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Socio-ecological gestures of mathematics education

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Abstract
In this theoretical article, we argue that the imminent collapse of earth systems that sustain life forms calls for mathematics education as a field to reflect on and re-evaluate its priorities and thus practices. We consider both what ecological collapse means for mathematics education and whether mathematics education might offer meaningful gestures in response. We explore how the relationship between the social and the ecological is conceptualised in mathematics education (and other relevant) research and what this implies for mathematics education. We read, in this scholarship, a growing focus on the ecological and conceptualisations of socio-ecological relations between existing entities that are dialectical, or mutually dependent. More rarely, are they seen as entangled and monist, and it is in this thought that we locate our contribution of multi-layered gestures of mathematics education. We describe these, in terms of three broad practices: listening for socio-ecological entanglement; attending to the scales of socio-ecological entanglements; and living entanglement as mathematics educators. We exemplify these gestures through examples of curriculum innovation. This article, a socio-ecological gesture in itself, is written in the spirit of opening a conversation into which we invite others.

Keywords Socio-ecological · Socio-political · Mathematics curriculum · Climate change · Methodology · Philosophy

1 Introduction

Contemporary world events offer stark evidence of the inseparability of social, ecological, health, spatial and political issues such as climate changes, chemical pollution, water and food insecurity, biodiversity loss, health pandemics, poverty, inequality, migration, totalitarianism, and loss of voice. This is a world characterised by complexity, uncertainty, vulnerability, movement, and informality, with the pace of change outstripping our knowledge. World events threaten all of life on the planet and challenge us to consider, in Latour's (2004) words, "Are we not like those mechanical toys that endlessly make the

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same gesture when everything else has changed around them?" (p. 225). We conceptualise (human) mathematics education as making "gestures", by which we mean practices that produce research, recontextualised knowledge, curriculum organisations, learning materials, professional development opportunities, and anything that becomes visible in the context of teaching and learning. We are particularly interested in gestures which relate to possible futures for mathematics education which do respond to what has changed around us. Bateson (2000) highlights the human tendency to overreach in its efforts to control and self-regulate the world. This is a world which we increasingly format as "calculable" using a supposedly neutral and universal mathematics, valued for its descriptive, categorical, and predictive possibilities (Mbembe, 2021; Skovsmose, 2011), in which mathematics education is commonly and unquestioningly considered a necessary individual and social "good". There is an urgency to (re)evaluate the "reach" of these practices of mathematics education and consider both what ecological collapse means for mathematics education and whether mathematics education might have a meaningful response. Prompted by Latour, we ask: what might be the "gestures" of a reconceptualised mathematics education; where and from what position might they be made, and for and with whom? In this article, we offer what we label a socio-ecological practice.

We do not see the socio-ecological practices/gestures we propose as replacing other mathematics education perspectives (which we map in Section 2), but as working with them. Thus, we proceed by firstly exploring a range of perspectives on the social and ecological, each bringing different conceptualisations. In Section 3, we offer our own definition and describe a series of layered gestures, exemplified with reference to mathematics curriculum innovation. We end in Section 4 by further theorising our proposal and inviting others to engage in on-going conversation and collaboration.

2 Trends in thinking about the socio-ecological

Mathematics education has long drawn on ideas from psychology (Lerman, 2000), which is where we begin our review of past work. We then focus on how socio-political research in mathematics education relates to socio-ecological thinking (2.2). Finally, we consider work from a range of perspectives within mathematics education that propose some form of entanglement of the social and the ecological and introduce our own entangled and monist perspective (2.3).

2.1 Socio-ecological psychology

Socio-ecological psychology, broadly, is research and practice that considers "how natural and social habitats affect human mind and behavior and how human mind and behavior in turn affect natural and social habitats" (Oishi & Graham, 2010, p. 356). Traced to Lewin's (1936) field theory, these ideas have moved in and out of focus in psychology since then (Oishi & Graham, 2010) and influenced related fields of organisational psychology (e.g. Boons, 2013) and ecological systems theory (e.g. Burns et al., 2015). A common thread involves locating individual behaviour and development in a manner that goes beyond the immediacy of the individual to take into consideration interaction across multiple settings (Uskul & Oishi, 2020). For example, a key perspective was Bronfenbrenner's (1977) socio-ecological model of human development (see Fig. 1), which comprised five concentric and interconnected circles, with the individual at the centre, surrounded by a microsystem

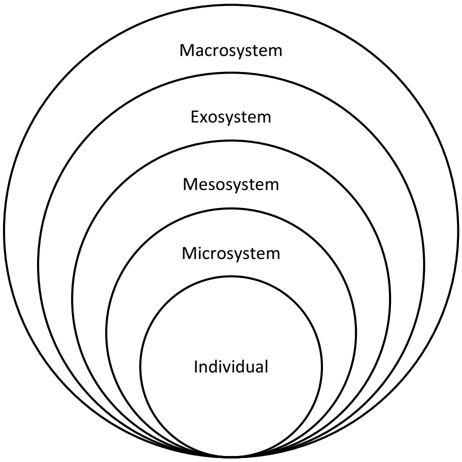


Fig. 1 An image of Bronfenbrenner's (1977) socio-ecological model

(family, peers), a mesosystem (relations between settings), an exosystem (local politics, industry, mass media), and a macrosystem (attitudes and ideologies of the culture).

Building on ecological systems theory, Louie and Zhan (2022) adapt Bronfenbrenner's model for thinking about mathematics education research and practice on equity. Concerned that mathematics education research commonly focuses on learning and development at the level of the individual student or teacher, they use the model to discuss contributions to a 2022 journal special issue. They identify, in several articles (e.g. Battey et al., 2022; van Es et al., 2022), aspects of a socio-ecological system which can sometimes be ignored and argue for the importance of mathematics education "coordinating attention across multiple layers of social activity" and to the power relations within and across these layers (p.367).

We see promise in thinking about a dialectic between the individual and multiple levels of the social and ecological, for instance, in terms of shifting away from the pathologising and ranking of mathematics knowledges and knowers (Louie & Zhan, 2022). Our reading is that socio-ecological psychology perspectives largely centre the individual human knowing subject (a point also made by Louie & Zhan, 2022). As shown in Fig. 2 (acknowledging the necessary over-simplification of any schematic diagram), we suggest the work cited in this section broadly takes the ecological as *context* for the social, that is, the interrelationship is one of ecological systems acting on the social and the individual (shown by the bold line). A dialectical, two-way relationship is acknowledged (the dotted line), for example, Louie and Zhan viewing the macrosystem as both forming and formed by inner layers; however, we view Fig. 1 as exemplifying a sense of the primary focus of analysis being the individual human (a focus we represent in Fig. 2).

Such conceptualisations surface important questions and are productive when research concerns are primarily about individuals and their relationships. We are also interested in conceptualisations of the living world that do not centre human concerns or actors, nor focus, for instance, on minimising the damage of humans on the environment (Ungar, 2002). In search of alternative ideas, we next move to work in mathematics education, in particular socio-political perspectives, which have gained traction in recent decades (following Valero, 2004; Gutiérrez, 2013). Such perspectives conceptualise mathematics and mathematics education as historical, social, and political practices. We discuss (not mutually exclusive) named areas of work within the socio-political. We acknowledge that the

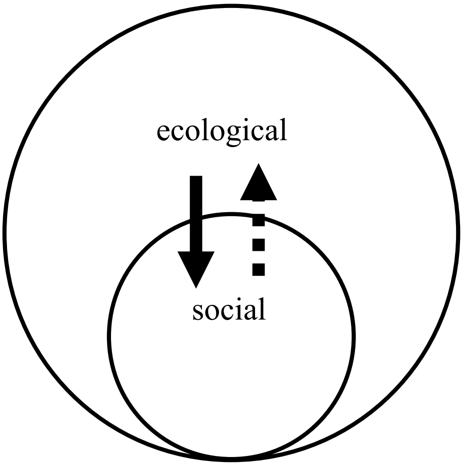


Fig. 2 The ecological as predominantly the context for the social, within a dialectical relationship

definitions of conceptual areas and their relations are contested. We will be suggesting that, in socio-political research, ecology can sometimes be taken as fixed background to social and cultural concerns, a position increasingly questioned.

2.2 Relations to the ecological in traditions of socio-political mathematics education

Critical mathematics education (CME), commonly raised from within the dominant Euro-modern knowledge and education structures (Vithal & Skovsmose, 1997), addresses how mathematics and mathematics education (re)produce, or "write", the world, knowledges, and subjectivities. CME is also concerned with mathematics and mathematics education for understanding, or "reading", the world, aspiring to the possibilities of (re)writing for a more democratic, socially just world (e.g. Alrø et al., 2010; Andersson & Barwell, 2021). CME has informed perspectives variously named as mathematics for social justice/peace/democracy. In the past 15 years, CME has demonstrated the potential of mathematics (education) to "write" and "read" the contemporary ecological condition of the world, or "climate change", in what might be called a mathematics for environmental sustainability (e.g. Barwell, 2013; Coles et al., 2013). From a relative lack of attention to the ecological in CME scholarship in those early years (Boylan & Coles, 2017), we note a growth in CME in the past 5 years towards conceptualising the social world (in particular, technology and mathematics-in-action) as having an impact on the ecological (e.g. Coles & Helliwell, 2023; Hauge & Barwell, 2017; Ödmo et al., 2023; Steffensen et al., 2023). Skovsmose (2023) now proposes "environmental justice", alongside "social justice" as a foundational concern of CME.

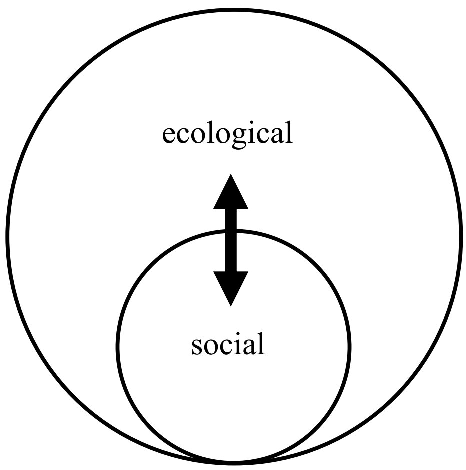
Socio-critical modelling, described as an "emancipatory perspective" (Kaiser & Sriraman, 2006, p. 304) of mathematical modelling for a critical understanding of the world, has strong links to CME. Educationally, socio-critical modelling centres "students' ability to be critical modellers and [to] recognize their power", rather than their mathematical understanding and skills (Abassian et al., 2020, p. 61). While little developed, the approach is growing, for example, students being given opportunities to "critically analyze problems that surround them" (Rosa, 2014, p. 233) by modelling the (inequitable) social impact and effects of human action on ecologies of the planet, in phenomena such as our carbon footprint (e.g. Steffensen & Kacerja, 2021), global warming (e.g. Basu & Panorkou, 2020), waste management (Villarreal et al., 2015), and fish stocks (Yanagimoto & Yoshimura, 2013).

Some areas of socio-political mathematics education have thought from the "outside" of the dominant canon (Vithal & Skovsmose, 1997) and been shaped by the marginalised positions in which they emerge. For example, ethnomathematics (e.g. Powell & Frankenstein, 1997; Rosa et al., 2016), the focus of which is "the cultural aspects of mathematics" (Rosa et al., 2016, p.1), is based on a view that there are diverse ways of knowing and doing mathematics and that these are socially and culturally situated. Its naming, *ethno*mathematics, points to a focus on the mathematics of "distinct groups identified by cultural traditions, codes, symbols, myths, and specific ways of reasoning and inferring" (Rosa et al., p. 2). Importantly, mathematics is used in context, "to explain, understand, and manage social, cultural, environmental, political, and economic environments" (p. 2, citing D'Ambrosio, 2007). D'Ambrosio (2015) long called for attention to environmental realities in ethnomathematics, something being addressed in recent scholarship (e.g. de Mattos & de Mattos, 2020) and proposals for an *ethno-biomathematics* (e.g., Eglash, 2023). Learnings from ethnomathematics are used to inform culturally responsive mathematics education which focuses on curriculum, pedagogy and assessment for groups whose cultural knowledges, interests, experiences, and languages are marginalised (e.g. Averill et al., 2009; Greer et al., 2009; Nicol et al., 2013).

Both ethnomathematics and culturally responsive mathematics education, with their focus on groups marginalised by coloniality and neoliberal globalisation, may position themselves as perspectives that "decolonise" mathematics education. Some mathematics education scholarship draws explicitly on traditions of "post-"/"de"/"anti"-colonial thinking and/or critical race theories to both highlight the co-constitution of racial (and related) difference in mathematics education in processes of coloniality and make visible historically subaltern mathematics knowledges and knowers (e.g. Martin, 2019; Swanson & Chronaki, 2017). Such scholarship may focus on both the ecological and socio-political, for example, climate change and systemic racism (e.g. Madden, 2019).

We value the prompting of the scholarship reviewed here, to question what kinds of mathematics may be productive in our contemporary world and to motivate for epistemic pluralism, i.e. recognising multiple ways of knowing (mathematics). We read in this work conceptualisations of the relationship between the social and ecological as a two-way dependency and influence (see Fig. 3). The ecological context affects socio-political concerns and socio-political

Fig. 3 The social and ecological as interdependent and mutually influencing



decisions have significant impacts on local and global ecologies. Both can be conceptualised as emerging from processes of becoming.

However, a conceptualisation such as Fig. 3 can mean the ecological is taken as fixed, or is obscured, at least for the purposes of a specific research project. At a simplistic level, research on language in classrooms is generally silent on the quality of air students are breathing, in order to speak. In the past, we (as researchers) have been able to take air quality as a fixed and benign aspect of the context, something rarely now the case (e.g. Evans, 2023; Tuchin, 2020; Tuñez, 2017). We believe there is a need for scholarship which acknowledges the agency and mutability of the ecological.

2.3 Entanglement of the socio-ecological

A move towards a more entangled view of the social and ecological (than Figs. 2 and 3), which works against fixedness and binaries of self/other, system/environment, and culture/nature, is discernible in some recent work in mathematics education and related fields. Scholarship on Indigenous ways of knowing, also positioned as a form of "decolonising" mathematics education, commonly foregrounds the ways of knowing, acting, and being of variously named Indigenous communities which are traditionally marginalised in dominant mathematics education practices (e.g. Lunney Borden, 2021; Nicol et al., 2020; Parra & Valero, 2021). Such scholarship shifts focus to epistemological concerns of mathematics education (indeed, questioning the very binary of Western/Indigenous mathematics) and signals the presence of entangled relationships across culture/nature, or the social and ecological (e.g. Anania & Stiglitz, 2023; Gutiérrez, 2017; Kulago et al., 2021).

A related shift is discernible within political ecology (linked to ideas of cultural ecology (Steward, 1937, 1955)), which is concerned with relations between ecological degradation and social marginalisation, and in particular how and why economic structures and power relations drive environmental change in an increasingly interconnected world (Karlsson, 2015; Roberts, 2020). In terms of ontology, the mutuality, but also independent existence of the social and ecological features strongly in political ecological thought, for example, Velasco (2017) proposes: "'Natural' and 'social' things are not opposites, but rather […] are networks made of interwoven processes that are simultaneously human and natural, real and fictional, mechanical and organic" (p. 16, translation by second author).

Recognition, by mathematics educators, of entangled ways of acting/knowing/being has been inspired by other theoretical traditions outside of the field of mathematics education, such as language and literacy studies, (eco)feminist, ecocritical, ecojustice, new materialist, and posthumanist ideas. Examples include re-imagining mathematics: for "the commons", which makes space and time for lands and all peoples (Chronaki & Lazaridou, 2023); for multispecies flourishing, in kin with all species (Khan, 2020); as a dialogic epistemology and ethics (Barwell et al., 2022); in the Anthropocene (Coles, 2017); and for the development of ecological selves and relational knowing and being (Boylan, 2017). Included in this scholarship are conceptualisations of assemblages or sets of material relations between the human body, technology, mathematical concepts, and the natural world, all viewed as actors that structure one another (e.g. Boylan, 2017; de Freitas & Sinclair, 2014; Khan et al., 2022; Rubel & Nicol, 2020).

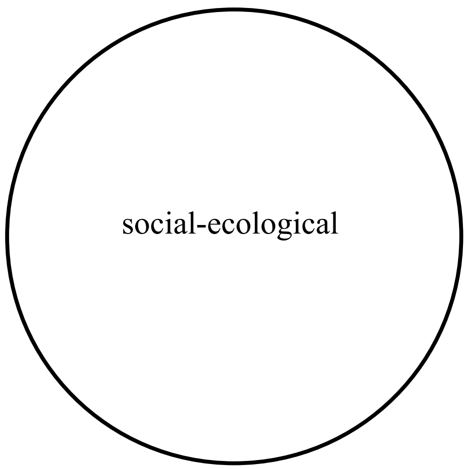
It is in such recent moves that we locate our proposals for *socio-ecological gestures* of mathematics education. We picture the socio-(political) and the ecological as entangled, as one, in a monist ontology (see Fig. 4). And while such a view may seem like a radical departure for many Western traditions, as Tuana (2023) notes, there are many

Indigenous world views which have never engaged in a pathological and often racialised divide between culture and nature, or the social and ecological.

By *entangled*, we want to indicate a mutual arising of human and other-than-human (natural and inanimate), involved in an intimate, layered folding in which each retains depths of the others, including in pasts, presents, and futures (Nuttall, 2009). Mbembe (2021) alludes to these relations, arguing that old boundaries are being redrawn (p. 21), suggesting therefore that critical thought (in our case mathematics education) needs to "think its possibility outside of itself, aware of the limits of its singularity, within the circuit that always connects itself to an elsewhere" (p. 228). Such a call challenges our thinking and our practices to inhabit the intersections of social constructs such as gender, class, language, and race; bodies; in-animate things; ecologies; space; territory; the symbolic (including mathematics) and the material; and practices (including mathematics education)—while also attending to the singularity, the independent existence, yet incompleteness of each (Nyamnjoh, 2020). It is through such relations that meaning is made, including the meaning of socio-ecological practices or gestures of mathematics education.

While Fig. 4 is perhaps easy to picture, the meaning and implications are not at all straight-forward to draw out or enact. And while, for convenience, we continue to use the words social and ecological separately on occasion in the writing that follows (in which case they largely map on to the words "culture" and "nature"), we insist on their entanglement. The idea of entanglement means that social and ecological concerns are not actually separable—every social concern has an indivisible ecological aspect and every ecological concern has an indivisible social aspect. The "socio-ecological" as a phrase points to the totality of relations involving human, non-human, living, and non-living actors. Given the scholarship reviewed in this section, and with the conceptualisation of the socio-ecological we propose, we now turn to our key questions: what ecological collapse means for mathematics education, and what gestures we (i.e. mathematics educators in the broadest sense) might be called on to offer in response?

Fig. 4 The social and ecological as entangled



3 Proposed socio-ecological gestures of mathematics education

The word "gesture", for us, operates at multiple levels or layers of mathematics education and, indeed, this article as a whole could be viewed as a socio-ecological gesture. In this section, we describe three broad practices, each of which we view as a gesture (and each of these broad practices involves specific gestures). In other words, gestures for us can operate at a range of layers of abstraction or generality. The three broad gestures we describe are: listening for socio-ecological entanglement; attending to multiple scales of socio-ecological entanglements; and living entanglement as mathematics educators. We aspire to these gestures in our own practices of mathematics education.

3.1 Listening for socio-ecological entanglement

At its simplest we propose, in this article, that all of us engaged in mathematics education might, in our thinking, usefully listen for and become more attuned to, the vital, complex socio-ecological entanglement, in which mathematics and mathematics education are actors. Such thinking provides the grounds for considered choices. On the one hand, we might consciously choose, at a particular time, that there are benefits to focusing on a topic while taking the ecological as bracketed or fixed. There is no doubt much still to be learnt from a focus on, e.g., mathematical discourse in classrooms, or on students' constructions of proof, while bracketing broader ecological concerns. And, on the other hand, we want to argue for the benefit in recognising and positioning such topics in intimate relations with (research on) what is not in focus, such as air quality in classrooms, or the impacts of war and other socio-ecological issues. The current time urges an expansive, complexifying gesture that recognises mathematics, and knowledge about teaching and learning of mathematics, as existing in relations of wider mutuality. And while we might choose to bracket that complexity for specific purposes, narrowing our focus, or zoom (Lerman, 2000), we want to argue for the importance of at least acknowledging that a narrow focus represents a necessarily incomplete singularity.

The gestures we include under the label of listening are ones of widening our gaze to deal with the (complex) socio-ecological problems communities and schools face, considering the complexity of both socio-ecological issues and educational systems. For example, considering the broad field of curriculum innovation in relation to listening for the socio-ecological might push us towards cross-curricular work and seeking out the local, regional, and global socio-ecological issues of pressing concern to the communities in which we work.

What may be required is more than putting mathematics education to work in critique of existing global and national institutional structures, but activism towards more just futures, for instance, instigating discussions and working with communities to resist current structural forms. Sitas (2023) argues that academic scholars have traditionally been good at critique but that we no longer have the luxury of just that. We may need to recognise multiple centres of concern and make space for them in mathematics classrooms, alongside attention to the mathematical.

3.2 Attending to multiple scales of socio-ecological entanglements

In past work (le Roux et al., 2022; Solares-Rojas et al., 2022), we have identified particular tensions in the expansive gesture of listening for entanglement, as described in Section 3.1.

These are tensions across *scales* of entanglement, from the local, to regional, to global, in pasts, presents, and futures. We recognise the need for reflexivity about the question, from where and at what scales are we looking/thinking/listening? Attending to our gaze necessitates some reflection on how we are conceptualising ourselves in relation to others (human and non-human), for instance, whether any of Figs. 2, 3, or 4 represent our perspective.

We offer an example from our own, collaborative, socio-ecological mathematics education practices to help think through this section. In Solares-Rojas et al. (2022), there is a report on a curriculum innovation project which took place in a community in Mexico, through which flows one of the most polluted rivers in the region. One of the outcomes of the project (which co-emerged as an aim, through the process of the project) was creating a "Memorial Museum" to a polluted river (the Atoyac). The Museum was created by students, teachers, and the project team of researchers, in one primary school, 1 km away from the river. The museum was erected temporarily for the community to visit. It had three galleries, one looking to the past (trying to capture oral histories of how the river had been), one the present (considering current pollution levels and their effects), and one to the future of the river–community relationship (including activism towards a healthy river). The work has since travelled in its documentation, both becoming a virtual Museum (see https://redcomunidadcienciaeducacion.org/index.php/museo/) and, in its physical manifestation, as an itinerant Museum that has been taken to around 30 different communities in the Atoyac region. The gesture of the Museum is a form of activism, moving beyond the classroom walls and aiming to raise consciousness. Community groups conceptualise the river as "living" and there is an explicit intention, in the Museum, to move away from viewing the river's state of pollution as fixed or inevitable (a key to our thinking about socio-ecological relations). Invoking temporal scales of past and future allows us to view the present as part of a process and to explore possible alternative, entangled futures.

Reflecting on the scales of socio-ecological entanglements, the Atoyac River Project made space for the *river*, and the community experiences thereof (in the past, present, and imagined future). The curriculum activities also made space for memory, imagination, fear, and multiple representations (gestures also called for in 3.1). We identified the potential to use school curriculum mathematics to study biodiversity loss over time, the distances and distribution of danger zones in risk maps, or the rate of toxicity of industrial chemicals present in the river waters. Yet we also needed to make space, beyond purely disciplinary knowledge of mathematics, for some of the more reflexive aspects of the work (long called for within CME), such as attention to the uses of mathematics, the assumptions behind mathematical models, and the ways mathematics is used to communicate (Skovsmose, 2023). Crucially, not all engagements with the socio-ecological problem were calculable. The project embodied an invitation, a gesture, to curriculum mathematics education, arising out of wider intradisciplinary and transdisciplinary projects which addressed the pressing concerns of communities. For us, our gesture of humble listening for the vital, complex socio-ecological entanglement in the project required that we, as mathematics educators, sit with the precarity and vulnerability of being on the margins, of being incomplete, and hence in need of mutual relations beyond mathematics (education).

Regarding scales, we recognise a temptation to propose that a curriculum innovation such as a Memorial Museum could be done in any community facing socio-ecological challenges, that is, that the local tends to the regional and global. And yet, in discussing the Atoyac work with others in South America, it is clear that a number of quite special, even singular circumstances existed, which allowed the museum to be developed. Firstly, there is a community group, active in the area for over 25 years, campaigning for a healthy river ("Coordinadora por un Atoyac Con Vida"). Secondly, there is a non-governmental organisation (NGO) working in the region for over 20 years and continuing to engage in a range of social and ecological justice projects ("Centro Fray Julián Garcés Derechos Humanos y Desarrollo Local"). Thirdly, a group of scientists have been researching the river health for over 18 years (e.g. Arellano-Aguilar et al., 2015; García-Nieto et al., 2011), work they are continuing. And finally, all three groups were happy to collaborate and co-operate with a group of educationalists to develop a curriculum innovation. In the absence of one of more of such circumstances, we have little sense of how successful a Memorial Museum project in a local area might be, while also wanting to offer the idea as one concrete curriculum proposal that could be explored in other contexts and that inevitably involves attending to temporal (and other) scales of entanglement.

Socio-ecological gestures need to grapple with the translocal in terms of connections and entanglements, as well as disconnections and singularities. In the example of the Atoyac Project, it was evident that companies from other localities are largely responsible for polluting the Atoyac River (Méndez Serrano et al., 2017). These companies were encouraged into the region with economic incentives and, of course, do provide employment. Thus, the Atoyac River Project surfaced the questions of how socio-ecological precarities are experienced in different but specific contexts, and by whom, and what paradoxes and racialised inequalities these precarities expose. The impact of having exceeded planetary limits (Rockström et al., 2009) is not felt homogeneously in the different regions of the planet, as the geopolitical South is facing not only the impact of natural phenomena as a result of climate change, but, additionally, the environmental consequences of historical and enduring exploitation of their natural resources, and economic and social crises (Comaroff & Comaroff, 2012; Martinez-Alier et al., 2016). Thus, with the folding of pasts, presents, and futures, the specific characteristics of the regions affected cannot be explored in isolation from their historical and contemporary translocal relations and related structures, such as global economic structures (Menton et al., 2020). In doing such work, we are challenged not to (re)produce essentialist and deficit descriptions of certain locals (Adler & Lerman, 2003). However, if the only way forward is to find a common world, then we find energy in arguments that thinking from geopolitical South locals may offer "privileged insight" into this world (Comaroff & Comaroff, 2012, p. 1). Hence our deliberate choice, in this paper, is to exemplify our ideas with learnings from the Atoyac River Project.

3.3 Living entanglement as mathematics educators

In this section, we begin our description of further gestures for mathematics education with some of the philosophical implications of adopting an entangled or monist position about the social and the ecological. We are concerned in this section to draw out how our view of social-ecological relations, as in Fig. 4, might differ from, for example, Figs. 1, 2, and 3. This section is the most purely philosophical of the article, and we are equally at pains to draw out specific implications for mathematics education. We consider first questions of ontology and epistemology.

Ontologically, we take the gesture of living entanglement, as aligning with several theoretical positions that propose a relational ontology (e.g. de Freitas & Sinclair, 2014; Haraway, 2019). The entanglement of the social and ecological means, for instance, the entanglement of subject and object. Rather than thinking about an encounter as a meeting of two pre-existing entities, a relational ontology implies the view that it is *through encounter* that subject and object inhabit an identity (for the duration of that encounter). Rather than asking, for example, "who acts?" (a question which presupposes an already existing subject), a more relational question would be, “how is it that such a subject is able to act in this way?” (Benjamin, 2015, p.87). The latter question invites attention to the existing webs of relations that allow action in the first place. And, if we take the social and ecological as one, these webs of relations include culture, politics, history, biology, ecology, and more. In other words, subjectivity is an after-effect of the socio-ecological relations that allow its emergence, not a pre-condition of those relations. We observed, for example, in the Memorial Museum (Solares-Rojas et al., 2022), an emergence of students’ relations with the Atoyac River. By interviewing older members of the community, the students gathered information about the animal population near the river in the past. They analysed the data by constructing tables and bar graphs and drew conclusions on the loss of biological diversity over time. We read in this work an encounter of socio-ecological entities—the river and students—in entanglement relations: children engaging with the river (through the stories of their grandparents and their biodiversity analysis using mathematical tools) and ecological changes of the river affecting children’s lives.

At the same time as finding resonance with ideas of relational ontology, we are also sympathetic to philosophical moves towards an “object-oriented ontology” (Bryant, 2011). Bryant, draws on a phrase of Bateson (2000) to state one of the principles of an object-oriented ontology, namely that “there is no difference that does not make a difference” (2011, p.263). For Bateson, the equivalent phrase was that information is “a difference that makes a difference”. In order for a change to become noticeable to an organism, the change has to be a difference which makes a difference to that organism. Changes which are too small, or happen too slowly, are inaccessible and hence provoke no new information. Bryant’s re-working of that phrase points to the kind of expansion of thought that we want to associate with the socio-ecological—if a change (in, say, pollution levels of a river) is imperceptible to a human, the difference does not somehow disappear—it will make a difference, for instance, to the quality of soil nearby the river, no matter how slight the change. A human-centric perspective can obscure such differences and relations.

Bryant proposes that all objects “translate” other objects in their interactions, where an “object” includes a mathematical concept. The translation of objects points to the manner in which, from an object-oriented perspective, the essence of any object is “withdrawn” from its interactions with any other object and yet objects are changed by interactions. So, a mathematical concept such as “risk” (used in disease risk maps during the Atoyac Project) is one that is never fully known. And, human entanglement with the non-human points to the mutual and recurrent relations within which we live. As humans (and we are also objects), we encounter “risk” and the concept encounters us, both potentially changing as a result.

It may seem contradictory to be considering relational and object-oriented ontologies together. We suppose socio-ecological gestures to tend towards the relational, and yet, we find ideas from object-oriented ontology helpful to think about the nature of the objects which arise from relations. As social scientists, we take it that our convictions about ontology are primarily metaphorical, in helping us direct our attention to particular areas or places of concern. As such, we see no contradiction in drawing on both relational and object-oriented ideas, and indeed, the two traditions help alert us to both movements of connection and to movements of disconnection or singularity, as pointed to in Section 3.2.

In terms of epistemology, we recognise the need for humility, by recognising that ways of knowing other than those seen as translocal and universal may be needed to address problems of the socio-ecological. Taking the social and ecological as entangled, involves recognising that all knowledges are incomplete (Nyamnjoh, 2020). In the socio-ecological gestures we consider in this section, what seems important is not to foreclose our awareness of the complexity of multiple relations in which we are entangled and, rather, to sit with the vulnerability of not knowing, recognising the inadequacy of instrumental knowledge of apparently linear cause and effect relations. We are attempting to take a view of all relations as folded, as ending up in loops that cycle and become iterative (Bateson, 2000). Such thinking makes us wary, for example, of any mathematics curriculum that lays out predefined learning paths for all students and attempts to flatten the diversity of experiences and relations.

In thinking of the socio-ecological as one, as mutually entangled, we concur with Kirby's (2011) collapse of the separation between culture and nature. Bateson (2000), following Korzybski (2005), counselled that the map is not the territory, and, for Bateson, there was a clear separation between the world of information (culture) and the world governed by laws of mechanics (nature). Bateson was surely right to warn of the dangers of taking the map to be the territory, in the sense that it can be a form of psychosis to confuse symbols for objects, with the objects themselves. And also, as Bateson noted, such confusions are built into rituals such as swearing allegiance to a flag. However, a socio-ecological practice might want to add that, while there is a difference between map and territory, *the map is also part of the territory*. Another way of saying this is to reject any sense of a blind or passive or mechanical "nature" out of which symbols and information emerge as human, "cultural" creations. To think the socio-ecological as one means to take the "translations" (Bryant, 2011) of objects as a form of symbolising work done by all objects in relation to other objects. The river is symbolised by the land it carves. Might we even think of advances in mathematics as entailing concepts doing such work on each other, e.g. the invention of imaginary numbers changing the concept of real numbers (in provoking the need for a distinction between them).

In the Atoyac River work, it became clear how the materiality of the river (and its current level of pollution) is entangled with the information contained in, e.g. plans of the industrial processes used by factories located on the river, and also with the cultures of communities sited nearby. The entanglement of the river and human lives could be observed in multiple, layered pathways of influence, e.g. human actions having led to the toxicity of the river, which has then led to deleterious health outcomes for humans living nearby. Again, focusing on implications for mathematics curriculum innovation, we recognise, in the work of many others, potential of living entanglement, e.g. through focusing on air quality (Brown, 2022), waste (Savard, 2017), incarceration (Wolfmeyer & Lupinacci, 2017), and social security algorithms (Skovsmose, 1994). In each case, there is potential for a curriculum to support becoming attuned to the entangled relations of human, non-human, and mathematics. Living entanglement implies re-thinking what mathematics is, that is, viewing the subject itself in a relational, communal manner. Moves by Gutiérrez (2017) to propose the idea of a living mathematics (mathematx), marked by Indigenous concepts that invoke vibrancy, reciprocity, and collectivity, or by Barwell et al. (2022) to invoke a dialogic mathematics, are good examples of what an entangled mathematics might mean. A communal mathematics education would move away from concern with individual attainment and towards collective action or activism. We might also consider mathematics beyond the human. On the one hand, this might mean recognising the potential agency of mathematical concepts, as we have touched on, mathematics as a subject that humans "hit up against" (Coles, 2022, p.224). On the other hand, this might also mean recognising the mathematics done by other species and acknowledging their intelligence and wisdom.

4 Discussion

We began this article by noting that the documented and experienced state of the planet—including the role of mathematics and mathematics education practices therein—provokes an urgency to (re)evaluate the priorities of mathematics education, specifically, the “reach” of our practices. Such a reconsideration involves considering both what ecological collapse means for mathematics education and whether mathematics education might offer meaningful gestures in response. We began by thinking with and from past and contemporary mathematics education scholarship and relevant approaches in other knowledge fields. We read, in this scholarship, socio-ecological relations that are dialectic or mutually dependent (Figs. 1, 2, and 3), which tend to take “the ecological”, or “nature”, as relatively fixed. We also recognise (and want to endorse) work which adopts an entangled and monist conceptualisation of the social and ecological (Fig. 4).

Latour’s (2004) notion of a gesture invokes a mechanical toy. Our view of the endeavour of mathematics education entails that humans are not mechanical creatures and that there are possibilities of acting otherwise. The three broad gestures we offered are each attempts to elaborate what an entangled view of the social and ecological might imply. The first set, listening to entanglement, focuses on where we place our attention as mathematics educators and what comes to the centre of attention. Of course, our concerns include the vital importance of both mathematics and education. And, we have been arguing that such concerns can no longer ignore the wider sets of relations within which mathematics education takes place. Listening to entanglement will necessarily take our attention away from a sole focus on mathematics and on education and, we believe, can enrich both in the process. By attending to the multiple relations within which mathematics and education exist, they can gain added meaning and relevance, as we illustrated briefly with the Atoyac River Project.

Our second set of gestures involved reflexivity about scales and temporalities, about bringing to awareness from where we are standing, when we attend to aspects of the landscape (both literal and academic), as mathematics educators. We noted the disjunctures that can be experienced between scales, when the historical accumulation of local actions has had a global impact, and yet, we still live with the seeming powerlessness of the individual to effect change when faced with global challenges. The third set of gestures, living entanglement as mathematics educators, pointed to the ontological and epistemological implications of taking seriously what is implied in Fig. 4—that the social and ecological cannot be separated.

From any of the socio-ecological gestures described above, new questions arise as potential starting points for a curriculum, such as those which guided the Memorial Museum for the Atoyac River: How is the river remembered? How do the river pollution levels vary over time, and what is the impact? What is the route back to a healthy river? These are questions for which learning mathematics is not the sole or primary aim. We acknowledge the extent of such a shift, given that mathematics education is, for many of us, the place that “we love and work” (Khan et al., 2022, p. 173). And, we are not wanting to set up new binaries (mathematics at the centre of attention versus mathematics not at the centre), no doubt what is centred in any practice shifts over time and even moment by moment. From our perspective, a body of pure mathematics emerges as a socio-ecological entity but we point here to questions which may sensitise us to entanglements.

The gestures we have outlined point to areas for further research in terms of curriculum innovation. What is the role of disciplinary knowing and thinking, in relation to transdisciplinary and relational competencies? What mathematics and mathematical fields (e.g.

systems theory, non-linear dynamics) could be offered to address socio-ecological questions? And how? What kinds of organisation(s) allow a centring of non-human, ecological concerns? With what kinds of inter-relations? What kinds of roles? We view these as urgent questions for further research.

Methodologically, if we believe in the inseparability of the socio-political and the ecological, this has implications for researching (and teaching). Our gestures (above) suggest practices of: listening and recognising asymmetries in the extent of vulnerability and precarity; seeking multiple narratives of the world; de-centring our attention from isolating and individualizing concern (be it humans, mathematics, education); paying attention to the various scales and times at which actions occur(ed); critical, reflexive questioning of any thinking that tends towards boundedness, human individualism, and linear progress; and recognising the particularity of mathematics educational responses, and for these to be linked to activism. We also recognise the need for school as a place for potential critique of dominant community assumptions.

In more philosophical considerations, we take it that life exists in intimate, layered, and folded relations across time, participants, and space, including historical and contemporary power asymmetries in these relations. A socio-ecological practice implies that we recognise our need for the other (human and non-human). We come to know ourselves as individual humans, to know and to act, in relations with, and through recognition by, others. What we are describing here is consistent with a recent articulation of a dialogic ethics (see Barwell, 2023). As argued in more detail in Coles (2023), a socio-ecological practice implies not taking the individual as a fixed and stable site of learning, but rather recognising our dependence on the relations from which we individualise.

Sitting with and responding to the precarity of the current socio-ecological condition of the world requires an acceptance of loss, in the sense that there is no pristine condition of the world to which we can return (Purdy, 2015). But for mathematics education, this may also be a sense of loss of certainty and centrality (Khan et al., 2022).

Before closing this article, we want to acknowledge what we have not discussed. In particular, we have only touched on questions of ethics, which we view as quite central to the gestures we are proposing, nor mentioned the role of technologies. Furthermore, we have only drawn out briefly implications for views of learning mathematics (e.g. pointing to ideas of a communal mathematics, linking to community and individual activism) and questions of pedagogy (e.g. dialogic forms of teaching, or possibilities exemplified in the Atoyac River Project). We propose these as important areas of future debate and research.

We recognise this article is only the start of what we hope will become an on-going conversation around what it means to enact and recognise socio-ecological practices of mathematics education. One thing of which we feel convinced is that, in the face of potential socio-ecological collapse, a *business-as-usual* approach to (human) mathematics and mathematics education is not a sustainable position. Our thinking, from our particular sites of action, needs to be in conversation with thinking in other contexts. We implore further research and dialogue to conceptualise and exemplify socio-ecological gestures, and how and what mathematics might follow.

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Data Availability Data from The Atoyac River Project, referenced in this study, are available from the authors upon request.

Declarations

Ethics approval Ethical approval for the Atoyac River Project was gained through the University of Bristol.

Conflict of interest The authors declare no competing interests.

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