic behavior (equality between crude natality and mortality rates) will thus allow the US population to grow.

Finally, China's aging population will be the strongest among the emerging economies. The evolution of aging will be reflected in the expected labor share. In 2100 African countries will hold the majority of working-age population, while in advanced countries a constant 10% will be maintained (Fig. 13c). Among the emerging countries, the deep decline in the working-age population in China is striking, and it will decrease rapidly compared to the current percentage of 70%.

Figure 14 shows the evolution of the education attainment as estimated by Barro and Lee (2015) in the period 1960–2015 and projected forward. Among advanced countries, Germany shows increasing human capital, as the tertiary level is much higher than in other countries. In Italy, this level will be much lower due to the persistence of secondary education and a low tertiary education level. Among the emerging countries, China has increasing levels of tertiary education over time. The USA dominates over all countries. As we will see, this information will be taken into account when calculating the human capital index.
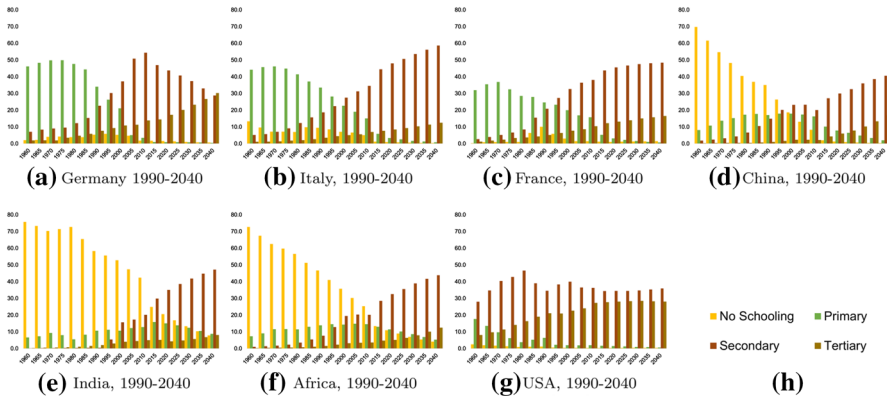
Fig. 14 Human Capital: Attained levels of education *Source*: Our elaborations on Barro and Lee (2015) data

There is much discussion in the literature as to which makes a greater contribution to growth–the number of school years achieved or the quality of human capital. Breton (2011) shows empirically that when the nation's average test scores and average schooling attainment are included in a national income model, both measures explain income differences, but schooling attainment has greater statistical significance. Kaarsen (2014) finds that there are significant differences in the quality of human capital: one year of schooling in the USA is equivalent to three or more years of schooling in several low-income countries. Rossi (2018) finds that traditionally, economists have concentrated on years of schooling and other measures of educational attainment. However, several pieces of evidence suggest that this focus on the quantity of schooling misses important dimensions of the variation in human capital, both at the individual and aggregate level. As a consequence, recent research gives a more central role to schooling quality, on the one hand, and human capital accumulated outside of the schooling system, on the other. The consequences for productivity measures are evident for our set of countries. The statistical relationship between GDP level and schooling attainment is positive (Fig. 15). In terms of total factor productivity (Feenstra et al. 2015) the US benchmark has been overtaken by the EU countries. In the case of Italy, growth reduction has been evident since the 1990s. Emerging countries such as India and China have started to converge, slightly reducing the gap with the advanced countries. This convergence will continue to increase with the number of patents per capita in China, which has reached the same level as in the United States. On the other hand, labor productivity in Italy is in severe decline.

The trends in technology have important consequences for factor concentration and movements, that is, for global integration outcomes. To appreciate the evolution of integration in trade and capital flows, we used network analysis to discuss the topic. The trends in trade and capital movements can be described as set out in Fig. 16. According to Cepeda-López et al. (2017), the Great Recession broke the world trade structure. The network topology in the figure shows the rise of China
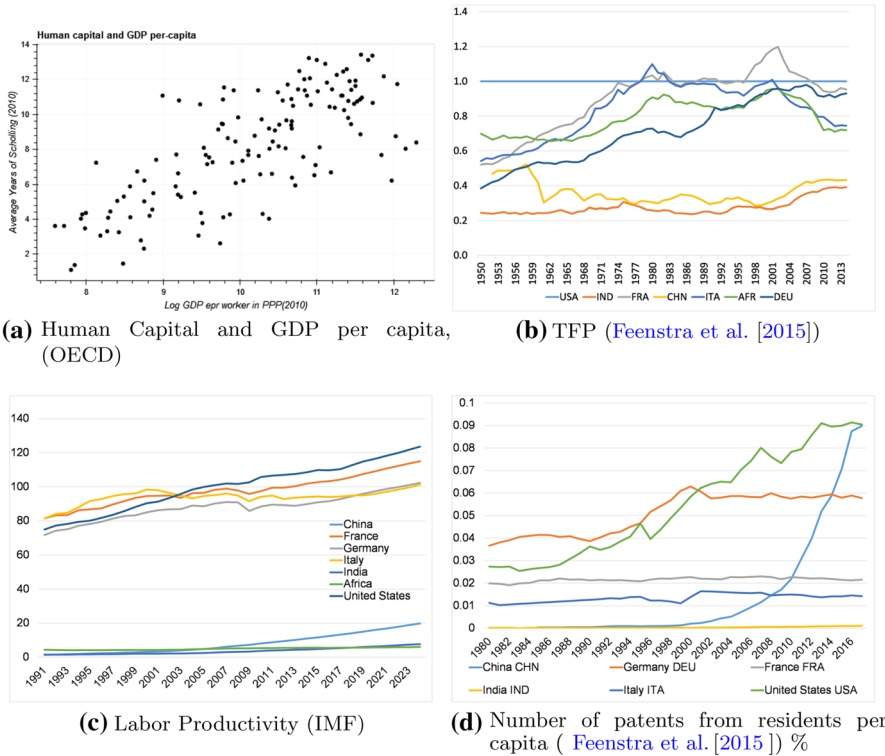
(a) Human Capital and GDP per capita, (OECD)

(b) TFP (Feenstra et al. [2015])

(c) Labor Productivity (IMF)

(d) Number of patents from residents per capita (Feenstra et al. [2015]) %

Fig. 15 Trends in technology

as a central node for trade in 2007, with the USA losing some of its leadership role during the last decade. The figure shows a trend of increasing dispersion of trade relations that came to a halt in 2010, when trends seem to have changed direction compared to the previous decades (Fig. 16a–c). Over the same period, the enormous growth in financial integration, expressed as the sum of financial liabilities and GDP assets, stabilized over the last decade, in line with trade behavior (Fig. 17).

Appendix 2: Equilibrium

The dynamic general equilibrium of the economy in the time period $[t_0, t_1]$ is defined as a set of micro variables $\left\{c_{t-s,i,j}, l_{t-s,i,j}, a_{t-s,i,j}\right\}_{t=t0}^{t1}$, aggregate variables $\left\{C_{j,t}, L_{j,t}, K_{j,t}, A_{j,t}\right\}_{t=t0}^{t1}$, and wage and interest rate $\left\{w_{j,t}, r_{j,t}\right\}_{t=t0}^{t1}$ such that:

1. Household utility in Eq. (2) is maximized subject to equations (3) and (3.1)
2. Factor prices equal their marginal productivities satisfying Eq. (12).
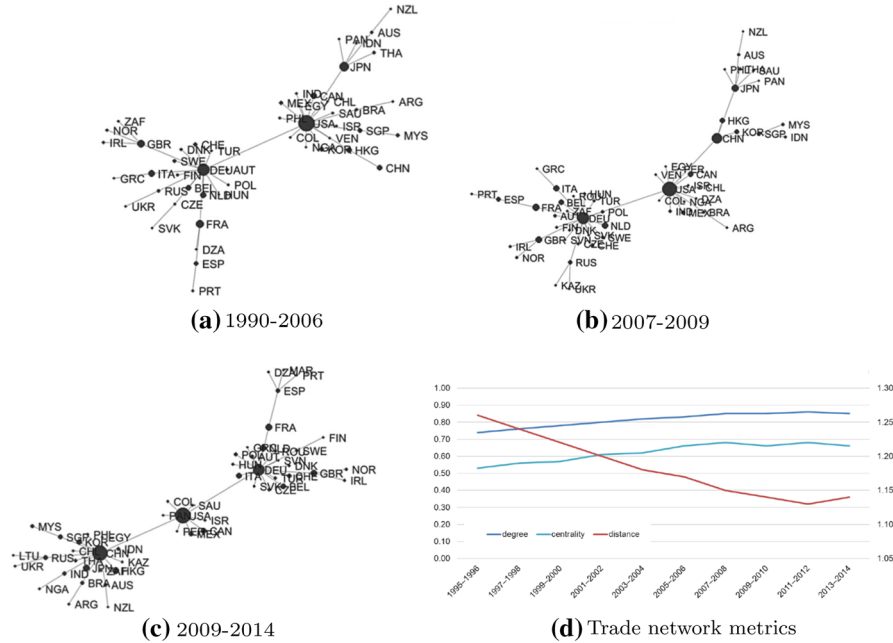3. Domestic labor and capital markets clear satisfying equations (14)-(16).

(a) 1990-2006

(b) 2007-2009

(c) 2009-2014

(d) Trade network metrics

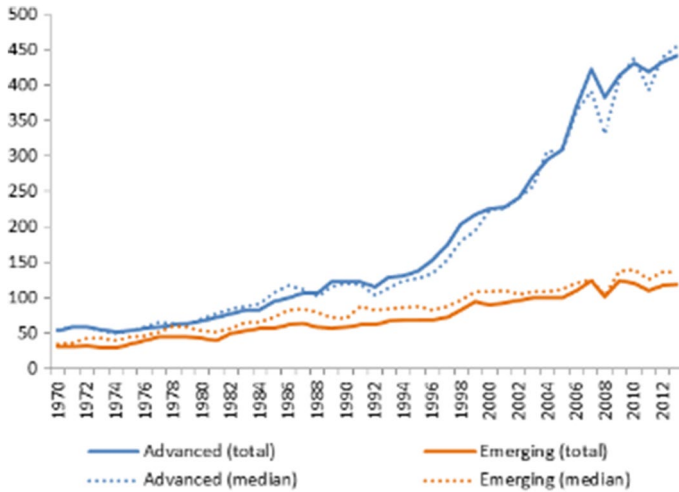**Fig. 16** Trade network evolution. *Source*: Cepeda-López et al. (2017)

Fig. 17 International financial integration. *Source*: IIP

4. Public sector issues debt to satisfy Eq. (13).
5. Domestic goods markets clear:

$$TFP_{j,t}K_{j,t}^{\beta}L_{j,t}^{1-\beta} + r_t F_{j,t} = \sum_i \sum_s c_{t-s,i,j} P_{t-s,i,j} + I_{j,t},$$

which implies:

$$Y_{j,t} = \sum_i \sum_s (q_{t-s}a_{j,t+1-s} - a_{j,t-s})P_{t-s,i,j} +$$
$$+ \sum_i \sum_s c_{t-s,i,j} P_{t-s,i,j} - r_t F_{j,t},$$

where $I_{j,t} = K_{j,t+1} - K_{j,t}(1 - \delta)$ is private sector investment, and δ is the depreciation rate.

6. In the open economy framework, market clearing on the international market requires the rate of return on capital in a single country to be such that the sum of all foreign assets in the system equals zero:

$$r_{j,t} : \sum_{j=1}^{J} F_{j,t} = 0 \quad \forall j = 1, ..., J \tag{18}$$

where J is the number of countries in the model.

7. World output is:

$$Y_t = \sum_{j=1}^{J} Y_{j,t}. \tag{19}$$

8. Foreign assets are defined as the difference between total assets and home assets, such that

$$F_{j,t} = A_{j,t} - B_{j,t} - K_{j,t} \tag{20}$$

and international capital flows are the difference between national savings and investment

$$CA_{j,t} \equiv \Delta F_{j,t} = S_{j,t} - I_{j,t}$$
$$= Y_{j,t} + r_t F_{j,t} - C_{j,t} - I_{j,t}, \tag{21}$$

where $Y_{j,t} + r_{j,t}F_{j,t}$ denotes total national income (from domestic and foreign sources).

Appendix 3: Calibration

Appendix 3.1: Demography and human capital

To address the demographic impact on economic growth, we use historical population data and projections provided by the UN (United Nations 2019) (see Sect. 1) for the period 1960–2100, with cohorts aged between 0 and 100 denoted by $P_{t-s,j}$

for each country j, with $t - s$ denoting the age of each cohort. In order to evaluate the role of human capital accumulation and labor productivity in the economies, we build a human capital index for each country based on the education levels provided by Barro and Lee (2015): data on education from 1950 to 2010 are grouped into three education levels: primary (LS), secondary (HS), and tertiary (TS).

To take account of the saturation of entry to higher education, we allow the evolution of education to slacken in the last 30 years of projection: we assume that people older than 30 in period t retain the education level of their previous age in period $t - 1$.

The human capital H_t is computed as a Törnqvist index:

$$H_t = \sum_i h_{i,t} = \frac{1}{2} \sum_{i=1}^{I} \Delta p_{i,t} \left(\frac{\lambda_t P_{i,t}}{\sum_{i=1}^{I} \lambda_{i,t} P_{i,t}} + \frac{\lambda_{t-1} P_{i,t-1}}{\sum_{i=1}^{I} \lambda_{i,t-1} P_{i,t-1}} \right), \qquad (22)$$

where I is the total number of education types (i.e. 3 for LS, HS, and TS) and $h_{i,t}$ is the human capital per education level i which enters household labor income; $P_{i,t} = \sum_s P_{t-s,j,i}$ denotes people with education level i in year t; $\lambda_{i,t}$ is a quality index for education level i (i.e. 8 years of study for level LS, 13 years for HS, and 18 for TS). $\Delta p_{i,t}$ is the variation in the number of people with education i in year t compared with the base year t_0, i.e. $\Delta p_{i,t} = ln\left(\frac{P_{i,t}}{P_{i,t_0}} \right)$. The index measured in Eq. 22 is intended to measure the per worker ability to provide productive labor services to firms. The index captures the capacity of education and training systems to qualify the working capacity of individuals.

Appendix 3.2: Exogenous and endogenous total factor productivity

Figure 2b shows the dynamics of the total residual productivity of the factors (i.e., the unexplainable component of the TFP, which is the variable used as an instrument to match the dynamics of the GDP with actual data). To match historical GDP data we calculate an exogenous index of TFP^{ex} (Eq. 8) which minimizes the distance between the simulated ($G\hat{D}P$) and the actual data ($G\bar{D}P$) given by the following equation:

$$TFP_t^{ex} = \frac{G\hat{D}P_t}{G\bar{D}P_t} = \frac{T\hat{F}P_t \hat{K}_{t-1}^{\beta} \hat{L}_t^{1-\beta}}{G\bar{D}P_t}. \qquad (23)$$

Given the exogenous TFP_t^{ex}, we simulate GDP according to Eq. 8 and minimize the norm of errors $\epsilon_t = G\hat{D}P_t - G\bar{D}P_t$ in the time-span $t = [1960, 2018]$ and considering it as the metric for a global numerical procedure.[7] For all countries, this component stabilizes toward the end of the simulation period, while from 1960 onwards, the total exogenous productivity of the factors greatly increases, signaling the presence of factors not captured by the model. Growth factors linked to demand have characterized the first decades of growth for all advanced and emerging countries.

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[7] We use genetic algorithm where the genes population is the TFP_t^{ex} to be used to match historical GDP.

Table 6 Random coefficients estimation

Countries	z^*	g^*	z	g	σ_z	σ_g
Germany	0.307	0.030	− 0.093	2.898	0.004	0.132
Italy	0.016	0.001	0.005	− 0.169	0.001	0.080
France	0.138	0.0115	− 0.038	1.396	0.002	0.103
China	0.476	0.041	− 0.047	1.928	0.002	0.136
India	0.088	0.0073	− 0.018	0.873	0.000	0.060
Africa	0.816	0.082	− 0.118	3.650	0.003	0.121
USA	0.014	0.048	− 0.008	0.063	0.000	0.074

It should be noted that the model, on the other hand, provides a good explanation of the growth process of the African continent.

An additional source of growth is represented by an endogenous TFP process. We estimate the parameter of endogenous TFP (see Sect. 3.2) by means of a simple linear relationship to find the long-run relationship between the capital-to-worker ratio and human capital quality. Endogenous TFP is defined as (10): $TFP = \left(\frac{K}{N}\right)^g H^z$. We estimate the parameters g and z in the long-run relation

$$\log(TFP) = g\log(K/N) + z\log(H) + \epsilon_{tfp}.$$

using capital stock data provided by Berlemann and Wesselhöft (2014) and data for labor, human capital, and TFP provided by Penn World Trade.[8] Fixed effects panel estimates lead to $g = 0.17$ and $z = 0.24$ (standard deviation 0.013 and 0.058, respectively), while estimation of random coefficients (Swamy 1970) leads to different values between countries, as shown in Table 6. For advanced countries, the relationship between capital per labor unit and human capital is significant and positive, except for Italy. The coefficient is almost zero, thus preventing Italy from benefiting from the positive effects of increasing return to scale. In the other countries, on the other hand, the sum of the global coefficients of the production function, including the role of human capital, generates a regime of increasing returns to scale that allows for related phenomena (trade, specialization, capital accumulation) to be inflated. To introduce heterogeneous TFP among countries, we apply the following procedure. We take the estimates from the fixed-effects approach to represent the case of homogeneous TFP among countries. We introduce heterogeneity through different country-specific coefficients. These coefficients (z^* and g^* in Table 6) are obtained as a deviation from the fixed effects estimates. In particular, we apply to the fixed effects coefficients the percentage difference between the random estimates for individual countries (z and g in Table 6) and the average coefficient resulting from the technique of random coefficients.[9]

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[8] Feenstra et al. (2015), available for download at www.ggdc.net/pwt.

[9] The model's assumptions is

$$y_i = X_i\beta_i + \epsilon_i \tag{24}$$

with $\beta_i = \beta + v_i$. The average estimator is $\hat{\beta} = \sum_i W_i b_i$ and singular estimators b_i is the $i - th$ OLS standard estimator and W a weghted matrix. See (Poi 2003) and (Swamy 1970).

Table 7 Projections of public expenditure on pensions (% of GDP) *Source*: OECD

Countries	2013–15	2050
Germany	14.9	12.8
Italy	10.0	12.5
France	15.7	14.8
China	7.7	8.1
India	4.9	5.9
Africa(*)	1.0	1.0
USA	2.2	3.3

(*) The data for Africa refer to South Africa

Table 8 Government expenditure on education, total (% of GDP) *Source*: World Development Indicators

Countries	%
Germany	4.81 (2015)
Italy	4.08 (2015)
France	5.46 (2015)
China	1.89 (1999)
India	3.84 (2013)
Africa*	4.00 (2016)
USA	4.99 (2014)

In parenthesis, the last year available

*Sub-Saharan Africa. In parenthesis, the last year available

Appendix 3.3: Technological obsolescence, labor share, fiscal trends

To calibrate the capital share β we use the labor income share data provided by the International Labour Organization (ILO).[10] We obtain a time profile of the capital share in production that is slightly increasing in the period 1991–2013 in all countries (Fig. 5).

We calibrate the pension benefit $pens_t$ in Eq. (13) in order to match the OECD projections of public expenditure on pensions over the period 2013–50 as in Table 7:

The debt-to-GDP ratio is calibrated to match data provided by IMF from 2000 to 2023, as shown in Fig. 5. Government spending on education SC in Eq. (13) is calibrated according to WDI data, shown in Table 8.

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[10] For sub-Saharian Africa we refer to Trapp, K. Measuring the Labour Income Share of Developing Countries: Learning from Social Accounting Matrices. UNU-WIDER, Helsinki, Finland (2015) 22 pp. [WIDER Working Paper No. 2015/041].
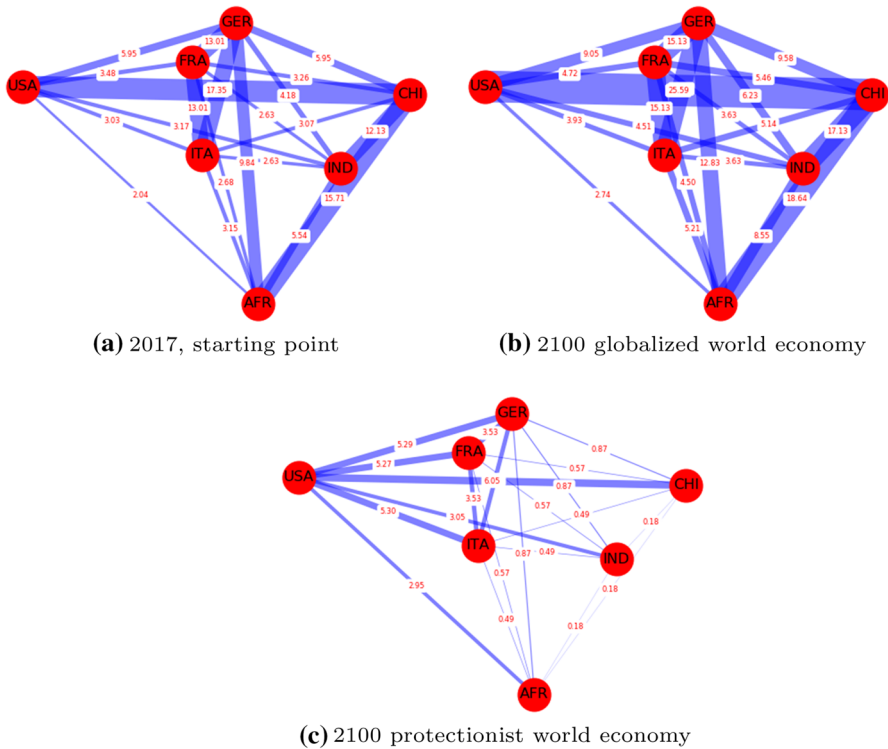
(a) 2017, starting point

(b) 2100 globalized world economy

(c) 2100 protectionist world economy

Fig. 18 Integration matrices. *Source*: Our calculations based on WITS data (World Bank)

Appendix 3.4: Capital market integration

We assume an increasing degree of economic openness that allows us to take into account the effects of slow capital market integration affecting the whole simulated economy and to obtain endogenous capital movements between countries.[11] We assume a non-linear trend[12] in the degree of openness/integration θ_t from 1970 to 2017 which affects the capital return set according to equation 12, which we report here:

$$r_{j,t} = \theta_{jj,t}(TFP_t f'_{K,t} - \delta) + \sum_{i=1}^{n} \theta_{ji,t} r_{i,t}$$

[11] This procedure allows us to account for the core-periphery dualism for the European countries (see Catalano and Pezzolla 2016).

[12] We assume a multivariate-logistic (replicator dynamics) evolution that has as an initial condition the diagonal matrix (complete closeness for all countries), converging in time (2017) to the calibrated matrix.

where $f'_{K,t}$ is the marginal productivity of capital, and $\theta_{jj,t}$ represents the degree of openness in 2017, computed as the sum of exports and imports to GDP for each country j, using World Bank bilateral data.[13] If $\theta_{jj} = 1$, the domestic capital stock is equal to the domestic saving ($K = A$), and the interest rate is determined only by domestic marginal productivity, while $\theta_{ij} = \theta_{ji} \ \forall j, i$ (i.e., with uniform matrix a global uniform interest rate applies). Figure 18a, b show the evolution of capital market integration between 2017 and 2030 (which stabilizes until 2100), based on trends in the period 2000-2017. What emerges is growing integration among emerging and advanced countries and, at the same time, greater integration between European countries. On the other hand, Fig. 18c shows the result of a process of increasing protectionism among all countries until 2030.

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[13] We use the obtained bilateral coefficients $\theta_{ji,2017}$ to feed a $n \times n$ matrix Θ_{2017}, which represents the bilateral degree of integration among countries in 2017. Starting from a system characterized by closed economies in 1812 (i.e., Θ_{1812} is an identity matrix $n \times n$) the matrix converges gradually to the Θ_{2017}. This implies that there is a process of gradual integration among countries.

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