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Donald E. Carlson (1938–2010), A Diminutive Giant

To the Memory of Our Longtime Colleague, Mentor, and Friend

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1 Introduction

Donald Carlson's connection with the *Journal of Elasticity* dates back to its founding in 1971, when he joined the editorial board at the request of Marvin Stippes, the founder and Editor-in-Chief as well as Carlson's onetime teacher, mentor, and subsequent colleague at the University of Illinois at Urbana-Champaign. In 1978, he succeeded Stippes, first as Acting Editor-in-Chief, then co-Editor-in-Chief with Richard T. Shield (1979 through 1981), and finally as Editor-in-Chief from 1982 through the end of 1997. Carlson continued to serve on the editorial board until 2006.

Carlson had the rare combination of having both a fine mind and character, which made him a joy to be around. To highlight Carlson's significant contributions to mechanics as well as his exemplary teaching, service to the profession, and personality, we herein present an overview of his life, which enriched all who came in contact with him.

2 Early Life and Education

Donald Earle Carlson was born in Tampico, Illinois,[1] became valedictorian of his high school class, and entered the University of Illinois in 1956 as a freshman in Mechanical Engineering. Thus began Carlson's almost fifty year long association with this institution and

[1] Ronald Reagan was also born in Tampico. Carlson's mother and aunt went to grade school with him and his older brother Neal.

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especially the Department of Theoretical and Applied Mechanics (TAM), which established a new undergraduate curriculum in Engineering Mechanics (EM) in 1958. In that same year, Carlson switched to EM as a junior after becoming frustrated with the ad-hoc presentation and lack of rigor of his class in thermodynamics [1]. This is the same view held by Gurtin [2], who has said that he had detested the subject since his undergraduate days since it was synonymous with steam tables. If, as Wordsworth [3] says, the child is father of the man, Carlson's lifelong passion for the intellectual rigor offered by mechanics can be seen in this decision to change undergraduate majors. His talent for mechanics also manifested itself early on; upon graduating in 1960 with the first EM class, he was awarded Bronze Tablet honors, the university's highest academic achievement for undergraduates. Marvin Stippes exerted an early influence on Carlson, who stayed on in Urbana for another year as a National Science Foundation fellow and teaching assistant and completed a master's degree in TAM under Stippes in 1961.

The early 1960s were an especially active period in American science in general and, commencing in the 1950s, mechanics had begun a resurgence. This was marked by the founding of the *Journal of Rational Mechanics and Analysis* by Clifford Truesdell in 1952 and its successor, the *Archive for Rational Mechanics and Analysis* in 1957, as well as the flood of seminal papers in these journals and others, which appeared during this period and put mechanics on a firm, rational basis. The Division of Applied Mathematics at Brown University was a center of this renaissance, and it was into this stimulating environment that Carlson entered as a doctoral student in 1961 with a fellowship. Carlson had planned on studying with Eli Sternberg, who was leaving for a sabbatical at the time, and thus eventually chose Richard Shield as his advisor. Carlson had also been accepted to MIT and made the "mistake" of telling some of his relatives in Illinois about this opportunity. Since Brown was an unknown quantity to them, they couldn't understand why anyone accepted at MIT wouldn't attend [1].

During his three years at Brown, apart from Shield, Carlson interacted with Morton Gurtin, Allen Pipkin, Ronald Rivlin, and others, but it was Eli Sternberg who had perhaps the most profound influence on Carlson's thinking and approach to mechanics throughout his career. Although in New England, Carlson remained a Midwesterner at heart. On a road-trip home once with his young family, as they came once again to the cornfields of Illinois, Carlson rolled down the window and delighted in the familiar and welcome smell. Back in Providence, he worked on completing his thesis [4] on the use of successive approximations for solving problems in finite elasticity for incompressible, isotropic, homogeneous, elastic bodies. With the assumption that the stress and displacement fields admit power series expansions in a small expansion parameter ϵ, Carlson found that linear equations result for successive terms in the expansions. If the boundary conditions are also similarly expanded, each equation corresponds to a linear boundary value problem for a particular order of ϵ, with the first-order theory being that for classical linear elasticity. Furthermore, for the special case of plane strain solutions satisfying a reciprocal property, terms of even powers of ϵ in the displacement field depend on lower order ones and can therefore be computed without having to solve the corresponding boundary value problem. In this context, rigid inclusions were thoroughly investigated, including the derivation of the respective second- and third-order solutions for elliptic and circular inclusions. Carlson concluded by studying the case of the shear of a cylindrical annulus, presented its fourth-order solution for a Mooney material, and compared it with the exact solution.

Upon giving Shield his thesis, Carlson commented to him that it was like Lagrange's *Méchanique Analitique*[2] in that it contained no figures. Lagrange wanted to distill mechanics to formulæ and, in the preface [5], writes

> On a déja plusieurs Traités de Méchanique, mais le plan de celui-ce est entiérement neuf. Je me suis proposé de réduire la théorie de cette Science, & l'art de résoudre les problêmes qui s'y rapportent, à des formules générales, dont le simple développement donne toutes les équations nécessaires pour la solution de chaque problême . . .
>
> On ne trouvera point de figures.

Shield replied to Carlson that the lack of figures was the only similarity his work shared with Lagrange's tome [1]. The comment can perhaps be seen in a different light when considering the view of Truesdell, who wrote that much of the *Méchanique Analitique* was derived either from earlier work of Lagrange or that of Euler and others [6]. So, it is fitting that Carlson's first paper, based on his thesis, appeared in the *Archive* and was communicated by Truesdell himself [7].

After finishing up at Brown, Carlson asked Shield what the next step should be. Shield then inquired whether Carlson was independently wealthy, which he replied was not the case [1]. So, an academic job appeared to be natural. Thus in 1964, at the invitation of TAM Department Head Thomas J. Dolan at Illinois, Carlson moved back to his alma mater and began his academic career as an assistant professor in TAM alongside his former mentor Marvin Stippes. Shield would later join them and serve as department head of TAM from 1970 to 1984. Carlson quickly moved up the ranks, becoming an associate professor in 1967 and a full professor in 1970. One of the perks of promotion was a hole punch. When Carlson first began at Illinois, he requested one, which, no sooner delivered, was taken away [1]. Evidently, assistant professors were undeserving of such indulgences.

3 Research

Carlson's training at Brown and his thesis topic set the stage for a varied and outstanding research career, during which he investigated topics in depth. Carlson's work on complete, approximate, and controllable solutions in mechanics problems were themes to which he would repeatedly return in different contexts. Also, among other contributions in thermoelasticity, his monograph on linear thermoelasticity [42] remains a classic. Always with an eye to the solution of practical problems, Carlson also investigated basic questions of mathematical methods and proved important results in both dimensional and tensorial analysis. Furthermore, he made advances in other areas of elasticity by examining the consequences of geometrical constraints, dead loading, and new materials such as nematic elastomers.

Carlson never forgot the applied in his work. The subjects he tackled are of significant practical interest, and his papers are filled with examples. At various points throughout his work, e.g., in [42], he indicates formulæ that can be used to measure coefficients, or shows simplifications for the practical solution of problems. This approach is seen elsewhere as well. In [35] and [41], for example, he relates the topic to a real problem encountered by an experimentalist and notes in [28] that the presented method may be more amenable to numerical techniques than earlier proposed procedures.

[2] This spelling was used in the first edition and was subsequently replaced by *Mécanique Analytique*.

We present an overview of Carlson's research contributions and have divided them into broad themes so as to highlight both the main thrust of a paper and its interconnectedness to others. Since many of his articles deal with multiple aspects of a topic, the appearance of a work under a particular heading does not exclude it from another.

3.1 Complete and Exact Solutions

The late 1960s and early 1970s were a productive time for Carlson. He first turned to the solution of the stress equations of equilibrium by stress functions and investigated whether such functions are complete, i.e., that all solutions, or a subclass thereof, could be written in terms of the stress functions. An illuminating and related series of papers followed. Adapting an argument of Günther [8] and the insight of Rieder [9], Gurtin [10] had established that, in the absence of body forces, the Beltrami representation of every solution of the stress equations of equilibrium for a continuum, whose boundary consists of one or more closed surfaces, is complete in that no solutions exist in any other form only if the solution is totally self-equilibrated, i.e., the corresponding resultant force and moment vanish on each of the component boundaries. Gurtin also derived a generalized Beltrami representation for solutions that are not necessarily totally self-equilibrated. For the original Beltrami solution, Carlson [11] subsequently arrived at the same result somewhat more simply using a generalization of the proof of the existence of a vector potential function.

Carlson [12] extended his treatment and examined stress functions for plane problems for bodies not subject to extrinisic body forces or couples but capable of supporting couple stresses, e.g., Cosserat continua, for which he showed completeness of the derived stress functions, interpreted them in terms of the resultant force and moment transmitted across an arc in the body, and gave the conditions for the stress functions to be single-valued. Carlson further showed that the stress function solution found by Mindlin [13] for linear elasticity was a special case. Then in [14], Carlson derived stress functions and proved their completeness for both a Cosserat continuum subject to extrinisic body forces and body couples as well as for the dipolar case of Green and Rivlin's [15] formulation of multipolar continuum mechanics. He also noted therein and in a subsequent refinement [16] that Günther's solution [8] for a Cosserat material was incomplete. However, due to the simplicity and use of Günther functions as the basis of dislocation theories, Carlson [17] addressed the question of which class of stress fields they covered. He showed that Günther's representation is complete for every totally self-equilibrated stress field and, for those not totally self-equilibrated, further derived a generalization of Günther's solution and proved its completeness. This proof also served as a basis for further work [18] on Beltrami stress functions.

In a note [19] in this journal, Carlson again returned to the subject of completeness of solutions. Here, Sternberg's influence is clearly seen since Carlson acknowledges Sternberg's comments on a draft; the topic was also one on which Sternberg [20] had written. Writing the displacement equations of equilibrium for a homogeneous, isotropic, linearly elastic body not subject to extrinisic body force in terms of the dilatation and rotation, Marguerre [21] had observed that the solution in the axisymmetric case could be expressed in terms of two displacement potentials. Carlson established the completeness of such a solution and used his proof as a springboard to demonstrate further the completeness of both Boussinesq's and Love's solutions in the same context of axisymmetric elasticity. Though these two solutions had previously been shown to be complete, Carlson's completeness proof of the latter was considerably shorter than others. Furthermore, he insightfully noted that the displacement potential solution of Knowles and Horgan [22] for the exponential decay of stresses in circular elastic cylinders subjected to axisymmetric self-equilibrated loads was related to Marguerre's solution and thus complete.

Carlson was also involved in other work related to solutions of problems in elasticity. For an incompressible, isotropic, homogeneous, elastic body in plane strain, Adkins [23] had established the reciprocal property by which an equilibrium state for one material provides an inverse deformation that is an equilibrium state for certain other materials in the class. In some instances, the inverse deformation can be identical to the original one, and Carlson had used this fact in his thesis [4] to ease the solution of the problems under consideration by successive approximations. In [24], Shield extended Adkins's result to finite deformations of homogeneous, elastic materials not subject to extrinisic body forces in which the strain-energy density depends only on the deformation gradient, i.e., a material of grade one. With Shield, Carlson [25] derived the same result variationally and extended it to include elastic materials of any grade, for which the strain-energy density can depend on all gradients of the deformation up to some given order.

With his fifth doctoral student, Wayne K. Murphy, Carlson investigated situations when solutions to thermoelastic problems are possible. In a joint paper [26], they examined the mixed problem of dynamic linear thermoelasticity in which the displacement and temperature fields are coupled in the two governing equations. One of these can be transformed so that the temperature appears as the only unknown, but the coupling remains in the other. If the curl of the curl of the displacement field vanishes, as it does for uniaxial and spherical fields, for example, Murphy and Carlson showed that the displacement then completely decouples from the temperature and can be expressed in terms of it. Further, they proved that any solution to the resulting initial-value boundary problem for the temperature is unique and that the corresponding thermoelastic process satisfies the initially set mixed problem for the uniaxial case. In other cases, the process does not provide a full solution to the mixed problem, since the tangential components of the traction boundary condition are not met. Murphy and Carlson demonstrated, however, that this is not a major hindrance. The complete solution can be obtained by superposing the uncoupled one with that to the residual problem, for which there is no temperature field and the displacement satisfies a conventional initial-value boundary problem for the vector wave equation. Again with a view to the solution of real problems in mechanics, their method was applied in [27] to the problem of a flat plate initially at rest and at a given reference temperature. One side of the plate is kept at the reference temperature and immediately subjected to a uniform and maintained pressure, while the other is fixed to a rigid nonconducting surface. Thus, here and in so much of Carlson's work, the theory is not just an end unto itself.

3.2 Approximate Solutions

Also related to his thesis work, Carlson wrote a joint paper [28] with his second doctoral student, Conway Chan, in which they employed the method of successive approximations to derive the governing second-order equations for the problem of second-order torsion of an incompressible, isotropic, homogeneous, hyperelastic cylinder. Through changes of dependent variables, they presented a solution procedure that importantly reduces this problem, and others not necessarily limited to torsion, to ones in two-dimensional linear elasticity with no body forces. An initial second-order solution is chosen to satisfy the second-order inhomogeneous condition of incompressiblity, and the surface tractions generated from it are calculated. These tractions are then used as a boundary condition for the solution to the classical problem with homogeneous incompressiblity, whose displacements and stresses are respectively added to those from the initial solution. For this second problem to have a solution, it is necessary that both the resultant force and moment from the surface tractions be totally self-equilibrated. This requirement echoes a similar one Carlson investigated for the completeness of Beltrami [11] and Günther [17] stress functions. Chan and Carlson then implemented their method for the case of second-order torsion, computed results for the practical example problem of torsion of a rectangular cross-section, and compared them with existing work.

In the context of nonlinear thermoelasticity, in which the coupled, nonlinear equations preclude the exact solution of specific boundary-value problems, Carlson together with his third doctoral student, Paul Yarrington, also used successive approximations in [29]. As a result of the assumptions of incompressibility and isotropy, the resulting equations are found to be linear at each order and completely uncoupled. Yarrington and Carlson then determined the second-order solution to the combined loading of an incompressible, isotropic, homogeneous, elastic, annular cylinder under a combined axial mechanical and radial thermal loading.

In the method of successive approximations, all field equations are expanded in Taylor series. To obtain more detailed solutions, the field equations can be kept exact, while the material class is instead limited and its corresponding response functions approximated by polynomials in the appropriate invariants. Thus, the material is characterized by the coefficients of the polynomials rather than functions. In the context of second-order approximations for elastic heat conductors, Carlson took this approach in [30] with his fourth doctoral student Stephen E. Martin. In their solution of the simple extension of an incompressible, isotropic, prismatic bar with uniform temperature, they compared their results with those from experiments on rubber and found good agreement. Also, they derived a method to determine the second-order dependence on temperature of any known elastic solution and applied this procedure to the case of simple torsion of an incompressible, isotropic, right-circular cylinder with uniform temperature. Martin and Carlson further considered problems in which the temperature field is nonuniform and examined the simultaneous extension and shear of a cylindrical annulus with radial heat flow.

3.3 Controllable States

For any combination of initially prescribed fields, e.g., deformation, electric, and dielectric displacement, a controllable state corresponds to a solution of the field equations for a constitutively defined class of materials that is independent of the particular material within that class. The solution is controllable in that it can be effected by the application of appropriate Cauchy tractions on the boundary of the body for any material within the class. In the search for controllable states, an a priori knowledge of the form of the material response functions for a particular class of materials is needed to determine the set of such states for it. Once these states have been derived, experiments can be designed to measure the response functions, since the state can be achieved for any material in the class by the application of suitable boundary conditions. This method thus serves as an alternative to the determination of these functions by successive approximations.

Carlson and his first doctoral student, Henry Petroski, initially focused on controllable states for thermoelastic materials. A consistent thread can be seen between this work and the previous research into the completeness of stress functions, since Carlson was concerned with the applicability of solutions in both subjects. Also, in both areas, he derived new results for hitherto unconsidered materials. Drawing on work of Ericksen [31, 32] and Singh and Pipkin [33], Petroski and Carlson examined controllable states in both rigid [34] and elastic [35] heat conductors achievable by the application of suitable surface tractions and heat fluxes. The identification of such states involves finding positive-definite, symmetric deformation tensors and temperature fields, that respectively satisfy the balances of momentum and energy. For a rigid conductor in the absence of an extrinisic heat supply and for an explicit dependence of the heat flux on the temperature, they showed that a steady-state temperature field is controllable if and only if it is uniform. For an elastic conductor with no extrinsic body force or heat supply, they found one, trivial controllable state for the compressible case and two, non-trivial ones for the incompressible case. For these last two, and in the context of specific geometries and boundary conditions, Petroski and Carlson [36] then arrived at the physical interpretation of the first as an axial elastic conductor and the second as an elastic heat siphon.

Returning to Ericksen's [31] problem of finding controllable deformations in homogeneous, isotropic, hyperelastic bodies and extending Marris and Shiau's [37] treatment, Carlson and Martin [38] examined the special class for which two principal stretches of the deformation tensor are equal and at least one of its principal invariants is not constant. In a new result, they then showed that this highly constrained case precludes any controllable states.

Again concerned with the universal nature of solutions, Carlson turned to a topic of interest to the experimentalist and demonstrated once more in a series of three papers in this journal [39–41] that he was not just concerned with theory. Carlson established the necessary and sufficient conditions for the three-dimensional solutions of the equations of elastostatics for a homogeneous, isotropic, linear elastic body to be independent of the Poisson's ratio ν or shear modulus μ. For the case of elastodynamics, in addition to ν and μ, he also found such conditions for solutions to be independent of the mass density ρ. Interestingly, for the Poisson's ratio analysis in elastostatics, the difference state between two solutions for separate values of ν satisfies the equations of steady-state thermoelasticity [39].

3.4 The *Handbook of Physics*

Carlson's magnum opus [42], his treatise on linear thermoelasticity in the *Handbook of Physics*, appeared in 1972. The longlasting worth and importance of this work can be seen by the over 250 citations [43] it has received since its publication almost forty years ago. So, as fate would have it, despite, or perhaps as result of, his undergraduate experience with thermodynamics, Carlson is probably best known for it. Carlson takes Gurtin's [44] book-length monograph on linear elasticity, appearing directly before in the same volume, as the basis for his presentation and development of the subject. In fact, in a footnote at the beginning, Carlson mentions that the article was planned with Gurtin as an extension of his work and that the second chapter was essentially written by him.

Their collaboration was not new, and Carlson credits Gurtin, for example, in one of his earlier papers [14] on stress functions. During their work together, the two would talk by phone, and when Carlson called, Gurtin already knew who it was by the time he said "hello". Carlson asked Gurtin why it was so easy to identify him, and the latter replied that it was simple because Carlson was the only person he knew without an accent [1]. Subsequently, during the writing of his 1981 book *An Introduction to Continuum Mechanics* [45], Gurtin regularly consulted with Carlson [46].

The *Handbook* article presents a full treatment of the theory of linear thermoelasticity beginning with the basic balance and imbalance laws of mechanics and thermodynamics and their consequences for the nonlinear theory, which are then specialized to the linear case. The equilibrium theory of linearly elastic heat conductors is then presented, and Carlson uses the body force analogy to allow him to bring the results from isothermal elastostatics to bear on the equilibrium theory of thermoelasticity. Although the analysis is general, he examines the relevant special case of a homogeneous and isotropic body in which the temperature difference field induces either a displacement-free or stress-free state. The final chapter covers the coupled dynamic theory of linear elastic heat conductors and includes a statement of the mixed problem, its characterization in terms of the displacement and temperature for both a general three-dimensional body in addition to the special case of a homogeneous and isotropic one, and a theorem and associated proof of the uniqueness of solutions. Carlson concludes with a discussion of variational principles as well as harmonic, plane, progressive wave solutions for homogeneous and isotropic bodies with neither extrinsic body force nor heat supply. The comprehensive list of references he provides at the end not only shows the depth of scholarship involved in the preparation of the work but also serves as an excellent resource for researchers. Also for linear thermoelasticity, in [47], Carlson and Martin presented the solution for the equilibrium temperature reached by a mechanically deformed insulated body, a practically relevant problem.

3.5 Mathematical Work

Of aid to researcher and practictioner alike was Carlson's work on dimensional analysis presented in two papers in the *Archive* [48, 49]. In [48], he laid out a rational basis for the theory of the subject and dedicated it to Truesdell on the occasion of his sixtieth birthday. This paper included the introduction of the concept of a fundamental system of units, the investigation of measures and dimensions relative to such a system, as well as the associated transformations of units, measures, and dimensions. Finally, and perhaps most importantly, Carlson gave a precise definition of the notion of dimensional homogeneity. This concept serves as the starting point of dimensional analysis and relies on the fact that the form of a dimensionally homogeneous equation is invariant under transformations of fundamental systems of units. Continuing on in a paper communicated by Truesdell [49],[3] he examined the behavior of the dimensional matrix of several physical quantities under general transformations of units. Carlson further showed that a fundamental system of units for the quantities under consideration can always be chosen such that the number of units equals the rank of the dimensional matrix and, in so doing, justified an early assumption of many. Then, after formulating dimensional homogeneity with respect to general transformations of units rather than just the traditional changes of scale, he established a generalized pi theorem that, unlike its predecessors, was not restricted to strictly positive measures. Concluding his characteristically in-depth treatment, Carlson showed that quantities with fixed zero measure as well as those which are dimensionless may be safely ignored with regard to dimensional homogeneity; these last results thus provided the theoretical underpinning for the widespread practice of tacitly ignoring such quantities.

In a series of four papers [51–54], Carlson and his tenth doctoral student, Anne Hoger, investigated various properties of tensors arising in mechanics and derived fundamental results therein. In the first of them in this journal [51], they examined the material time derivatives $\dot{\mathbf{V}}$ and $\dot{\mathbf{U}}$ of the left and right stretch tensors $\mathbf{V}$ and $\mathbf{U}$, respectively defined as the square root of the left and right Cauchy–Green strain tensors $\mathbf{B}$ and $\mathbf{C}$. Guo [55] had derived expressions for $\dot{\mathbf{V}}$ and $\dot{\mathbf{U}}$ by using the polar decomposition theorem and had thus avoided taking the derivative of the square root of a tensor. Using the Cayley–Hamilton theorem, Hoger and Carlson computed a formula for such a derivative in terms of invariants, used it to calculate $\dot{\mathbf{V}}$ and $\dot{\mathbf{U}}$ directly, and suggested additional methods apart from Guo's, whereby $\dot{\mathbf{V}}$ and $\dot{\mathbf{U}}$ could also be obtained without requiring a tensor square root to be differentiated.

[3] The value of this work is attested by the second author of this biography for it helped him considerably in [50].

Because of their definitions via square roots, the computation of $\mathbf{V}$ and $\mathbf{U}$ presented difficulties. However, in [52], Hoger and Carlson again applied Cayley–Hamilton to calculate $\mathbf{U}$ from $\mathbf{C}$ without using square roots; the left stretch tensor can then be easily found from the polar decomposition theorem with the inverse $\mathbf{U}^{-1}$ of the right stretch tensor. In an intermediate result, they also derived the inverse of a tensor comprised of the sum of a positive-definite tensor and a positive multiple of the identity tensor. Furthermore, formulæ for $\mathbf{U}^{-1}$ in terms of the invariants of both $\mathbf{U}$ and $\mathbf{C}$ and for the invariants of $\mathbf{U}$ as functions of those of $\mathbf{C}$ were presented for the cases of two and three dimensions. Continuing with their results on invariants in another paper [53], Carlson and Hoger derived expressions for the derivatives of the principal invariants of a second-order tensor on an underlying vector space of any dimension.

Carlson and Hoger also examined the derivative of a tensor-valued function of a tensor in [54]. By writing such a function as the product of a spectral function of the tensor argument's distinct eigenvalues and their associated eigenprojections and summing over these eigenvalues, they avoided problems of uniqueness encountered in other representations arising from repeated eigenvalues. Such an insight allowed Carlson and Hoger to proceed by taking the derivative of the spectral decomposition. For the two-dimensional case, they computed results when the eigenvalues of the argument are either distinct or repeated. They extended their results to n dimensions for the case when the eigenvalues are distinct and presented an explicit formula for the derivative. For repeated eigenvalues in three dimensions and beyond, Carlson and Hoger employed a newly presented lemma from Ball [56] to compute the derivative and thus covered all possible cases.

3.6 Internal Constraints

Any constraint, such as that of incompressibility, places a limit on aspects of material response in that all processes must be compatible with the constraint. To deal with constraints in continuum mechanics, Truesdell and Noll [57] made an a priori decomposition of the stress into an active part given by a constitutive equation and a reactive one assumed to be powerless for all motions consistent with the constraint. This approach was derived from Ericksen and Rivlin [58], who, in the context of constrained hyperelasticity, showed that the stress is automatically decomposed when the constitutive equations for it and the internal energy satisfy the balance of energy in all motions allowed by the constraint. In [59], Carlson and his colleague in Mechanical Engineering, Daniel Tortorelli,[4] derived Ericksen and Rivlin's result using a simple geometrical argument about the constraint manifold of the deformation gradient and thus avoided their a priori decomposition.

In terms of research compatibility, the arrival in 1995 of Eliot Fried on the TAM faculty gave Carlson a kindred spirit in the department for the first time since Marvin Stippes's death some fifteen years earlier. Fried's presence reinvigorated Carlson's passion for mechanics and led to a most successful collaboration during Fried's seven years at Illinois. In a contribution to the collection of essays and papers assembled in this journal to Truesdell's memory, Carlson, Fried, and Tortorelli [60] expanded on Carlson and Tortorelli's paper [59] and looked more broadly at the consequences of geometrically-based constraints beyond hyperelasticity. Therein, investigating internal constraints geometrically, they took the view that since such constraints apply to broad classes of materials, they are more basic than constitutive equations. For the constrained case, the fact that the deformation gradient belongs to a constraint manifold was combined with the projection theorem to prove that any

[4]During his graduate studies at Illinois, Tortorelli also took several courses from Carlson.

second-order tensor can be uniquely decomposed in terms of the tangent and normal spaces of the constraint manifold. Thus, the stress tensor admits such a representation, with an automatically powerless tangential (reactive) component and a constitutively determined normal (active) component appearing in the free-energy inequality. This result mirrors that found by Truesdell and Noll but is more natural in that the decomposition is a geometrical consequence and not an initial assumption. After making the appropriate elastic constitutive Ansätze for the free-energy and stress and requiring that the free-energy inequality be satisfied for every process consistent with the constraints, Carlson, Fried, and Tortorelli then recovered the theory of constrained hyperelasticity. So, their treatment is as general as Truesdell and Noll's but less restrictive. They further showed that, as for unconstrained hyperelasticity, the moment of momentum balance in constrained hyperelasticity does not have to be taken as an axiom since it results from material frame-indifference.

3.7 Nematic Elastomers

With their jointly advised doctoral student, David Anderson, Carlson and Fried [61] presented a continuum-mechanical theory of incompressible and microstructurally inextensible nematic elastomers. These rubber-like solids are formed by the cross-linking of polymeric fluids that include nematic molecules either as elements of their main-chains or as pendant side-groups. The development of such a theory represented a significant advance in the attempt to model a material with microstructure. In addition to the classical deformation, they introduced the orientation, which accounts for the evolution of the nematic microstructure. The introduction of this kinematic variable then led naturally to orientational forces, which expend power over the time-rate of the orientation and appear in the orientational momentum balance, an additional balance to the usual one for deformational forces. Also, since the problem was restricted to a purely mechanical setting, an energy imbalance served in lieu of the first and second laws of thermodynamics. It was here that Anderson, Carlson, and Fried first used the geometrical argument formalized and developed in [60] to obtain the decomposition into active and reactive components of the deformational stress tensor, the internal orientational body force density, and the orientational stress tensor. They then assumed constitutive equations for the active components, required them to be consistent with the energy imbalance in all processes, and thereby determined the active components in terms of the response function for the energy density. Finally, after formulating the requirements of observer independence and material symmetry for the energy-density response function, they specialized it and derived an expression comprised of separate elastic and nematic contributions as well as a joint term accounting for the coupling between the two.

In a further article in this journal [62], Carlson, Fried, and Sellers expanded on the work discussed above and presented a continuum theory of nematic elastomers allowing for the polymeric microstructure to be either isotropic, uniaxial, or biaxial and incorporated the neo-classical free-energy density of Warner et al. [63]. This free-energy density depends on the deformation gradient only through the relative strain of the microstructure to the continuum. For these materials, they examined experimentally relevant force-free states and soft elasticity, in which large deformations are possible with the application of small forces, and found a continuous spectrum of such states. Carlson and his co-authors then derived a similar result for a nematic elastomer described by a generalized free-energy density depending isotropically on the relative strain. The existence of such states for which nontrivial deformations are possible with little energy cost then led them to propose that the free-energy density should also depend on the macroscopic strain so as to avoid such unrealistic situations. Such a conclusion, based on both theoretical results and physical insight, is a hallmark of Carlson's work.

3.8 Linear Elasticity

With Chi-Sing Man in an article in the *Archive* [64], Carlson investigated the traction problem of dead loading in three-dimensional linear elasticity with initial stress. As opposed to controllable states, in which the tractions are applied on the boundary of the possibly evolving region occupied by the body in the spatial configuration, dead loading corresponds to the case for which the extrinsic body force and traction fields acting on the body are prescribed in the reference configuration independent of the ensuing deformation. As in the nonlinear theory, they found that a physically significant example of dead loading occurs when neither extrinsic body force nor surface tractions are present in the current configuration, i.e., zero loading. Further, in the linear context, Man and Carlson demonstrated that pressure loading corresponds to dead loading if there is no initial traction and live loading otherwise. After presenting the weak formulation of the problem, of practical use in the finite-element method, for example, they showed that for a sufficiently small initial stress and the assumption that the incremental elasticity tensor is positive-definite, a unique solution exists up to the addition of an arbitrary displacement in a finite-dimensional space. For the classical problem of zero initial stress, this space corresponds to the set of infinitesimal rigid displacements, but the question arises as to its characterization for non-zero initial stress. Man and Carlson addressed this issue and importantly gave sufficient and explicit conditions for which the space also corresponds to one of infinitesimal rigid displacements.

In his final paper [65], dedicated to his friend and classmate from Brown, Michael Carroll of Rice University, Carlson returned full circle to one his heroes, Eli Sternberg. Sternberg had always emphasized that a solution to a problem in linear elasticity is valid only if the infinitesimal strain does not exceed the proportional limit and thus lies in the linearly elastic regime. The linear theory is derived by assuming that the displacement gradient is small in magnitude. Also, the governing equations are generally combined so that the strain and stress are eliminated, resulting in the displacement formulation. In this context, Carlson examined the range of applicability of linearly elastic solutions and determined that in fact not only the first but also the (suitably non-dimensionalized) second gradient of the displacement must be small. This result also connects to his work on stress functions and controllable states since Carlson was also concerned therein with the range of solutions. As an example, he set the standard, undergraduate problem of a right, circular cylinder comprised of a homogeneous, isotropic, elastic material hanging under its self-weight. To solve the maximum allowable length for the cylinder before linear elastic behavior is lost, Carlson used both a one-dimensional strength of materials method and the full, three-dimensional linearly elastic equations, which include the Poisson's ratio effect. By neglecting the size of the second displacement gradient, he showed that the range of applicability of the linear theory is significantly exceeded in both solutions. Thus, as in his work on dimensional analysis, for example, in a clean and elegant way, Carlson forces us to question our assumptions and keeps us honest.

4 Teaching

Apart from his research, the importance of mechanics to Carlson can be seen in the time and care he spent educating and nurturing students. What shines through in all of Carlson's œuvre is his clarity and thoroughness of both thought and presentation. This approach extended to his teaching as well. His lectures were always well organized, contained neither superfluous details nor too little information, and attested to both much practice and preparation. To

Table 1 List of doctoral advisees with year of completion

Henry J. Petroski (1968)	Faitori M. Omer (1978)	John K. Reed* (1985)
Conway Chan (1970)	Fadil Santosa* (1980)	Randall S. Marlow (1989)
Paul Yarrington (1974)	Yensen Wu* (1981)	G. George Abatt (1994)
Stephen E. Martin (1975)	Glenn L. Bowers* (1982)	Amit Acharya (1994)
Wayne K. Murphy (1978)	Anne G. Hoger (1984)	David R. Anderson* (1999)

*denotes co-advisor

step into Carlson's office was to enter a world of knowledge completely filled with books. In fact, he could often be seen working there late at night, long after most (including graduate students) had called it a day. The results of these efforts can be seen in the meticulously crafted, handwritten notes with his impeccable script, which he distributed in his graduate classes and ultimately became prized possessions for his students. Carlson also pursued depth in his undergraduate courses and would, for instance, go beyond the usual, superficial definition of stress as force divided by area. He also always had time for students to drop by his office to ask questions or chat. His popularity among undergraduates can be seen in Carlson being named an honorary Knight of St. Pat, one of the signal awards bestowed by the College of Engineering's students on one of its faculty.

Carlson further supervised twenty-eight Master's students and the fifteen doctoral students listed in Table 1.

5 Service

Carlson was extremely active in the life of the university. He served as interim or acting department head on three occasions (1970–1971, 1988–1989, and 1991–1992). In addition, his committee work spanned the department, college, and university levels. He advised numerous undergraduates and was named an outstanding advisor by the college in 2000, 2003, 2004, and 2005. Further, Carlson came in contact with many graduate students outside of class by serving on their qualifying and preliminary examination boards and thesis committees.

Carlson fervently believed in the value of mechanics as an intellectual pursuit and its place in the academy. The decision by the trustees of the University of Illinois in 2006 which ended the Department of Theoretical and Applied Mechanics at Illinois as a free-standing department after a more than 100 year distinguished history was one against which Carlson courageously and indefatigably battled. His exemplary efforts involved writing letters, speaking with administrators, and supporting students and colleagues in their fight. His decision to retire in 2006 was certainly connected with the approval of the merger of TAM with Mechanical Engineering.

Carlson also was active in the broader mechanics world. He was a longtime member of the Society for Natural Philosophy and served as its secretary and on various committees. In addition, Carlson was a member of the board of governors of the NSF Institute for Mechanics and Materials at the University of California, San Diego from 1992 through 1998. Apart from his long service to this journal, Carlson served on the editorial board of the *Journal of Thermal Stress* from 1978 to 2000, and reviewed both proposals for the National Science Foundation and papers for a host of journals. Furthermore, he organized Festschriften

[66–69] in honor of the contributions of Ericksen, Knowles, Gurtin, and Fosdick. In his foreword for Gurtin, Carlson, in characteristic modesty, apologized for his "inattentiveness" in preparing the volume subsequent to Gurtin's birthday.

6 Legacy

Carlson received well-deserved recognition for his work. He spent the 1984–1985 academic year at the University of Minnesota as both a visiting professor in the Department of Aerospace Engineering and Mechanics and a senior visitor at the Institute for Mathematics and its Applications. In 1988, he was named a fellow of the American Academy of Mechanics and received its Lifetime Service Award in 2005. For his 65th birthday in 2003, Eliot Fried and one of us (J.W.P.) led the effort to organize a research conference in Carlson's honor. A dozen colleagues including Millard Beatty, Michael Carroll, Roger Fosdick, Morton Gurtin, James Knowles, and Walter Noll came to Urbana from across the country for the meeting. It was clear that Carlson was moved by this tribute, and at the closing session, simply and genuinely thanked everyone for coming.

In all his interactions, whether with students, faculty, or staff, Carlson treated everyone with respect, and despite his achievements, was neither pretentious nor arrogant. His modesty extended to the car reviews in the auto magazines he'd read: if Carlson couldn't afford the car, he figured there was no point in proceeding further [1]. He was always good company and could also be counted on for a joke, amusing story, or humorous remark. A few examples are worth mentioning. On introducing Timothy Healey of Cornell at a TAM seminar on chirality in nonlinear elastic rods, Carlson presented Healey with the traditional TAM coffee mug and dead-panned that it was definitely a right-handed mug because a left-handed one would have been sinister. After one of us (R.T.) commented that Carlson's mother's recent eye surgery had produced a sea change, he pointed out that it was literally a "see change". One Halloween, Carlson ran into an acquaintance who enquired whether Carlson was dressing up for the holiday. "No, I always look like this," he replied [1]. It is thus clear that Carlson, a man of short stature with self-described "overdesigned" knees, had an endearing sense of humor.

Truesdell and his wife apparently dressed formally for dinner every night [70]. In a similar way, whether at conferences or the classical concerts he regularly attended on campus, Carlson always dressed for the occasion and, against the relaxed, contemporary attitude in such matters, hewed to a high standard and showed his respect for all involved.

In his dedication in [65], Carlson wrote

> Dedicated to my friend and classmate Michael M. Carroll on the occasion of his 75th birthday in recognition of a long and distinguished career of research, teaching, and academic administration.

The description applies aptly to Carlson himself.

If a person is to be judged on what he makes of his life and the effect he has on those around him, then by these measures, Donald Carlson was without question exceptional and special. Although these qualities make his loss that much more difficult for those who knew him, we will remain forever enriched by having benefited from his scholarship, counsel, and friendship.

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