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RESEARCH ARTICLE

More buyers or more sellers: on marketing resource allocation strategies of competing two-sided platforms

Amit Kumar Bardhan[1] · **Saad Ashraf**[1]

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Abstract
Two-sided platforms enable and supplement transactions between buyers and sellers. We consider a decision problem facing two such platform firms competing in a market. Each firm needs to divide its budget of a planning period between promotion towards attracting new sellers and new buyers. We propose a generalized Nash equilibrium problem (GNEP) model to find optimal allocations. The GNEP approach provides an eloquent framework for analysis and theory development. An intuitive result from the interpretation of optimality conditions is that a firm's focus should be higher towards the group whose presence is less on the platform. This focus can shift depending on competitors' ability to dissuade new customers. Interestingly, the model recommends that a firm should focus on getting new sellers when its customer-focused promotion adversely impacts competitors' customer acquisition. Predatory promotion strategy adversely affects both. More useful implications can be drawn from the equilibrium analysis. We have assumed that a limited number of sellers are available in the market, whereas no restrictions are imposed on new customer acquisitions. This situation is typical during the entry phase of a two-sided platform.

Keywords Two-sided platforms · Platform competition · Resources allocation · E-commerce · Generalized Nash equilibrium

1 Introduction

Two-sided platforms enable transactions between two inter-dependent groups. These two groups, in most cases, are identified as buyers and sellers. Platforms add their value to the value offered by the sellers to the buyers and earn commission on business effectuated. Shopping malls, digital marketplaces, electronic payments systems,

✉ Amit Kumar Bardhan
amit-bardhan@fms.edu

[1] Faculty of Management Studies, University of Delhi, Delhi, India

hyperlocal delivery services, consoles for video games are prominent examples of such platforms. Today, as they become ubiquitous, their business models and strategies draw immense interest [8, 22, 48, 85].

New businesses have emerged that harness the proliferation of the internet and mobile technologies. Delivery service companies like *DoorDash*, *Uber Eats*, and *GrubHub* collect food items from restaurants and deliver them to customer locations. Today's largest companies in China are platform marketplaces, e.g., *Alibaba* and *JD.com*. However, building a platform business is very difficult. It needs substantial investment to start and has a high entry barrier [41]. The early stages are especially challenging. A few can grow rapidly due to network effects at the expense of others [47, 69].

Buyers and sellers demonstrate divergent behaviour on the platforms. While sellers compete among themselves, buyers are encouraged by other consumers' participation. The seller competition influences pricing and product offerings [8]. An increase in the number of customers can lead to exponential growth benefiting from the network effect. However, a platform's revenue growth remains stunted in the absence of a critical number of sellers. A platform becomes valuable to both buyers and sellers after it has grown big, i.e., when direct and indirect network effects can create value. Before this, it is a challenging time. A bad decision in the growth phase can be damaging [47, 58]. Platforms have to deal with the chicken-and-egg problem where they face the dilemma of pursuing both sides [14].

Platforms try to attract participants through promotion and by offering subsidies. They offer buyers additional discounts and promise faster fulfilment [10]. Sellers are offered minimal rent, technology upgradation, information sharing and better visibility [58]. Platform firms invest heavily in these activities, colloquially called 'cash burning', especially in their growth phase [80]. These investments are fraught with risk. Judicious deployment of resources among multiple activities is critical. At a fundamental level, the question is who gets the preference, new buyers or new sellers. The performance of a platform greatly depends on the dynamic equilibrium between buyers and sellers. It is vital to recognise which group to support more and when [81, 84].

Hänninen [35] has presented a review of articles on transaction platforms in marketing journals. Marketing plays a critical role in the growth of these platforms [64]. Mu and Zhang [57] discuss the marketing capability of different sellers on a platform competing with each other to attract customers. Hu and Zhou [37] study an e-commerce platform's information service and omnichannel retailing strategy in the presence of seller competition. It is observed that pricing has been an essential component of economic analysis while other marketing mix variables like promotion have not received adequate attention. Only a few studies discuss resource allocation problems for platform firms. Marketing resource allocation decisions are complex. For better decisions, managers seek better segmentation, explanatory mapping of relationships within and between segments, and predictive ability [70]. In this regard, the utility of model-based decision tools is well recognised [54, 86]. Zhou et al. [94] have discussed a budget allocation model where the platform allocates its promotional budget between sellers to maximize total sales on the platform. The promotion targeted at customers was not included in the study. Yang et al. [90]

model a competitive scenario where platforms are competing through advertising outlays. However, the advertising is focused on bringing in customers and does not consider advertising to sellers. Yoo et al. [92] discuss a competitive scenario where the platform competes with its own sellers. This results in a '*coopetition*', which is a mix of cooperation and competition. The research concludes that *coopetition* impacts relationship performance between the platform and its sellers.

Decisions on the apportionment of budget between the seller and customer are complex but essential. Surprisingly, we could not find substantial work on the topic in competitive settings. Mantrala et al. [49] have discussed a marketing investment problem to increase profits of the "dual revenue" market of newspapers with demands arising from both subscriptions and advertising space. Sridhar et al. [76] presented theoretical results and suggested an algorithm to find the temporally optimal allocation of the promotional budget for a two-sided platform. Li et al. [46] have discussed a resource allocation problem where investment needs to be made for both sides of a platform across multiple product categories. These papers do not consider the presence of a competitor platform. Ignoring competition has been attributed as one of the prominent reasons for the failure of platforms [91]. In this article, we demonstrate that competition significantly alters optimal decisions.

A platform operator must understand the relative contribution of the two groups in its growth trajectory. The knowledge will be effective in the development of targeted promotion strategies [58]. Parker and Van Alstyne [62] recommend the comparison of cross-sided elasticities to choose the side to support. Fath and Sarvary [28] suggest it is more efficient to favour buyers, while Li and Zhu [47] recommend more investment in the stronger group with weaker indirect network effect to grow the network as a whole. We show that in a competitive setup, the equilibrium policy depends on the relative strengths of the platforms, and it is not obvious at the outset which group should be preferred.

The literature on the growth of two-sided platforms has focused on inter-group and intra-group dynamics [19]. Cross-subsidization and pricing have been the dominant themes of research [34]. It has ignored some fundamental operational issues of platform firms. For instance, negative cross-platform external effects characterize the growth phase of platforms. Consumers can get attracted by higher discounts and effective competitor advertising and leave the platform. Models of platform competition have ignored this loss of customers. This paper intends to address this research gap. Agnostic exploration of the behaviour of buyers and sellers can provide actionable insights to managers.

This paper focuses on the competition between two platforms in their growth phase when both are trying to grow 'organically' by acquiring new buyers and sellers. We present a resource allocation decision problem facing these firms. Using a theoretical framework, we suggest a stylized model describing promotional activities' effectiveness. The platforms carry out targeted promotional activities, which attract new buyers and sellers. The elasticities of the two promotions are different. Moreover, a platform can lose buyers due to the promotion of its competitor. They both need to simultaneously decide the level of effort towards the buyers and sellers for the next planning horizon. Caillaud and Jullien [14] have found that it is efficient for the platform's market to have one of the two groups single-home. Single-homing sellers are considered when the platforms seek exclusivity in the market and are ready to spend more for this advantage. Gaming platforms like Xbox, PlayStation and Wii prefer that the games they provide on their platform are not available on other consoles [38]. Similarly, online streaming services buy or license content for their platform which is not available on other platforms.

In reality, instances of mixed-homing where customers (or sellers) can either single-home or multi-home are common [38]. Wu and Chamnisampan [88] have discussed different homing strategies to be adopted endogenously for different scenarios. However, literature on platform competition *abstracts* this scenario into a single exogenous possibility, that is, a side can either single-home or multi-home [38]. We assume that consumers can multi-home, while in the market there are a limited number of sellers, and they join one of the two platforms [34]. Though in the numerical experiments, we have tested instances where the number of sellers is very large, such that platforms cannot acquire all of them even with the combined budgets.

We have developed a generalized Nash equilibrium problem (GNEP) model for the above decision problem. We believe this is the first such attempt to use the GNEP approach in this context. In a GNEP model, the constraints of a player contain the decision variables of its competitors. Therefore, the satisfaction of constraints is affected by the strategies chosen by both players. Due to this, in addition to the utility function of a player being affected by the competitor, the feasible solution space of a player is also altered by the decision of the competitor [27]. Since sellers are single-homing in our problem, a seller acquired by one player cannot join the other. So, the strategy space of the other player gets modified with respect to the remainder of the sellers. To model the effect of single-homing sellers on a player's strategy, a GNEP approach is more suitable.

In this paper, we first prove the existence of a Nash equilibrium and investigate the uniqueness of equilibrium for the proposed GNEP model. Some important results found during the optimality analysis are as follows. First, a critical seller count is necessary for a two-sided platform when sellers are limited. Single-homing sellers will get greater attention when their sensitivity to promotion is high, or the platform does not have a sufficient number of sellers. Second, suppose a platform has already acquired a sizable number of sellers, or the customer sensitivity to promotion is low. In that case, more allocation should be made towards getting new customers to the platform. But this relationship may not always hold in the presence of negative cross-platform effects. If this effect is negligible, higher customer sensitivity to the promotion results in a greater focus on them. But the opposite happens when the negative cross-platform is non-negligible. That is, when the possibility of customer loss is very high due to negative cross-platform effects, the platform refrains from large investments on customer acquisition and instead invests in sellers.

The rest of the paper is organized as follows. After a brief review of background literature in Sect. 2, we present the problem formulation in Sect. 3. In this section, we also discuss the conditions for the existence of an equilibrium solution and, using the optimality conditions, deduce theoretical results on optimal strategies for platforms. In Sect. 4, after describing an algorithm for solving a GNEP model, we analyze results from a detailed numerical study. The next section discusses the implications and suggests how best to implement optimal strategies in practical scenarios.

2 Literature review

The terms "platform", "two-sided platform", or "multi-sided platform" are used to define any system, firm, or service acting as an intermediary between two or more user groups [66]. A group of users on a platform collectively form a network [65]. The platform provides each user with networks of other user groups to interact and transact with [14]. This interconnection allows a platform's interface to create value and charge the users for it.

Two-sided platforms usually provide intermediation by bringing together networks of sellers and buyers [14, 61]. The size of the network is a significant element of a platform's value. Growth in the size of a network can influence more users to join it [5]. Additionally, a more extensive network means user groups on the other side have more agents to choose from. This influences the network growth of all the groups on the platform. For two-sided platforms, the dilemma is which group to target and when, giving rise to the so-called 'chicken-and-egg problem'; [14]. Sellers will get attracted to the platform with more buyers. However, buyers will only join the platform if it has many sellers. Thus, the platform faces the dilemma of which side to attract first, which in turn will also attract the other side. Consequently, decisions by the platform for one user group impact other user groups as well. These decision-making problems range from functional decisions, such as pricing, quality of technology, bundling, etc., to corporate decisions, such as diversification, vertical integration, disintegration, etc. [65].

There is a tendency for markets to favour a single platform in the presence of network effects [41]. This phenomenon is referred to as tipping [68]. However, coexistence is possible when there is no single efficient platform [36]. Jullien and Sand-Zantman [41] mention that tipping can be avoided if platforms differentiate from competitors, or at least one user group on the platform can multi-home, or if the platforms find interoperability beneficial. Even then, a few platforms take control of the market, and the remaining competing platforms fight for small niches. Platforms often seek a "winner-takes-all" strategy to attempt the largest market share [65, 79]. However, the "winner-takes-all" strategy does not always work [17]. Allowing at least one side to multi-home restricts monopolization and enables firms to compete with each other [9]. This causes multi-homing by a user group, a frequently observed scenario in platform competition [41]. Due to tipping, it is essential to include the effect of competition when studying platform strategies.

Existing discussion on platforms and platform competition can be classified through the perspectives of information systems, management, and economics [32, 65]. Roson [67] has surveyed two-sided markets, focusing on pricing principles. McIntyre and Srinivasan [51] have reviewed strategic management issues for platforms. In recent times, digital platforms have gained more focus over offline ones. Discussion on the design, structure, and architecture of a platform as an information system is growing [7, 32] and has been reviewed by De Reuver et al. [24]. Jullien and Sand-Zantman [41] have reviewed the extant discussion on the platform economy and competition. Rietveld and Schilling [65] have also contributed with a systematic review of platform competition. The focal point when discussing platform competition remains network externalities [65]. Some important works in which the relationship between competition and network externalities for platforms were established, are Katz and Shapiro [43] and Church and Gandal [21]. Later research problems often use these models as their foundation [65].

Price competition between platforms has been the most prevalent topic of the chicken-and-egg problem [14, 41]. Rochet and Tirole [66] show that the value of the network externality is essential for reaching optimal pricing. Armstrong [5] suggests that higher elasticity in demand when indirect network effects exist causes a loss of sales if prices are increased. Although, the willingness of users to join a platform increases as its network size increases [13]. This means that the platform with a larger network can have greater pricing power. When this happens, other platforms with a smaller network can decrease their pricing and make a place in the market [13]. Pricing strategies for a user group also depend on whether it is allowed to multi-home or single-home [5, 9, 33]. Overall, arms-length pricing is a key strategy implemented to attract different user groups to a platform [11].

Competition based on other forms of platform differentiation such as bundling, information, and diversification has also been discussed [65]. Amelio and Jullien [3] show that bundling by a monopoly platform benefits both sides of a two-sided platform, where one side gets higher utility, and the other gets higher externality. Duan et al. [26] show that the privacy concerns of users towards a platform collecting their personal information affect the pricing policies of the platform. Collecting consumer information is necessary for personalized advertising. Yan et al. [89] show that in a duopolistic competition, the question faced by platforms is not whether to collect information or not but whether to use the information or not. In terms of service information provided by the platform to its customers, the competition between sellers determines optimal service levels [37]. Casadesus-Masanell and Hervas-Drane [15] discuss a competition based on information disclosure. Internet firms achieve vertical differentiation by opting for different levels of information disclosure. This allows the coexistence of different pricing and information models. Jullien and Pavan [40] discuss information management policies and conclude that dispersed information weakens competition. Li and Zhu [47] discuss a game-theoretic model where platforms aim to avoid multi-homing by their users. Reducing information transparency reduced multi-homing and increased the industry-level product variety. De Corniére and Taylor [23] take a competition-in-utilities approach which allows accommodation of multiple variables. They use it to study the impact of data on platform competition. Markovich and Yehezkel [50] outline a coordination problem for users in platform competition: one platform is high-quality, and the other is low-quality. Different reasons why the user group would prefer the low-quality platform or the high-quality one have been discussed.

Several other variables for platform competition have been considered [65]. However, a lack of extensive discussion on problems concerning promotional activities exists. A two-sided platform competing with other platforms faces the same chicken and egg problem for its promotional budget allocation. Which user side to target first for more intensive promotion? Should a bigger promotion be done for the single-homing side, multi-homing side, or both? How much promotion should be done for one side until it becomes redundant? Does promotion targeted at one side benefit the competitor more, and when? Existing literature shows that answers to these questions are not obvious, especially under competitive scenarios. The discussion from next section onwards engages with some of the above questions.

The GNEP modeling approach adopted in this study can facilitate more elaborate analysis, yet its application is limited in marketing and operations [25]. On the other hand, the usefulness of GNEP as a modeling approach has become well recognized in economic sciences, engineering and computer science [27]. Early management applications of GNEP approach can be found in the areas of transportation and inventory control [1, 78, 93]. Anselmi et al. [4] utilized the GNEP approach to model the SaaS/PaaS cloud service provisioning problem and compared the equilibria with the social optimum. Recently, Nagurney and Dutta [59] modeled a supply chain competition among blood service organizations as a GNEP. A competition for medical supplies during the COVID-19 pandemic was modeled as a GNEP [60]. An analysis of the peer-to-peer electricity market was conducted by [45] using the GNEP approach. The GNEP approach is useful in modeling a problem whenever the players compete for a limited resource. The decision problem we present next requires platforms to share a scarce resource, the single-homing sellers.

3 A generalized Nash equilibrium model

Consider two two-sided platform firms that connect buyers and sellers in an undifferentiated products market. They compete with each other in a duopoly. These platforms (identified as P1 and P2, and in notations with superscripts ν and $-\nu$ respectively) have fixed promotional budgets (B^{ν} and $B^{-\nu}$) for the next planning horizon.

Each firm will use its budget for targeted promotion to add new customers and grow the number of sellers on its platform. The two activities would be independent of one another. That is, the effect of promotion targeted at new buyers does not impact new sellers directly. However, while potential customers of a firm get positively influenced by its promotion, higher promotion of the rival discourage them. It is assumed that the customer demand grows (or degrows) linearly based on the promotional effort of the two firms.

Let r^{ν} and $r^{-\nu}$ be their fixed marginal utilities. They exist in the problem as exogenous parameters. Both platform firms face similar decision problems of allocating resources. Let x^{ν} (resp. $x^{-\nu}$) be the proportion of its budget P1 (resp. P2) allocates to promotion aimed at potential customers. Consequently, $(1 - x^{\nu})$ (resp. $(1 - x^{-\nu})$) is allocated by P1 (resp. P2) to seller acquisition.

We consider the non-competitive setting first, where only of player (say, P1) is present. P1 aims to find an $0 \leq x^{\nu} \leq 1$ which maximizes its total utility. With the assumption that the addition of a new seller to the platform has a multiplier effect [58, 66], P1's objective in the absence of P2 will be to maximize the following total utility.

$$\pi^{\nu}(x^{\nu}) = r^{\nu}(N_0^{\nu} + \alpha^{\nu} x^{\nu} B^{\nu})(R_0^{\nu} + \gamma^{\nu}(1 - x^{\nu})B^{\nu}) \tag{1}$$

In (1), N_0^{ν} and R_0^{ν} are the counts of customers and sellers of the platform at the beginning of the planning period. α^{ν} and γ^{ν} are the sensitivity of new customers and sellers to promotion respectively. If the promotions are effective, α^{ν} and γ^{ν} will have higher values. These parameter values are estimated externally using statistical methods [20]. There is no limit on the number of new buyers that the platform can get during the planning horizon. The function in (1) is concave, and its maximum is obtained for,

$$x^{\nu *} = \frac{1}{2} - \frac{N_0^{\nu}\gamma^{\nu} - R_0^{\nu}\alpha^{\nu}}{2\gamma^{\nu}\alpha^{\nu}B^{\nu}} \tag{2}$$

All parameter values in (2) are non-negative. Therefore, $x^{\nu *} < \frac{1}{2}$ when $\frac{N_0^{\nu}}{\alpha^{\nu}} > \frac{R_0^{\nu}}{\gamma^{\nu}}$, i.e. more resources should be allocated to new seller acquisition if the ratio between the number of buyers already on the platform and the sensitivity of new customers to promotion is greater than the ratio between the initial count of sellers and the sensitivity of new sellers to promotion. If the inequality is reversed, more efforts should be made toward new buyers. One of the two groups will get a lesser allocation of promotional budget if the initial count of the group is high and its new members are less sensitive to the promotional effort.

When we maximize (1) as an unconstrained problem and apply the terminal conditions of $0 \le x^{\nu} \le 1$ to (2), the following relationship between budget and model parameters is obtained.

$$B^{\nu} > \left| \frac{N_0^{\nu}}{\alpha^{\nu}} - \frac{R_0^{\nu}}{\gamma^{\nu}} \right| \tag{3}$$

Consequently, condition (3) requires that the budget be higher than a threshold value for the optimal solution to lie in the interior of $0 \le x^{\nu} \le 1$. Otherwise, the firm should allocate the complete budget to one of the two activities. The same holds true for P2.

If $x^{\nu} = 1$, then P1 will employ the entire budget on promotion targeted at potential customers, while $x^{\nu} = 0$ would imply a complete focus on getting new sellers. In the duopoly competition model described below, it will be shown that focusing on either activity or focusing on one group will be beneficial under different circumstances. Let parameter β^{ν} be P1's sensitivity of losing customers. A higher value of β^{ν} implies that customers get discouraged from joining the platform of P1, due to the customer focused promotion of P2. P2 will face similar circumstances in the presence of P1 with $\beta^{-\nu}$.

We consider that the market has a limited number of sellers $\overline{R}$ available in the market. It is further assumed that sellers are not allowed multi-homing, whereas customers do not have such restrictions. So, both P1 and P2 have to compete for sellers because it is a scarce resource. Additionally, due to the risk of losing customers to their competitor through β^{ν} or $\beta^{-\nu}$, P1 and P2 need to make their allocation choices carefully.

3.1 Duopoly model

The platform P1 seeks to determine its optimal division of budget by solving the following optimization problem (MODELOP1).

$$\max \pi^{\nu}(x^{\nu}, x^{-\nu}) = r^{\nu}(N_0^{\nu} + \alpha^{\nu} x^{\nu} B^{\nu} - \beta^{\nu} x^{-\nu} B^{-\nu})(R_0^{\nu} + \gamma^{\nu}(1 - x^{\nu}) B^{\nu}) \tag{4}$$

subject to,

$$R_0^{\nu} + \gamma^{\nu}(1 - x^{\nu}) B^{\nu} + R_0^{-\nu} + \gamma^{-\nu}(1 - x^{-\nu}) B^{-\nu} \le \overline{R} \tag{5}$$

$$0 \le x^{\nu} \le 1 \tag{6}$$

In MODELOP1, the objective function (4) maximizes the total utility for P1, and constraint (5) restricts the total number of sellers in the market to $\overline{R}$. This constraint is common for both platforms. A similar optimization model (MODELOP2) can be written for P2. Observe that the objective function and the constraints of the optimization models have decision variables of both the players.

MODELOP1 and MODELOP2 belong to the class of game theory models called the generalized Nash equilibrium problem (GNEP) [27]. Below, we formally define a GNEP and discuss its solution.

Definition 1 The GNEP is a game with $N \ge 2$ players. Let, a player i has n_i number of decision variables. It finds the optimal values of the variables $y^i \in \mathbb{R}_i^n$ by solving the optimization problem,

$$\max_{y^i} \theta_i(y^i, y^{-i}) \text{ subject to } g^i(y^i, y^{-i}) \le 0 \tag{7}$$

Here $\theta^i : \mathbb{R}^n \to \mathbb{R}$; $g^y : \mathbb{R}^n \to \mathbb{R}^{m_i}$; $n = n_1 + \cdots + n_N$ the total number of variables and $m = m_1 + \cdots + m_N$ the total number of constraints. The vector $y^{-i} = (y^1, \cdots, y^{i-1}, y^{i+1}, \cdots, y^N$ stands for the vector that consists of all the decision vectors except player ith decision variables.

In a GNEP, both the utility function and the strategy (feasible) space of a player get affected by the decision of the other player [27]. For example, when both players share a limited resource. In the model above, both share a constraint on the single-homing sellers. In MODELOP1 (similarly in MODELOP2), the outcome depends on the allocation decisions of both P1 and P2. The objective (4) and constraint (5) contain both platforms' decision variables. In (4), P1's utility depends on the action $x^{-\nu}$ taken by P2. Moreover, due to (5), P1's solution space for x^{ν} is restricted to the choice of $(1 - x^{-\nu})$ made by P2. Therefore it is a GNEP, and we state it as the following lemma.

Lemma 1 *The problem defined through MODELOP*1 *and MODELOP*2 *is a GNEP with two players having one decision variable each.*

The *strategy space* for player i is a set of all feasible solutions of a GNEP optimization model. For the GNEP defined above the strategy space is,

$$Y^i(y^{-i}) = \{y^i | g^i(y^i, y^{-i}) \leq 0\} \tag{8}$$

Using this description of the strategy space, we quote the theorem [27] on the conditions for the existence of equilibrium solution for a GNEP.

Theorem 1 *Let $Y^i(y^{-i})$ be the strategy space for the player i in a GNEP as defined in definition 1. If for each player i, $Y^i(y^{-i})$ is nonempty, closed and convex and Y^i as a point to set mapping is both upper and lower semicontinuous; and $\theta_i(., y^{-i})$ is quasiconcave on $Y^i(y^{-i})$, then a generalized Nash equilibrium exists.*

It is easy to verify from the second derivative condition that, the objective function in (4) is concave and therefore is also quasiconcave. The constraint defined through (5) and (6) are affine linear and they satisfy the condition on continuity, also they enclose a convex space. Applying theorem 1, for MODELOP1 and MODELOP2, a point $\bar{x} = (\bar{x}^\nu, \bar{x}^{-\nu})$ can be found, such that no player can improve its objective by changing its strategy unilaterally from this point. Then $\bar{x}$ is an equilibrium solution to our GNEP. We state the result as a lemma below.

Lemma 2 *A generalized Nash equilibrium exists for the two-players GNEP defined through MODELOP1 and MODELOP2.*

3.2 Equilibrium analysis

The uniqueness of a generalized Nash equilibrium when it exists is yet to be established [16]. Even, it is often tedious to find equilibrium solutions for which multiple approaches have been suggested in the past [77]. They include among others, applications of Karush-Kuhn-Tucker (KKT) conditions, Quasi Variational Inequalities (QVI) and Nikaido-Isoda (NI) function [27]. Though, obtaining a solution is not guaranteed, the analysis of solution steps often produce useful insights. With this goal, we examine the KKT necessary conditions for equilibrium.

It is known that when any standard constraint qualification holds, the equilibrium solution for any player of a GNEP satisfies the KKT conditions [25]. MODELOP1 (also MODELOP2) have only linear inequality constraints, and therefore the *linear independence constraint qualification* holds. We derive below the KKT necessary conditions for optimality for MODELOP1. The Lagrangian associated with the optimization problem of player ν can be written as follows. For the sake of simplicity we have assumed, $r^\nu = 1$.

$$\begin{aligned}\mathcal{L} :&= (N_0^\nu + \alpha^\nu x^\nu B^\nu - \beta^\nu x^{-\nu} B^{-\nu})(R_0^\nu + \gamma^\nu (1 - x^\nu) B^\nu) \\ &+ \lambda_1^\nu (\overline{R} - (R_0^\nu + \gamma^\nu (1 - x^\nu) B^\nu + R_0^{-\nu} + \gamma^{-\nu} (1 - x^{-\nu}) B^{-\nu})) \\ &+ \lambda_2^\nu x^\nu + \lambda_3^\nu (1 - x^\nu)\end{aligned} \tag{9}$$

Where, λ_1^ν, λ_2^ν and λ_3^ν are Lagrange multipliers. The KKT conditions are derived as follows.

$$\alpha^\nu B^\nu(R_0^\nu + \gamma^\nu(1-x^\nu)B^\nu) - \gamma^\nu B^\nu(N_0^\nu + \alpha^\nu x^\nu B^\nu - \beta^\nu x^{-\nu} B^{-\nu}) + \lambda_1^\nu \gamma^\nu B^\nu + \lambda_2^\nu - \lambda_3^\nu = 0 \tag{10}$$

$$\lambda_1^\nu(\overline{R} - (R_0^\nu + \gamma^\nu(1-x^\nu)B^\nu + R_0^{-\nu} + \gamma^{-\nu}(1-x^{-\nu})B^{-\nu})) = 0 \tag{11}$$

$$\lambda_2^\nu x^\nu = 0 \tag{12}$$

$$\lambda_3^\nu(1-x^\nu) = 0 \tag{13}$$

$$x^\nu, \lambda_1^\nu, \lambda_2^\nu, \lambda_3^\nu \geq 0 \tag{14}$$

Similar conditions can be derived for platform $-\nu$. An analysis of the KKT optimality conditions lead to the following results, which are presented as propositions. The proof of the propositions are given in the appendix. In the relationships discussed below, the optimal strategy of the competitor is assumed to be known. In the next section, we will discuss the dynamics of the simultaneous changes in the equilibrium strategies of both players in consequence of changes in model parameters.

Proposition 1 $\overline{x}^\nu = 0$, *if* $\frac{1}{\alpha^\nu}(N_0^\nu - \beta^\nu \overline{x}^{-\nu} B^{-\nu}) > \frac{1}{\gamma^\nu}(R_0^\nu + \gamma^\nu B^\nu)$.

If no promotion is done for new customers, at the end of the planning period, P1 will have $(N_0^\nu - \beta \overline{x}^{-\nu} B^{-\nu})$ customers and $(R_0^\nu + \gamma^\nu B)$ sellers on its platform. The inequality indicates that the platform will concentrate on the growth of sellers when it already has a large customer base, and the number of new customers it can get is much smaller than the benefit from new seller addition. P1 will be more likely to increase the value of x^ν if P2 makes higher provisions towards customers; here competition forces a change in strategy.

Consider the example of *Sony*'s gaming platform *PlayStation*. The *PlayStation* and *PlayStation 2* consoles were household names [74, 75] when *Microsoft* entered the market with *Xbox* in 2001 [52, 53]. In different planning periods, *Sony* invested on exclusivity from gaming studios *Santa Monica Studio* and *Naughty Dog* by ensuring they turn into first-party sellers. This resulted in the *God of War*, *Crash Bandicoot* and *Uncharted* series of games being released exclusively on the *PlayStation* platform [55, 56]. In the long run, this investment towards seller acquisition for exclusive content resulted in increased sales.

Proposition 2 $\overline{x}^\nu = 1$, *if* $\frac{\alpha^\nu}{\gamma^\nu} R_0^\nu \geq N_0^\nu + \alpha^\nu B^\nu - \beta^\nu \overline{x}^{-\nu} B^{-\nu}$ *and* $\lambda_1^\nu = 0$.

If a platform has a large seller base, it can completely focus on getting new customers during the planning period. The relationship in the proposition conveys this intuitive strategy. The right-hand side of the inequality gives the count of customers at the end of the planning period. If the competitor spends more on customers, the incumbent may follow. These results may not hold when both platforms can capture all remaining sellers from the market. β^{v} is one of the three components present on the right-hand side of the inequality. It determines the extent to which the competitor's promotion for customers can damage a platform's prospect in attracting them. This direct effect has a major influence on the competitive strategies towards multi-homing customers, also distinctly observed in the numerical experiments presented later in this paper.

Gaming platforms *PlayStation*, *Xbox* and *Wii* keep on introducing technologically updated versions of their consoles in the market [87]. For instance, *Sony* released *PlayStation 3* after *PlayStation 2* had become obsolete. After a particular console becomes popular, such as the *PlayStation 3*, it is not very difficult to acquire non-exclusive content from third-party sellers (gaming studios). In such planning periods, the platform focuses on improving the customer sales of the current console through marketing efforts [83]. However, while focusing on marketing efforts it must be watchful of competing gaming platforms offering similar non-exclusive content and services to customers [83].

Proposition 3 (Interior point solution) $\overline{x}^{v} > \frac{1}{2}$, *if* $\frac{N_0^{v}}{\alpha^{v}} - \frac{R_0^{v}}{\gamma^{v}} < \frac{\beta^{v}}{\alpha^{v}} \overline{x}^{-v} B^{-v}$

This proposition is about the platform firm's need to allocate promotional resources to both new customers and new sellers. The left-hand side of the inequality is the difference between the two ratios $\frac{N_0^{v}}{\alpha^{v}}$ and $\frac{R_0^{v}}{\gamma^{v}}$. These terms can demonstrate the existing customer power and seller power, respectively, for P1 i.e. they indicate the relative position of strength of the platform with the respective groups. The right-hand side can be visualized as the competitor's power to wean away customers. The result in the proposition conveys that if the customer power of P1 is low, more effort should be targeted at new customers. The optimal strategy will be to focus on getting new sellers when the platform has lesser seller power. It is interesting to observe that inequality changes depending on the competitor's ability to dilute P1's customer power.

When a new console is about to be released by a gaming platform, it becomes necessary to attract both sellers (gaming studios) and customers [83, 87]. Since the content from the previous generation console is not supported on the new release, it is essential to keep the old sellers and gain new ones for producing updated or fresh content. Platforms also advertise to convince customers to shift to the new console and invest in better or new content [83]. Thus, they must invest in both seller and customer acquisitions in such scenarios. Existing customer power and seller power will determine the budget allocation by the platform towards the two groups.

Proposition 4 *When the marginal utility of seller market expansion is high, the platforms will prefer new seller acquisition over new customer addition.*

The proposition states that competition for seller acquisition becomes intense when there are limited sellers and business from new sellers can bring substantial value addition to the platforms. Allowing x^{v} to take any value in [0, 1], the optimality condition (10) can be written as $\alpha^\nu[R_0^\nu + \gamma^\nu(1-x^\nu)B^\nu] - \gamma^\nu[N_0^\nu + \alpha^\nu x^\nu B^\nu - \beta^\nu x^{-\nu}B^{-\nu}] = \lambda_1^\nu \gamma^\nu B^\nu$ i.e. at equilibrium α^ν[Total sellers] $-\gamma^\nu$[Total customers] $= \lambda_1^\nu \gamma^\nu B^\nu$. We can interpret the Lagrange multiplier λ_i^ν as the marginal utility of incremental addition to $\overline{R}$, i.e. the benefit from an incremental expansion in the seller's market. When the marginal utility is high, the platform will prefer having a larger number of sellers on the platform. The relationship, though, has a moderating influence from the sensitivity of the two groups towards the targeted marketing efforts of the platform.

Steam and *Epic Games* are the two largest video game digital distribution platforms for PC gaming. *Steam* attracted a large seller base by promoting its free API *Steamwork* to potential sellers [2]. The API allowed easy integration of within-game transaction and support functions into the *Steam* platform. Around 2008, sellers would choose *Steam* due to this service, allowing *Steam* to capture a seller even before game development had reached completion. Almost every competitor of *Steam* had to close shop [12]. On the other hand, *Epic Games* promoted itself to potential sellers as the platform which took a very small sales commission from its sellers [30]. This attracted a lot of sellers to *Epic Games* for its $5-12\%$ commission over *Steam*'s $25-30\%$. Due to the ease in purchasing PC games online as compared to that for consoles, an increasingly large seller base to choose from is a major factor in sales.

At equilibrium, when constraint (5) is not tight, the marginal utility of the seller market expansion is zero. It is intuitive and easy to verify then that, at equilibrium, the number of customers and the number of sellers that each platform acquires will solely be determined by the relative sensitivities of the two groups towards the targeted promotional campaigns. This is stated in the following proposition. Actually, it is the solution of the corresponding Nash equilibrium problem (NEP) of the GNEP.

Proposition 5 *When there is no limit on the number of sellers, at equilibrium, the ratio of the number of customers and number of sellers of platform* ν (*resp.* $-\nu$) *is equal to* $\frac{\alpha^\nu}{\gamma^\nu}\left(resp. \frac{\alpha^{-\nu}}{\gamma^{-\nu}}\right)$.

Here, both the platforms will pursue independent strategies such that both maximize their respective utilities. They will invest the most in that group which responds to their campaign better.

Proposition 6 $\overline{x}^\nu \geq x^{\nu *}$, *if* $\lambda_3^\nu = 0$.

$\lambda_3^\nu = 0$ means that $x^\nu \geq 0$ and $\overline{x}^\nu \geq x^{\nu *}$. Thus, budget allocation towards customers in competition cannot be lower than when the platform is in a monopoly (with identical parameter values). This is because, in a monopoly, $x^{\nu *}$ is the optimal allocation for seller acquisition. Any deviation from this adversely affects the maximum utility. During competition, a platform would not wish to increase sellers beyond the optimal allocation. On the other hand, it may require to give more attention to customers otherwise it can lose some of them to its competitor.

4 Solution algorithm and model analysis

Finding solution of a GNEP is not easy. Yet some models due to their structure can be solved relatively more quickly. We adopt the following '*Nonlinear Jacobi-type*' method [27] for finding optimal solutions for MODELOP1 and MODELOP2.

Algorithm

Step 0 Seed the decision variables with starting solution, $x^0 = (x^{\nu 0}, x^{-\nu 0})$

Step 1 Let $x^k = (x^{\nu k}, x^{-\nu k})$ be the solution in step k. If $x^k = x^{k-1}$ STOP. Otherwise go to step 2.

Step 2 Compute the solution

$$x^{\nu k+1} \text{ of } \max_{x^\nu} \pi^\nu(x^\nu, x^{-\nu k}) \text{ subject to } x^\nu \in X_\nu(x^{-\nu k})$$

$$\text{and } x^{-\nu k+1} \text{ of } \max_{x^{-\nu}} \pi^{-\nu}(x^{\nu k}, x^{-\nu}) \text{ subject to } x^{-\nu} \in X_{-\nu}(x^{\nu k})$$

Step 3 Set $x^{k+1} = (x^{\nu k+1}, x^{-\nu k+1})$ and go to step 1.

It is not guaranteed that the above algorithm would converge to a unique solution. But it can be proved that if the sequence $\{x^k\}$ generated in the algorithm converges to $\bar{x} = (\bar{x}^\nu, \bar{x}^{-\nu})$, then $\bar{x}$ is a Nash equilibrium [27]. We implemented the algorithm on simulated data with a wide range of parameter values. Values of α, β, γ, N_0 and R_0 for both platforms were varied assuming normal and uniform distributions. The equilibriums were found in less than eleven iterations in all cases. The elapsed times were within 10 seconds (convergence criteria 10^{-8}) in a personal computer having a 2.40 GHz processor and 8 GB memory. A summary of performance, values of parameters, and decision variables are given in table 1. The choice of seed impacted the number of iterations, but we could not identify a definite pattern.

4.1 Numerical experiments

We conducted experiments in a simulated environment to understand the effect of model parameters on equilibrium solutions. As expected, changes in one player's parameter values influenced the optimal strategies of both. This section deliberates on the key observations from the experiments. We created multiple problem instances by randomly generating values of α, β, γ, N_0, and R_0 of the platforms assuming them to be random variates of uniform and normal distributions. Values of B^ν and $B^{-\nu}$ were selected in such a way that they satisfied (3). The values of $\bar{x}^\nu$ and $\bar{x}^{-\nu}$ varied between $0 \leq x \leq 1$. The propositions **1** to **5** could be verified from the results.

Table 1 Summary of numerical experiments

Criteria		Elapsed Runtime (Seconds)	Iterations	x^{ν}	$x^{-\nu}$	**Parameter**	**Parameter**
Normal distribution							
						α^{ν}	$\alpha^{-\nu}$
Effect of α	Min.	4.445596	5	0.5553	0.554	0.0004373	0.0002135
	Mean	–	6.28375	0.562875	0.5627	0.001967325	0.002010325
	Max.	5.905565	9	0.5917	0.6381	0.003511	0.00352
						β^{ν}	$\beta^{-\nu}$
Effect of β	Min.	5.763071	6	0.5614	0.5617	0.00001723	0.00001933
	Mean	–	7.13	0.56615	0.56605	0.000050515	0.000050065
	Max.	8.88816	10	0.5704	0.5697	0.00007552	0.00007778
						γ^{ν}	$\gamma^{-\nu}$
Effect of γ	Min.	6.388851	6	0.5444	0.5424	0.00001823	0.00002992
	Mean	–	7.09	0.56905	0.5665	0.000049775	0.000050575
	Max.	8.374883	9	0.6721	0.6088	0.00007215	0.0000769
						N_0^{ν}	$N_0^{-\nu}$
Effect of N_0	Min.	3.336151	5	0	0	38.47	69.52
	Mean	–	6.191	0.48194	0.4794	587.266	562.016
	Max.	5.573098	10	1	1	2161	2149
						R_0^{ν}	$R_0^{-\nu}$
Effect of R_0	Min.	3.738721	5	0	0	14.2	14.51
	Mean	–	5.682	0.628774	0.623154	30.264	29.94
	Max.	4.677518	9	1	1	47.43	46.33
Uniform distribution							
						α^{ν}	$\alpha^{-\nu}$
Effect of α	Min.	3.98447	5	0.5437	0.5411	0.0001175	0.000104
	Mean	–	6.2	0.564025	0.563475	0.002079725	0.002052875
	Max.	4.931846	9	0.6872	0.7807	0.003995	0.003991
						β^{ν}	$\beta^{-\nu}$
Effect of β	Min.	4.795599	6	0.5604	0.5604	0.00001004	0.00001031
	Mean	–	6.8875	0.56585	0.56615	0.000048365	0.000049655
	Max.	4.991652	9	0.5736	0.5749	0.00008986	0.00008995
						γ^{ν}	$\gamma^{-\nu}$
Effect of γ	Min.	4.649394	6	0.5351	0.5349	0.00001016	0.00001032
	Mean	–	6.91	0.59135	0.5757	0.00004885	0.000051055
	Max.	4.818142	9	0.8	0.8012	0.00008983	0.00008985
						N_0^{ν}	$N_0^{-\nu}$
Effect of N_0	Min.	3.404367	5	0	0	2	2
	Mean	–	6.212	0.509756	0.5114	509.428	506.444
	Max.	5.183514	9	1	1	3473.3	3484.7
						R_0^{ν}	$R_0^{-\nu}$

Table 1 (continued)

Criteria		Elapsed Runtime (Seconds)	Iterations	x^{ν}	$x^{-\nu}$	**Parameter**	**Parameter**
Effect of R_0	Min.	3.682262	5	0	0	5.057	5.025
	Mean	–	5.481	0.599378	0.607538	29.687	30.574
	Max.	4.539258	8	1	1	54.95	54.837

4.1.1 Competitive chicken-egg problem and Nash equilibrium

The *chicken and egg problem* presents compelling arguments for favoring either of the two groups. A platform can acquire many new customers by investing entirely in them. The single-homing sellers would prefer the platform with a larger customer network. This would force the competitor to spend more on acquiring new customers and focus less on sellers initially. However, customers can multi-home, so they can choose both platforms [14]. On the other hand, focusing entirely on seller acquisition will lead to a larger share of single-homing sellers. This would automatically attract more customers to the platform. Gaining more sellers can block the competitor's growth. However, it would be difficult to convince sellers to choose a platform without enough customers. The sellers can choose the competitor's platform for its large customer network even if it is not focused on sellers.

Effect of $\overline{R}$ *and single-homing of sellers* The limited number of sellers are restricted to single-home. Monopolization of sellers by a platform can occur, deterring the growth of its competitor [68]. Therefore, both platforms will attempt to acquire at least a critical number of sellers regardless of the elasticities of their parameters. While doing so, they will also lose some of their competitive power to wean away customers from their competitor.

Relationship of β *and utility* The platforms also have the opportunity to damage customer base of their competitor through β. When the players are unevenly matched, β allows the player's survival with a smaller resource. However, deterring the growth of the competitor does not always provide a competitive edge [41]. Understanding how the competitor will respond to diminished value from negative indirect network effects is important.

Nash Equilibrium GNE (and NE) models are useful to formulate problems where the competitor's response needs to be predicted. We showed that Nash equilibrium $\overline{x}$ exists for the promotional resources allocation GNEP model. Deciding on alignment with Nash equilibrium allows the players to avoid indirect network effects which can hurt them. The knowledge that a unique Nash equilibrium exists has practical utility as firms can pursue a clear decision path [39]. However, even if the existence of a unique Nash equilibrium is questionable, computing a Nash equilibrium is useful in understanding the competitive

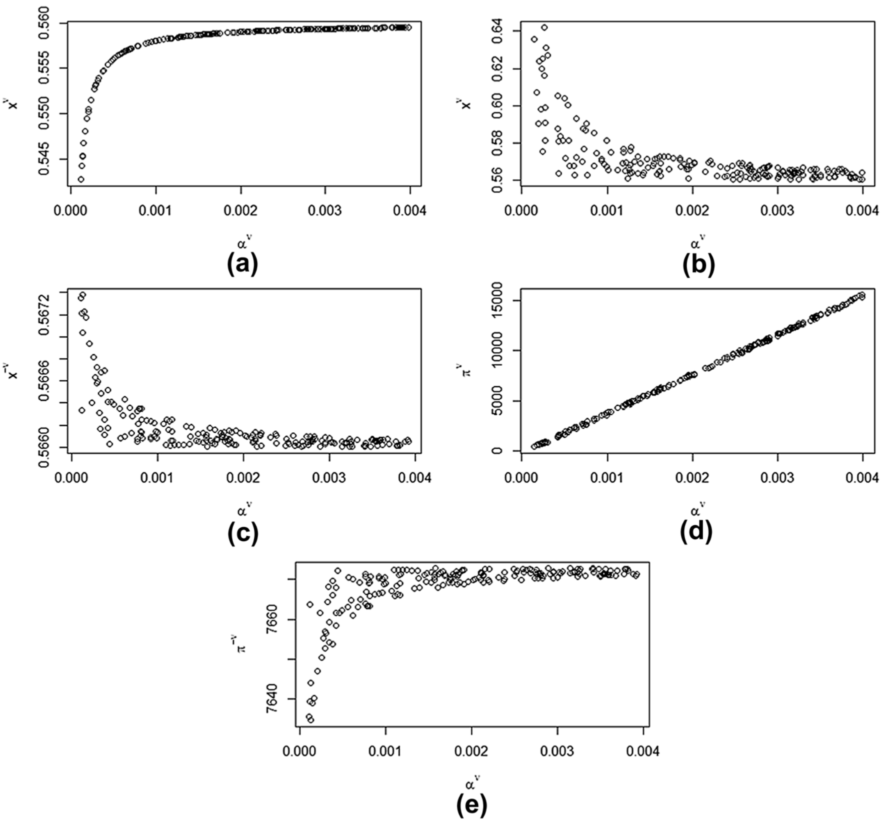

Fig. 1 Effect of α

game at play. If the strategy space, one's utility function, and rationality of both players are assessed accurately, the predictions a player can make are useful [31]. The numerical experiments break down how different parameters impact the equilibrium position of the players. The results outline what can be expected from the competitor in various situations and how to respond to it. Even if the competitor does not comply with its assumed rationality, the best responses can help design a realistic course of action. The numerical experiments suggest that the players prefer a mixture of chicken and egg.

4.1.2 Effect of α

If customer acquisition becomes more profitable due to increased α, the platform increases its focus on acquiring new customers. If the value of β is negligible, the net effect of α and β is influenced by the value of α. Unhindered customer acquisition is profitable in this scenario. So, the player increases x as α increases (Fig. 1a). Consequently, its utility π also increases.

Relationship of α with β As the value of β becomes significant, the equilibrium strategy differs completely from above. An influential β value can lead to an increased focus on seller acquisition even as α increases. Suppose, P1 increases x^v motivated by a high α^v. This also increases P1's power to damage P2's promotion for new customers. P2 would respond to this loss of customers by increasing x^{-v}. The response will increase P2's power to destroy the value of P1 as well. Therefore, it would not be beneficial for P1 to increase focus on customer acquisition despite a high α^v. However, an increased α^v provides P1 improved efficiency, as higher α^v requires lesser investment to maintain the same number of customers. If further customer acquisition is not beneficial, P1 uses this surplus towards seller acquisition. Therefore, we see instances where a high β^v causes an increase in investment towards seller acquisition. Figure 1b demonstrates the decreasing x^v as α^v increases. Figure 1d shows the increasing π^v as α^v increases and x^v decreases. We see that an increased α can benefit both sides for the same platform.

Effect of α on competition For a large α^v, P1 enjoys higher efficiency while promoting to customers. The surplus can be invested in customer acquisition or seller acquisition depending on the net effect of β^v and α^v. As P1 changes x^v, P2 responds by readjusting x^{-v}. When P1 increases x^v and P2 increases x^{-v} as a response, π^v increases because of α^v. P2 can face a negative effect on its π^{-v}. This occurs because higher promotion also activates a negative cross-platform effect. So, P2 can lose more customers when it is forced to adopt a retaliatory strategy.

As discussed earlier, P1 with a large α^v would move towards seller acquisition if it sees a decline in utility from increasing its x^v. By decreasing x^v, P1 wants P2 to respond with a decrease in x^{-v}. P1 loses some of its power to discourage new customer enlistment with P2. Due to this, P2 needs to invest less in customer acquisition to maintain its intended position. P2 can invest this surplus either in customer acquisition or seller acquisition to maximize π^{-v}. To adjust with P2's choice, P1 can also change its initial decision. If the decrease in x^v succeeds in decreasing x^{-v}, both P1 and P2 improve their utility, as seen in Fig. 1c and e. The graphs show that as α^v increases, P2 decreases x^{-v} and improves its π^{-v}. We see that one platform's increased customer sensitivity to promotion affects the decision-making of both players. It is also possible that both players benefit from the situation even while competing. Thus, we see that when α^v increases, P1 improves its utility in a Nash equilibrium. If P2 poses a large enough threat to destroy the value of P1, then P2 can also enjoy an improvement in utility in the Nash equilibrium resulting from an increased α^v.

4.1.3 Effect of β

An increased β improves the competitor's power to impact the value of a player's customer side adversely. As the rate of customer loss increases, the player would want to accelerate customer acquisition. If so, the value of x increases when β increases. When the effect of α dominates β, π increases to an increasing x. Fig. 2a and b demonstrate the effect of β on its player's x and π values when α dominates the overall effect. However, if β dominates the net effect, π may decline even after

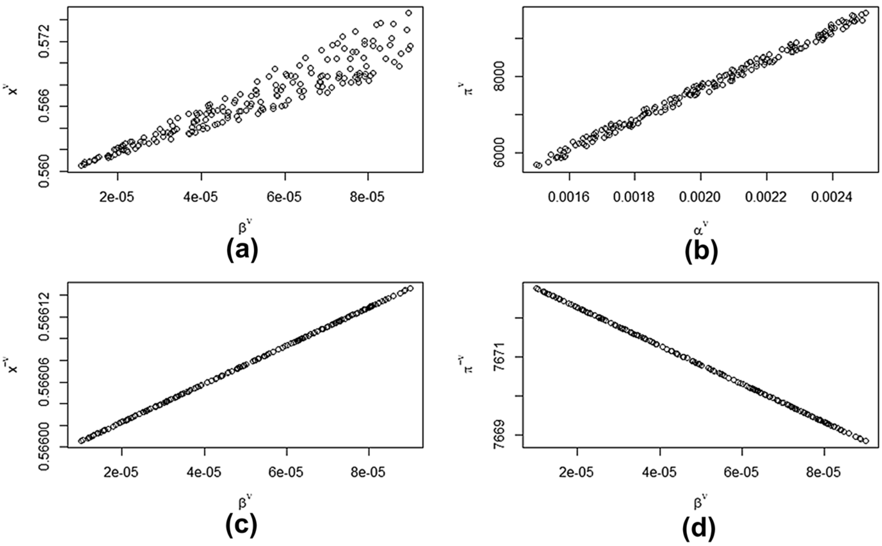

Fig. 2 Effect of β

increasing x due to the net loss in the number of customers. If increasing x with the current α can acquire enough customers, the player will continue increasing x. Otherwise, the player would prefer to shift towards seller acquisition. This forces the competitor to rethink its decision.

Effect of β on competition P1 can increase x^ν when facing an increasing β^ν. If so, P2 would need to increase $x^{-\nu}$ to cover the resulting loss of customers. In a Nash equilibrium influenced by β, players would have to choose the better direction between improving one's seller acquisition and destroying competitor's customer acquisition. Both P1 and P2 will lose utility if they end up increasing x^ν and $x^{-\nu}$ due to an increasing β^ν. We discussed earlier how P1 and P2 could both improve their utility by shifting towards seller acquisition in such cases. However, this will not always work. They can be worse off if they shift to seller acquisition without covering the loss of customers from decreasing x^ν and $x^{-\nu}$ when β^ν is increasing. The players would prefer losing less utility by focusing on customer acquisition if preferring seller acquisition makes them worse off. An increased β is supposed to help the competitor damage customer value from the platform. However, the platform's response can cause both of them to lose utility. P1 gets negatively affected due to P2 destroying its customer value. While doing this, P2 faces a negative effect on its utility due to P1's response. Fig. 2c and d show how an increasing β^ν causes P2 to lose utility value despite increasing $x^{-\nu}$. Thus, we see that β of one player can negatively affect both players.

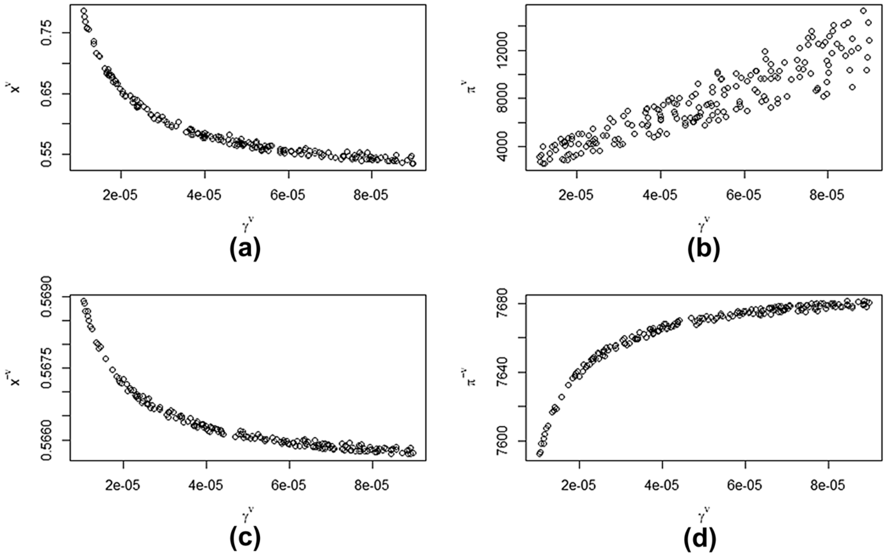

Fig. 3 Effect of γ

4.1.4 Effect of γ

The effect of γ on the platform's decision-making is straightforward. As γ increases, the platform moves towards seller acquisition if doing so improves utility. This is seen in Fig. 3a and b. The platform will remain at its initial position if changing its x provides no improvement in utility. A higher γ on its own can help achieve a preferred utility. Also, the competitor cannot respond to an increasing γ as it could do with α. We observe that despite the single-homing of sellers, a platform would not always move towards monopolization in a Nash equilibrium.

Effect of γ *on competition* We observe that when P1 can improve its utility due to γ^{ν}, P2 can also improve its utility in the resulting Nash equilibrium. As γ^{ν} increases, P1 is expected to decrease x^{ν}. If the net effect of this exchange makes P1 worse off from the loss of customers, P1 will not decrease x^{ν}. P2 can also increase its focus on seller acquisition, as it has to worry less about the impact of $\beta^{-\nu}$. Both will face smaller losses of customers as they move together towards seller acquisition. In Fig. 3c and d, we see that utility of P2 increases by increasing seller acquisition even though the γ of its competitor has improved.

4.1.5 Effect of N_0 and R_0

The benefit of higher initial customers and initial sellers can be observed from Figs. 4 to 5, respectively. When a platform holds a dominant position in either of the sections, the focus of its promotion moves towards the other. The values of N_0 and R_0 also influence how a platform reacts to the values of α, β, and γ. For instance,

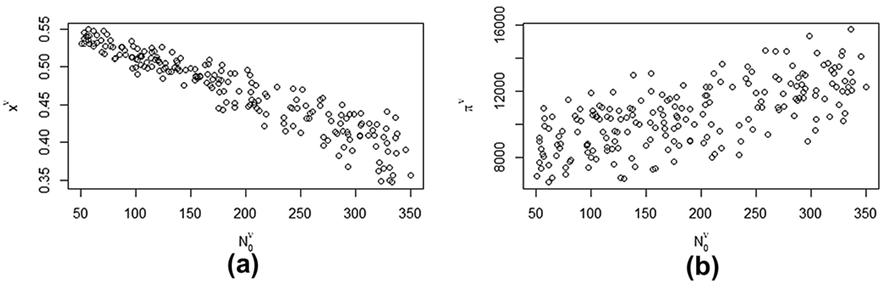

Fig. 4 Effect of N_0

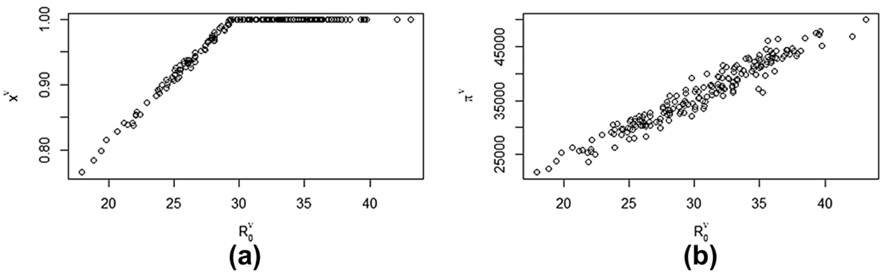

Fig. 5 Effect of R_0

say a platform has a very high R_0. It would be less sensitive towards an increasing γ because it wants to focus more on acquiring new customers.

4.1.6 Cross platform impact of P1 and P2's parameters

Parameter values of one platform can have complementary and contradictory impacts on its competitor's parameter values. For instance, increasing β^{ν} requires P1 and P2 to increase x^{ν} and $x^{-\nu}$. At the same time, say increasing $\alpha^{-\nu}$ requires P1 and P2 to decrease x^{ν} and $x^{-\nu}$. Then, β^{ν} will slow down the effect of $\alpha^{-\nu}$ and vice versa. The actual direction of the investments will depend on which parameter overwhelms the effect of the other. Similarly, x^{ν} and $x^{-\nu}$ will change faster if the parameters of P1 and parameters of P2 require them to move in the same direction. The spread of values across the trends in the graphs shows this cross-platform impact.

5 Theoretical and managerial implications

A rational player is expected to use only those strategies that are best responses to the competitor's strategies. This choice of strategy is based on the beliefs the player has about its competitor [63]. It is not always possible to assess a competitor's rationality beforehand in real-life situations. One way to do so is by analyzing competitors' behaviour in previous planning periods as in repeated games. Another way is to make assumptions about the behaviour of the competitor. It would be sensible to assume that the competitor aims to maximize its utility. A Nash equilibrium will be reached in real life if both platforms can predict this correctly [31].

Even if rationality is correctly assessed, it is still difficult to assess the equilibrium both platforms are predicting in cases where multiple Nash equilibria exist. Schelling's [71] theory of focal points suggests that platforms may be able to coordinate on a particular equilibrium. Focalness here depends on the platform's culture and past experiences. Coordination occurs if platforms interact with each other before the game [31]. However, Aumann [6] argues that interacting beforehand does not guarantee which strategy will eventually be played. So, a platform has two predictions to make in an uncoordinated real-life situation. One is to predict whether a competitor will play a best-response. The second is predicting which best-response will be played by the competitor.

Another point to note is that a game depends on more information than is presented in its model. For instance, utility in our model would not translate to profits in real life if sufficient customers are not acquired. Also, the cost for capacity enhancement may be irreversible while the promotional activities for customer acquisition can be reversed. The experience of platforms, their ability to make mistakes, their biases, and their cultural preferences are all examples of information that gets abstracted away from a game [31]. These factors become pertinent when devising the strategy. From this perspective, we discuss the managerial implications of the numerical results.

First, consider the situation when parameter values and other requirements can be assessed accurately such that a practical unique Nash Equilibrium is reached. A course of action should be devised using the equilibrium strategy. Relevant information from outside the game should also be considered while doing so. Second, consider situations when uniqueness does not exist (or cannot be proved) or an irrational competitor exists. A general best-response strategy to cover possible best responses from the competitor should be devised in such scenarios.

Best Response Strategy Acquire critical number of sellers. Allocate the remaining budget for customer acquisition.

The experiments showed that strategies of complete one-side focus are dominated by budget allocation strategies towards both customers and sellers. The complete one-side focus was only possible when the platform had already made substantial gains on the other side. We make arguments on how the proposed strategy would succeed in most cases. First, we define the critical number of sellers.

Critical Number of Sellers The platform should have a seller network such that demand from its existing customers N_0 can be satisfied. If N_0 is very high, the experiments show that the platform moves towards higher seller acquisition. We observed that increasing γ did not always lead to an increased investment on the seller-side. We concluded that the platform already has the critical number of sellers in such situations. The numerical results show that both platforms always aim to acquire some critical number of sellers in a Nash equilibrium. Only cases with a very high R_0 value have $\overline{x} = 1$, indicating that R_0 has reached the critical value in such scenarios.

Let R^ν_C be the critical number of sellers. From proposition (2), we know that when $\overline{x}^\nu = 1$, P1 does not need to acquire more sellers regardless of what P2 does. Note that, this is the situation where the player possesses the critical number of sellers and may not need to acquire more during the planning period.. We have $\frac{\alpha^\nu}{\gamma^\nu} R^\nu_0 \geq N^\nu_0 + \alpha^\nu B^\nu - \beta^\nu \overline{x}^{-\nu} B^{-\nu}$ from proposition (2). We can say, $R^\nu_0 \geq \frac{\gamma^\nu}{\alpha^\nu}(N^\nu_0 + \alpha^\nu B^\nu - \beta^\nu \overline{x}^{-\nu} B^{-\nu})$. Here, $R^\nu_C = R^\nu_0$. Therefore, critical number of sellers can be determined from the following relationship.

$$R^\nu_C \geq \frac{\gamma^\nu}{\alpha^\nu}(N^\nu_0 + \alpha^\nu B^\nu - \beta^\nu \overline{x}^{-\nu} B^{-\nu})$$

Now consider $\overline{x}^{-\nu} = 0$, i.e. P2 needs to increase the number of sellers on its platform. This is the condition when R^ν_C is maximum i.e. $R^\nu_C \geq \frac{\gamma^\nu}{\alpha^\nu}(N^\nu_0 + \alpha^\nu B^\nu)$.

Argument 1 The risk of monopolization by the competitor is high for the side which can single-home [41]. When there are a limited number of sellers, acquiring a critical number is necessary to stay afloat and meet customers' demands. Cabral [13] argues that dominance is temporary when monopolization cannot occur. So, focusing on stopping monopolization as a response would work better than monopolization or complete disregard for the monopolization threat.

Argument 2 Even if a platform monopolizes seller-side, the position of the monopolist can still be contested [41]. For instance, sellers can eventually switch if the competitor reduces switching costs and offers a better customer network. Moreover, Caillaud and Jullien [14] have discussed how monopolizing a single-homing side can lead to profit depletion. The platform usually refrains from focussing entirely on the seller-side in the experiments unless they have substantial initial customers.

Argument 3 There is no utility benefit to a platform when its promotion reduces the competitor's customer base through β. Customers can multi-home, so there is no direct competition for them [41]. Thus, a complete focus on forming a large customer network and influencing sellers through network benefits will not always work. However, customer-network of one platform has a negative effect on the other platform's customer-network due to the relationship between α and β. Increasing x for customer acquisition also increases the loss of competitor's customers and vice versa. Investing part of the budget in customer acquisition would be essential to minimise this loss of customers.

Argument 4 New sellers may fear competition from other sellers on the platform [41]. Going beyond a critical number of sellers can lead to congestion. This would cause some sellers to eventually move to the competitor's platform [42]. Single-homing sellers can prefer to go to a smaller platform to escape intense competition even if it comes at the cost of reduced demand [42]. Thus, a platform needs to acquire customers to provide business to existing sellers. The competitor can respond in two ways. If the competitor focuses on sellers, the platform has the opportunity to increase customer-network without the threat of competitors destroying its value. If the competitor focuses on customers, then the platform has no choice but to cover the increasing loss in utility by acquiring more customers. In both cases, having acquired a critical number of sellers, investing in customer acquisition would be useful if competitor's response cannot be predicted.

5.1 *Uber* India case

We present here a caselet as an illustration of the results presented above. When ride-sharing platform *Uber* entered India, it faced competition from local rival *Ola* and had to invest 1 billion dollars to gain market share. During this period, both *Uber* and *Ola* had strict policies requiring their drivers to not multi-home with both platforms. This tactic is common with ride-sharing platforms [88]. To attract drivers away from *Ola*, *Uber* India offered *'unprecedented incentives'* to drivers [18]. This was because a sufficient volume of available vehicles at that time was deemed necessary by *Uber* India to attract customers. After reaching a critical number of drivers, the company switched its incentive plan to a less costly one and started focussing more on customer acquisition [44].

To counter the growing market share of *Uber* India, *Ola* invested in advertising campaigns for customer acquisition. One of them backfired. The campaign turned into an internet meme praising *Uber* India instead [29]. Although *Ola* invested in the campaign to acquire customers, the benefit was reaped by *Uber* India. We observe that expenditure on customer acquisition will not always work. However, *Ola* retained its competitive edge even after expanding into other ventures such as e-bikes. While *Uber* claims to have a 50% share of the Indian market [72, 73], it required heavy expenses to do so. In terms of revenue share, *Ola* still has almost 75% of the market [82]. We see that despite *Uber* India spending huge amounts of money to monopolize the Indian market, it is finding it difficult to tip the market for its benefit. An important point to note here is that among other things, currently, *Ola* offers a better reward system to its drivers [44]. This will be more attractive during retention of existing drivers and acquisition of new ones. The winning strategy for the players should include relative evaluation during each planning period of the number of drivers and customers on their respective platforms. The results should be used in optimal allocation of resources as posited in this article.

5.2 Conclusion

This paper studied a budget allocation problem of two-sided platforms for attracting sellers and buyers in a competitive scenario. The problem was conceptualized as a GNEP, and an algorithm was suggested to solve it. The solution algorithm was effective in solving for Nash equilibrium. Beyond the convergence to Nash equilibrium, the results from this endeavour provided useful insights into how different parameters impact the behaviour of the platform and its competitor. This helped deconstruct the overall problem and analyze it scenario-wise. A seller network was found essential to be successful. Both platforms strive to ensure a critical number of sellers. However, the platforms avoided complete focus on sellers despite the threat of monopolization. Cross-platform effects were observed on the customer network. This caused the platforms to focus on customers from the beginning. We observed the existence of Nash equilibrium situations where both players benefited from the improved efficiency of one player. This occurred when platforms minimized the negative cross-platform effect on their customer networks by increasing seller acquisition together. We also discussed managerial insights that can help in real-life situations. A best-response strategy of acquiring critical sellers and investing remaining budget on customer acquisition works well for most situations. This strategy also tackles the difficulty in confirming the uniqueness of the Nash equilibrium.

The GNE model demonstrates with ease how both platforms are affected by the decision-making of the other. Due to competition, both shift their focus between sellers and customers depending on what their competitor is doing. A platform can effectively strategise the allocation of its funds using insights from the model. This study has a few limitations. Cross-platform effects have been successfully captured. However, the role of direct network effects on the platform's decision-making was not considered. Platforms at some stage would expect acceleration in customer growth from inter-personal communication powered by organic growth due to promotions. It can devise campaigns, especially for the imitators. Also, promotional activities towards buyers and sellers are recognizably different and often independent. Also, they have variable cost structures, which we have ignored. Another interesting area of inquiry would be to study the effect of budget allocation on the model parameters.

Appendix A

Proof of Proposition 1 For the KKT conditions and assumptions of the proposition, $\alpha^\nu B^\nu(R_0^\nu + \gamma^\nu B^\nu) - \gamma^\nu B^\nu(N_0^\nu - \beta^\nu x^{-\nu} B^{-\nu}) + \lambda_1^\nu \gamma^\nu B^\nu + \lambda_2^\nu = 0$ i.e. $\lambda_1^\nu \gamma^\nu B^\nu + \lambda_2^\nu = B^\nu\left[\gamma^\nu(N_0^\nu - \beta^\nu x^{-\nu} B^{-\nu}) - \alpha^\nu(R_0^\nu + \gamma^\nu B^\nu)\right]$. As the LHS of the equation is non-negative, the result follows.

Proof of Proposition 2 Following steps as above we get, $\lambda_1^\nu \gamma^\nu B^\nu - \lambda_3^\nu = R_0^\nu \alpha^\nu B^\nu - \gamma^\nu B^\nu(N_0^\nu + \alpha^\nu B^\nu - \beta^\nu x^{-\nu} B^{-\nu})$. No inference can be drawn when $\lambda_1^\nu > 0$. The result follows when $\lambda_1^\nu = 0$.

Proof of Proposition 3 On substitution of $\lambda_2^\nu = \lambda_3^\nu = 0$, in first order condition (10) we get, $\alpha^\nu B^\nu(R_0^\nu + \gamma^\nu(1-x^\nu)B^\nu) - \gamma^\nu B^\nu(N_0^\nu + \alpha^\nu x^\nu B^\nu - \beta^\nu x^{-\nu} B^{-\nu}) + \lambda_1^\nu \gamma^\nu B^\nu = 0$.

As $\lambda_1^\nu \geq 0$, $x^\nu \geq \frac{1}{2} + \frac{R_0^\nu \alpha^\nu + \beta^\nu \gamma^\nu x^{-\nu} B^{-\nu} - N_0^\nu \gamma^\nu}{2\gamma^\nu \alpha^\nu B^\nu}$. The result follows as all parameter values present in the expression are positive. Also, when $\lambda_1^\nu = 0$,$x^\nu = 1 - \frac{\overline{R} - (R_0^\nu + R_0^{-\nu}) - \gamma^{-\nu}(1 - x^{-\nu} B^{-\nu})}{\gamma^\nu B^\nu}$.

Proof of Proposition 6 On substitution of $\lambda_3^\nu = 0$, in first order condition (10) we get $\lambda_1^\nu \gamma^\nu B^\nu + \lambda_2^\nu = B^\nu\left[\gamma^\nu(N_0^\nu + \alpha^\nu x^\nu B^\nu - \beta^\nu x^{-\nu} B^{-\nu}) - \alpha^\nu(R_0^\nu + \gamma^\nu(1 - x^\nu)B^\nu)\right]$. As the LHS is non-negative, this gives $x^{-\nu} \leq \frac{\alpha^\nu}{\beta^\nu B^{-\nu}}[(\frac{N_0^\nu}{\alpha^\nu} - \frac{R_0^\nu}{\gamma^\nu}) + B^\nu(2x^\nu - 1)]$. Since $x^{-\nu} \geq 0$, we have $(\frac{N_0^\nu}{\alpha^\nu} - \frac{R_0^\nu}{\gamma^\nu}) + B^\nu(2x^\nu - 1) \geq 0$. Thus, $x^\nu \geq \frac{1}{2} - \frac{N_0^\nu\gamma^\nu - R_0^\nu\alpha^\nu}{2\gamma^\nu\alpha^\nu B^\nu}$. From (2), $x^{\nu*} = \frac{1}{2} - \frac{N_0^\nu\gamma^\nu - R_0^\nu\alpha^\nu}{2\gamma^\nu\alpha^\nu B^\nu}$. The result follows.

Appendix B Acronym and notations

Term	Definition	Term	Definition
NEP	Nash equilibrium problem	GNEP	Generalized Nash equilibrium problem
ν and $-\nu$	Specifies the Platforms P1 and P2 respectively in the notations	MODELOP1 and MODELOP2	Optimization problem of P1 and P2 respectively
B^ν	Promotional budget of P1	r^ν	Fixed marginal utility of P1
x^ν	Decision variable, i.e. proportion of B^ν to be allocated for customer acquisition	$(1 - x^\nu)$	Proportion of B^ν to be allocated for seller acquisition
N_0^ν	Number of customers of P1 at beginning of the planning period	R_0^ν	Number of sellers of P1 at beginning of the planning period
α^ν	Sensitivity of new customers towards P1's promotion	β^ν	Cross-platform external effect for P1
γ^ν	Sensitivity of new sellers towards P1's promotion	$\overline{R}$	Total number of sellers in the market
π^ν	Objective Function / Utility Function of P1	$\overline{x}$	Nash equilibrium profile of GNEP
$\overline{x}^\nu$	P1's action in $\overline{x}$	$\overline{x}^{-\nu}$	P2's action in $\overline{x}$

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Declarations

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