# Online sequential bundling: profit analysis and practice 

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Accepted: 14 December 2020 / Published online: 5 January 2021
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#### Abstract

We investigate monopoly profit under a new online tying strategy, namely sequential bundling. This method allows customers to buy additional products at discounted prices immediately after purchasing one or some of the available products. This strategy has been practiced by Walmart and others but has not been modelled to date. We use microeconomics analysis to conduct a comparison of the gained profit with the three commonly used bundling strategies: no bundling, pure bundling and mixed bundling. The main result shows that the sequential bundling strategy yields higher profits in comparison to all three classic strategies. In particular, for the symmetric cost case, our model provides a useful tool for practitioners such as common online shops.


Keywords Tying and bundling • Price discrimination $\cdot$ Sequential bundling • No bundling • Pure bundling • Mixed bundling

## 1 Introduction

The field of impulse buying has attracted the attention of researchers and practitioners for the past sixty years [18]; within this field, bundling is a very popular salespromotion tool [32]. Indeed, Holzweber [14] argued that tying (synonymic bundling) has been significantly modified doctrine of competition law in the digital age.

Tying refers to a situation where some of the products in the package may be bought individually, whereas bundling refers to the situation in which products can be purchased in a package only. Since the economic characteristics of these variants

[^0]are similar, they are usually analyzed together [14, 20]. In this paper we refer to tying and bundling interchangeably. To date, three types of bundling have been discussed in the literature (e.g. [25]): no bundling, when the two products (or services) are purchased separately; ${ }^{1}$ pure bundling, when two products are sold jointly only, making it impossible to acquire the products individually; and mixed bundling, when the bundled goods are offered separately as well as in a package. The seller offers a financial incentive to buy the package since the sum of the components' prices is higher than the price of the package.

After more than four decades since Adams and Yellen [1] published their classic paper about commodity bundling, we focus on a new tying method. This new method has been used by companies such as Walmart, which in August 2016 bought Jet, a fast-growing innovative e-commerce provider, in order to upgrade its ability to compete with Amazon. The appeal of Jet's to Walmart was its real time pricing algorithm, which tempts customers with lower prices if they add more items to their basket. ${ }^{2}$ Evidently, it helped Walmart to fend off Amazon's rapid rise, and acquiring Jet has paid off as Walmart's e-commerce sales grew by $37 \%$ during $2019 .{ }^{3}$

However, this practical technique has not been characterized mathematically. The motivation of this paper is to analyze this new technique, namely sequential bundling. Prior research explored classic bundling techniques such as pure bundling and mixed bundling [12]. These strategies are shown to yield lower profits in relative to sequential bundling. Another comparable technique, namely sequential pricing [2-4], is based on monitoring customer behavior which may violate privacy regulation and may not be affordable for sellers. Our research provides a practical tool for small as well as big e-commerce sellers without the use of prior customer behavior.

Advances in information technologies enable pricing decisions for bundled products to be sequential rather than simultaneous. Aloysius et al. [4] noted that the retail pricing applications of these new technologies has been under-researched. We compute the profit from sequential bundling, assuming that sellers gain no information from monitoring customer behavior, and compare it with the profits related to the three commonly used marketing techniques. To the best of our knowledge, no mathematical analysis has been conducted to compare the profits gained by sequential bundling, although this new online pricing strategy is receiving increased attention.

The paper is organized as follows: After the literature review, in Sect. 3 we present the model of sequential bundling and computes the general form of a firm's profit. Sections 4 presents the three commonly used bundling strategies-no bundling, pure bundling and mixed bundling. Sections 5 presents comparisons of the profits resulting from sequential bundling with these three classic bundling

[^1]strategies. The last section elucidates the conclusions obtained from our analysis. The "Appendix" contains mathematical proofs of our results.

## 2 Literature review

Product bundling is a pervasive marketing strategy designed to maximize profits under different market environments and it has been a popular research issue in economics [32]. Tying and bundling practices are particularly rewarding for marketdominant undertakings in digital markets [14]. Using Post-Chicago School concepts, Holzweber argued that digital markets are particularly vulnerable to tying and bundling practices. These practices are more prevalent in the digital setting and they also tend to be more harmful for competition than in brick-and-mortar-markets. Generally, the doctrine of tying and bundling is applied in all cases where consumers are nudged to demand a supplementary product. A pioneer in offering bundling in digital markets was the music industry, Bodily and Muhammed [6] asserted that the bundling strategy helps music companies maintain returns on their investment in new artist development; hence, offering a package of songs created by different artists benefits both consumers and the seller(s). While the potential benefit seems to be obvious for the sellers, bundling may result in dissatisfaction. That is indeed the case of "product and logistics" bundling, for which Niu et al. [21] showed that the logistic service industry provider's profits might be negatively affected by serving two competing retailers, and on the other hand customers may not prefer their contracted logistics service provider.

In view of customers' satisfaction bundling may provide an advantage for consumers not only by offering lower prices but also by easing the perceived burden of buying decisions. Sarin et al. [27] showed that bundling facilitates consumers' buying decisions, e.g., tying a new high-tech product with an existing technology helps customers reduce the perceived risk associated with the purchase of the new high-tech product.

Previous research of price theory refers to the advantages of bundling to sellers in elevating profitability. Schmalensee [28] and Mathewson and Winter [17] analyzed the profitability of tying a competitively-supplied good to a monopolized good. The analysis of a bivariate normal distribution of consumer preferences [29] shows that under symmetry the strategy of pure tying increase profits and reduce consumer surplus because it decreases the effective dispersion of tastes. The distribution of preferences was generalized by McAfee et al. [19] who ranked the profitability of each tying strategy under monopoly and duopoly market structures. Geng et al. [13] analyzed the profitability of pure tying when consumer valuations of goods are additively separable. Furthermore, Fang and Norman [11] analyzed symmetric log-concave distributions of valuations, and Dansby and Conard [9] suggested criteria for setting the boundaries of lawful tying.

Under the assumption that consumers perceive goods as either substitutes or complements, Lewbel [15] and later Venkatesh and Kamakura [31] analyzed the gained tying profit. Salinger [26] provided a novel analysis of the combined effects of cost and demand. He showed that when tying lowers costs, it tends to be more
profitable if demands for the components are positively correlated and component costs are high. Based on an example from the newspaper industry, Pierce and Winter [23] compared data related to two newspaper firms one of which applies mixed tying whereas the other applies pure tying.

The procedure of sequential bundling was apparently described first by Palfrey [22] who showed that the monopolist's pure tying decision is strongly influenced by the number of buyers when the monopolist uses first- and second-price auctions, based on imperfect information about consumer valuations with no production costs. Similarly, in reference to tree-structured auctions, Carlsson and Andersson [7] analyzed prices in multi-commodity markets, where it is possible to express the demand for each commodity separately, and also express the demand for the bundle of these commodities.

From a marketing point of view, consumers practically choose multiple products sequentially, acquiring them one by one [24]. A general bundling choice model with heterogeneous products in multiple product categories was suggested by Chung and Rao [8] who showed how this model can be used to find market segments for bundles, and to determine the optimal bundle prices for different market segments.

A structural multivariate probit model was presented by Li et al. [16] to who investigated how customer demand for multiple products evolves over time and its implications for the sequential acquisition patterns of naturally ordered products. The stream of literature related to product bundling is a complex and important research area in marketing science which raises an interesting research question regarding the effect of the time sequences on the choices conducted by consumer with asymmetric preferences [24].

### 2.1 Sequential pricing

Stremersch and Tellis [30] identified two key dimensions in classifying bundling strategies: focus and form. The focus of bundling can be either the price or the product, while the form of bundling can be none, pure, or mixed bundling. In other words, from the perspective of classification, sequential pricing focuses on the price while sequential bundling focuses on the product. While focusing on price, Aloysius et al. [2-4] introduced the method of sequential pricing and analyzed the optimal pricing strategy for sellers who can monitor a customer's initial purchase decision. In practice, the problem of conditional sequential pricing lies in being able to identify the order in which prices are observed, as well as the customer's action(s), and then exploiting this information. They also conducted simulations for a range of distributions of buyer values, to compare sequential pricing with mixed bundling.

In recent years, digital marketing has enabled efficient collection not only of order data, but also of browsing and online shopping data, and the exploitation of browsing data has increased. Browsing data provides marketers with information on the consumers' decision-making processes, rather than only the final buying decisions [32]. They found a significant advantage for sellers in making decisions on product bundling based on integrating both browsing and actual shopping data in comparison to the use of either order data or browsing data separately.

The adoption of recent technological advances such as radio frequency auto-identification enable sellers to price discriminate based on a customer's revealed purchasing intentions. Aloysius et al. [4] explored the monopoly profit of different bundling strategies and showed that sequential pricing can increase profits relative to mixed bundling or pure bundling of multiple products. They also found that when a customer's values for the bundled goods are highly positively correlated and sellers can condition the second good's price on the buyer's decision to purchase the first good, sequential pricing increases profits. Their research of price discrimination refers to sellers exploiting information gained from monitoring customer behavior within their shopping experience.

The process of sequential pricing is based on information about which products the customer wishes or intends to buy, while in the process of sequential bundling the customer shows interest only in the first product. Consequently, the difference between sequential bundling and sequential pricing is that in sequential bundling the seller does not need prior information regarding customer buying behavior. Sequential bundling offers a practical method to set prices which maximizes profit with respect to the consumers' preferences.

### 2.2 Comparison to prior bundling strategies

A model of mixed tying in digital markets was developed by Akcura and Altinkemer [5] for the case of multiple products. They characterized a model that considers the firm and customer preferences that allow the firm to maximize profits by offering the most attractive package in accordance with the customer's income. They showed that when costs do not increase in relative to the bundle valuation, firms find it beneficial to limit the number of bundles offered in the market.

Gayer and Shy [12] developed a model to compute consumers surplus, profit and welfare under the commonly used marketing techniques (no tying, pure tying and mixed tying). Another paper that analyzed these three marketing techniques is Razeghian and Weber [25], who examined the effect of changes in peer-trade propensity on the design of consumption bundles, where consumption bundling is intertemporal, corresponding to renting (temporal unbundling), selling (temporal bundling), and the coexistence of these product offers (mixed temporal bundling).

In comparison to previous research $[1,5,12,25]$ that discussed the different bundling strategies, we focus on a new bundling strategy, namely sequential bundling. This research compares the different strategies under similar conditions. However, since the sequential process entails actions in two stages, it is assumed that customers are myopic, meaning that their buying decisions do not consider strategies that impact on future purchasing of other goods. We show that sequential bundling is the most profitable strategy in comparison with the strategies presented in prior research.

## 3 The model

Our model does not assume exploitation of the information gained from monitoring customer behavior. Besides the entailed cost, online customers dislike being monitored and tend to avoid it by incognito browsing. Recently, some authorities constrain it by regulating auto-identification techniques due to privacy protection and ethical considerations (cf. Draper [10]). Hence our model compares online tying strategies while offering the same set of prices to all customers at each stage.

### 3.1 Potential buyers

The model refers to a monopoly firm which is assumed to sell two goods ( either products or services) labeled as $X$ and $Y$ to heterogeneous buyers.

Buyers are uniformly distributed on the $k^{2}$ square $[0, k] \times[0, k]$ with unit density, where $k>0$. Let $(a, b) \in[0, k] \times[0, k]$ index a specific potential buyer. Consumers buy at most one unit of $X$ and at most one unit of $Y$. If consumer buys both goods, it is considered a purchase of a basket of goods.

The index $(a, b)$ also measures the gross benefits derived from the consumption of good $X$ and $Y$ which is the degree of satisfaction in monetary terms. Formally, the (net) utility of a consumer indexed by $(a, b) \in[0, k] \times[0, k]$ is given by:

$$
U(a, b)= \begin{cases}a+b-p_{X}-p_{Y} & \text { if buys } X \text { and } Y  \tag{1}\\ a-p_{X} & \text { if buys good } X \text { only } \\ b-p_{Y} & \text { if buys good } Y \text { only } \\ 0 & \text { if does not buy any good. }\end{cases}
$$

### 3.2 Production costs

Unit costs of producing $X$ and $Y$ are denoted by $c_{X}$ and $c_{Y}$, respectively. The cost of producing a basket is $c=c_{x}+c_{y}$ which is the sum of the two-unit costs (cost of producing one unit of $X$ and one unit of $Y$ ). The additive cost structure rules out economies of scale that may result from joint production of the two goods. Assuming that the unit production cost of each component is bounded by $0 \leq c_{X}<k$ and $0 \leq c_{Y}<k$, we ensure that the cost of producing a basket is lower than the highest consumer valuation, meaning that $0 \leq c<2 k$. Otherwise, no consumer would purchase any good.

### 3.3 The proposed asymmetric costs model: sequential bundling

In the first stage of sequential bundling-before the initial buying, the seller sets prices $p_{X}^{1}$ and $p_{Y}^{1}$. In the second stage-after the initial buying, the seller sets two new discounted prices- $p_{X}^{2}$, where $p_{X}^{2}<p_{X}^{1}$ and $p_{Y}^{2}$, where $p_{Y}^{2}<p_{Y}^{1}$-these new prices are only available to consumers who only purchased a single product. In particular, $p_{X}^{2}$ (

Fig. 1 Consumption choice under sequential bundling

$\left.p_{Y}^{2}\right)$ is only available to the consumers who purchased the product $Y(X)$ in the first stage only. The utility function implies that the set of consumers who buy $X$ (buy $Y$ ) is determined from $a-p_{X}^{2}>a-p_{X}^{1} \geq 0,\left(b-p_{Y}^{N}>b-p_{Y}^{1} \geq 0\right)$. Figure 1 illustrates the set of consumers who purchase one of the following six options: 1 . Buy $X$ only-for a price of $p_{X}^{1}$. 2. Buy $Y$ only-for a price of $p_{Y}^{1}$. 3. Buy both goods in the first stage-for a total price of $p_{X}^{1}+p_{Y}^{1}$. 4. Buy $X$ in the first stage and $Y$ in the second stage-for a total price of $p_{X}^{1}+p_{Y}^{2}$. 5 . Buy $Y$ in the first stage and $X$ in the second stage-for a total price of $p_{Y}^{1}+p_{X}^{2}$. 6. none.

In view of Fig. 1, the producer sells $q_{X}=q_{X}^{1}+q_{X}^{2}$ units of $X$ and $q_{Y}=q_{Y}^{1}+q_{Y}^{2}$ units of $Y$, where $q_{X}^{1}=\left(k-p_{X}^{1}\right) k, q_{X}^{2}=\left(p_{X}^{1}-p_{X}^{2}\right)\left(k-p_{Y}^{1}\right), \quad q_{Y}^{1}=\left(k-p_{Y}^{1}\right) k$ and $q_{Y}^{2}=\left(p_{Y}^{1}-p_{Y}^{2}\right)\left(k-p_{X}^{1}\right)$. The seller chooses prices $p_{X}^{1}, p_{Y}^{1}, p_{X}^{2}$ and $p_{Y}^{2}$ that maximize profits from the sale of each good. Formally, the seller maximizes

$$
\begin{align*}
\pi_{S B}= & \left(p_{X}^{1}-c_{X}\right)\left(k-p_{X}^{1}\right) k+\left(p_{X}^{2}-c_{X}\right)\left(p_{X}^{1}-p_{X}^{2}\right)\left(k-p_{Y}^{1}\right)+\left(p_{Y}^{1}-c_{Y}\right)\left(k-p_{Y}^{1}\right) k \\
& +\left(p_{Y}^{2}-c_{Y}\right)\left(p_{Y}^{1}-p_{Y}^{2}\right)\left(k-p_{X}^{1}\right) \tag{2}
\end{align*}
$$

The first-order conditions yield the two 2-top-index (after discounting the prices) best-response functions $p_{X}^{2}=\frac{p_{X}^{x}+c_{X}}{2}$ and $p_{Y}^{2}=\frac{p_{Y}+c_{Y}}{2}$.

Substituting the best response functions into the profit function (2) yields

$$
\begin{align*}
\pi_{S B}= & \left(p_{X}^{1}-c_{X}\right)\left(k-p_{X}^{1}\right) k+\left(\frac{p_{X}^{1}+c_{X}}{2}-c_{X}\right)\left(p_{X}^{1}-\frac{p_{X}^{1}+c_{X}}{2}\right)\left(k-p_{Y}^{1}\right) \\
& +\left(p_{Y}^{1}-c_{Y}\right)\left(k-p_{Y}^{1}\right) k+\left(\frac{p_{Y}^{1}+c_{Y}}{2}-c_{Y}\right)\left(p_{Y}^{1}-\frac{p_{Y}^{1}+c_{Y}}{2}\right)\left(k-p_{X}^{1}\right) \tag{3}
\end{align*}
$$

From (3) we have:

$$
\begin{align*}
\pi_{S B}= & \left(p_{X}^{1}-c_{X}\right)\left(k-p_{X}^{1}\right) k+\left(\frac{p_{X}^{1}-c_{X}}{2}\right)^{2}\left(k-p_{Y}^{1}\right)+\left(p_{Y}^{1}-c_{Y}\right)\left(k-p_{Y}^{1}\right) k \\
& +\left(\frac{p_{Y}^{1}-c_{Y}}{2}\right)^{2}\left(k-p_{X}^{1}\right) \tag{4}
\end{align*}
$$

The first-order conditions yield the following two 1-top-index (before discounting the prices) best-response functions:

$$
\begin{align*}
& k^{2}-2 p_{X}^{1} k+c_{X} k+\left(\frac{p_{X}^{1}-c_{X}}{2}\right)\left(k-p_{Y}^{1}\right)-\left(\frac{p_{Y}^{1}-c_{Y}}{2}\right)^{2}=0  \tag{5}\\
& k^{2}-2 p_{Y}^{1} k+c_{Y} k+\left(\frac{p_{Y}^{1}-c_{Y}}{2}\right)\left(k-p_{X}^{1}\right)-\left(\frac{p_{X}^{1}-c_{X}}{2}\right)^{2}=0 \tag{6}
\end{align*}
$$

From (5) and (6) we have:

$$
\begin{align*}
& p_{X}^{1}=\frac{-\left(p_{Y}^{1}\right)^{2}+2\left(c_{X}+c_{Y}\right) p_{Y}^{1}+4 k^{2}+2 c_{X} k-\left(c_{Y}\right)^{2}}{6 k+2 p_{Y}^{1}}  \tag{7}\\
& p_{Y}^{1}=\frac{-\left(p_{X}^{1}\right)^{2}+2\left(c_{X}+c_{Y}\right) p_{X}^{1}+4 k^{2}+2 c_{Y} k-\left(c_{X}\right)^{2}}{6 k+2 p_{X}^{1}} \tag{8}
\end{align*}
$$

Substituting (7) into $p_{X}^{2}=\frac{p_{X}^{1}+c_{X}}{2}$ yields

$$
\begin{equation*}
p_{X}^{2}=\frac{-\left(p_{Y}^{1}\right)^{2}+2\left(2 c_{X}+c_{Y}\right) p_{Y}^{1}+4 k^{2}+8 c_{X} k-\left(c_{Y}\right)^{2}}{12 k+4 p_{Y}^{1}} \tag{9}
\end{equation*}
$$

Substituting (8) into $p_{Y}^{2}=\frac{p_{Y}^{1}+c_{Y}}{2}$ yields

$$
\begin{equation*}
p_{Y}^{2}=\frac{-\left(p_{X}^{1}\right)^{2}+2\left(c_{X}+2 c_{Y}\right) p_{X}^{1}+4 k^{2}+8 c_{Y} k-\left(c_{X}\right)^{2}}{12 k+4 p_{X}^{1}} \tag{10}
\end{equation*}
$$

In order to identify the extremum points for sequential bundling under this asymmetric costs case we conduct the following numerical simulation. Table 1 displays

Table 1 Prices and profits under sequential bundling for 7 representative pairs of $c_{x}(k)$ and $c_{y}(k)$, where $c_{x} \leq c_{y}$

| $c_{x}$ | $C_{Y}$ | $p_{X}^{1}$ | $p_{X}^{2}$ | $p_{Y}^{1}$ | $p_{Y}^{2}$ | $\pi_{S B}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0.4 k | $0.5283 k$ | $0.2641 k$ | $0.7006 k$ | $0.5503 k$ | $0.3707 k^{3}$ |
| 0.2 k | 0.2 k | $0.6181 k$ | $0.409 k$ | $0.6181 k$ | $0.409 k$ | $0.3527 k^{3}$ |
| 0 | 0.8 k | $0.5152 k$ | $0.2576 k$ | $0.876 k$ | $0.838 k$ | $0.2681 k^{3}$ |
| 0.2 k | 0.6 k | $0.6162 k$ | $0.4081 k$ | $0.7972 k$ | $0.6986 k$ | $0.2122 k^{3}$ |
| 0.4 k | 0.4 k | $0.7104 k$ | $0.5552 k$ | $0.7104 k$ | $0.5552 k$ | $0.1937 k^{3}$ |
| 0.4 k | 0.8 k | $0.7069 k$ | $0.5534 k$ | $0.8952 k$ | $0.8476 k$ | $0.1031 k^{3}$ |
| 0.6 k | 0.6 k | $0.8048 k$ | $0.7024 k$ | $0.8048 k$ | $0.7024 k$ | $0.084 k^{3}$ |

the profit-maximizing prices for representative pairs of $c_{X}$ and $c_{Y}$ in the ranges $c_{X} \in[0, k)$ and $c_{Y} \in[0, k)$ respectively, where $c_{X} \leq c_{Y}{ }^{4}$ and $k>0$.

### 3.4 Sequential bundling under symmetric costs

In order to obtain the profit function under sequential bundling, the analysis relies on the following simplification:

Assumption 1 The unit production cost of component $X$ equals that of component $Y$. Formally, $c_{X}=c_{Y}$.

Assumption 1 implies that we can consider $c_{X}=c_{Y}=c / 2$, where $c$ is the cost of producing a basket containing one unit of each good. It also implies that the firm will choose to price the components equally so that $p_{X}^{1}=p_{Y}^{1} \stackrel{\text { def }}{=} p_{1}$ and similarly $p_{X}^{2}=p_{Y}^{2} \stackrel{\text { def }}{=} p_{2}$, where $p_{2}=\frac{p_{1}+c / 2}{2}=\frac{2 p_{1}+c}{4}$, yields that $q_{X}^{1}=q_{Y}^{1} \stackrel{\text { def }}{=} q_{1}$ and similarly $q_{X}^{2}=q_{Y}^{2} \stackrel{\text { def }}{=} q_{2}$, where $q_{1}=\left(k-p_{1}\right) k$ and

$$
q_{2}=\left(p_{1}-p_{2}\right)\left(k-p_{1}\right)=\left(p_{1}-\frac{2 p_{1}+c}{4}\right)\left(k-p_{1}\right)=\frac{2 p_{1}-c}{4} \cdot\left(k-p_{1}\right) .
$$

This way the complexity of the profit-maximization problem is reduced.
From (5), (6) and Assumption 1, the solution to the profit-maximization problem yields the following first-order condition:

$$
\begin{equation*}
k^{2}-2 p_{1} k+\frac{c k}{2}+\left(\frac{p_{1}-\frac{c}{2}}{2}\right)\left(k-p_{1}\right)-\left(\frac{p_{1}-\frac{c}{2}}{2}\right)^{2}=0 \tag{11}
\end{equation*}
$$

From (11) we have:

$$
\begin{equation*}
3\left(p_{1}\right)^{2}+(6 k-2 c) p_{1}-\frac{16 k^{2}+4 c k-c^{2}}{4}=0 \tag{12}
\end{equation*}
$$

[^2]From (12) we have that the unique ${ }^{5}$ equilibrium price $p_{1}$ is

$$
\begin{equation*}
p_{1}=\frac{-6 k+2 c+\sqrt{84 k^{2}-12 c k+c^{2}}}{6} \tag{13}
\end{equation*}
$$

From (4) and Assumption 1 we derive:

$$
\begin{equation*}
\hat{\pi}_{S B}=\frac{1}{8} \cdot\left(2 p_{1}-c\right)\left(k-p_{1}\right)\left(2 p_{1}-c+8 k\right) \tag{14}
\end{equation*}
$$

where $\hat{\pi}_{S B}$ is the profit of sequential bundling under symmetric costs.
Substituting (13) into the profit function (14) yields

$$
\begin{align*}
\hat{\pi}_{S B}= & \frac{1}{432} \cdot\left(-6 k-c+\sqrt{84 k^{2}-12 c k+c^{2}}\right) \cdot\left(12 k-2 c-\sqrt{84 k^{2}-12 c k+c^{2}}\right) \\
& \cdot\left(18 k-c+\sqrt{84 k^{2}-12 c k+c^{2}}\right) \tag{15}
\end{align*}
$$

Assumption 2 The costs of the products- $c_{X}, c_{Y}$ are linear functions of $k$. Formally, $c_{X}=\gamma_{X} k$ and $c_{Y}=\gamma_{Y} k$, where $0 \leq \gamma_{X}, \gamma_{Y}<1$.

From Assumption 2 we can derive that $c=c_{X}+c_{Y}=\left(\gamma_{X}+\gamma_{X}\right) k$. Denote $\gamma=\gamma_{X}+\gamma_{Y}$, where $0 \leq \gamma<2$, yields that $c=\gamma k$.

Substituting $c=\gamma k$ into (13) yields

$$
\begin{equation*}
p_{1}=\frac{-6+2 \gamma+\sqrt{84-12 \gamma+\gamma^{2}}}{6} k \tag{16}
\end{equation*}
$$

Substituting $c=\gamma k$ and (16) into $p_{2}=\frac{2 p_{1}+c}{4}$ yields

$$
\begin{equation*}
p_{2}=\frac{-6+5 \gamma+\sqrt{84-12 \gamma+\gamma^{2}}}{12} k \tag{17}
\end{equation*}
$$

Substituting (16) into $q_{1}=\left(k-p_{1}\right) k$ yields

$$
\begin{equation*}
q_{1}=\frac{12-2 \gamma-\sqrt{84-12 \gamma+\gamma^{2}}}{6} k^{2} \tag{18}
\end{equation*}
$$

Substituting $c=\gamma k$ and (17) into $q_{2}=\frac{2 p_{1}-c}{4} \cdot\left(k-p_{1}\right)$ yields

$$
\begin{equation*}
q_{2}=\frac{1}{72} \cdot\left(-6-\gamma+\sqrt{84-12 \gamma+\gamma^{2}}\right) \cdot\left(12-2 \gamma-\sqrt{84-12 \gamma+\gamma^{2}}\right) k^{2} \tag{19}
\end{equation*}
$$

Substituting $c=\gamma k$ into (15) yields

[^3]Fig. 2 Consumption choice under no bundling


$$
\begin{align*}
\hat{\pi}_{S B}= & \frac{1}{432} \cdot\left(-6-\gamma+\sqrt{84-12 \gamma+\gamma^{2}}\right) \cdot\left(12-2 \gamma-\sqrt{84-12 \gamma+\gamma^{2}}\right)  \tag{20}\\
& \cdot\left(18-\gamma+\sqrt{84-12 \gamma+\gamma^{2}}\right) k^{3} .
\end{align*}
$$

## Result 1

Under Assumptions 1 and 2-

1. The prices of no bundling are homogenous in the first degree with respect of $k$.
2. The market shares of no bundling are homogenous in the second degree with respect of $k$.
3. The profit of no bundling is homogenous in the third degree with respect of $k$.

From Result 1 we derive that if we double the size of $k$ then under no bundling-the prices will be doubled as well; the market share will be multiplies by 4 ; the profit will be multiplied by 8 .

## 4 Three classic bundling strategies

### 4.1 No bundling

In no bundling ${ }^{6}$, the seller sets prices $p_{X}^{N B}$ and $p_{Y}^{N B}$, where superscript $N B$ denotes no bundling.

In view of Fig. 2, the producer sells $q_{X}^{N B}=\left(k-p_{X}^{N B}\right) k$ units of $X$ and $q_{Y}^{N B}=\left(k-p_{Y}^{N B}\right) k$ units of $Y$. The seller chooses prices $p_{X}^{N B}$ and $p_{Y}^{N B}$ that maximize profits from the sale of each good. Formally, the seller solves

$$
\begin{equation*}
\pi_{N B}=\left(p_{X}^{N B}-c_{X}\right)\left(k-p_{X}^{N B}\right) k+\left(p_{Y}^{N B}-c_{Y}\right)\left(k-p_{Y}^{N B}\right) k . \tag{21}
\end{equation*}
$$

The unique equilibrium prices, sales levels, and profit under no bundling are as follows:

$$
\begin{align*}
& p_{X}^{N B}=\frac{k+c_{X}}{2}, p_{Y}^{N B}=\frac{k+c_{Y}}{2} \text { and } q_{X}^{N B}=\frac{k-c_{X}}{2} k, q_{Y}^{N B}=\frac{k-c_{Y}}{2} k \text { and } \\
& \quad \pi_{N B}=\frac{1}{4}\left[\left(k-c_{X}\right)^{2}+\left(k-c_{Y}\right)^{2}\right] k . \tag{22}
\end{align*}
$$

The following analysis relies on the simplification of Assumption 2.
Substituting $c_{X}=\gamma_{X} k$ and $c_{Y}=\gamma_{Y} k$ into (22) yields

$$
\begin{align*}
& p_{X}^{N B}=\frac{1+\gamma_{X}}{2} k, p_{Y}^{N B}=\frac{1+\gamma_{Y}}{2} k \text { and } q_{X}^{N B}=\frac{1-\gamma_{X}}{2} k^{2}, q_{Y}^{N B}=\frac{1-\gamma_{Y}}{2} k^{2} \text { and } \\
& \quad \pi_{N B}=\frac{1}{4}\left[\left(1-\gamma_{X}\right)^{2}+\left(1-\gamma_{Y}\right)^{2}\right] k^{3} . \tag{23}
\end{align*}
$$

Result 2

Under Assumption 2-

1. The prices of no bundling are homogenous in the first degree with respect of $k$.
2. The market shares of no bundling are homogenous in the second degree with respect of $k$.
3. The profit of no bundling is homogenous in the third degree with respect of $k$.

### 4.1.1 No bundling under symmetric costs

In order to compare between the prices under no bundling and under sequential bundling (see later on part 4.2), the following analysis relies on the simplifications of Assumption 1 and Assumption 2-meaning that $c_{X}=c_{Y}=c / 2$ and $\gamma_{1}=\gamma_{2}=\gamma / 2$ respectively.

Substituting $\gamma_{1}=\gamma_{2}=\gamma / 2$ into (23) yields

[^4]Fig. 3 (a) Consumption choice under pure bundling with high production cost. (b) Consumption choice under pure bundling with low production cost



$$
\begin{equation*}
p_{X}^{N B}=p_{Y}^{N B}=p_{N B}=\frac{2+\gamma}{4} k \text { and } q_{X}^{N B}=q_{Y}^{N B}=q_{N B}=\frac{2-\gamma}{4} k^{2} \text { and } \pi_{N B}=\frac{1}{8}(2-\gamma)^{2} k^{3} . \tag{24}
\end{equation*}
$$

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### 4.2 Pure bundling

In pure bundling ${ }^{7}$, the firm does not sell individual units of $X$ and $Y$ separately. Instead, the firm sells a basket consisting of one unit of $X$ and one unit of $Y$. We denote this basket by $X Y$ and it price by $p_{B}$, where subscript $B$ denotes pure bundling.

### 4.2.1 Pure bundling under high costs

For the high-cost case, the seller sets price $p_{B H}$.
In view of Fig. 3a, the producer sells $q_{B H}=\left(2 k-p_{B H}\right)^{2} / 2$ units of basket $X Y$. The seller chooses price $p_{B H}$ that maximize profits from the sale of the basket. Formally, the seller solves

$$
\begin{equation*}
\pi_{B H}=\left(p_{B H}-c\right)\left(2 k-p_{B H}\right)^{2} / 2 \tag{25}
\end{equation*}
$$

The unique equilibrium price, number of baskets sold, and profit under pure bundling are as follows:

$$
\begin{equation*}
p_{B H}=\frac{2(k+c)}{3} \text { and } q_{B H}=\frac{2(2 k-c)^{2}}{9} \text { and } \pi_{B H}=\frac{2(2 k-c)^{3}}{27} \tag{26}
\end{equation*}
$$

where subscript $B H$ denotes tying under high production cost. Note that $k \leq p_{B H}<2 k$ under the assumed high cost $(0.5 k \leq c<2 k)$.

The following analysis relies on the simplification of Assumption 2.
Substituting $c=\gamma k$ into (26) yields

$$
\begin{equation*}
p_{B H}=\frac{2(1+\gamma)}{3} k \text { and } q_{B H}=\frac{2(2-\gamma)^{2}}{9} k^{2} \text { and } \pi_{B H}=\frac{2(2-\gamma)^{3}}{27} k^{3} . \tag{27}
\end{equation*}
$$

Result 3

## Under Assumption 2-

1. The price of pure bundling under high production costs is homogenous in the first degree with respect of $k$.
2. The market share of pure bundling under high production costs is homogenous in the second degree with respect of $k$.
3. The profit of pure bundling under high production costs is homogenous in the third degree with respect of $k$.

### 4.2.2 Pure bundling under low costs

For the low-cost case, the seller sets price $p_{B L}$.

[^5]Fig. 4 Consumption choice under mixed bundling


In view of Fig. 3b, the producer sells $q_{B L}=k^{2}-\left(p_{B L}\right)^{2} / 2$ units of basket $X Y$. The seller chooses price $p_{B L}$ that maximize profits from the sale of the basket. Formally, the seller solves

$$
\begin{equation*}
\pi_{B L}=\left(p_{B L}-c\right)\left[k^{2}-\left(p_{B L}\right)^{2} / 2\right] . \tag{28}
\end{equation*}
$$

The unique equilibrium price, sales level, and profit under pure bundling are as follows:

$$
\begin{gather*}
p_{B L}=\frac{\sqrt{c^{2}+6 k^{2}}+c}{3}, q_{B L}=\frac{6 k^{2}-c \sqrt{c^{2}+6 k^{2}}-c^{2}}{9},  \tag{29}\\
\pi_{B L}=\frac{\left(c^{2}+6 k^{2}\right) \sqrt{c^{2}+6 k^{2}}+c^{3}-18 c k^{2}}{27}, \tag{30}
\end{gather*}
$$

where subscript $B L$ denotes tying under low production cost. Note that $p_{B L}<k$ under the low-cost assumption ( $c<0.5 k$ ).

The following analysis relies on the simplification of Assumption 2.
Substituting $c=\gamma k$ into (29) yields

$$
\begin{equation*}
p_{B L}=\frac{\sqrt{\gamma^{2}+6}+\gamma}{3} k, q_{B L}=\frac{6-\gamma \sqrt{\gamma^{2}+6}-\gamma^{2}}{9} k^{2} . \tag{31}
\end{equation*}
$$

Substituting $c=\gamma k$ into (31) yields

$$
\begin{equation*}
\pi_{B L}=\frac{\left(\gamma^{2}+6\right) \sqrt{\gamma^{2}+6}+\gamma^{3}-18 \gamma}{27} k^{3} . \tag{32}
\end{equation*}
$$

## Result 4

Under Assumptions 1 and 2-

1. The price of pure bundling under low production costs is homogenous in the first degree with respect of $k$.
2. The market shares of pure bundling under low production costs is homogenous in the second degree with respect of $k$.
3. The profit of of pure bundling under low production costs is homogenous in the third degree with respect of $k$.

### 4.3 Mixed bundling

In mixed bundling ${ }^{8}$, the firm sells tied baskets $X Y$ along with selling the individual components $X$ and $Y$ separately. We denote the price of a basket by $p_{X Y}^{M B}$ and the components' prices by $p_{X}^{M B}$ and $p_{Y}^{M N}$, where superscript $M B$ denotes mixed bundling. Clearly, while consumers buy the basket it must follow that $p_{X Y}^{M B}<p_{X}^{M B}+p_{Y}^{M B}$. That is, the price of the basket is lower than the sum of the prices of the two components.

The utility function implies that the sets of consumers who prefer buying the basket over buying $X$ only and $Y$ only are determined from $a+b-p_{X Y}^{M B} \geq a-p_{X}^{M B}$ and $a+b-p_{X Y}^{M B} \geq b-p_{Y}^{M B}+0-$, receptively.

In view of Fig. 4, the quantity sold of each component separately and the number baskets sold are given by

$$
\begin{gather*}
q_{X}^{M B}=\left(k-p_{X}^{M B}\right)\left(p_{X Y}^{M B}-p_{X}^{M B}\right), q_{Y}^{M B}=\left(k-p_{Y}^{M B}\right)\left(p_{X Y}^{M B}-p_{Y}^{M B}\right),  \tag{33}\\
q_{X Y}^{M B}=\left[( k - ( p _ { X Y } ^ { M B } - p _ { X } ^ { M B } ) ] \cdot \left[\left(k-\left(p_{X Y}^{M B}-p_{Y}^{M B}\right)\right]-\frac{1}{2}\left(p_{X}^{M B}+p_{Y}^{M B}-p_{X Y}^{M B}\right)^{2} .\right.\right. \tag{34}
\end{gather*}
$$

The seller sets three prices, $p_{X}^{M B}, p_{Y}^{M B}$, and $p_{X Y}^{M B}$, that solve

$$
\begin{equation*}
\pi_{M B}=\left(p_{X}-c_{X}\right) q_{X}^{M B}+\left(p_{Y}-c_{Y}\right) q_{Y}^{M B}+\left(p_{X Y}-c_{X}-c_{Y}\right) q_{X Y}^{M B} . \tag{35}
\end{equation*}
$$

The solution to the profit-maximization problem yields the following three firstorder conditions:

$$
\begin{align*}
0=\frac{\partial \pi_{M B}}{\partial p_{X}} & =\left(k-2 p_{X}+c_{X}\right)\left(p_{X Y}-p_{X}\right)-\left(p_{X}-c_{X}\right)\left(k-p_{X}\right)  \tag{36}\\
& +\left(p_{X Y}-c_{X}-c_{Y}\right)\left(k-p_{X}\right) .
\end{align*}
$$

[^6]Table 2 Prices and profits under mixed bundling for 7 representative pairs of $c_{x}(k)$ and $c_{y}(\mathrm{k})$, where $c_{x} \leq c_{y}$

| $c x$ | $C Y$ | $p x$ | $p Y$ | $p x Y$ | $\pi M B$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0.4 k | $0.5546 k$ | $0.8 k$ | $1.085 k$ | $0.3589 k^{3}$ |
| 0.2 k | 0.2 k | $0.6803 k$ | $0.6803 k$ | $1.0822 k$ | $0.341 k^{3}$ |
| 0 | 0.8 k | $0.5083 k$ | $0.9333 k$ | $1.3363 k$ | $0.2626 k^{3}$ |
| 0.2 k | 0.6 k | $0.6238 k$ | $0.8444 k$ | $1.3109 k$ | $0.2063 k^{3}$ |
| 0.4 k | 0.4 k | $0.7383 k$ | $0.7383 k$ | $1.304 k$ | $0.1877 k^{3}$ |
| 0.4 k | 0.8 k | $0.7066 k$ | $0.9154 k$ | $1.544 k$ | $01012 k^{3}$ |
| 0.6 k | 0.6 k | $0.8149 k$ | $0.8149 k$ | $1.5311 k$ | $0.082 k^{3}$ |

$$
\begin{align*}
0=\frac{\partial \pi_{M B}}{\partial p_{Y}} & =\left(k-2 p_{Y}+c_{Y}\right)\left(p_{X Y}-p_{Y}\right)-\left(p_{Y}-c_{Y}\right)\left(k-p_{Y}\right)  \tag{37}\\
& +\left(p_{X Y}-c_{X}-c_{Y}\right)\left(k-p_{Y}\right) . \\
0=\frac{\partial \pi_{M B}}{\partial p_{X Y}}= & \left(p_{X}-c_{X}\right)\left(k-p_{X}\right)+\left(p_{Y}-c_{Y}\right)\left(k-p_{Y}\right) \\
& +\left(k-p_{X Y}+p_{X}\right)\left(k-p_{X Y}+p_{Y}\right)-\frac{1}{2}\left(p_{X}+p_{Y}-p_{X Y}\right)^{2}  \tag{38}\\
& +\left(p_{X Y}-c_{X}-c_{Y}\right)\left(p_{X Y}-2 k\right) .
\end{align*}
$$

From (36), (37) and (38) we have:

$$
\begin{align*}
p_{X} & =\frac{1}{6}\left\{c_{X}-c_{Y}+2 k+3 p_{X Y}\right) \\
& \left.-\sqrt{\left(c_{X}\right)^{2}+\left(-2 c_{Y}+4 k-6 p_{X Y}\right) c_{X}+\left(c_{Y}\right)^{2}+\left(8 k-6 p_{X Y}\right) c_{Y}+4 k^{2}-12 p_{X Y} k+9\left(p_{X Y}\right)^{2}}\right\} .  \tag{39}\\
p_{Y} & =\frac{1}{6}\left\{-c_{X}+c_{Y}+2 k+3 p_{X Y}\right) \\
& \left.-\sqrt{\left(c_{Y}\right)^{2}+\left(-2 c_{X}+4 k-6 p_{X Y}\right) c_{Y}+\left(c_{X}\right)^{2}+\left(8 k-6 p_{X Y}\right) c_{X}+4 k^{2}-12 p_{X Y} k+9\left(p_{X Y}\right)^{2}}\right\} .  \tag{40}\\
p_{X Y} & =\frac{1}{3}\left\{c_{X}+c_{Y}+4 k\right) \\
& -\sqrt{10 k^{2}+2\left[\left(c_{X}+c_{Y}-6\left(p_{X}+p_{Y}\right)\right] k+\left(c_{X}\right)^{2}+\left(c_{Y}-3 p_{X}\right) c_{X}+\left(c_{Y}\right)^{2}+\left(c_{X}-3 p_{Y}\right) c_{Y}+9\left[\left(p_{X}\right)^{2}+\left(p_{Y}\right)^{2}\right]\right\} .} \tag{41}
\end{align*}
$$

In order to identify the extremum points for mixed bundling under this asymmetric costs case we conduct the following numerical simulation. Table 2 displays the profit-maximizing prices for representative pairs of $c_{X}$ and $c_{Y}$ in the ranges $c_{X} \in[0, k)$ and $c_{Y} \in[0, k)$ respectively, where $c_{X} \leq c_{Y}{ }^{9}$ and $k>0$.

[^7]

Fig. 5 (a) The excess profit of sequential bundling over the profit of no-, pure- and mixed-bundling, for $\mathrm{c}=0.4 k$. (b) The excess profit of sequential bundling over the profit of no-, pure- and mixed-bundling, for $\mathrm{c}=0.8 k$. (c) The excess profit of sequential bundling over the profit of no-, pure- and mixed-bundling, for $\mathrm{c}=1.2 k$

## 5 A comparison of sequential bundling with classic strategies

## The main result

Sequential bundling is more profitable than no-, pure- and mixed-bundling.
Formally: 1. $\pi_{S B}>\pi_{N B}$. 2. $\pi_{S B}>\pi_{B}$. 3. $\pi_{S B}>\pi_{M B}$.
The main result is plotted in Fig. $5 \mathrm{a}-\mathrm{c}$, which display the excess profit under sequential bundling, for symmetric ( $c_{X}=c_{Y}$ ) and asymmetric ( $c_{X}<c_{Y}$ ) cost structure, relative to all three commonly used bundling strategies.

### 5.1 A comparison of prices between sequential bundling and no bundling

Denote by $r_{1}$ the ratio between $p_{1}$-the price under sequential bundling in stage 1 and $p_{N B}$-the price under no bundling, minus one. Formally:

$$
\begin{equation*}
r_{1}=p_{1} / p_{N B}-1=\left(p_{1}-p_{N B}\right) / p_{N B} . \tag{42}
\end{equation*}
$$

Equivalently, denote by $r_{2}$ the negative ratio between $p_{N B}$ and $p_{2}$-the price under sequential bundling in stage 2 , plus one. Formally:

$$
\begin{equation*}
r_{2}=1-p_{2} / p_{N B}=\left(p_{N B}-p_{2}\right) / p_{N B} . \tag{43}
\end{equation*}
$$

From (42) and (43) we derive that $p_{1}=\left(1+r_{1}\right) p_{N B}$ and $p_{2}=\left(1-r_{2}\right) p_{N B}$ respectively, meaning that if we increase (decrease) the price under no bundling by $r_{1}\left(r_{2}\right)$ percent then we will get the price under sequential bundling in stage 1 (2).

Substituting (16) and (24) into (42) yields

$$
\begin{equation*}
r_{1}=\frac{-18+\gamma+2 \sqrt{84-12 \gamma+\gamma^{2}}}{3(2+\gamma)} \tag{44}
\end{equation*}
$$

Substituting (17) and (24) into (43) yields

$$
\begin{equation*}
r_{2}=\frac{12-2 \gamma-\sqrt{84-12 \gamma+\gamma^{2}}}{3(2+\gamma)} \tag{45}
\end{equation*}
$$

Result 5

Under Assumptions 1 and 2-when the production costs $c$, where $c=\gamma k$, increase:

1. The difference between the price under sequential bundling in stage 1 and the price under no bundling decreases with respect of $\gamma$. Formally: $\frac{d r_{1}}{d \gamma}<0$ for all $0 \leq \gamma<2$.
2. The difference between the price under no bundling and the price under sequential bundling in stage 2 decreases with respect of $\gamma$. Formally: $\frac{d r_{2}}{d \gamma}<0$ for all $0 \leq \gamma<2$.

Intuitively, Result 5 describes the weakening of the monopoly power to discriminate prices due to the increase in the production costs.

### 5.2 A numerical example

As an example for the proposed model under symmetric costs, we assume that $k=10$, $c_{X}=c_{Y}=4$, where $c_{X}$ and $c_{Y}$ are the unit production costs of components $X$ and $Y$ respectively. We derive that $c=c_{X}+c_{Y}=8$.

Substituting $k=10$ and $c=8$ into the following prices and profits functions -
Under no bundling (22) yields $p_{X}^{N B}=p_{Y}^{N B}=\frac{2 \cdot 10+8}{4}=7 \quad$ and $\pi_{N B}=\frac{1}{8}(20-8)^{2} \cdot 10=180$.

Under pure bundling (26) yields $p_{B H}=\frac{2(10+8)}{3}=12$ and $\pi_{B H}=\frac{2(20-8)^{3}}{27}=128$.
Under sequential bundling ( 10 and 11 respectively) yields

$$
\begin{aligned}
p_{X}^{1} & =p_{Y}^{1}=p_{1}=\frac{-60+16+\sqrt{8,504}}{6}=7.104, p_{X}^{2}=p_{Y}^{2} \\
& =p_{2}=\frac{p_{1}+c / 2}{2}=\frac{2 p_{1}+c}{4}=\frac{2 \cdot 7.104+8}{4}=5.552
\end{aligned}
$$

and

$$
\begin{aligned}
& \pi_{S B}=\hat{\pi}_{S B}=\frac{1}{432} \cdot(-60-8+\sqrt{8,504}) \cdot(120-16-\sqrt{8,504}) \\
& \quad \cdot(180-8+\sqrt{8,504})=193.7
\end{aligned}
$$

Another way to find these prices is by $p_{X}^{N B}=p_{Y}^{N B}=p_{N B}, r_{1}$ and $r_{2}$ (44 and 45):

$$
\begin{aligned}
r_{1}\left(\gamma=\frac{c}{k}=\frac{8}{10}=0.8\right) & =\frac{-18+0.8+2 \sqrt{84-12 \cdot 0.8+0.8^{2}}}{3(2+0.8)}=0.0149 . \\
p_{1} & =\left(1+r_{1}\right) p_{N B}=1.0149 \cdot 7=7.104 . \\
r_{2}(\gamma=0.8) & =\frac{12-2 \cdot 0.8-\sqrt{84-12 \cdot 0.8+0.8^{2}}}{3(2+0.8)}=0.2069 . \\
p_{2} & =\left(1-r_{2}\right) p_{N B}=0.7931 \cdot 7=5.552 .
\end{aligned}
$$

Under mixed bundling, from Table 2 we have:

$$
p_{X}^{M B}=p_{Y}^{M B}=7.383, p_{X Y}^{M B}=13.04 \text { and } \pi_{M B}=187.7
$$

In summary, we can see that in our example $\pi_{S B}>\pi_{M B}>\pi_{N B}>\pi_{B}$. That is, under sequential tying there are higher profits relative to the other three methods-no bundling, pure bundling and mixed bundling, for $k=10$ and $c_{X}=c_{Y}=4$.

## 6 Discussion and conclusions

This paper compares the profits resulting from a new bundling method, sequential bundling, with three widely used tying strategies-no bundling, pure bundling and mixed bundling. Various bundling strategies discriminate prices to different extents. We show that under sequential bundling, where price discrimination is similar among all consumers according to their initial purchase, sequential bundling yields higher profits relative to the other three methods for all possible production costs.

In comparison to sequential pricing, sequential bundling is a convenient method used by practitioners since it does not consider customers' browsing information which may not be affordable for small sellers, and due to some regulations may entail constraints for sellers. Sequential pricing, as suggested by Aloysius et al. [2-4], who based their research on monitored customer behavior, offers the most suitable bundle at the highest possible price, meaning that sellers can exploit customers by discriminating prices according to their preferences.

This study analyzes, for the first time, the strategy of sequential bundling, which was explored empirically in previous studies [8, 16, 24]. Bundling strategies with no regard to sequential bundling have also been analyzed in in recent years [12, 30, 32]. Our results demonstrate the advantages of the sequential bundling in comparison to the commonly used techniques of no tying (Fig. 2), pure tying (Fig. 3a,b) and mixed tying (Fig. 4) that where previously analyzed by Gayer and Shy [12] under the assumptions that buyers are uniformly distributed on the unit square, with symmetric production costs for the mixed tying case. Sequential bundling yields the highest profit for sellers in comparison to the three alternative strategies. Under constant production costs $\left(\bar{c}=c_{X}+c_{Y}\right)$, when the production cost dispersion decreases $\left(\left|c_{X}-c_{Y}\right| \downarrow\right)$, the difference in terms of excess profit: (a) increases relative to both no bundling and mixed bundling, (b) decreases relative to pure bundling. In addition, under symmetric costs ( $c_{X}=c_{Y}$ ), when the production costs increase ( $c \uparrow$ ), the difference in terms of excess profit: (a) decreases relative to both no bundling and mixed bundling, (b) increases relative to pure bundling.

This study has several contributions. First, we provide a theoretical model to the literature of online consumer behavior which characterizes this new technique of sequential bundling. Second, we show that this new bundling method is superior to the three classic bundling methods in terms of profitability. Finally, for practitioners, this study highlights the advantage of sequential bundling which outperform other techniques and can be easily applied in digital marketing.

Future research can extend the model for a monopoly that can offer more than one additional product. Another interesting extension of the model would be to analyze oligopolistic and competitive markets. Further development of this model will explore the potential advantage of multi-sequential tying of multiple products, following the initial tying proposition offered to the customer. This kind of analysis may provide a dynamic view of sequential bundling. Further research may also examine other demand side factors such as losing the myopic property
of customers. An empirical study can compare consumer behavior as related to different market segments in different time periods. For example, analyzing e-commerce consumers' preferences prior and upon the COVID-19 pandemic. This perspective may shed light on the influence of sequential bundling mechanism on profitability as a result of changes in consumer behavior.

The implications of this model are useful for practitioners while suggesting to sellers how to determine prices to maximize their profits. We provide numerical examples of symmetric and asymmetric cost structures that demonstrate the excess profit gained by applying sequential bundling in both cases. The strategy of sequential bundling is clearly a better choice than the other bundling techniques.

## Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

## Appendix

Proof of Result 1 We can derive all parts of Result 1 immediately from (16)-(17), (18)-(19) and (20) respectively.

Proof of The main result part I The highest profit level under no bundling occurs when $c_{X}=0$ or $c_{Y}=0$ (maximum production cost dispersion). In this case the highest profit is $\max \pi_{N B}=\left[(k-c)^{2}+k^{2}\right] / 4$. Subtracting the maximum profit of no tying from profit of sequential bundling under symmetric costs yields

$$
\begin{aligned}
\hat{\pi}_{S B}-\pi_{N B} & =\frac{1}{432} \cdot\left(-6 k-c+\sqrt{84 k^{2}-12 c k+c^{2}}\right) \cdot\left(12 k-2 c-\sqrt{84 k^{2}-12 c k+c^{2}}\right) \\
& \cdot\left(18 k-c+\sqrt{84 k^{2}-12 c k+c^{2}}\right)-\left[(k-c)^{2}+k^{2}\right] / 4>0,
\end{aligned}
$$

for any $k>0$, and for any given $c$, where $0 \leq c<2 k$.
Proof of Result 2 We can derive all three parts of Result 2 immediately from (23).

Proof of The main result part IIa Recall from (26) that the profit from bundling under high production cost is $\pi_{B H}=2(2 k-c)^{3} / 27$. Subtracting the profit of bundling under high production cost from profit of sequential bundling under symmetric costs yields

$$
\begin{aligned}
\hat{\pi}_{S B}-\pi_{B H} & =\frac{1}{432} \cdot\left(-6 k-c+\sqrt{84 k^{2}-12 c k+c^{2}}\right) \\
& \cdot\left(12 k-2 c-\sqrt{84 k^{2}-12 c k+c^{2}}\right) \\
& \cdot\left(18 k-c+\sqrt{84 k^{2}-12 c k+c^{2}}\right)-2(2 k-c)^{3} / 27>0,
\end{aligned}
$$

for any $k>0$, and for any given $c$, where $0.5 k \leq c<2 k$.

Proof of Result 3 We can derive all three parts of Result 5 immediately from (27).

Proof of The main result part llb Recall from (27) that the profit from tying under low production cost is $\pi_{B L}=\frac{\left(c^{2}+6\right) \sqrt{c^{2}+6}+c^{3}-18 c}{27}$. Subtracting the profit of tying under high production cost from profit of sequential bundling under symmetric costs yields $\hat{\pi}_{S B}-\pi_{B H}=\frac{1}{432} \cdot\left(-6 k-c+\sqrt{84 k^{2}-12 c k+c^{2}}\right)$.
$\left(12 k-2 c-\sqrt{84 k^{2}-12 c k+c^{2}}\right) \cdot\left(18 k-c+\sqrt{84 k^{2}-12 c k+c^{2}}\right)-\frac{\left(c^{2}+6 k^{2}\right) \sqrt{c^{2}+6 k^{2}}+c^{3}-18 k^{2}}{27}>0$, for any $k>0$, and for any given $c$, where $0 \leq c<0.5 k$.

Proof of Result 4 We can derive all parts of Result 4 immediately from (31) and (32).

## Proof of Result 5

1. Differentiating $r_{1}$ (44) with respect to $\gamma$ yields the following formula: $\frac{d r_{1}}{d \gamma}=\frac{4\left(4 \gamma-48+5 \sqrt{84-12 \gamma+\gamma^{2}}\right)}{3 \sqrt{84-12 \gamma+\gamma^{2}}(2+g)^{2}}$, where $\frac{d r_{1}}{d \gamma}<0$ for all $-34 / 3<\gamma<2$. Since $\gamma$ is bounded within the range [0,2), we conclude that $\frac{d r_{1}}{d \gamma}<0$ for all $0 \leq \gamma<2$.
2. Differentiating $r_{2}$ (45) with respect to $\gamma$ yields the following formula: $\frac{d r_{2}}{d \gamma}=\frac{8\left(\gamma-12+2 \sqrt{84-12 \gamma+\gamma^{2}}\right)}{3 \sqrt{84-12 \gamma+\gamma^{2}}(2+g)^{2}}$, where $\frac{d r_{2}}{d \gamma}<0$ for all $\gamma$, and hence, we conclude that $\frac{d r_{2}}{d \gamma}<0$ for all $0 \leq \gamma<2$.

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[^1]:    ${ }^{1}$ While no bundling is not an actual bundling strategy, following Adams and Yellen [1] who compared bundling strategies to the strategy of no bundling, it is common in the literature to refer to no bundling as one of the bundling strategies.
    ${ }^{2}$ Boxed-in unicorn, Walmart buys Jet.com. The Economist, August 13, 2016.
    ${ }^{3}$ Walmart winds down Jet.com four years after $\$ 3.3$ billion acquisition of e-commerce company. CNBC, May 19, 2020.

[^2]:    ${ }^{4}$ Without loss of generality and by symmetry-we assume that $c_{X} \leq c_{Y}$.

[^3]:    ${ }^{5}$ The second solution of the quadratic equation is negative.

[^4]:    ${ }^{6}$ See also Gayer and Shy [12] for the special case in which $k=1$.

[^5]:    ${ }^{7}$ See also Gayer and Shy [12] for the special case in which $k=1$.

[^6]:    ${ }^{8}$ See also Gayer and Shy [12] for the special case in which $k=1$ and $c_{X}=c_{Y}$.

[^7]:    ${ }^{9}$ Without loss of generality and by symmetry-we assume that $c_{X} \leq c_{Y}$.

