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"I'm a bit out of place here."—Preservice teachers' positioning in the figured world of university mathematics

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Abstract
Preservice mathematics teachers are sometimes trained in programs so that they have both their own courses and joint courses with mathematics majors from the beginning of their studies. While this is thought to provide them with both deep mathematical knowledge and teaching-specific content right from the start, many of them report disaffection with and disengagement from mathematics during their very first semester. Current approaches often frame this from an individual's perspective, investigating cognitive and affective individual differences among students. In contrast, we aim to understand this issue from a sociocultural perspective, examining underlying social processes. In this study, we thus explored preservice teachers' experiences of the mathematics component of their training, using Holland and colleagues' theory of figured worlds as a theoretical lens. Three group interviews with 14 preservice higher secondary teachers in a common mixed setting in Germany (one course specific to preservice teachers, one general mathematics course together with major students) were analyzed. Our findings displayed that the preservice teachers experienced two dichotomous figured worlds of mathematics and mathematics teaching. Our further analysis of their positioning between these worlds provided new insights to explain their disaffection, disengagement, and consequent learning behavior. We discuss practical implications, focusing on different teaching systems and interventions in teacher education.

Keywords Preservice teachers' education · University mathematics · Sociocultural perspective · Participation · Mixed courses

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1 Introduction

Mathematics teacher education at the university level is widely discussed. Various models can be distinguished, according to whether preservice teachers (PSTs) first complete a mathematics degree, followed by a teaching degree, or study both mathematics and teaching-specific content from the beginning (Musset, 2010). In all approaches, PSTs attend regular mathematics courses to improve their knowledge and acculturation in this subject (Bishop, 2002; Cooney & Wiegel, 2011). Although experts assume that in-depth content knowledge is beneficial for teaching (Capraro et al., 2005; Loewenberg Ball et al., 2008), we know that many PSTs report dissatisfaction with their mathematics studies, question the relevance of the content, and describe a lack of practical experience (Gildehaus & Liebendörfer, 2021; Tatto et al., 2012). To accommodate PSTs and accelerate their training, recent programs include teaching-specific content from the beginning, while offering general mathematics degree courses as well (Murray & Star, 2013). This may result in mixed enrollment courses, which are jointly taken by PSTs in a teaching program and students majoring in mathematics. Such a setting is common in higher secondary PST education (for teachers who may also teach 16–19-year-old students in grades 11–13) in Germany. However, in such mixed courses in Germany, PSTs have been observed to develop disaffection with and disengagement from mathematics (Geisler et al., 2023; Liebendörfer, 2018). Despite minimal differences in cognitive achievements at the beginning of their studies (Blömeke et al., 2011), PSTs perform worse than mathematics majors at the end of their first semester (Göller et al., 2021). PSTs think about dropping out more often than mathematics majors (Blömeke, 2009), more strongly lose interest during the first semester, specifically when they struggle with abstract formalism and proofs in mathematics (Rach, 2014; Rach et al., 2018; Ufer et al., 2017), and report a weaker sense of belonging to mathematics than major students (Guse et al., 2023a). Some of them seem to focus on "just passing" the content-based courses (Gildehaus & Liebendörfer, 2022, p. 1745). Thus, PSTs copy homework from their classmates more often (Liebendörfer & Göller, 2016), use more surface-learning strategies, and apply proof strategies less often (Liebendörfer et al., 2021). Lecturers report PSTs' lack of motivation to leave their school-related student role and be open to new mathematics, especially regarding proof-based problem solving (Hilgert, 2016).

In current research, these findings are explained by differences in affective characteristics between PSTs and mathematics majors (Guse et al., 2023b). For example, PSTs start their studies with clear career aspirations (strengthened by enrolling in a specific program in the mixed models), thus valuing the teaching job itself higher than the subject(s) in which they choose to specialize (Fray & Gore, 2018). Thus, PSTs' lower interest in mathematics compared to that of mathematics majors partly explains PSTs' lower satisfaction (Kosiol et al., 2019). However, this explains neither the PSTs' loss of interest during the first semester nor the increase in performance differences between these two groups. In general, PSTs' disaffection with and disengagement from mathematics in mixed courses do not appear to be clear results of either cognitive abilities or of affect. Current perspectives thus run the risk of a somewhat deficit view on PSTs not being motivated to learn mathematics. They ignore that PSTs' disengagement may have its roots in their social environment. For example, some studies hint at PSTs' feelings of being excluded from the mathematics community and less valued in mixed settings (Carstensen et al., 2021; Liebendörfer, 2018), indicating that social processes occurring in the mixed courses may play a relevant role that has not yet been investigated. Building on this sporadic evidence, we aim to shift

our perspective in the following sections. While the current perspectives on cognition and affect take an individualistic, psychologist view, we aim to investigate the disaffection and disengagement phenomenon in terms of the underlying social processes. To this extent, we take a sociocultural perspective, which we explain in the next section.

2 Theoretical perspective

An inherent rationale of a sociocultural perspective is that learning can be understood as situated and as becoming a participant in a certain community. Thus, knowledge is not only gained on an individual level but also developed in interactions. Such interactions underlie social structures that may position and (hierarchically) relate people to one another in day-to-day practices (Skott, 2019). Thus, a sociocultural perspective enables us to investigate PSTs' positioning and to gain a better understanding of the sources of their perception of exclusion from the mathematical community. Furthermore, we can focus on the students' own sense-making of their actions and themselves in a complex situation (Wenger, 1998). While recent research broadly investigated PSTs' motivation and participation in terms of an anticipated disengagement, as stated in the introduction, there is less knowledge about PSTs' stories of how they experience their learning environment.

To integrate both perspectives of underlying social structures and individual sense-making, we use Holland and colleagues' (1998) theoretical lens in the following analysis. They state that identity and agency are mainly developed in *figured worlds* that are "socially and culturally constructed realms of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al., 1998, p. 52). Newcomers to a figured world, such as incoming students enrolled in university mathematics courses, learn which actions or outcomes are valued over others and which are the significant markers of positionality in everyday events that relate people to one another in a hierarchic sense. Newcomers thus figure their own positionality in a figured world as perceptions of their greater or lesser access to spaces and activities (Holland et al., 1998, p. 128) and to certain outcomes and acts in this world. Inclusionary/exclusionary work can then simply be done by keeping certain people from learning what is taken as indices of social positions/significant markers. "Hence, even in situations where all students are admitted to the area of learning, learning is likely to become unevenly distributed in its specifics" (Holland et al., 1998, p. 135), such as when students are addressed with different expectations.

Furthermore, Holland et al. (1998) describe how stories circulating within figured worlds play a crucial role in shaping individuals' sense-making of themselves, e.g. many PSTs seem to integrate stories about the irrelevance of mathematical content at university for PSTs into their sense-making. Through storytelling, individuals learn about the roles, expectations, and values associated with different identities within their community. These stories contribute to the construction of one's own stories in terms of what Holland et al. (1998) frame as figurative or narrativized identities.

Bringing together these two aspects of an individual's positionality and figurative identity, Holland et al. (1998) draw on Bakhtin's (1981) concept of dialogism and self-authoring. Self-authoring describes the process of actively and consciously creating one's own story. Broadly speaking, one is telling oneself and others who one is by making sense of one's own perceived positionality and integrating and negotiating circulating stories, narrativized into a story about oneself.

Following this framework, we can investigate PSTs' positioning in an institutionalized two-group setting (e.g., who takes up which position in the courses, who has access to which outcomes, who is included in or excluded from significant actions) yet remain with their individual perspectives and specifically analyze how they figure the setting, position themselves in it, and make sense of themselves. We thus pose the following research questions (RQs):

RQ1: How do first-semester, higher secondary PSTs figure the world of university mathematics in terms of valued outcomes and recognized characters?
RQ2: How do first-semester, higher secondary PSTs position themselves in relation to others in this figured world?

3 Methods

The context of this study is the German teacher education system, in which higher secondary PSTs study two subjects in a mixed-course setting. PSTs attend subject-specific courses (some jointly with mathematics majors), didactics of their subjects, as well as general pedagogical courses. Our study was conducted at a medium-sized university, where two mathematics courses are obligatory for PSTs in the first semester. The Linear Algebra (LA) course offered lectures twice a week and tutorials once a week (4 plus 2 teaching hours per week). On average, 150 students took the course per semester; 70% of them were PSTs, 20% comprised mathematics majors, and 10% consisted of computer science majors who had chosen mathematics as their minor. Students had to earn at least 33% of the total points in their weekly homework to be eligible to take the final exam. The second course, Introduction to Mathematical Thinking (IC), was a bridging course,[1] specific to PSTs only. Lectures and tutorials were each given once a week (2 plus 2 teaching hours per week). Students had to pass two short exams during the semester, as well as obtain 50% of the total points in their weekly homework to be eligible to take the final exam. Both courses were purely based on subject content and did not address pedagogical content knowledge. As usual at this university, both courses were taught by mathematics professors. LA tutors were mainly mathematics major students in more advanced semesters; IC tutors were PSTs in advanced semesters. For both courses, voluntary office hours in the learning support center[2] were available twice a week. All mathematics students of that university could voluntarily participate in a pre-course.[3]

Three group interviews with 4–5 PSTs per group (n = 14; 8 females, 18–23 years old) were conducted in the 2019/2020 winter term, two months after the first semester had started. The interviews were held in person after PSTs voluntarily answered an open call. The call addressed self-formed groups of PSTs to allow a free discussion and to limit the interviewer's influence and power. They were guaranteed anonymity and had no further contact or relation with the interviewer (the first author). The participants represented PSTs taking diverse second subjects (e.g., sciences, foreign languages, sports, or social science).

[1] Bridging courses aim to facilitate the transition to university-level mathematics. See Liebendörfer et al. (2023) for further explanation.
[2] See Schürmann et al. (2021) for an overview of mathematics learning support centers.
[3] See Biehler et al. (2018) for a short introduction to pre-university mathematics courses.

The interviews had freewheeling topics to give the PSTs the opportunity to set their own themes. The interviews started with a question about their experiences and impressions during the first months of their studies. Further questions included their daily routines for mathematics, current challenges, as well as their current feelings about and levels of satisfaction with their studies. Perceived differences (e.g., between school and university or the courses) were *not* elicited directly by the interviewer but were brought up by the PSTs in all interviews. Follow-up questions then included how they would describe such perceived differences to other people and how they identified themselves with mathematics at university. The interviews were video recorded and fully transcribed in German. The segments reported here were translated into English by the authors. We provide the original data as well.

The data analysis consisted of several iterations. Our theoretical position led us to notice the social construction and interpretation of the figured worlds of mathematics, which might guide the PSTs' positioning in relation to one another. Since this might concern rather sensitive issues that the PSTs might not necessarily discuss openly, we used a discourse analysis approach that "can illuminate problems and controversies" (Gee, 2014, p. 10) and thus reveal the PSTs' implicit statements. First, we identified the PSTs' descriptions of the figured worlds in university mathematics teaching (as operationalized in Table 1), coding segments about their figuring of valued actions or outcomes, as well as characters. To foster reliability, this step was performed independently by the first two authors and later cross-checked collectively (J.W. Creswell & J.D. Creswell, 2018). We then delved deeper into how these descriptions were connected to social formatting of power and how the PSTs positioned themselves in relation

Table 1 Operationalization of figured worlds for our analysis

Concept	Operationalized definition and example
Figuring valued actions or outcomes	References to what seemed relevant or important in a given context. For example, in the description, "Here [in the LA course], no matter how hard you work, if you don't really understand it or don't have access to it, it's really extremely difficult," we assumed that *really understanding* was figured as a valued outcome of the course
Figuring characters and their access to valued outcomes of the world	References to recognized characters in the figured world and their perceived access to valued outcomes or actions of the world. For example, in the following descriptions, *mathematics majors* are figured as characters with *access* to some specific (abstract) thinking and understanding, enabling them to perform certain acts, such as joking with the professor: "They [mathematics majors] are more into this abstract thinking, which we somehow…" "They are more on the professor's level than on ours." "Well, they always get along really well with him, and he also jokes with them."
Positioning oneself in relation to others	References to one's perceived access to spaces and activities, as well as one's positioning (as either an insider or an outsider) in relation to others. In this example—"So you really notice, the lecture is tailored to mathematicians. (…) and you as a preservice teacher are just like (…), 'Okay, somehow, I'm a bit out of place here.'"—the student perceived herself as an *outsider in relation to the "mathematicians"* who were also attending the lecture

to these formats. We investigated their positioning in terms of how they described themselves in relation to others, as operationalized in Table 1. To foster validity, this step was performed collectively by all three authors, who come from various backgrounds in terms of gender, culture, and teaching/major background. Extensive discussions among the authors were held to greatly capture the original meaning (J.W. Creswell & J.D. Creswell, 2018).

4 Findings

We first present our findings regarding RQ1, showing how most of the PSTs figured the courses of IC and LA as containing dichotomous valued actions and offering different opportunities for engagement. Then, we examine the PSTs' positioning in relation to these two dichotomous worlds in RQ2. We conclude with a case study of one student who managed to better navigate the two worlds and positioned herself as an insider of both.

4.1 RQ1: Figuring LA and IC as two dichotomous sub-worlds

All interviewed PSTs described university mathematics as a distinguished world with two different sub-worlds; some of them did so from the beginning of the interview. For example, in her first sentence in interview 3 (I3), Marie_I3 differentiated between the courses they attended:

> We have IC and LA at the moment, and in IC, I think we are all doing quite well, can follow, and such. But especially now in LA, you realize, okay, maybe you are not yet on this level of thinking.
> *Wir haben ja jetzt einmal EmDA und einmal LinA im Moment und in EmDA laufen wir, glaube ich, auch alle ganz gut mit, [kommen] ganz gut zu recht so. Aber halt vor allem jetzt so LinA, da merkt man halt schon so, okay, man ist einfach vielleicht auch noch nicht auf dieser gedanklichen Ebene.*

This distinction became more apparent in the interviews in terms of what was figured as valued in these sub-worlds. Since the PSTs mainly based their distinctions on the different courses, we use their own words in the following analysis.

4.1.1 Valuing different mathematics in the IC and LA sub-worlds

In general, the PSTs described the mathematics of the figured world at university as very different from how they figured mathematics in school. However, they perceived differences in how distant they figured the valued mathematics in the courses they knew from school and to what extent this mathematics was still aligned with or continued mathematics in school. The LA course was figured as "completely different from [the mathematics taught] in school." (Kira_I2) -*„komplett anders als in der Schule."* and unconnected to anything they knew. It was perceived as a "foreign language, you only speak half." (Kira_I2) -*„eine fremde Sprache, die man nur so halb spricht."* The PSTs figured the value of abstract formalization instead of imagination in this sub-world but struggled to deeply discuss what was meant by this value. When asked what exactly made mathematics in the LA course so challenging, they agreed that it was "this abstract [thinking], those unimaginable [objects]" (Tim_I1), -*„dieses Abstrakte, dieses Unvorstellbare"*, without further explaining "this."

In contrast, "in IC, it is not like that" (Thomas_I2 -*„In EmDA ist das nicht so"*, responding to Kira's "foreign language" comment). "In IC, you get along nicely." (Kira_I2) -*„In EmDA kommt man super mit."* Mathematics in the IC course was perceived as less abstract in the sense that they could still somehow imagine the mathematical content. It was figured as aligned with the pre-course (which itself was figured as aligned with and connected to mathematics in school). As Tina_I3 described, "IC still reminds me of the pre-course. There are elements [from the pre-course] that reappear [in the IC lecture] and where you can imagine something again." -*„EmDA erinnert mich immer teilweise noch so an den Vorkurs. Da sind so Elemente [aus dem Vorkurs], die dann halt wieder auftauchen [in der Vorlesung] und wo man sich halt wieder etwas vorstellen kann."*

However, the PSTs' figuring of these different courses also distinguished the IC course as somehow a "weakened form" (Marc_I1) -*„abgeschwächter Form"* of the LA course, stating that in the figured world of university mathematics, the LA course was perceived as of higher value in terms of providing *real* mathematics, not the kind of formal mathematics that could still be aligned with school mathematics. In contrast, the IC course content was regarded as more relevant for future teachers.

4.1.2 Valuing different ways of teaching mathematics in the IC and LA sub-worlds

In accordance with figuring "different mathematics" in the two sub-worlds, their content was also figured as being *taught* differently. In the IC course, mathematics teaching was strongly figured as *delivering mathematics*. This included breaking things down and providing example-based and imaginable explanations, as well as step-by-step solutions, as Milena_I3 described, "breaking things down to this not completely mathematical thinking (…) that you just have a simple explanation, like you can roughly imagine this [referring to a formal mathematics definition] as this, and that's how it works." -*„Und das einfach mal herunter zu brechen auf dieses nicht komplett mathematische Denken. (…) dass man einfach noch eine einfache Erklärung dazu hat, [und so] das und das kann man sich ungefähr da drunter vorstellen, so funktioniert das."* Delivering mathematics also included flexible, individual, and student-oriented reactions to difficulties, as Marie_I3 explained:

> In IC, I think, there are only people [tutors] who are really prospective teachers, and you can say that they (…) really try to teach you and really try their best (…). If they notice that in the exercises, maybe more difficulties occur in a task, they calculate it again somehow on the blackboard or explain it somehow again in general.
> *In EmDA sitzen, glaube ich, ja auch nur Leute, die auch wirklich auf Lehramt sind und wo man merkt, die (…) versuchen halt wirklich, es dir beizubringen und versuchen halt wirklich ihr Bestes (…). Und, ja, versuchen halt, wenn sie merken, dass in der Übung vielleicht vermehrt irgendwie Schwierigkeiten mal in einer Aufgabe vorkommen, das noch mal irgendwie an der Tafel zu rechnen oder es irgendwie noch mal allgemein zu erklären.*

As Marie stated, the IC teaching staff was figured as taking a kind of (school-) teacher role that showed strong alignment with what was often figured as valued teaching in school—delivering mathematics to students and being aware of whether they could follow and understand or failed to do so. This was in line with the PSTs' figuring of the IC course as being nicely structured throughout its content, making it easier for them to look up specific definitions and ask questions about them. ("In IC, the syllabus is really well structured, and you know roughly where you can look, or at least where you can ask." Marie_I3

-*„In EmDA ist das Skript bei uns auch richtig gut strukturiert und da weiß man ungefähr, wo man nachgucken kann, oder zumindest wo man nachfragen kann."*)

The LA course was strongly figured in contrast to the IC course. It was perceived as *not* breaking things down, strictly adhering to a formal presentation of mathematics, *not* providing example-based or imaginable explanations, and specifically *not* providing step-by-step solutions. The teaching staff was figured as not reacting flexibly to students' needs and difficulties. For example, Kevin_I2 noted:

> In LA, we usually don't have any examples. 'Cause our professor says, 'Yes, we haven't defined numbers at this time, so we can't use numbers' [strictly maintaining a formal representation, not providing an example]. And in IC, just, 'Yes, use numbers.' Then [the professor says,] 'you do this, this, this' [providing a step-by-step solution]. And all of a sudden, everybody understands. In LA, they (…) don't use an example. (…) no one understands.
> *Bei LinA haben wir auch meistens keine Beispiele dabei. Dann sagt unser Professor halt: ‚Ja, Zahlen haben wir zum Beispiel noch nicht definiert gehabt zu dem Zeitpunkt, also dürfen wir ja keine Zahlen verwenden.' Und in EmDA [sagt der Professor] einfach: ‚Ja, benutzt doch Zahlen. Dann macht ihr das, das, das'. Und auf einmal haben es alle verstanden. In LinA (…) wird kein Beispiel verwendet (…) keiner versteht es.*

Actual descriptions of the way that mathematics was taught in the LA course were rare if they were not contrasted to the IC course. Kevin_I2 tried to explain how he perceived the teaching in the LA lecture, framing it as *simply copied*: "It [the content] is simply copied [from the syllabus] (…) to the blackboard, without somehow *really* explaining something about it." -*„Es [die Inhalte] wird einfach stumpf [vom Skript] (…) an die Tafel geschrieben, ohne irgendwie was großartig da noch zu erklären."*

Thus, the LA teaching staff was figured as *not* aligning with any kind of valued teaching actions from school, refusing to *deliver mathematics* to the students but *simply copying* it for them. This included not rephrasing explanations and thus being perceived as inflexible and unwilling to respond to difficulties: "You ask again, and then it's explained exactly in the same wording (…). And then you're like, 'Yeah, I didn't understand that before'." (Ronja_I3) -*„Man fragt noch mal nach, und dann wird es genau im gleichen Wortlaut genauso noch mal erklärt. (…) Und dann denkt man sich so:‚Ja, das habe ich ja schon nicht verstanden'."* Finally, the LA syllabus was perceived as less structured—and in this sense—less student-oriented.

4.1.3 Valuing different ways of learning mathematics in the IC and LA sub-worlds

The different values of mathematics and its teaching further found their way into the PSTs' perceptions of the valued ways of learning mathematics in the two courses. Most of the PSTs' learning occurred around their homework.

In the IC course, homework was viewed as connected to the exercises of the weekly tutorials, valuing reproduction and procedural applications, as expressed by Marie_I3:

> But I also think that the homework of IC and [that of] LA are different. (…) in IC, it's like [you can look at what] we do in the tutorial, and then look again, yes, okay, maybe I can derive something from that for the homework. Or you have done something similar and can apply it in the same way.

> *Ich finde aber auch, dass die Hausübungen von EmDA und LinA sich noch mal unterscheiden. (…) in EmDA ist das halt meistens so, dass man halt ähnliche Aufgaben macht zur Hausaufgabe und dann kann man halt manchmal noch mal gucken, okay, was haben wir in der Präsenzübung gemacht, und dann noch mal gucken, ja, okay, vielleicht kann ich daraus etwas ableiten zur Hausaufgabe. Oder man hat etwas Ähnliches gemacht und kann das halt genauso anwenden.*

Such applications strongly aligned with how the PSTs figured mathematics learning in school. "There [in school mathematics] was a structure, and I learned it once, and then I could actually apply it." (Marie_I3) -*„Es gab eine Struktur und (…) ich habe das einmal gelernt und dann konnte man es anwenden."*

While the PSTs described IC homework as not too easy, it was nevertheless figured as very approachable. For example, Louisa_I1 stated that the learning support center was "not necessary for IC," -*„für Emda nicht so nötig"* and her peers agreed. Thus, the IC course was figured as offering some kind of guided learning, where structure and guidance were provided through similarities and alignment and where the PSTs seemed protected from struggling. However, they figured their learning outcome as highly dependent on the teaching, leaving themselves with less agency to enact in learning.

In contrast, LA homework was perceived as highly demanding, perhaps even excessively so. Most PSTs figured the LA sub-world as requiring immense effort yet sometimes remained inaccessible, as Marc_I1 observed:

> So sometimes, there are moments when you (…) don't understand a single word of the stupid task, except for 'prove' or 'show'. (…) you really go into it, hoping that it's going to be good, and then [it is like] you get a slap in the face.
> *Also es gibt auch manchmal so Momente, wo man (…) kein einziges Wort aus der blöden Aufgabenstellung versteht, außer 'beweisen Sie', oder 'zeigen Sie', (…), wo man echt mit der Hoffnung ran geht so, boah, das wird gut und dann kriegt man so einen verbalen Schlag ins Gesicht.*

The content of the tutorial was not perceived as being aligned with the homework. "I think that in LA, we don't do anything in the tutorial (…) that has anything to do with the homework" (Marie_I3) -*„Aber ich finde halt, in LinA, also wir machen halt in der Präsenzübung, (…), gar nichts, was mit der Hausübung zu tun hat."* LA homework was figured as demanding high creativity; "everything requires a new idea" (Marie_I3) -*„alles erfordert eine neue Idee"* instead of reproduction. To handle the homework, the PSTs figured real understanding as relevant, as Marie_I3 explained: "Here [in the LA course], no matter how hard you work, if you don't really understand it or don't have access to it, it's really extremely difficult." -*„Hier [in LinA], egal wie fleißig man ist, wenn man halt irgendwie es nicht so richtig versteht oder nicht so den Zugang dazu findet, ist es echt extrem schwierig."* At the same time, the PSTs could not figure out how to acquire "real understanding." Understanding remained mysterious, even though it was observed as occurring. For example, Lisa_I2 described how she recognized a student correcting the professor during the lecture: "And then you think to yourself, wait, why did he even understand that?" -*„Wo man sich denkt, wieso hat der das jetzt verstanden?"* Most PSTs figured the teaching style of *simply copying* as the main source of their own misunderstanding, as Milena_I3 stated: "In order for us to be able to deliver it [mathematics] reasonably, it must first be delivered to us reasonably." -*„Damit wir das vernünftig beibringen können, muss es erst mal uns vernünftig beigebracht werden."*

Thus, the faculty staff was figured as denying access to understanding, and the responsibility for successful learning was attributed to the staff.

Following this (lack of) understanding yet its perceived relevance for homework, it was a great struggle for the PSTs to earn points. Copying homework from others was highly restricted in the LA course, resulting in zero points for all students with similar solutions. Neither was the learning support center perceived as directly helpful for doing homework, as Marie_I3 mentioned: "We are always told that the learning support center is not there to ask questions about homework." -*„Uns wird aber immer gesagt, das Lernzentrum ist nicht dafür da, um zu Hausaufgaben Fragen zu stellen."*

In conclusion, mathematics learning in the LA sub-world was figured as unapproachable, unguided, and mysterious in its demands, mostly culminating in "getting the homework points." "Understanding" in LA was figured as a necessary requirement for success in general and for achieving the homework goal in particular. Nonetheless, how this "understanding" could be achieved remained elusive for the PSTs and at times, considered simply impossible.

4.2 RQ2: PSTs' positioning in relation to others in these sub-worlds

Following these different perceptions of which mathematics is valued and how it is taught and learned, we further analyze the PSTs' positioning in these figured sub-worlds.

4.2.1 Insiders of the IC sub-world

The interviewed PSTs clearly aligned themselves with everything that was valued in the IC sub-world and positioned themselves as insiders of this world. They described how they "always enjoyed doing mathematics in school" (Marie_I3) -*„ich habe Mathe immer gerne gemacht in der Schule"* and thought that "IC would also be really fun if you had more time (…) if you weren't sitting on LA for so long." (Tina_I3) -*„EmDA würde, glaube ich, auch richtig Spaß machen, wenn man halt vielleicht da mehr Zeit hätte(…) wenn man nicht so lange an LinA sitzen würde."*—showing alignment with less abstract and more imaginable mathematics. They further referred to their own teaching experiences in private tutoring, affirming that they "nicely delivered mathematics to children" (Marie_I3) -*„So, gut [den] Kindern etwas beizubringen"*, thus identifying themselves with the idea of delivering mathematics. Regarding their own learning, they described themselves as diligent and structured, valuing the guided learning in the IC sub-world.

We did not gain much insight into the PSTs' positioning in relation to the teaching staff in the IC course. However, they consistently stated how IC "gives a good feeling" (Louisa_I1) -*„gibt da ein gutes Gefühl"*, and their descriptions of their learning actions were consistent with how they figured teaching in the IC course. Thus, we can assume that they positioned themselves as not only valued and engaged learners in the IC course but also recipients of knowledge that were depended on the teacher, as mentioned earlier.

The PSTs also positioned themselves in relation to others in the IC sub-world. They figured themselves as a rather homogeneous, social, and not *too* smart group: "So, we [PSTs] are on the same page; we all understand about the same." (Ronja_I3) -*„So, dass wir [Lehramtsstudierende] halt auf einer Wellenlänge sind, sodass wir alle ungefähr gleich viel verstehen."* This same level of "understanding"—or in the LA course, "not understanding"—was of great importance. Sharing their lack of understanding with one another helped them cope with the situation:

Thomas_I2: "If we get stuck, then we definitely help each other. (…) If we all get stuck, then we somehow get through it together."
Kira_I2: "(…) It's shared stupidity."
Thomas_I2: "It is good that you are not alone at that moment."
Thomas_I2: „Wenn wir mal nicht weiterkommen, dann helfen wir uns auf jeden Fall gegenseitig. (…) Wenn wir alle nicht weiterkommen, dann stehen wir das irgendwie zusammen durch."
Kira_I2: „(…) Es ist geteilte Dummheit."
Thomas_I2: „Es tut gut, dass man nicht alleine ist dann in dem Moment."

This social aspect was developed further when it was valued as creating a safe space for one another, where it was perceived as helpful to discuss and explain the lessons together for their mutual learning. It was also deemed important to have an even distribution about this learning, underlining both the social aspect and the necessity of being on the same level of understanding:

I think it would also be difficult if everything is super obvious and super simple for one person. Sure, they can help the others, but it doesn't help them that much. (…) I think it is important in learning groups to be on a similar level, to avoid [the case where] one [person] always helps the other, but it doesn't balance out. (Ronja_I3)
Aber ich glaube, es wäre auch schwierig, wenn jetzt für einen das alles super ersichtlich ist und super einfach. Klar, der kann den anderen helfen, aber ihm selber bringt das ja auch nicht so viel. (…) Ich glaube, das ist auch schon teils wichtig für so Lerngruppen, dass sie halt einen ähnlichen Stand haben, bevor halt wirklich nur der eine immer dem anderen hilft, aber sich das nicht so ergänzt. (Ronja_I3)

The other side of this figuring meant that *too* much mathematics knowledge, in relation to the LA course, was not valued in the IC sub-world because it conflicted with an equal contribution of everyone and a safe space of "not understanding."

4.2.2 Outsiders of the LA sub-world

In contrast to the IC course, almost all PSTs figured themselves as outsiders of the LA sub-world, as Ronja_I3 explained, "So you really notice, the lecture is tailored to mathematicians. (…) and you as a preservice teacher are just like (…), 'Okay, somehow, I'm a bit out of place here'." -*„So, dass man wirklich merkt, dass die Vorlesung auf die Mathematiker zugeschneidert ist. (…) Dass man als Lehrämtler da (…) sich so denkt, ‚Ja, okay, irgendwie bin ich hier so ein bisschen fehl am Platz'."*

However, this did not mean that the PSTs could not align themselves with what was valued in the LA sub-world; instead, they figured themselves as not having access to the valued outcomes. While the PSTs criticized the too formal and abstract content of the LA course, as well as teaching by simply copying, they strongly aligned themselves with the valued learning outcome of understanding. As Milena_I3 stated, "I am a person who has to understand (…). I want to understand. But in math at the moment, at least in LA, I do not understand anything anymore." -*„Ich bin halt auch die Person, die sagt, ich muss die Sachen verstehen (…). Ich möchte die verstehen. Und in Mathe ist halt im Moment zumindest in LinA der Fall, ich verstehe nichts mehr."*

The PSTs figured themselves as less valued by the LA teaching staff compared to the IC staff. This was underlined by their perception of not being directly addressed in this course, such as when the professor referred to the Analysis course, which the

mathematics majors attended instead of the IC course. Moreover, the interviewed PSTs perceived themselves as addressed by what was valued in the IC sub-world (teaching/delivering mathematics) but restrained from what was valued in the LA sub-world (understanding/solving problems):

> Milena_I3: "You always have the feeling that he [the LA professor] doesn't really think much of preservice teachers, either. He doesn't say that precisely, but always (…)"
> Gina_I3: "So, you always notice that you're talked down to."
> Milena_I3: "Like, 'Yes, we do that [referring to the content of the lecture] so that the future teachers sitting here can teach their students that, so they can then solve the math problems of tomorrow.' And then you think to yourself, are we ourselves too stupid for that, really?"
> *Milena_I3: „Aber man hat auch immer so das Gefühl, dass der von Lehrämtlern auch nicht wirklich viel hält. Also das sagt der nicht präzise, aber immer (…)"*
> *Gina_I3: „Also man merkt immer, dass man so heruntergeredet wird."*
> *Milena_I3: „So: ‚Ja, das machen wir, damit die zukünftigen Lehrer, die hier sitzen, das irgendwann mal ihren Schülern beibringen können, und damit die dann die Matheprobleme von morgen lösen können.' Und da denkt man sich so, sind wir zu dumm dafür oder was?"*

Similarly, their positioning in relation to other students in the LA course was a distanced and contrasted one, as Marie_I3 described: "So, computer scientists and mathematicians always sit together, and preservice teachers always sit together somehow. And I think that we just don't come together at all."—*„Also, Informatik und Mathe sitzt immer zusammen und Lehramt sitzt halt immer irgendwie zusammen. Und ich finde, das kommt halt fast gar nicht zusammen."*

Those major students were figured as having access to the valued outcome of understanding. They were perceived as people who "all understand it right away" (Ronja_I3) -*„die [Fachstudierenden] verstehen das auch alle sofort"*, which enabled them to struggle less with homework points and establish a closer relationship with the LA course professor:

> Milena_I3: "They [mathematics majors] are more into this abstract thinking, which we somehow (…)"
> Ronja_I3: "They are more on the professor's level than on ours."
> Milena_I3: "Well, they always get along really well with him, and he also jokes with them."
> Gina_I3: "But we just don't understand it."
> *Milena_I3: „Also die sind schon in diesem abstrakten Denken drin, wo wir anscheinend so (…)"*
> *Ronja_I3: „Die sind schon eher auf Niveau des Professors als so bei uns."*
> *Milena_I3: „Also die verstehen sich auch immer total gut mit dem und der macht auch Witze mit denen."*
> *Gina_I3: „Aber wir verstehen es halt nicht."*

In line with the valued teaching actions in the LA sub-world, the major students were also figured as refusing to explain/break things down, as Milena_I3 mentioned: "And then, you think to yourself, 'Yes, nice for you [that you understand that exercise], but maybe you could explain it to me?!' But then, they (…) don't really manage to break it down." -*„Und dann denkt man sich so: ‚Ja, das freut mich für dich [dass du die Aufgabe verstehst],*

vielleicht erklärst du das einem mal.' Aber dann (…) die schaffen es auch nicht, dass herunter zu brechen."

PSTs thus figured major students as clearly aligned with what was valued in the LA sub-world and, in line with that, figured themselves as being less aligned, less valued and having less access to valued actions and outcomes, in relation to the majors (even though they did align with some of the valued actions such as that of understanding, for example). In relation to their classmates in the LA course, the PSTs positioned themselves mainly as *not* being major students, and in this regard, *not* understanding. For example, being a real mathematician was reserved for the major students, who had (greater) access to understanding and homework points, as Marc_I1 stated: "I would never call myself a mathematician." -*„Also ich würde mich niemals als Mathematiker bezeichnen."*

4.2.3 Insider of both sub-worlds: Louisa's case

In this section, we present the case of Louisa, a PST who not only positioned herself distinctly from her peers but also figured the mathematics world quite differently. First, she did not dichotomize the sub-worlds, as her peers did. Although she considered the IC course slightly easier, she did not perceive its mathematics differently in any way. She clarified that "studying any kind of STEM is not easy and is different from school," -*„naturwissenschaftliches Studium ist nicht einfach, ist anders als Schule."*, figuring both courses as containing formal and abstract mathematics. However, in contrast to her peers, referring to her overall studies, she stated at the beginning of the interview, "it is tiring but doable." -*„es ist anstrengend, aber es geht."* Thus, she did not figure LA as some kind of unapproachable or mysterious course but a challenging yet somehow manageable one, including the homework points. In general, she did not figure homework unreasonable but a task that would help her structure and regulate her learning: "Without this kick in the ass, you have to hand it in now, (…) you would certainly do a little less (…). It [the homework] definitely helps that you keep on top of things regularly." -*„Klar würde man, ohne diesen Arschtritt, du musst das jetzt abgeben, (…), sicherlich eine Ecke weniger machen (…). Es [die Hausübungen] hilft auf jeden Fall, dass man halt regelmäßig am Ball bleibt."*

Instead of talking about the course professors and tutors, she mainly discussed her own learning actions (e.g., how she was using computer algebra systems to check solutions or searching for relevant definitions in the syllabus). While her peers were talking about how abstract and formal the LA course was, she remained mostly silent. Only at one point in the interview did she agree with her peers about a specific lesson (sets and powersets) of the LA course that she also did not understand at all, mainly because "we never saw an example [about that]." -*„weil wir da nie ein Beispiel [zu] gesehen haben."*

She regarded the tutors of *both* sub-worlds as helpful supporters of her learning. For example, she shared her experience in the learning support center, where "there was a tutor from linear algebra (…); I also chatted with him, [and] he also helped me." -*„der Tutor war da von Lineare Algebra, (…) da habe ich auch mit ihm geschnackt, [und da] hat er mir auch geholfen."* She explained how she was handling her insecurities in doing the homework, with the tutors' help:

> I often write questions on the exercise sheet, and (...) they've [the tutors who correct the homework] written three or four sentences. Because if you don't write a question, just the correction is usually not that helpful. So of course, it usually takes forever; they have limited time, but when you write a question, I've always been lucky so far.

Oft schreibe ich dann auch auf das Übungsblatt Fragen und (…) sie [die korrigierenden Tutor:innen] haben dann doch drei, vier Sätze dazu geschrieben. Weil wenn man keine Frage schreibt, nur die Korrektur ist meistens nicht so hilfreich. Also klar, dauert ja sonst auch ewig, die haben ja auch begrenzte Zeit, aber wenn man eine Frage schreibt, hatte ich bis jetzt immer Glück.

As in many other statements, Louisa integrated the perspective of the tutors, or more generally, of the faculty staff, into her reasoning, considering it understandable that tutors might have limited time for a lot of students. In contrast to her peers, she figured tutors and faculty staff as experts who acted reasonably and in accordance with the study program's goals.

Louisa's positioning aligned with the way she figured the world of university mathematics. She positioned herself as an active learner within the figured world who engages in learning for her own goals and enjoyment: "I've always found it interesting to learn new things (…) [and] well, you do it [studying mathematics teaching] because you basically enjoy it." -*„Ich fand schon immer, es ist interessant, Neues zu lernen (…) [und] man macht es [Mathematik-Lehramt studieren] ja schon, weil man prinzipiell Spaß daran hat."* This perspective again stood in contrast to her peers' usual statement about their main goal: "to become a teacher." (Ronja_I3) -*„Lehrerin werden."*

Notably, Louisa did not clearly designate herself as becoming a teacher in the future:

Well, I decided on math and computer science [teaching degree] because in school, I thought that those subjects were great, but I also see the chances of doing something outside of school (…). So, I don't know yet whether I will eventually become a computer scientist student instead of a preservice teacher. I wouldn't [want to choose either one]. Why can you only be one; why can't you be both?
Und ich habe mich dann für Mathe und Informatik [auf Lehramt] entschieden, weil ich die Fächer eben in der Schule toll fand, aber auch da eben die Chancen sehe, was Außerschulisches zu machen (…) Deshalb weiß ich noch nicht so, ob ich dann vielleicht irgendwann doch Informatikerin bin statt Lehrämtlerin. Ich würde auch nicht [wählen wollen müssen]. Warum kann man nur das Eine sein, warum kann man nicht alles sein?

Here, we observe an explicit resistance to figuring the two sub-worlds (in terms of teaching and computer science) as mutually exclusive. Moreover, Louisa questioned the designation of this program's graduates as being either computer scientists or PSTs. This questioning emerged after all the other group members had identified themselves clearly as teachers-to-be, highlighting the uniqueness of Louisa's voice.

In alignment with this duality, Louisa also positioned herself in various ways as a participant of the two sub-worlds (although, at least according to the voices of her friends, these sub-worlds were sometimes mutually exclusive). This dual positioning demanded from Louisa a complex dance between the valued actions of the two worlds. While positioning herself as mostly oriented to the LA sub-world (e.g., she did *not* question the relevance of formal and abstract mathematics), she also aligned herself with the IC sub-world, mainly by being social and emphasizing that she was not one of the *really* good people. For example, she explained how the necessary homework points in LA were split into two halves throughout the semester:

So, we need a third of the points in the first half and in the second half, (…) so that the people who are really, really good are not already through it [received all necessary points] in the first half. (…) well, that wouldn't have been our problem

(laughs).
Also wir brauche in der ersten Hälfte ein Drittel der Punkte und in der zweiten Hälfte (…) damit nicht die Leute, die richtig, richtig gut sind eben in der ersten Hälfte dann das [die notwendige Punktzahl] schon durch haben (…) Ja, das wäre nicht unser Problem gewesen (lacht).

Furthermore, she made it clear that her enjoyment of her studies partly came from her peers ("I always find it really fun with us." -*„Ich finde es immer ganz lustig bei uns."*) While she was the one who often shared solutions for the homework, she valued the others' participation as well, underlining an equal contribution: "I have no problem sending the entire exercise sheet in there [the WhatsApp group of her learning group], especially because it's really varied well so far." -*„Also habe ich auch kein Problem damit, das ganze Übungsblatt da rein zu schicken [in die WhatsApp Gruppe ihrer Lerngruppe], gerade weil es sich echt bis jetzt gut abwechselt auf jeden Fall."*

On one hand, while Louisa described her general study situation as "very good" -*„sehr gut"* and was quite satisfied overall, her double participation had consequences for both her alignments. As stated earlier, she seemed to silence herself in most of her peers' discussions about the "too formal LA course." On the other hand, as noted, Louisa always described herself as "lucky" to receive helpful answers from the tutors, acknowledging these as somewhat unexpected— perhaps due to her, as a teacher student being part of a "teacher positioning."

5 Discussion

Previous studies observed how PSTs are often dissatisfied with and disengaged from their mathematics courses (Geisler et al., 2023; Kosiol et al., 2019; Rach et al., 2018), specifically when these courses are shared with mathematics major students (Gildehaus & Liebendörfer, 2021; Guse et al., 2023a). However, current approaches, focusing on hindering cognitive or affective attributes of PSTs, can only partly explain their dissatisfaction and disengagement (Fray & Gore, 2018; Geisler et al., 2023; Guse et al., 2023b; Kosiol et al., 2019) and remain on an individualistic level, which may provide a deficit perspective of PSTs. Thus, based on a sociocultural approach and the framework of figured worlds, we further investigated the *processes* giving rise to these observations. We found that PSTs figured the university mathematics world in a mixed institutional setting as dichotomous, in contrast to current findings about students in homogeneous courses, described as a highly exclusive, united math club (Beccuti et al., 2023). Our interviewed PSTs in the mixed setting figured the different courses they attended as mutually exclusive sub-worlds, where valued outcomes of mathematics, teaching, and learning were not only figured differently but in contrast to one another. While the IC course, specific to PSTs, was figured as some kind of continuation from school mathematics, the shared LA course was figured as rather inaccessible for the PSTs. They perceived themselves as lacking access to the significant marker of positionality—understanding. We unraveled how this was connected to an unfavorable positioning of PSTs as insiders of the IC course but outsiders of the LA course, where they perceived themselves less valued and unable to engage in successful learning. Our findings do not seem related to the specific content of the course (i.e., LA) since the challenges reported by the students are encountered in other courses as well (Biehler, 2019; Liebendörfer, 2018). We thus believe that similar findings could have been obtained in settings with other courses such as real analysis.

The perceived dichotomy of positioning between the two courses was resisted by only one student in our group of participants—Louisa. However, she also figured the sub-worlds differently, perceiving them as less contrasted. Nonetheless, her participation in both sub-worlds required a complex negotiation between the conflicting valued actions. While Louisa admitted some uncertainty about her desire to become a teacher, her case highlighted the general problem of the dichotomous sub-worlds of PSTs and mathematicians. Furthermore, her situation emphasized figuring and positioning as inseparable (Holland et al., 1998). Her positioning as a "double insider" was consistent with her figuring of the sub-worlds as less dichotomous. In contrast, the other PSTs' figuring was mainly based on their positioning as future teachers, again supporting their perception of the PST specialized courses as contrasting the mathematicians' world.

The implications that can be drawn from this analysis are that if we want to change PSTs' positioning, we also need to be aware of their figuring of the world in which they engage. In this context, the mixed-course setting accentuated and enabled contrasted rather than continuous figuring. Institutional structures that were believed to help PSTs find their way into mathematics provided a space to align themselves with a position contrary to that of the mathematics course. On one hand, specific PST courses may reinforce practices that leave PSTs in a world closely aligned with school mathematics and their role as school students. On the other hand, in mixed courses, we need to be aware that students from different programs may position themselves in relation to each other as dichotomous groups, which may not benefit all of them.

If we relate these findings to PSTs' observable disengagement and disaffection, such sentiments seem unsurprising, given their figuring and positioning. However, their disengagement can no longer be viewed as "one way," following unfavorable personal attributes. Moreover, PSTs' strong alignment with the valued outcome of understanding indicates this is not (only) about hindering motivation to engage in practices valued by the mathematical community. The social structures in the mixed setting showed that more than alignment is necessary to gain access. They have prepared the ground for PSTs' positioning in terms of what they need, what they are capable of, and who is responsible for their learning. This seems to help PSTs position themselves as outsiders of a (too) challenging mathematics world.

Louisa's case shows that fluidity between positions and an insider position in both sub-worlds are possible, but it has also unraveled the difficulties in achieving this, for example, in attributing success to herself. The fluidity may go hand in hand with a less clear career aspiration, which seems contrary to the idea of offering mixed courses as an approach that equips future teachers with knowledge about teaching *and* mathematics from the beginning.

Thus, for efficient teacher education in mixed settings, much more alignment between the sub-worlds is needed for the contrasting figures of these worlds to soften and blur. For example, the figuring of mathematics majors understanding everything right away, may seem contradictory, given that many of them struggle with mathematics in the transition as well (Gueudet, 2008). Nonetheless, this contradiction emphasizes that practical implications may need to address the "lack of understanding" as part of the mathematics world that its practitioners face most regularly in their research. Similarly, explaining may need to be addressed as a valued outcome among all students, not limited to PSTs. Integration of explanation tasks on homework could intervene here.

It may be noticed that PSTs' positioning as future teachers was built on strong ideas about how to teach and learn mathematics. This finding is in line with current research about PSTs feeling quite at ease with how to teach (Hine & Thai, 2019). We might assume

that if they would not experience any kind of rupture that would make them rethink this positioning (Holland et al., 1998, p. 141) in their subsequent studies (e.g., their pedagogical courses), they would possibly align with these ideas throughout their education, reproducing the rather traditional concept of delivering mathematics to students. Similar assumptions could be made about their positioning as knowledge receivers, as we observed in relation to both the IC and the LA courses. To address these issues, it seems not enough to "make PSTs feel more at home with mathematics" but rather to make them reflect further on their own positioning as mathematics learners as well as future teachers.

Concerning this study's limitations, our findings are based not only on a particular and small student group but on a very specific context and situation. PSTs' descriptions of the figured worlds may change in the next semesters, specifically when attending pedagogical courses. PSTs other than those interviewed may have positioned themselves differently in the worlds, as well as figured them in various ways. Future research should be conducted to generalize our findings and to further inquire about the reasons why certain PSTs, such as Louisa, manage to blur the differences between the sub-worlds, while others dichotomize them. We believe that our current study may build an important basis for such further research.

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