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A long-term shift to include students' first language in the mathematics teaching practice: socialization events and learning opportunities

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Abstract
The notion of multilingual students' first language has been advocated as a resource in mathematics learning for some time. However, few studies have investigated how implementing students' L1 in the teaching practice impacts multilingual students' mathematics learning opportunities. Based on a 9-month-long ethnographic study conducted in Iran, we investigate what a long-term shift from mathematics teaching in the language of instruction (Persian) to mathematics teaching that includes students' first language (Turkish) may mean in terms of learning opportunities. In language positive classrooms, students' socialization into mathematics and language includes using students' first languages and paying explicit attention to different aspects of language use in mathematics. Among other things, socialization events provide possibilities to share explanations of mathematical thinking. The results of this study suggest that using students' first languages may reinforce other language positive socialization events and provide mathematics learning opportunities during individual assignment activities. Furthermore, the results suggest that the conceived value of mathematics education in the local communities changed with the introduction of students' L1 in the teaching practice. Consequently, this study indicates that using students' first languages in mathematics classrooms may be a key issue in multilingual contexts.

Keywords Multilingualism · Language as a resource · Socialization · Mathematical discourse

1 Introduction

The notion of using multilingual students' first language (L1) as a resource in mathematics learning has been advocated to establish equitable mathematics learning opportunities for multilingual students (e.g., Chronaki et al., 2022; Ryan & Parra, 2019). As a guiding

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principle for mathematics instruction, the use of first language as a resource means that multilingual students can use a wider range of their language repertoires than what is possible when monolingual norms that privilege the language of instruction (L2) prevail in the classroom (Planas & Setati Phakeng, 2014). In mathematics classroom practices, the flexible use of students' L1 is not disconnected from other dimensions of language use (Barwell, 2020). Therefore, principles for mathematics instruction that enhance language from perspectives other than the flexible use of students' L1s are also relevant in multilingual contexts (Barwell, 2020; Erath & Prediger, 2021). Despite a growing body of mathematics education research on language-related issues and multilingualism, de Araujo et al. (2018) and Erath et al. (2021) concluded that few studies have attended to how the implementation of suggested language-enhancing principles may change teaching practices and influence students' opportunities to learn mathematics. There is a particular lack of studies that report on what a long-term shift from mathematics teaching in the language of instruction (L2) to the inclusion of students' L1 in teaching practices means in terms of multilingual students' learning opportunities (de Araujo et al., 2018; Erath et al., 2021; Schüler-Meyer, 2017). Moreover, many studies about language and language use rely on findings from Western contexts. However, cultural norms influence what principles may support students who learn mathematics in different contexts (Barwell, 2020; de Araujo et al., 2018; Xu & Clarke, 2019). Therefore, to understand how implementing students' L1 impacts mathematics learning opportunities across a wide range of classrooms, studies conducted in non-Western contexts could provide important dimensions and perspectives (Erath et al., 2021; Xu & Clarke, 2019).

Consequently, previous research has identified (a) few studies attend to shifts to include students' L1 in the mathematics teaching practice, (b) few long-term studies have been conducted, and (c) the contexts in which multilingualism has been studied need to be broadened. The present study addresses these limitations by reporting on a 9-month-long ethnographic study conducted in two Iranian secondary schools where the use of students' L1 (Turkish) was implemented as part of the teaching practice.

To investigate what a long-term shift from mathematics teaching in the language of instruction (Persian) to include the students' L1 (Turkish) may mean in terms of learning opportunities, we draw on the notion of language positive classrooms developed by Barwell (2020). In language positive classrooms, students' socialization into mathematics and language includes paying attention to different aspects of language use, which could be supported by the implementation of language-enhancing principles. Following Barwell (2020), we view learning mathematics as socialization into the discourse of mathematics. Hence, specific events when socialization into the discourse of mathematics may occur—socialization events—are mathematics learning opportunities. We asked the following questions in this study:

1. How are mathematics learning opportunities enhanced with the inclusion of students' L1 in the teaching practice?
2. How did teachers and students experience the shift to include students' L1 in the mathematics classroom in terms of learning opportunities?

The remainder of this paper is organized as follows. Section 2 reviews the literature on learning opportunities in multilingual mathematics classrooms. Section 3 presents our theoretical considerations. Sections 4 and 5 attend to multilingualism in the Iranian educational context and methodology, respectively. In Section 6, we present the results, which are discussed in Section 7.

2 Enhancing learning opportunities in multilingual mathematics classrooms

Flexible use of the language of instruction and students' L1s in hybrid language practices is an essential strategy to support mathematics learning for multilingual students (de Araujo et al., 2018; Erath et al., 2021; Planas & Seteti Phakeng, 2014; Schüler-Meyer, 2017).

By flexible use of language, we refer to functional and dynamic flow among the language repertoires available to the speaker. The notion of translanguaging (García & Li, 2014) captures how multilingual speakers' language repertoires function as social resources without clear boundaries. Translanguaging highlights socio-political dimensions of flexible language use that position all languages as equally valuable (Garcia & Li, 2014).

Students' active participation in mathematical discourse is often regarded as an indicator of learning opportunities (Erath & Prediger, 2021). Some studies have reported on how a temporary implementation (one or a few lessons) of flexible language use changed multilingual students' participation in mathematical discourse in terms of increased student activity and agency. And how the implementation made positionings for multilingual students as knowing subjects available (Chronaki et al., 2022; Ryan et al., 2021; Schüler-Meyer et al., 2019), which positively influenced the sense of self (Chronaki et al., 2022; Langer-Osuna & Nasir, 2016; Ryan et al., 2021).

A prerequisite for increased student participation when implementing flexible language practices is that teachers manage to create a safe "zone of comfort" where multilingual students feel free to express themselves and experiment with their language resources as they participate in mathematical discourse (Mackinney, 2022). A short-term intervention study, in which a monolingual (German-only) classroom practice was shifted into a multilingual practice (German and Turkish), found that multilingual Turkish-speaking students were initially reluctant to use Turkish in mathematical discourse. To establish flexible language use in the classroom, the teacher had to invest in Turkish and a hybrid language use to show that Turkish was valued and "safe" to use before the students started using Turkish themselves (Schüler-Meyer et al., 2019). Similarly, other studies have shown that merely "allowing" multilingual practices in the mathematics classroom is not enough to enhance learning opportunities; teachers must plan for and enact in-the-moment moves to support students in leveraging their language resources to make meaning of mathematics (DiNapoli & Morales, 2021).

A shift to include students' L1s in mathematics instruction alone does not necessarily provide more and richer learning opportunities that socialize multilingual students into the discourse of mathematics (Barwell, 2020; de Araujo et al., 2018; Erath et al., 2021). However, flexible use of the L2 and students' L1 supports the enhancement of opportunities where students can participate in activities that involve making statements, taking part in discussions, and making descriptions to engage in their own or other students' reasoning, which supports their conceptual understanding (Moschkovich, 2018). However, as pointed out by Xu and Clarke (2019), the advocacy of students' participation in mathematics classroom discussions and dialogue draws on studies conducted in Western contexts based on Western ideals. Xu and Clarke (2019, p. 144) argued that suggested teaching principles ought to "be sensitive to the constraints and affordances that culture places on practice".

In multilingual mathematics classrooms, the use of embodied mathematical pedagogies can play an important role (Chikiwa, 2021; Liu & Takeuchi, 2023). The results of a case study conducted in South Africa showed that all forms of teacher gestures that indicate and refer to real objects and locations may support multilingual students' socialization into the discourse of mathematics (Chikiwa, 2021). Another study, conducted in a multilingual elementary

classroom in Canada, investigated a shift from a "normalized" pedagogy to an embodied pedagogy as part of a professional development intervention (Liu & Takeuchi, 2023). That study found that when the multilingual students had the opportunity to use their embodied resources, the students expanded their agency for mathematics learning (Liu & Takeuchi, 2023).

Including students' L1 in the teaching practice requires that teachers invest in the value of students' L1s so that students feel safe to use their L1 (Mackinney, 2022; Schüler-Meyer et al., 2019) and plan for and enact in-the-moment moves to support students' flexible use of L1 and L2 resources (DiNapoli & Morales, 2021). A shift to include students' L1 could increase student participation and agency (Chronaki et al., 2022; Ryan et al., 2021; Schüler-Meyer et al., 2019), which, in turn, can provide a positive sense of self (Chronaki et al., 2022; Langer-Osuna & Nasir, 2016; Ryan et al., 2021). Discursive (Barwell, 2018, 2020; Erath & Prediger, 2021; Moschkovich, 2018) and embodied (Chikiwa, 2021; Liu & Takeuchi, 2023) dimensions also need to be recognized to enhance learning opportunities.

3 Theoretical considerations

The present study investigates how the shift to include students' L1 in mathematics classrooms may provide learning opportunities for multilingual students. To tease out what the shift means regarding learning opportunities, we use the notion of socialization events that Barwell (2020) identified as significant moments of explicit socialization into the discourse of mathematics. The discourse of mathematics includes discursive dimensions of diverse argumentation, such as conjecturing, making explanations, posing problems and questions, and reasoning (Barwell, 2018, 2020; Erath & Prediger, 2021; Moschkovich, 2018).

Mathematics classroom cultures that actively pay attention to languages (L1 and L2) and mathematical practices may provide grounds for socialization into the discourse of mathematics (Barwell, 2020). In classroom cultures, which Barwell (2020) named language positive classrooms, the use of students' L1 is encouraged. In language neutral classrooms, by contrast, little or no attention is paid to socialization into language practices. To study moments of significance in socialization processes in language and mathematics that characterize language positive (and language neutral) mathematics classrooms, Barwell (2020) used the notion of socialization events. Moments that may be of particular significance for socialization into the discourse of mathematics, such as participating in discussions that involve argumentation and/or the use of signs or gestures, are referred to as socialization events (Barwell, 2020). Since participation in socialization events offers rich possibilities to learn mathematics, we see socialization events as mathematics learning opportunities. Consequently, we consider moments of explicit socialization (learning opportunities) as instances when students actively and productively engage in the discourse of mathematics (Barwell, 2020).

In practice, classroom cultures are seldom or perhaps never solely language positive or language neutral. Rather, at different moments, they comprise different proportions of socialization practices that characterize the classroom culture (Barwell, 2020).

How the socialization events unfold in the classroom relates to multilingual students' opportunities to engage in mathematical discourse. Following Barwell (2020, p. 172), we present the seven socialization events and how they may unfold in language positive classroom practices (Table 1).

Barwell (2020) exemplified attention to features of mathematical discourse as teachers reformulating students' utterances, while teachers prompting students to use "generically appropriate forms"—such as when explaining the solution of a problem—would

Table 1 The seven socialization events and practices in language positive classrooms (adapted from Barwell, 2020)

Socialization event	Practices
Students' use of home languages	Home languages are regularly heard; students refer to or use their home language during mathematics discussion
Occurrence of nonstandard accents, pronunciation, or orthography	Students' different "nonstandard" accents, pronunciation, spelling, and punctuation in L2 are presented and are explicitly related to standard norms through various socialization practices
Explicit attention to features of mathematical discourse	Second-language learners actively participate in socialization through the use of various features of mathematical discourse; relations between more formal and informal mathematical discourses are made visible through a variety of socialization practices
Encounters with mathematics classroom genres	Engagement with mathematical genres is inclusively supported through specific socialization practices
Use of gestures in mathematical interaction	Gestures are actively used by second-language learners and teachers in socialization processes; explicit links are made between gestures and other aspects of mathematical discourse
Explaining mathematical thinking	Second-language learners are supported by various socialization practices to jointly develop explanations or accounts of mathematical thinking
Moments of reduced or active participation	Reduced participation. Silence; short responses to closed questions; display of hesitation or reluctance to participate. Active participation. Students actively participate in mathematics classroom interaction, taking extended turns or sequences of turns and initiating exchanges with the teacher or other students

refer to mathematics classroom genres. In practice, reformulating a student utterance may also mean prompting students to use "generically appropriate forms." The choice of utterances constitutes the discourse and vice versa. Hence, in practice, it is difficult to distinguish between mathematical genre and mathematical discourse. Therefore, we consider the notion of "encounters with mathematics classroom genres" as part of the socialization event "explicit attention to features of mathematical discourse."

According to Barwell (2020), socialization is usually considered in terms of communicative and interactive activities. However, there are also situations where students' participation is low and they are unwilling to participate in interactive activities. We use the socialization event "moments of reduced participation" to consider students' reduced or active participation in socialization events.

4 Multilingualism and the Iranian educational context

Iran has historically been a multilingual and multicultural society, with Persian as a dominant part of Iranian culture. Iran is the motherland of different ethnic and linguistic groups that speak languages including Persian, Turkish, Lorish, Arabic, Kurdish, and Balochi. However,

when modernization policies began with the founding of a European-influenced Ministry of Sciences in 1858, Persian became the sole language of instruction in schools despite the linguistic diversity of the population (Hoominfar, 2014; Kalantari et al., 2020). Although Article 15 of the Iranian constitution, so-called "ethnic equality," explicates, "The people of Iran, regardless of their ethnicity and tribe, have equal rights and the race, language, and the like will not be a privilege in itself", there is no systematic program that gives a role to indigenous languages to be taught in the educational system (Hoominfar, 2014). However, multilingualism is often overlooked in conventional teaching and learning methods in Iranian education, although it is an essential factor influencing student performance. On average, according to the TIMSS studies (1995–2007), 37% of Iranian eighth-graders reported that they either occasionally speak Persian or not at all. Reported data in the 2007 TIMSS showed that more than half (57%) of students at low-achieving schools confirmed that their L1 was different from the language used in the TIMSS assessment, while 89% of individuals from the high-achieving schools reported that their L1 was Persian (Mohammadpour et al., 2015).

The present study was conducted in a Turkish-speaking context, the largest ethnic minority in Iran. In Iran, non-Persian speaking students have lower academic success in years of schooling. For example, 50 percent of students with Turkish as L1 fail reading and writing courses in Persian (Hoominfar, 2014), and non-Persian students suffer from insufficient communication skills (Kalantari, 2012). Although some studies have indicated that this is caused by the monolingual policy (Jahani Asl, 2007), little research has been done to investigate multilingual education for Iranian students.

5 Methodology

The research design was based on an ethnographic study to investigate social activities (James & Prout, 2003), such as those that connect to socialization practices in mathematics classrooms. We attempted to collect detailed information by involving, interacting, and engaging in direct dialogue with the teachers (Reza and Saeed) and students over 9 months (that is, one academic year) in a natural environment.

5.1 Participants

The study was conducted at two secondary schools in Zanjan province, Iran. Both public schools are in rural areas and include grade 8. Two mathematics sessions were held every week in both classes. The students in the two classes were aged 12 and 13. There were eight students in class A (four boys and four girls) and 11 in class B (seven boys and four girls). All students were born where they had their education. They came from families with similar socioeconomic status and similar historical and traditional roots in Zanjan, with their L1 being Turkish. Seven of the students were also fluent in Persian (L2). None of the parents had a university education. In this study, all personal names have been replaced by pseudonyms to preserve confidentiality.

This study included all students (19) in the two classrooms. Consent was obtained from students' principals and parents. Before asking for consent, the research aims, methods, estimated time, etc. were explained to the participants. The teachers participating in this study, Reza and Saeed, were multilingual, with Turkish as their L1 and Persian as their L2. Reza has a PhD in mathematics and nine years of mathematics teaching experience in multilingual classrooms. Saeed holds a bachelor's degree in mathematics with 24 years of teaching experience in multilingual classrooms. Before the shift, their teaching practices included Persian only.

5.2 Ethical considerations and the shift to include L1

The main motivation for conducting this research was to develop an intervention to reduce the high academic dropout in multilingual regions compared to monolingual settings in Iran (Fardinpour, 2011; Hoominfar, 2014; Mohammadpour et al., 2015) and how to alleviate it. The second author of this study is a multilingual (Turkish as L1, Persian as L2) Iranian teacher-researcher. He was born and raised in Zanjan province and has been a student in multilingual environments for 12 years and has 13 years of experience teaching mathematics in multilingual settings. The other authors are researchers interested in how multilingual students could use their L1 in teaching and learning mathematics. This ethnographic study began with a 2-day workshop with mathematics teachers. The second author talked with 17 multilingual mathematics teachers about the challenges commonly faced in mathematics classes like the ones they were teaching. The teachers mentioned challenges with teaching and learning mathematics in multilingual classes and disengaged students. For example, some teachers, students, and parents face challenges with textbooks written in Persian as many people in Zanjan province are not fluent in Persian.

5.3 Making the shift to include students' L1s

Most of the teachers emphasized the need to make shifts in the teaching practice and the need to include the L1 in the classrooms. Reza and Saeed, who appear in the present study, were particularly interested in developing teaching practices that support multilingual students. Therefore, the second author and the two teachers prepared to implement an intervention based on introducing L1 in the teaching practice. This shift included providing opportunities to increase communication in L1, applying ordinary and concrete examples in L1, using and encouraging visualizations, creating situations for reasoning and guessing using equivalents to explain difficult Persian terms, and installing posters on the walls of the two classrooms in L1 with phrases such as "we will be able to learn mathematics well in Turkish" and "mathematics and its learning are beautiful, especially with Turkish" (Fig. 1). The practices were introduced step-by-step and performed during the research period.

Fig. 1 Posters installed on the walls of the classrooms in L1 (Turkish)

Before the shift to include students' L1, the students were informed that this was an attempt to enhance participation and enjoyment in mathematics learning to provide learning opportunities. Before the shift (the first 3 months of the 9 months), the focus was mostly on identifying the current state; after the shift (the last 6 months of the 9 months), the focus was mainly on identifying the consequences and experiences of the shift. Also, in the last 3 months of data collection and after recording two excerpts, each student was invited to participate in a recorded audio interview.

5.4 Data collection

During the 9-month period, data were collected continuously from classroom observations, photos, audio and video recordings of interactions and communications in the mathematics classrooms, and semi-structured interviews with the students and their mathematics teachers (Table 2). Audio and video recordings were used to capture student–student and teacher–student interactions in small groups or whole-class mathematics activities. The audio and video recordings were selectively transcribed verbatim. The interviews with teachers were conducted in L1. Also, both languages were used in conversations and interviews with students so that students could use any language they preferred to express their opinions, experiences, and suggestions. All interviews and conversations with the teachers and the students were made by the second author. Most of the questions were the same for both teachers in four interviews and for each student in 19 interviews. With the permission of the teachers, the second author worked and interacted with the students either individually or during small-group work. He often worked with small groups of students on various language-related and mathematical problem-solving activities. After a short period of time, the students felt comfortable with his presence in their class and students often asked him to help and contribute to their conversations. The second author was an active "participant observer" and engaged in classroom life by teaching, listening, observing, and asking questions to gather data that would connect to the research questions and support the students' learning. In this way, he interacted with the students as they participated in their mathematics lessons.

Table 2 Overview of data material

Material	Activity	Quantity
Field notes	Classroom interaction	43 (pages)
Field notes	Teachers' workshop comments	14 (pages)
Field notes	Conversations (informal interviews) with two teachers (Reza and Saeed)	30 (pages)
Field notes	Conversations (informal interviews) with the students	50 (pages)
Photos	Students' activities and discussions	39
Video recordings	Student group discussions	4 (60–70 min each)
Video recordings	Student–teacher discussions	7 (30–40 min each)
Audio recordings	Interviews with students	19 (10–15 min each)
Audio recordings	Interviews with two teachers (Reza and Saeed)	4 (15–20 min each)

5.5 Data analysis

To trace how the shift to using students' L1 in mathematics in the two classrooms may provide learning opportunities for the students, we coded the data material using Barwell's socialization events. We operationalized learning opportunities as instances when students actively engaged in mathematics classroom interaction. The socialization event moments of participation refers to this dimension of a language positive mathematics classroom. The other six events relate to how socialization events support participation. We began the coding process by paying attention to the socialization event moments of participation because those moments inform whether the shift to include students' first language influenced their participation. We coded instances of student participation during observations (including field notes, photos, and video recordings) during the 9 months. We also coded teachers' and students' talk in interviews about participation in mathematics classroom interaction. We used the codes "active participation" and "reduced participation." By quantifying the number of times the codes occurred informs whether the shift to include students' L1 changed participation. We proceeded with the analysis by using the other six socialization events (codes used were "the use of L1, pronunciation, mathematical discourse, gestures, and explaining") to refine (if possible) how the observed or described participation unfolded through the other six socialization events that Barwell (2020) identified as indicators for language positive classrooms. After that, we chose two transcribed recordings of class interactions (one transcribed recording from each class) published previously (Malaki et al., 2022) and re-analyzed them to understand how the shift to include students' L1 in the teaching practice shaped learning opportunities. We chose the two transcribed video recordings for two reasons. First, they represent two mathematics topics that the teachers in this study recognized as challenging for their multilingual students: algebra and geometry, more specifically, congruent shapes (two shapes that fit precisely on each other). Since the teachers found these two topics challenging, it was fascinating to investigate how a shift to include students' L1 may influence the emergence of and students' participation in socialization events. The second reason is that, based on the whole-data material analysis, these two video recordings are typical of students' interactions at the end of the 9 months.

6 Results

We start this section by focusing on shifts in moments of participation that the use of students' L1 generated. We then consider how the teachers and the students experienced the shifted moments of participation in learning opportunities. Finally, we provide an in-depth analysis of how the socialization events unfolded during two whole-class interactions at the end of the 9 months.

6.1 Shifts in moments of participation

Before the shift, the language of teaching was limited to Persian, as that is the language of mathematics textbooks. Reza and Saeed believed that using Persian commonly leads to reduced participation in the classrooms, assignments, and mathematical discourses.

One of the authors carefully checked students' attendance and absence checklists. Before the shift, students had frequent absences, which gradually decreased after the inclusion of Turkish. Some students who attended mathematics class often sought permission from the

teacher to leave the class for various reasons. After the shift, this process decreased significantly. It is important to note that the last 3 months of the 9 months coincided with a busy agricultural season in the Zanjan province, during which students are frequently absent from school to help with farming. Therefore, a higher frequency of absences compared to the first 3 months of the project would be normal. Despite that, as Table 3 indicates, the frequency of absences is lower after the shift.

Before the shift, some students in two classrooms copied answers from mathematics solution books for both in-class and at-home assignments (Fig. 2). After the shift, they discussed, solved the assignments, and wrote their solutions themselves (Fig. 3). The students themselves noticed this change. For example, Mehran said, "Previously, many students and I did not solve the assignments by ourselves. Now, by using Turkish, we discuss with the teacher and each other about different solutions." Ali said, "Before using Turkish, I would copy the answers of the assignments from my classmates or textbook solutions. Now, I don't and solve them with the help of my friends and the teacher."

After the shift, close-knit friendships were formed among students and their teachers. Mahta and Negin shared that their friendship developed due to having used Turkish, and Armin was happy with the formation of friendly relations. The number of questions asked and explanations to the teacher were improved for the students before and after the shift. Table 4 shows changes in the number of asked questions and explanations provided

Table 3 Change in the number of absences and permission to leave the class before and after the shift

	Names	Number of absences in the first 3 months of the project before the shift	Number of absences in the last 3 months of the project after the shift	Number of permissions to leave the class in the first 3 months of the project before the shift	Number of permissions to leave the class the last 3 months of the project after the shift
Class A	Hasan	3	1	5	1
	Mohammad	5	1	10	3
	Kasra	3	0	6	2
	Ali	2	1	1	0
	Rojin	4	0	1	1
	Sara	3	0	2	1
	Negin	5	1	2	0
	Shadan	3	0	1	0
Class B	Amir	4	2	6	3
	Leila	1	1	1	0
	Yashar	6	1	9	4
	Hosein	2	0	0	0
	Yegane	2	0	2	0
	Farhad	5	2	7	2
	Armin	4	1	4	0
	Mehran	3	1	2	0
	Arsalan	4	2	5	1
	Mahta	1	0	1	0
	Parmin	3	1	2	0

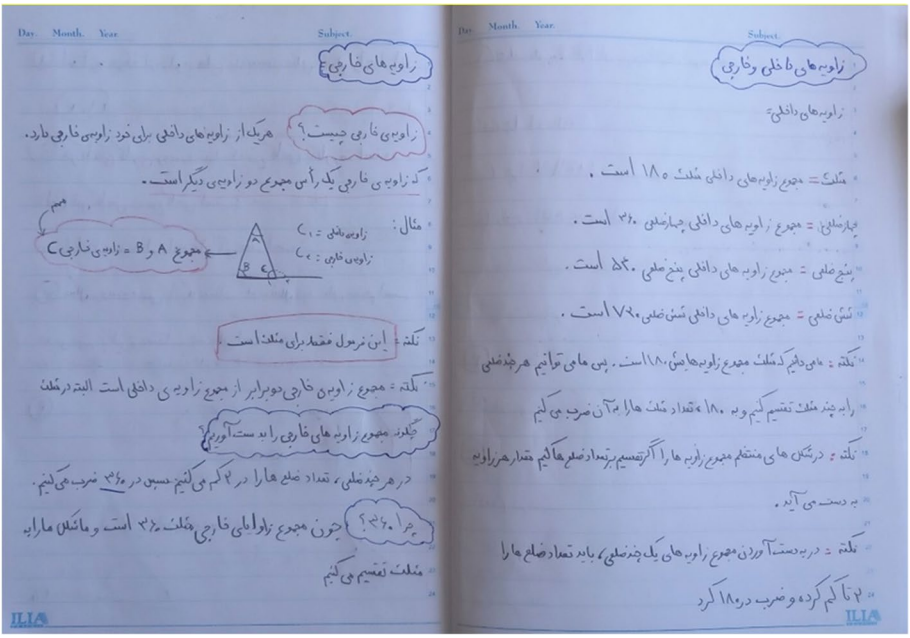

Fig. 2 A picture of the assignments copied by Hamed from the mathematics solution books before the shift

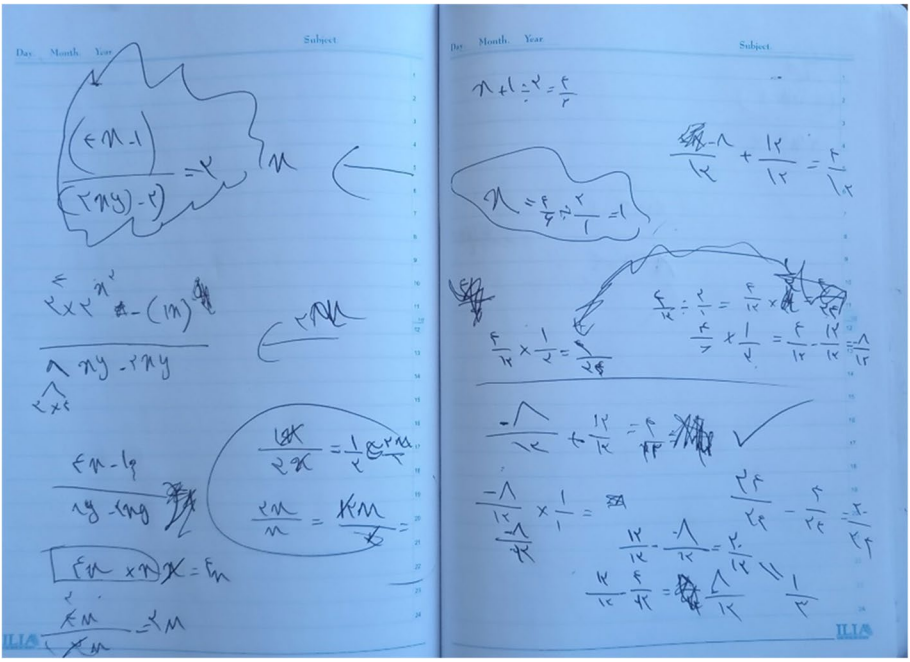

Fig. 3 A picture of the assignments done by discussion by Hamed after the shift

Table 4 Change in the number of asked questions and explanations provided voluntarily before and after the shift in classes A and B

	Names	Number of asked questions before the shift (the first 3 months out of 9 months)	Number of asked questions after the shift (the last 3 months of 9 months)	The number of explanations provided voluntarily before the shift (the first 3 months out of 9 months)	The number of explanations provided voluntarily after the shift (the last 3 months of 9 months)
Class A	Hasan	13	24	3	7
	Mohammad	4	21	0	5
	Kasra	11	26	3	6
	Ali	16	29	5	7
	Rojin	11	15	1	5
	Sara	13	23	2	8
	Negin	7	22	2	7
	Shadan	9	20	1	4
Class B	Amir	9	13	2	3
	Leila	21	23	8	8
	Yashar	5	19	0	4
	Hosein	19	29	6	7
	Yegane	12	19	4	6
	Farhad	9	20	1	4
	Armin	10	18	1	3
	Mehran	13	22	3	7
	Arsalan	10	15	2	4
	Mahta	17	21	7	9
	Parmin	11	24	3	7

voluntarily before and after the shift in classes A and B. Note that all the questions asked before and after the shift are related to mathematics.

Based on the comments of 14 students, we found that, to ask a question or present their opinions, these students had to make their sentences in Turkish and then translate them into Persian. For example, Yashar said, "I make the question in my mind in Turkish and then translate it into Persian to ask." Amir said, "I get tired of constantly translating my questions and sentences from Turkish to Persian. I prefer not to speak."

Before the shift, it was observed on several occasions that students were not interested in presenting their mathematical thinking. They lowered their heads or preferred to hide behind the person in front of them. In one of the sessions, when the teacher asked the students to recapitulate a discussion about interior angles, the students did not respond.

6.2 Shifts in experiences of learning opportunities

Before the shift, Reza, Saeed, and 13 teachers (workshop teachers) indicated that the teaching strategies that they planned in Persian usually did not generate learning opportunities. For example, students were not interested in participating in discussions conducted in

Persian. After the shift to include students' L1, Reza stated that he never expected such an outcome regarding students' participation, the number of questions asked, and the number of students volunteering to present.

In the interviews, many students emphasized the opportunities that the use of their L1 opened for them to engage in the discourse of mathematics with peers and teachers. For example, Mohammad said, "Previously, we could not discuss, ask questions and say what we did not understand, but today we can pour out freely our views to the teacher or others in the classroom." Farhad said, "My fear and anxiety about asking questions has decreased."

For some students, the learning opportunities provided by the shift enhanced experiences of being mathematically knowledgeable. For example, Kasra pointed to the writings on the whiteboard and told his teacher, "That part of the writing is my idea; let me explain more." In another position, Mohammad, after explaining his mathematical thinking, said, "Did I explain well? I can solve more complicated examples. When the teacher or classmates ask me to explain something to them, and I explain it well, I get a sense of pride, and this makes me very happy." After the shift, the students were encouraged to think about comments and give feedback when discussing with each other. They thought about their classmates' comments and reflected on their contributions. For example, in item 14 of Excerpt 1, Ali used Kasra's comment and explained his mathematical thinking. Then Kasra improved his explanations with the help of Ali's comment, and finally, Ali completed these explanations correctly. Mehran said, "Turkish helped us work together as a team and enjoy ourselves while doing mathematics." Sara stated, "Turkish allowed us to discuss, present our comments and points of view, and learn more from peers."

Students' experiences in shifted learning opportunities are also related to the emotional experiences of mathematics class. For example, Arsalan said, "Because teaching mathematics in Persian is dull and monotonous to non-Persian speaking students, using Turkish changes the learning environment to be amusing and fun."

Parmin reported, "I was not interested in mathematics, but now I enjoy participating in mathematics discussions with Turkish words." For some students, the enjoyment was related to success. Rojin stated, "Turkish made me see mathematics as enjoyable and easy to learn." It is essential to mention here that several students emphasized the use of Persian (L2) alongside Turkish (L1); they believed that both Persian and Turkish should be used simultaneously in the classroom and realized that using only one language can be problematic. For example, Leila said, "We should use Persian alongside Turkish because when we enter universities in Persian-speaking cities, we have to use Persian." Shadan said, "In the future, we can find a suitable job if we speak Persian well."

6.3 In-depth analysis of socialization events at the end of the 9 months

We conducted an in-depth analysis of two whole-class interactions to illuminate how socialization events unfolded at the end of the 9 months; see Excerpts 1 and 2.

6.3.1 Students' use of home languages

After including Turkish in the teaching practice, Turkish was used frequently and flexibly, although the prescribed language of instruction was Persian. Excerpt 2 demonstrates the flexible use of Turkish and Persian in the mathematical discourse after the shift. In item 1, the teacher seems to follow Amir's use of Turkish to encourage the students to use the

mathematical concept of Yerbeyer. Hosein (item 3) responds to the teacher's question in Turkish. After that, the teacher uses Persian except for the word Yerbeyer (items 4, 7, 9, 14, 18). Hosein follows this practice (item 11), while Leila uses Turkish only in items 13 and 15. In item 16, the teacher introduces the Persian word for congruent (Hamnehesht) by paying attention to translation practices that the students seem to be familiar with. This seems to prompt the students Yashar (item 19), Leila (item 20), and Hosein (item 21) to use the Persian word Hamnehesht. It is interesting to note that before the teacher introduced the Persian word Hamnehesht, Leila used Turkish only (items 13, 15) while after the introduction of Hamnehesht, she used Persian only (item 20). Hosein, on the other hand, used only Persian except for the concept Yerbeyer before the teacher had introduced the Persian word Hamnehesht (items 11, 21).

In Excerpt 2, the flexible use of students' L1 provided a moment of significance for socialization into the discourse of mathematics because the students participated in discussions that involved argumentation about congruent shapes. Moreover, there are sequences of turns (e.g., items 10–13 and 19–21) where students interact without the teacher's support. This indicates some level of autonomy in participation in the discourse of mathematics.

6.3.2 Occurrence of nonstandard accents, pronunciation, or orthography

In class A, the teacher wrote the algebraic expression A + D + C on the whiteboard and then asked the students to pronounce A, D, and C. The students accurately pronounced A (/ə/), D (/d/), and C (/sē/) in English. Using English in connection to algebraic expressions is a common practice in Iran. The students' pronunciation of A (/ə/), D (/d/), and C (/sē/) provided a moment of participation because it provided an opportunity for them to orally engage in classroom interactions in contrast to keeping silent. The pronunciation event transformed into a socialization event that highlighted features of mathematical discourse when the teacher suggested A as Alma (apple), D as Dash (stone), and C as Soghan (onion), which was repeated by Hasan (Table 5, item 1). Hasan substituted the algebraic symbols A, D, and C with three words that have meaning in students' L1, that is, Alma, Dash, and Soghan. The substitution was facilitated by the pronunciation of the letters A, D, and C in English is the same as the pronunciation of the first letters of Alma, Dash, and Soghan. Here, the socialization event of pronouncing A, D, and C according to standard norms extended beyond students' accurate pronunciation. By substitution, the pronunciation activity transformed into an activity where A, D, and C came to represent objects. The pronunciation activity that illuminated moves between specific objects and the symbolic representation in algebraic expressions provided opportunities for the students to be socialized into the practice of using algebraic expressions. Here, it is necessary to acknowledge that in the discussion of algebra, the use of the beginning letters of actual objects has been noted as a problematic way of introducing variables (MacGregor & Stacey, 1997); however, in the classrooms, the use of objects has helped to open up conversations.

6.3.3 Attention to features of mathematical discourse

In class A, the teacher invited students to participate in mathematical discourse by presenting key terms and symbols; for instance, by saying, "Let us make an assumption. Assume that A is Alma, D is Dash, and C is Soghan. With this assumption, tell me your ideas about A + D + C." This assumption made a relationship between concrete/informal and abstract/

Table 5 Excerpt 1: the algebraic addition (class A)

No	Names	Original (Turkish in blue, Persian in black)	English translation (from Turkish in blue, from Persian in black)
1	Hasan	Agar A Alma ola, D Dash ola, va, C Soghan ola, bouyolaroum…	If A is apple, D is stone, and C is onion, in this case…
2	Kasra	Agar ma jam konim A, D, C, bizim varoumuz olagakh, ouch dana Alma, Dash, va Soghan	If we add up A, D, and C, we will have three apples, stone, and onion
3	Ali	Yani az har kodam seta darim, Kasra?	Does it mean that we have three of each, Kasra?
4	Kasra	Na, man nemidoonm az har koodom chanta darim	No, I don't know how many we have of each of them
5	Mohammad	A, yanoo necha dana Alma?	A, means how many apples?
At this moment, the teacher said, "Give me your ideas about 2A + D + C + A + C according to the previous assumptions: A is Alma (apple), D is Dash (Stone), and C is Soghan (onion)"			
6	Ali	Moallem ghoft A + D + C, epteda	First, the teacher said A + D + C
7	Mohammad	Alan da moallem dedi 2A + D + C + A + C, va dedi ghabakhdeki farzlarinan	Now again, the teacher said 2A + D + C + A + C with the same assumptions
8	Ali	Dar in lahze, ma darim 2A, D, C, A, va C	Currently, we have 2A, D, C, A, and C
9	Kasra	Ma bayad inhara jam konim, manim nazarima 2A yanoo 2 Alma	We must add these up, in my opinion, 2A means 2 apples
10	Kasra	Ya inke begim 2A yani A + A, doroste?	Or we can say 2A means A + A, right?
11	Teacher	Bale, doroste. Hala mani va mafhoome D chi mishe?	Yes, correct. Now, what's the meaning and the concept of D?
12	Ali	D yano bir dana D, aya midoonid chera?	D means one D; do you know why?
13	Teacher	Chera?	Why?
14	Ali	Az anja ke Kasra ghoft 2A = A + A va chon ma farz kardim ke A, Alma bashad, pas dar natije, 2 Alma olar, Alma + Alma ya bir Alma + bir Alma, pas ma mitounim be jaye A haman 1A benevisim	As Kasra said, 2A = A + A, and as we supposed A is apple, so 2 apples are apple + apple or one apple + one apple, so we can put 1A instead of A
15	Teacher	Dar in mored mitounim begim 1D be jaye D	In this case, we can say 1D instead of D
Kasra said in Turkish, while laughing, "now visualize having a tray of two apples, one stone, one onion, another apple, and another onion". Then, Ali says in Turkish while drawing a dish in his notebook, "Now; we have three apples, one stone, and two onions in the dish"			
16	Hasan	Man bildim	I got the point
17	Hasan	Aya ejaze midid man biyam va mafhoome 2A + D + C + A + C ra tozih bedam	Let me come to the board and explain the concept 2A + D + C + A + C
18	Teachers	Aya shoma mitooni tozih bedi, Mohammad?	Can you explain, Mohammad?
19	Mohammad	Man? Man biyam? Farsi ya Turki?	Me? Should I come? In Persian or Turkish?
20	Teacher	Har joor rahati	As you wish

Table 5 (continued)

No	Names	Original (Turkish in blue, Persian in black)	English translation (from Turkish in blue, from Persian in black)
At this time, Mohammad drew a dish on the board with two apples, an onion, a stone, another apple, and another onion in it. Then, he drew the second dish with three apples, a stone, and two onions next to each other			
21	Mohammad	Avvalki ghab ela 2A + D + C + A + C olajec, va ikimji ghab da 3A + D + 2C olajec, va choon boo iki ghabdekilar birdilar, pas diya billik, 2A + D + C + A + C = 3A + D + 2C	The first dish is 2A + D + C + D + C, and the second dish is 3A + D + 2C. Since the things in both dishes are equal, we conclude that, 2A + D + C + A + C = 3A + D + 2C
22	Teacher	Chokh chokh ghashagidir	Very very beautiful
23	Hasan	Aya momken ast mesale digari benevisid ta ma an ra hal konim?	Can you write down another example for us to solve it?

Table 6 Excerpt 2: the congruent shapes (class B)

No	Names	Original (Turkish in blue, Persian in black)	English translation (from Turkish in blue, from Persian in black)
1	Amir	Birdilar	They are the same
2	Teacher	Ya namana?	Or what?
3	Hosein	Yerbeyer dilar	They are congruent
4	Teacher	Aya dasthayam bar rooye ham Yerbeyer hastan? Chera?	Are our hands congruent with each other? Why?
5	The students	Bale	Yes
6	Yashar	Chon ki boular kamelan bir birinin oustouna doushoular	Because they cover each other completely
7	Teacher	Hala, har kodam az shoma do shekle Yerbeyer mesal bezanid	Now, everyone gives two examples of congruent shapes
At this moment, each student immediately gave examples of congruent (Yerbeyer) shapes. It seemed that they were able to give more examples of congruent (Yerbeyer) shapes			
8	Yegane	Ein do mosaic (kashi) Yerbeyer hastan	These two mosaic tiles are congruent
9	Teacher	Chera?	Why?
10	Yegane	Choon aga biz boo iki mosaici (kashi), kelasoon yerinnan chighardakh va bir birinin oustouna ghoyakh, mesle dasthaye shoma, bar rouye ham gharar khahand gereft	Because if we take the mosaic tiles and put them on each other, like your hands, they will fit exactly on top of each other
11	Hosein	Saghfe kelas ba kafe kelas Yerbeyer hastan	The ceiling and the floor of the class are congruent
12	Yashar	Rast doovar, chap doovaroonan Yerbeyer dilar	The right-hand wall is congruent with the left-hand side
13	Leila	Manim riyazi kitaboom, Hoseinin reyazi kitabinan Yerbeyer dilar	My mathematics book is congruent with Hosein's book
14	Teacher	Faghat ba ketabe Hosein?	Only with Hosein's book?
15	Leila	Faghat Hosein kinan yokh, ehtemal hamoo riyazi ketablaroonan Yerbeyer ola	Not only with Hosein's book but presumably, it is also congruent with all the students' mathematics books in the class
16	Teacher	Hala begid be man ke be jaye kalameye Su (means water in L1) va be jaye kalameye Chorak (means bread in L1) che kalemati dar Farsi estefade mishe?	Now, tell me, which words do we use instead of water and Bread in Persian?
17	The students	Ab (means water in L2) and Nan (means bread in L2)	Water and Bread
18	Teacher	Be hamin tartib, ma be jaye kalameye Yerbeyer (means congruent in L1), az kalameye Hamnehesht (means congruent in L2) dar Farsi estefade mikonim. Aya shoma mitoonid do shekle Hamnehesht nam bebarid?	Likewise, we use congruent (Hamnehest) in Persian instead of congruent (Yerbeyer) in Turkish. Can you name two congruent shapes?
19	Yashar	Rast doovar, chap doovaroonan Hamnehesht dilar	The left-hand wall of the class is congruent with the right-hand side

Table 6 (continued)

No	Names	Original (Turkish in blue, Persian in black)	English translation (from Turkish in blue, from Persian in black)
20	Leila	Ketabe riyaziye man ba ketabe riyazeye ham kelasihayam Hamnehesht hast	My mathematics book is congruent with my classmates' mathematics books
21	Hosein	Saghfe kelas ba kafe kelas Hamnehesht hastan	The ceiling and the floor of the class are congruent

formal explicit. The students drew on the informal/formal relationship throughout the discussion about algebraic expressions (see Table 5). Consequently, the practice of making a relationship between the informal and the formal explicit by means of students' L1 gave the students the opportunity to be socialized into the discourse of algebraic expressions. Further, the practice of making a relationship between the informal and the formal explicit by means of students' L1 moved the interaction from being highly teacher-structured to student-led participation (see, for example, Table 5, item 14).

In class B, after the teacher put his two hands together as a sign of the same shape and the students called them Yerbeyer, he first asked the students to provide examples of Yerbeyer shapes (Table 6, items 8–13). In the next step, the teacher used some ordinary Turkish words that the students knew the Persian equivalents that supported their understanding of Yerbeyer as Hamnehesht (Table 6, items 16–18). The teacher then asked the students to provide examples of Hamnehesht shapes. In this part, the students repeated the same examples and called them Hamnehesht shapes (Table 6, items 19–21). By means of students' L1 and later their L2, the teacher brought attention to features of mathematical discourse about congruent shapes. This teaching practice provided opportunities for socialization into the discourse of mathematics because it allowed the students to actively participate in the activity about congruent shapes.

6.3.4 Use of gestures in mathematical interaction

In addition to explaining, discussing, and writing, the teachers and students used gestures. For example, in class B, the teacher put his left palm on his right palm and asked, "What is the relationship between my left and right hand?" By doing so the teacher made a link explicit between the gesture of putting his hands together and the mathematical notion of congruity. As shown in Excerpt 2, the teacher's gesture appeared to be an entry to participation in mathematical discourse and prompted students to give examples of congruent shapes (Yerbeyer shapes) (Table 6, items 7–8). In addition, the notion of the two hands completely overlapping and hence being congruent was used by Yegane (Table 6, item 10) to justify her claim about the mosaic tiles being congruent. Yegane confidently pointed to two mosaic tiles with her hands and said, "Because if we take the mosaic tiles and put them on each other, similar to your hands, they will fit exactly on each other" (items 8–10). She demonstrated her mastery by showing two mosaic tiles with her hands and persuaded her teacher well by means of her oral explanations together with her hand gestures. Further, by using gestures in the teaching practice, the teacher concurrently licensed gestures as part of mathematical discourse. In both classes, the students used gestures to point to various objects, whiteboards, and their own and each other's writings. For example, as shown in Excerpt 2 (item 13), one of the students pointed to two mathematics books with her hands and said, "These two books are congruent." Another student pointed to two windows with his hands and said, "Both class windows are congruent" (Fig. 4). The socialization event use of gestures in mathematical interaction appeared to motivate the students to reason and dispute their classmates' viewpoints. Consequently, the use of gestures appeared to provide opportunities for socialization into forms of mathematical argumentation, which is part of the discourse of mathematics.

Fig. 4 Amir points to two windows with his hands and explains, "Both class windows are congruent"

6.3.5 Explaining mathematical thinking

In class A, in the practice of teaching and learning algebraic addition, the teacher used formal algebra symbols such as A, D, and C. To re-contextualize the general formal symbols, he used verbal representations of ordinary objects beginning with the same letter as the formal symbols. He said, "Assume that A is Alma, D is Dash, and C is Soghan. With this assumption, tell me your ideas, …." He invited the students to present their comments, ask their questions, and explain their mathematical thinking (Table 5). Using Turkish, they could explain their mathematical ideas, ask questions, volunteer to present, participate in mathematical interactions, and so on. The re-contextualization practice together with the use of students' L1 provided opportunities for the students to jointly develop explanations about algebraic expressions. Table 5 shows how each student was able to explain their mathematical thinking in Turkish (items 1, 2, 9, 14, 21).

In class B, one of the obstacles faced by students was the complex and unfamiliar term congruent. When the teacher put his left palm on his right palm to allow the students to engage with the concept of congruent, he asked [Daste chapam nesbat be daste rastam nagoordo?] using both Persian and Turkish. Using L1 and gestures prompted the students to explain their mathematical thoughts in their L1 (Table 6, items 8–13). The teacher then used equivalents to encourage the students to express in Persian what they had explained in Turkish (Table 6, items 19–21). To answer questions, the students constantly looked around, looking at each other and at different objects as if trying to find something. For example, Leila said in Turkish, "Manim riyazi kitaboom, Hoseinin reyazi kitabinan Yerbeyer dilar" [My mathematics book is congruent with Hosein's]. After the teacher used the equivalent words, Leila said in Persian, "Ketabe riyaziye man ba ketabe riyazeye ham kelasihayam Hamnehesht hast" [My mathematics book is congruent with my classmates' mathematics books] (Table 6, items 13–15, 20). Consequently, the practices of using gestures, objects, students' L1, and claims and/or questions that invited students to provide mathematical explanations shaped socialization events where the students developed their mathematical thinking together.

7 Discussion and conclusion

This 9-month-long ethnographic study, situated in a multilingual educational setting in Zanjan in Iran, where the language of instruction is Persian, contributes with knowledge to three under-researched dimensions (see de Araujo et al., 2018; Erath et al., 2021; Schüler-Meyer, 2017) of how a shift to include students' L1 (here Turkish) in the teaching practice may enhance learning opportunities.

First, the present study shows how a shift to include students' L1 in the teaching practice may unfold. The results confirm that the inclusion of students' L1s in teaching practices can enhance learning opportunities in the classroom for multilingual students (de Araujo et al., 2018; Erath et al., 2021; Planas & Seteti Phakeng, 2014; Schüler-Meyer, 2017). Although the changed teaching practice focused on the inclusion of the students' L1, many of the socialization practices that Barwell (2020) identified in language positive classrooms emerged when the inclusion of students' L1 was made. This finding may suggest interconnected relations between the seven socialization events and their language positive practices. Previous studies have demonstrated that a shift to include students' L1s in mathematics instruction alone does not necessarily provide more and richer learning opportunities (Barwell, 2020; de Araujo et al., 2018; Erath et al., 2021). Our results show how language positive socialization events may be reinforced by the implementation of the socialization event "use of students' home languages." For example, "explicit attention to mathematical discourse" was reinforced by using the Turkish words Alma, Dash, and Soghan. In the same vein, gestures reinforced the discourse of mathematics when the teacher put his two palms together to demonstrate the meaning of the concept 'congruent' while verbally explaining the Persian word Hamneheshti is "congruent" with the Turkish word Yerbeyer. This result indicates that using students' L1s could reinforce other language positive practices, which suggests that using students' L1s is a key issue. Further studies are required to clarify the relation between implementation of flexible language use of students' full language repertoires and other language positive practices. In addition, this finding suggests that it is worthwhile for teachers to invest time in facilitating zones of comfort (Mackinney, 2022; Schüler-Meyer et al., 2019), where multilingual students feel free to use the full range of their language repertoires. This is important particularly since many students in the study experienced increased learning opportunities due to the shift to include their L1 in the teaching practice. However, little is known about how to support teachers in establishing such classroom practices.

Second, the present study provides a long-term perspective on a shift to include students' L1, which is a perspective that is scarce in current research (de Araujo et al., 2018; Erath et al., 2021; Schüler-Meyer, 2017). The present study confirms findings from short-term studies (e.g., Chronaki et al., 2022; Ryan et al., 2021; Schüler-Meyer et al., 2019) that the inclusion of students' L1s enhances learning opportunities over longer periods. While a short-term consequence of a shifted teaching practice may be visible in the classroom since they are connected to teaching more directly, out-of-school activities that relate to mathematics learning opportunities may be less explicit in immediate classroom interaction. One such change may concern engagement with assignments. Here, after the shift, students engaged with their assignments in productive ways that supported learning and allowed for mathematical discussions in the classroom. Another dimension, which little is known about, is how a shift to include students' L1 in the teaching practice influences students' attendance in mathematics class. Based on the long-term perspective, the

present study shows that students and their parents value mathematics education higher in relation to the value of, for example, labor in the agricultural sector when mathematics teaching employs flexible language practices. This is an important finding because it shows how the conceived value of mathematics education in local communities may change with the introduction of students' L1 in the teaching practice.

Third, the present study broadens the contexts in which multilingualism has been studied. In the Iranian context (and in many other contexts), multilingualism has historically been the norm, but this changed to the benefit of a monolingual norm due to colonialism (Hoominfar, 2014; Kalantari et al., 2020). We suggest that a systematic program that recognizes indigenous languages in the educational system (Hoominfar, 2014) enhances mathematics learning opportunities for students in these contexts.

The present study was conducted in a classroom where teachers and students shared the same L1, which may make a shift to include students' L1 in the teaching practice appear quite easy. Considering the positive influences that the implication of students' L1 in mathematics teaching practices appears to have in this context, further investigations should focus on principles that can be employed to enhance students' L1s in contexts where classrooms comprise a broad range of L1s which may diverge from the teachers' L1. Such investigations would honor the socio-political dimensions of flexible language that, for example, Garcia and Li (2014) emphasized to position all languages as, at least a bit more, equally valuable.

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