Educational Studies in Mathematics (2024) 115:353–385
https://doi.org/10.1007/s10649-023-10295-0

Using positioning theory to think about mathematics classroom talk

Ove Gunnar Drageset[1] · **Fiona Ell**[2]

Accepted: 23 December 2023 / Published online: 20 February 2024

Abstract

This article aims to connect two research areas by using positioning theory to review the literature on talk moves, teacher interactions, and discourse patterns in mathematics education. First, a conceptual review identified 44 articles with 94 concepts describing interactions and discourse patterns. Similar concepts were grouped in a process that developed five categories, each describing one teacher position (a teacher who tells, a teacher who supports, a teacher who uses students' ideas to create learning, a teacher who orchestrates, and a teacher who participates). Related to each position, we describe rights, duties, and communication acts. We suggest that these five teacher positions represent three transcendent storylines (teachers are providers of insight, teachers are facilitators of learning, and teachers are participants in learning). Using positioning theory enables us to understand the underlying powers that shape the classroom in relation to transcendent storylines, rights, and duties. We use this to explore what the implications are of these storylines and positions for equity and access to important mathematical ideas. This article contributes to our understanding of the complexity of classroom interactions and how transcendent storylines might play a role in subverting or promoting particular classroom communication patterns.

Keywords Positioning theory · Storylines · Talk moves · Interactions · Discourse patterns

1 Introduction

The mathematics classroom is at the heart of mathematics education research, and for many, discourse is the most central element of the mathematics classroom (Erath et al., 2021; Krummheuer, 2011; Sfard, 2008). Thus, a large body of research on mathematics classroom discourse has emerged over the recent decades (Erath et al., 2021; Xu & Clarke, 2019). Mathematics classroom discourse is, of course, inextricably entwined with

✉ Ove Gunnar Drageset
ove.gunnar.drageset@uit.no

Fiona Ell
f.ell@auckland.ac.nz

1 UiT The Arctic University of Norway, Tromsø, Norway

2 University of Auckland, Auckland, New Zealand

mathematics itself. The nature of what is to be learned in a mathematics classroom shapes how it might be approached and how it might be spoken about.

For some mathematics education researchers, classroom discourse *is* the students' learning (e.g., Lavie et al., 2019; Sfard, 2008); for others, discourse enables learning (e.g., da Ponte & Quaresma, 2016; Drageset, 2014b) or opens a window on students' thinking (e.g., Stein et al., 2008). Furthermore, changing discourse is seen as a way to improve learning, moving from the 'chalk and talk' stereotype to a more participatory model where learners share ideas and talk to each other about their thinking (Kazemi & Hintz, 2014). However, despite a growing consensus, over a period of many years, on the importance of shifting classroom interaction patterns so that students talk more and talk about mathematical concepts and problems (Alrø & Skovsmose, 2004; Chapin et al., 2013; Kazemi & Hintz, 2014; Mercer & Littleton, 2007; Stein et al., 2008), change in classrooms seems slow and difficult to sustain (Heyd-Metzuyanim et al., 2016; Xu & Clarke, 2019).

When discourse is centered as a critical mechanism for learning mathematics, access to discourse becomes an equity issue (Erath et al., 2021; Gutiérrez, 2009; Hunter, 2010; Tait-McCutcheon & Loveridge, 2016; Vogler et al., 2018). Mathematics classroom discourse occurs at the intersection of personal factors, wider social structures, and the discipline of mathematics. These influences shape what is said, how it is said, and by whom (Yackel & Cobb, 1996). If access to discourse is access to learning, it is essential to understand who participates and how they do so (Krummheuer, 2011). Patterns of classroom discourse are held in place by the views of rights and responsibilities held by the discourse participants (Barwell, 2013; Harré & Moghaddam, 2003; Tait-McCutcheon & Loveridge, 2016). Understanding how this works increases the possibility of changing towards more equitable and inclusive practice. Learning mathematics in classrooms can be seen as learning the content of mathematics and how to participate in that learning (Barwell, 2013; Erath et al., 2021; Krummheuer, 2011). Researchers have used close analysis of classroom discourse to reveal patterns of participation and inequities that might arise from not knowing how to participate, or participating in nonroutine ways, in the mathematics classroom (Davies & Hunt, 1994; Krummheuer, 2011; Tait-McCutcheon & Loveridge, 2016). As well as shaping what is learned and how it is learned, classroom discourse can also be seen as a means of establishing and perpetuating relationships that frame what is possible to learn (Tait-McCutcheon & Loveridge, 2016). A theoretical framework that foregrounds how discourse emerges from, and shapes relations between people in particular situations could therefore help mathematics educators to consider how discourse patterns might empower or marginalize learners in the mathematics classroom (Hunter, 2010; Tait-McCutcheon & Loveridge, 2016). One framework that might do this is positioning theory (Harré & Moghaddam, 2003; Tait-McCutcheon & Loveridge, 2016). This conceptual review considers whether thinking about classroom interaction patterns highlighted in recent mathematics education research through the lens of positioning theory can help us think about the relationships among teachers, students, and mathematics and the implications of these for equity in mathematics learning.

The purpose of this review, therefore, is to re-examine work on discourse in mathematics classrooms using positioning theory (Davies & Harré, 1990). Positioning theory is interested in understanding social phenomena by considering the positions and associated storylines created as people interact with each other (Harré & Moghaddam, 2003). These positions and storylines can help us to see how discourse patterns in mathematics classrooms might impact participation and opportunities to learn (Hiebert & Grouws, 2007). Thus, equity of access to mathematical ideas considers what mathematics is available to learners as they take up or reject the positions offered in mathematics classroom discourse.

1.1 Positioning theory

Positioning theory (Davies & Harré, 1990) was developed to explain the dynamic nature of evolving social relationships and the fact that the roles people adopt are not fixed; rather, they emerge through interaction with others. This focus on the immanent nature of positioning through discourse is helpful in considering the mathematics education literature on discourse patterns because positioning theory pays attention to the same things as these analyses do: the way that discourse unfolds and the consequences of that unfolding (Wagner & Herbel-Eisenmann, 2009). This is not unproblematic, in part because of the multilayered complexity of classroom discourse, which emerges moment by moment but is also shaped by longer-term forces that intersect classrooms—for example, common tropes about what it is to teach and learn mathematics (Herbel-Eisenmann et al., 2015). Acknowledging the difficulty of combining an immanent view of positioning and the existence of important transcendent discourses (of learning and of mathematics), positioning theory nevertheless offers a way to consider work on talk moves, teacher interactions, and discourse patterns in a common frame.

In its original form, positioning theory proposes a three-part framework for understanding discourse, comprising positions, speech acts, and storylines (Davies & Harré, 1999). The three parts function to assign rights and responsibilities among the participants. Positions are "the cluster of rights and duties to perform certain actions with a certain significance" (Harré & Moghaddam, 2003, p. 5). For example, if you are positioned as the teacher in the traditional sense, you have the right to lead the classroom, but you also have the duty to help students learn.

Storylines are the "loose cluster of narrative conventions" (Harré & Moghaddam, 2003, p. 6) that emerge as the participants interact. These storylines are about the positions that are being created in the interaction, but in the case of classroom interactions, they are also about learning and about mathematics itself. Storylines are created jointly by participants, although past experience and longer-term storylines might mean that rather than creating a joint storyline, participants are simultaneously in different storylines but do not realize this (Wagner & Herbel-Eisenmann, 2009).

Speech acts are "the socially significant actions, movements or speech" (Harré & Moghaddam, 2003, p. 6) that comprise the interaction. The idea of speech acts has been extended to include all forms of communicating (Herbel-Eisenmann et al., 2015), so the term 'communication acts' is often used instead. Communication acts build storylines as they proceed, and reciprocally, storylines invoke rights and duties, shaping communication acts, positioning the participants in interaction relative to one another, and, in the case of classroom interaction, relative to the subject matter and how it is learned.

At any time, multiple storylines can be evoked by communication acts and their associated positionings. Davies and Harré (1990) describe this as the "braided development of several storylines" (p. 50). Things that happen at the moment in a mathematics classroom are about what is happening and simultaneously invoke wider storylines. Herbel-Eisenmann et al.'s (2015) model uses the idea of scale to show how narrative layers are nested in each other, with broader storylines shaping more local ones. In this way, broader discourses about capability, access, and equity enter moment-by-moment classroom interactions. Using positioning theory to think about discourse patterns as communication acts allow us to think about the rights and duties, and storylines, that come with different ways of communicating in mathematics classrooms.

A final layer of positioning theory is a three-part framework to explain the differences in taking up positions through interaction: willingness, capability, and power (Davies & Harré, 1999; Huang & Wang, 2021). This framework is highly relevant to teacher–student interactions, where long-established dynamics shape discourse patterns. 'Willingness' describes whether or to what extent participants are prepared to position others and be positioned themselves. 'Capability' describes the extent to which participants are able to take up the positions offered, which is bounded by their temperament and personal experiences. Finally, and significantly for classroom processes, 'power' is defined as how "individuals are permitted, allowed or encouraged to perform positions" (Harré & Moghaddam, 2003, p. 125).

Taken together, the three aspects of positioning theory (positions as rights and duties, storylines, and communication acts), coupled with the consideration of willingness, capability, and power, provide a way to consider work on discourse patterns in mathematics classrooms and to connect what is said in micro-interactions to longer-term storylines about mathematics teaching and learning (Anderson, 2009; Tait-McCutcheon & Loveridge, 2016), thereby enhancing our ability to understand how power is working through interaction, and what this means for access to mathematical ideas and recognition of learner identity (Gutiérrez, 2009).

This review looks across an indicative range of typologies, descriptions, and labels used in exploring mathematics classroom talk to see whether thinking about what positions are established through assigning rights and duties, what storylines are enacted, and how power is used or shared, can provide insights into this literature that are otherwise implicit or underexplored. It seeks to answer the question: *What storylines and associated positions can be identified in mathematics education literature on talk moves, teacher interactions, and discourse patterns, and what are the implications of these for equity in mathematics learning?*

Figure 1 maps this research question onto the positioning theory triangle of communication acts, storylines, and positions (Van Langenhove & Harré, 1993). The triangle shows how the three aspects of positioning theory interrelate and co-determine each other (Harré & Moghaddam, 2003). Beginning at the bottom left, the figure shows how the questions that guide this review arise from the relationships in the positioning triangle. The three research questions are bold and marked 'RQ'. Taking a body of research on classroom interaction and discourse, we consider whether there are recurring patterns. For these recurring patterns we ask: What positions and storylines might arise from these patterns? The answers to these questions form the findings of the study. The two evaluative questions

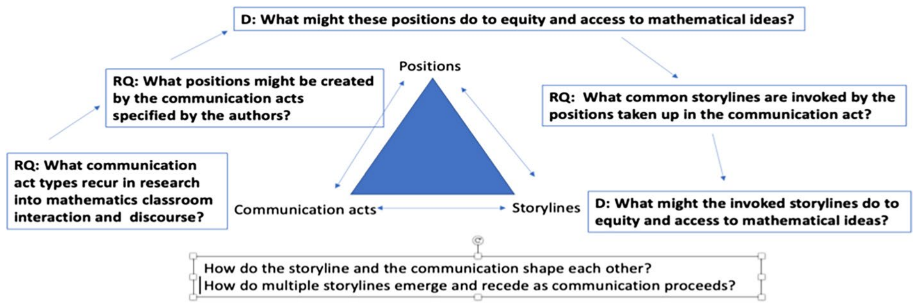

Fig. 1 How the conceptual review questions arise from positioning theory

about equity and access to ideas form the basis of the discussion and are bold and marked 'D' in Fig. 1. The remaining two questions (in plain text) are beyond the scope of this analysis because data about the mutual constitution of storylines and how interaction proceeds were not available in all the studies we reviewed.

2 Method

This conceptual review of 44 articles centers on a body of work that has a family resemblance conceptual structure (Podsakoff et al., 2016). Works on classroom discourse, interactions, and talk moves use different terms but share key 'family' characteristics: a focus on the centrality of talk to learning in the mathematics classroom, with concepts describing interactions and discourse patterns as the unit of analysis. The 44 articles are not intended to provide a comprehensive survey of the field of work on classroom discourse, interactions, and talk moves, rather they form an indicative body of work that is used to explore the question of whether using positioning theory to consider their findings can reveal patterns in positions and storylines that help us think about equity in mathematics learning. Thus, this study is searching for ways with which to organize and understand our shared knowledge rather than describing a current state of the art, thus being explorative and not offering a fully representative review of the research literature.

2.1 How the articles were selected and analyzed

There were two phases to the article selection. First, a hand search of articles published from 2010 to 2020 was undertaken in two prominent mathematics education journals: *Journal of Mathematics Teacher Education* and *Educational Studies in Mathematics*. The keywords 'talk moves,' 'teacher interactions,' and 'discourse patterns' were used to identify relevant work. The articles were read in full by the first author using the rules for inclusion presented in Table 1. The Table 1 list of inclusion rules reveals that we chose to use teacher interactions as a starting point as there were few student interactions (partly caused by the keywords). Second, references from these works were followed, searching for concepts describing talk moves, teacher interactions, and discourse patterns. This also included using Google Scholar to find articles that cited the articles already selected. This process continued until new concepts describing discourse no longer appeared and data saturation was reached. This process yielded 29 additional articles, resulting in a total of 44 articles and 94 concepts (see Appendices 1 and 2).

Table 1 Rules for inclusion and exclusion of concepts (and consequently articles)

Included when	Excluded when
The article suggests a name (concept) to describe a type of interaction or discourse pattern	Concepts that do not describe a type of interaction or discourse pattern
The concept is elaborated upon (by example, argument, synthesis of other concepts, or definition)	Concepts used from other authors were excluded, but the references to these concepts were added to the list of articles to consider for selection
The concept describes a teacher interaction (or one that could be both teacher and student interaction)	Concepts that describe a student interaction that does not involve a teacher interaction

There were two main steps to the analysis. First, the definitions were compared to identify clusters of related concepts. Clusters were determined by the function of the communication acts, which was thought of as assigning positions. This process yielded five clusters describing the positions of teachers and students in relation to each other and the mathematics they were working on. An example of cluster formation is given in the paragraph below:

> The concept of *inviting* (Da Ponte & Quaresma, 2016) was grouped with the concepts of *probing for facts* and *probing for understanding* (Bennett, 2010), *probing students to explain their thinking* (Boaler & Brodie, 2004, *eliciting student thinking* (Fraivillig et al., 1999) and *teacher as elicitor* (Lobato et al., 2005) as they all described interactions where the teacher seeks to access student thinking. This group was first called *access*, but seeing concepts like *providing students' opportunities to share their solutions* (Teuscher et al., 2016), we found that *accessing* student thinking related to *sharing*, particularly during the plenary discourse. Gradually adding more concepts, this formed a group that was named *Access and share*. Further, the latter was then set with two other groups to define a teacher position called *A teacher who uses students' ideas to create learning* (see teacher Position 3 and Appendix 1 for further detail). An important part of this process was to constantly review the groups, regrouping concepts, and redefining group names and definitions based on the included concepts.

Second, both authors considered the clusters and their associated positions as a group to identify overarching storylines (as described by Herbel-Eisenmann et al., 2015). Three storylines encompassed the five positions identified through clustering the communication acts. Positioning theory was used to consider the results of the analysis in terms of rights and duties assigned through the communication acts, and the implications of the teacher's positioning for student positioning. This connection between teacher positions, student positions, and rights, duties, and communication acts is summarized in one table for each teacher position, presented in Tables 3, 4, 5, 6, 7 in the Findings Sect. 2.2 below. As the analysis is based on teacher interactions, it focuses on describing the positions and storylines arising from the literature about teacher interactions, and the consequences these might have for student positioning. Since positioning clearly goes both ways, is an ongoing negotiation, and positions are not always accepted, this complexity is addressed in the discussion. Thus, the findings describe teacher positions and the possible consequential positioning of students. A final iteration was done by compiling a table as summary (Appendix 1) and using this as a validity check that all concepts correspond to their communication acts, positions, and storylines.

2.2 Findings: teacher positions in the classroom

This section presents the storylines, teacher and student positions within these storylines, and the interactions and discourse patterns that illustrate both the positions and the storylines, found in 94 interaction types in the 44 reviewed articles. Our unit of analysis is not the data or examples from these articles. Rather, it is the concepts authors developed to describe interactions in the classroom. Examples of these concepts can be found in each reference (Appendix 2).

Central to considering these findings is the nature of positions, communication acts and storylines. While they are presented here in tables in order to describe the patterns that emerged from our analysis, they are not fixed. As described above, positions, communication acts, and storylines are constantly emerging as discourse unfolds and are contingent on what has come before. Positions may shift utterance by utterance, and certainly, over longer periods of time, positions will be negotiated through communication acts, accepted and rejected, and create storylines that will change and evolve. Although we present our findings as contrasts in order to make the patterns clear, this should not be taken as ascribing permanence or dichotomizing. We aim to illustrate the potential of different positions and communication acts for providing access to ideas and developing willingness, capability, and power in participation. Moment-by-moment choices by teachers and students are immanent and contingent, while at the same time, they connect to, and reproduce, storylines that come from beyond that particular moment in time. Our analysis seeks to capture that idea to help us think about the link between discourse and wider storylines about teaching and learning mathematics while not suggesting that one teacher holds one position in any permanent sense.

As we considered the rights and duties, communication acts, and storylines in the identified collection of research, we recognized the role that mathematics itself played in shaping the positioning triangle in mathematics classrooms. There are many ways to view what it is to learn mathematics, and these views shape how you might communicate about it. Discourse is shaped not only by the interlocutors but also by the nature of what is to be communicated. Thus, we added inferred descriptions of the nature of mathematics for each storyline to Tables 3, 4, 5, 6, 7, and this idea is taken up further in the discussion.

Table 2 provides an introductory overview of the storylines, positions, and communication acts found in the review. The findings are then presented by suggested storyline, and by teacher position within these storylines. Evidence from the reviewed research is presented and then summarised in a table for each teacher position (Tables 3, 4, 5, 6, 7). The

Table 2 An overview of storylines, positions, and communication acts found in the review (further detail is provided in Appendix 1, which presents which concepts and references each communication act is based on)

Storyline	Position	Communication acts
(Storyline 1) Teachers as providers of insight	(Position 1) A teacher who tells	Telling to initiate student work Telling how to do Telling about connections
	(Position 2) A teacher who supports	Reducing complexity Assessing Progressing student thinking
(Storyline 2) Teachers as facilitators of learning	(Position 3) A teacher who uses student ideas to create learning	Providing access and sharing Pointing out Reasoning with a student idea
	(Position 4) A teacher who orchestrates	Managing Developing ideas Focusing on peer thinking
(Storyline 3) Teachers as participants in learning	(Position 5) A teacher who participates	Collaborating Acting in role

Table 3 A teacher who tells and a student who listens

	Teacher	Student
Position	A teacher who tells	A student who listens
Rights	To be allowed to tell and be listened to To decide what information to share, and how and when to share it To decide how and what to do	To receive useful and accurate information To receive clarification when requesting it
Duties	To initiate student work To be clear on expectations To know the mathematics they are teaching To be clear, and give sound and accurate explanations and connections	To pay attention To ask when something is not understood
Communication acts	• Telling to initiate student work • Telling how to do • Telling about connections	• Passive listener • Active listener that requests clarifications and elaborations
The nature of mathematics	A set of established relationships and processes to be mastered by students	

Table 4 A teacher who supports and a student who is receptive to help and feedback

	Teacher	Student
Position	A teacher who supports	A student who is receptive to help and feedback
Rights	To decide when to support and when not to	To be given support
Duties	To help students find an answer To help students understand complex procedures, concepts, and connections To assess students' work To give feedback that progresses students' thinking	To be receptive to help and feedback To give the teacher access to work and thinking To engage in development of their own thinking
Communication acts	• Supporting by reducing complexity • Supporting by assessing • Supporting by progressing students' thinking	• Asking for help or assessment • Accepting feedback • Engaging in development of thinking
The nature of mathematics	A set of established relationships and processes to be mastered by students	

Table 5 A teacher who uses students' ideas to create learning and a student who owns the ideas and is willing to share them

	Teacher	Student
Position	A teacher who uses students' ideas to create learning	A student who owns the ideas and is willing to share them
Rights	To access students' ideas To point out, suggest, and challenge ideas To select which ideas to work on	To be the owner of ideas To suggest and challenge ideas
Duties	To be receptive and willing to consider different ideas	To give access to their own ideas To share haring ideas To engage in the development of ideas
Communication acts	• Accessing and sharing • Pointing out • Uptake (using students' thinking)	• Giving teacher access to ideas • Sharing ideas • Engaging in joint development of their own and others' ideas
The nature of mathematics	A logically connected body of ideas, concepts, and methods to be explored, connected, and developed	

Table 6 A teacher who orchestrates and a student who contributes

	Teacher	Student
Position	A teacher who facilitates	A student who contributes
Rights	To set the rules for the discourse To allocate time for thinking and short discussions with a partner To choose who responds to ideas	To share ideas To suggest new approaches
Duties	To give students time to develop ideas To set rules and develop norms that focus on respect and build confidence	To ask questions for clarification To evaluate ideas
Communication acts	• Managing • Developing ideas • Focusing on peer thinking	• Sharing ideas • Asking questions • Suggesting new approaches • Assessing ideas
The nature of mathematics	A logically connected body of ideas, concepts, and methods to be explored, connected, and developed	

Table 7 A teacher who participates and a student who participates

	Teacher	Student
Position	A teacher who participates	A student who participates
Rights	To collaborate on equal terms To not know To play a role	To collaborate on equal terms To explain and argue To decide based on arguments
Duties	To open a space for students' arguments by avoiding power claims To collaborate on equal terms To play a role that contributes to students' participation and learning	To aim at power based on arguments To collaborate on equal terms
Communication acts	• Collaboration • Teacher-in-role	• Collaboration • Role-play
The nature of mathematics	A set of (realistic/important) problems that requires a joint development of knowledge to be solved and logical arguments to be evaluated (assessed/decided)	

tables include the position, associated communication acts, storylines, rights, and duties, and inferred positions for students for each proposed teacher position.

2.3 Storyline 1: Teachers as providers of insight

The 'teachers as providers of insight' storyline is based on the idea of someone learning from another who knows. This might be connected to several ways of learning, such as transfer of knowledge, apprentice and master, or Vygotsky's et al. (1978) zone of proximal development with the teacher as the main support. Two main teacher positions are related to this storyline: a teacher who tells and a teacher who supports. Both teacher positions are based on a teacher who claims power and consequently positions students as subordinate.

2.3.1 Teacher Position 1: A teacher who tells

Teachers typically know more mathematics than students and might position themselves as someone who knows more than others. The communication act that makes this position visible is the teacher sharing the knowledge by telling students about mathematics. But teacher telling has acquired a bad reputation and is often seen as negative. Lobato et al. (2005) suggest that teaching as telling is undesirable when it minimizes the opportunity to learn about students' ideas, focuses only on procedural aspects, emphasizes the teacher's power and students as subordinate rather than developing students' judgment, minimizes cognitive engagement, communicates only one solution path, and prematurely closes mathematical exploration. To avoid these drawbacks, Lobato et al. (2005) suggest reformulating telling in three ways: focusing on the function rather than the form, the conceptual rather than the procedural, and connection rather than isolated actions. In this framework, it is not the telling that is significant, it is what one tells.

In addition, Lobato et al. (2005) reformulate telling by connecting it to more student-centered interactions such as initiation, defined as stimulating students' mathematical construction, and eliciting, which occurs when the teacher's actions draw out students' ideas and reasoning. One main observation is that there is qualitative variation in telling related to differences between how the telling relates to procedures versus concepts and isolated actions versus interrelated actions (Lobato et al., 2005).

Looking at the concepts describing interactions and discourse patterns in research literature that describes teacher telling, these fall into three types. The first type of telling is telling to initiate student work, as described above by Lobato et al. (2005), but da Ponte and Quaresma (2016) also emphasize telling as initiation by describing how teachers inform and suggest. This is also visible in the discourse pattern described as setting the scene (Haavold & Blomhøj, 2019), which describes how the teacher engages and inspires students to start out on a mathematical inquiry.

The second type of telling is telling how to do, either by explaining (Henning et al., 2012), demonstrating how to solve a task or use a method (Drageset, 2014b), advising a new strategy (Drageset, 2014b), or modeling how an expert thinks (Fukawa-Connelly, 2012). The list of drawbacks to telling (Lobato et al., 2005) is mainly connected to telling how to do, which is where the teacher might claim the power to focus on procedures and consequently limits student participation. Telling how to do is also typical of discourse patterns such as unidirectional communication (Brendefur & Frykholm, 2000), where the teacher dominates the discussion by lecturing and asking closed questions, and conventional textbook culture (Wood et al., 2006), where the major interaction pattern is 'initiation–response–evaluation (IRE)' (Cazden, 1988; Mehan, 1979). However, telling what to do might also be a vital part of introducing new methods and explaining, and cannot always be related to procedural thinking and lack of student participation.

A third type is telling about connections between methods or concepts, as both Lobato et al. (2005) and Rowland et al. (2005) illustrate. This type is closely related to what it means to understand the logic of mathematics. During a dialogue, this is often seen when the teacher points out or emphasizes important aspects.

A 'teacher who tells' positions students as listeners, either as passive listeners or as active listeners with requests for clarifications and elaborations. Table 3 uses the positioning theory constructs of rights and duties (positions) and communication acts to summarize the findings about a teacher who tells and a student who listens.

2.3.2 Teacher Position 2: A teacher who supports

A teacher might take a position as a supporter of students' learning, which still maintains the teacher's power and the students as subordinate. A vast number of concepts describe different communication acts in supporting students' work with mathematics, including both interactions and discourse patterns. Based on the organization of these concepts into groups, three types of teacher-supporting students are suggested.

The first type of teacher support is reduction of complexity, which is about making it easier for students to reach an answer by what Boston and Smith (2009) called 'lowering cognitive demand'. This is achieved by either adding information or asking one question for each step until the answer is reached. Adding information can be done by giving cues (Henning et al., 2012), hints and highlights (Conner et al., 2014), or direct contributions to arguments (Conner et al., 2014). Such added information might also be called 'simplification' (Drageset, 2014b) due to its effect. On some occasions, when the added information changes the entire task, it is referred to as a 'Topaze effect' (Brousseau & Balacheff, 1997). When making it easier by asking one question for each step, it is about breaking problems down into smaller parts by directed guidance (Warshauer, 2015); the questions are typically closed (Drageset, 2014b), and the discourse pattern might be called 'guided algorithmic reasoning' (Lithner, 2008). Such patterns are also related to the proceduralization (Stein et al., 1996) and routinization (Boston & Smith, 2009) of mathematics by emphasizing that knowing mathematics is knowing methods and rules, step by step. Reduction of complexity is visible in several discourse patterns, such as the conventional problem-solving culture (Wood et al., 2006) where the main interaction pattern is the teacher giving hints, and funneling (Steinbring, 1989; Wood, 1998), where one specific method or answer is wanted and all other answers are rejected or funneled into the 'right' path. A strong emphasis on reduction of complexity might be the foundation for what Lavie et al. (2019) called "ritual student participation," where the learner is satisfied with completing routines without independent decision making or coherence between subroutines, or what Lithner (2008) called imitative reasoning, where students search for examples to copy or rules to follow. While reduction of complexity might sound like something that affects student learning negatively, this depends on the balance with other types of interaction. After all, a teacher who never adds information to help students who are stuck and never guides them through a new method would probably end up with frustrated students. Such a balance is visible when da Ponte and Quaresma (2016) describe actions in task solving that support and guide students, both explicitly and implicitly, towards a path that they can follow. Similarly, Drageset (2014a) suggests that teacher guidance might include advice, corrections, and pointing out.

The second type of teacher support is the assessing of students' work. One way to do this is by verifying and validating (Conner et al., 2014) the correctness of a contribution or by confirming (Henning et al., 2012) that the student is on the right track. Naturally, such positive support is an important part of motivating or helping students move forward toward solutions or understanding. But at other times, the teachers might reject (Henning et al., 2012), correct directly (Conner et al., 2014), or correct through questions (Drageset, 2014b). Arguably, these corrections are as important as the positive support, as they help students change direction or might even enable thinking and new understanding.

The third type of teacher support is to progress students' thinking to reach an answer or understanding. At its core, progressing students' thinking is about supporting the development of students' ideas for how to solve a task without providing teacher hints or assessment. This can be done by promoting and encouraging (Conner et al., 2014), by asking open questions that do not simplify or direct students (open progress initiatives; see Drageset, 2014b), or by probing guidance (Warshauer, 2015), asking for reasons and justifications, seeking explanations, and asking for written work showing students' thinking. Fraivillig et al. (1999) also describe how a teacher progresses students' thinking by reminding them of conceptually similar situations, providing background knowledge, and encouraging them to request assistance.

These three types of teacher support—reduction of complexity, assessing, and progressing students' thinking—suggest three ways of enacting the teacher as taking a supporting position. A danger of only using reduction of complexity is that students may become reliant on teacher support and we might see what Lavie et al. (2019) call "ritual student participation." As reliance on a teacher is a natural part of student development, such as described in Vygotsky's et al. (1978) zone of proximal development, the ultimate goal of any education is empowerment; and even if different ways to reduce complexity might be important tools for a teacher to use, other types of interaction are also needed to be able to empower students, perhaps most importantly by progressing their thinking until they are independent of teacher support.

A teacher who supports positions a student as someone who is receptive to help and feedback and who engages in the development of one's own thinking. Table 4 uses the positioning theory constructs of rights and duties (positions) and communication acts to summarize the findings about a teacher who supports and a student who is receptive to help and feedback.

2.4 Storyline 2: Teachers as facilitators of learning

The 'teachers as facilitators of learning' storyline is based on the idea that to create learning, one needs to facilitate others' learning through reflection and communication. This might be based on personal reflection, social learning, or a combination of these. The core of this storyline is that it is the students' ideas that are the content of the learning process.

2.4.1 Teacher Position 3: A teacher who uses students' ideas to create learning

The third position, which is based on the largest number of communication acts from the literature, describes a position where the learning should start with the student's ideas.

The first type of teachers' use of students' ideas describes different ways to access and share students' ideas. Sometimes, getting access to their ideas and sharing them are separate processes, such as in five practices (Stein et al., 2008), where teachers monitor and select ideas during the students' work and then ask them to share the intentionally picked ideas afterward. In the articles that are the basis of our analysis, most concepts describe this as one process, where accessing students' ideas in a plenary is at the same time the sharing of these. Naturally, most interactions are based on questions. Gaspard and Gainsburg (2020) separate questions that anticipate predictable responses from those that anticipate unpredictable responses. Most concepts related to accessing students' ideas seem to be related to the latter, such as probing for facts or understanding (Bennett, 2010) or for students to explain their thinking (Boaler & Brodie, 2004), or requesting more details or justification (Drageset, 2014b). Also, eliciting or enlightening students' thinking in different ways (Conner et al., 2014; Fraivillig et al., 1999; Henning et al., 2012; Lobato et al., 2005) and generally inviting students or providing students with opportunities to share their solutions (da Ponte & Quaresma, 2016; Teuscher et al., 2016) seem to be about asking questions where the responses are difficult to anticipate.

The second way of using students' ideas is by pointing out students' thinking that the teacher wants to emphasize for the other students. Several authors describe how teachers reformulate, repeat, revoice, and clarify (Alrø & Skovsmose, 2002; Conner et al., 2014; Henning et al., 2012; Kooloos et al., 2020; O'Connor & Michaels, 1993), all of which are interactions where the function is to point out something important by emphasizing it or making the students' thinking clearer or more accurate. Also, summing up or recapitulating (Henning et al., 2012) uses the teacher's voice to point out what was important in an explanation or discussion. In addition, such pointing out might be done by orientating and focusing (Boaler & Brodie, 2004; Drageset, 2021; Wood, 1998) students before a task or by refocusing them (Conner et al., 2014) during work or discussions.

The third way of using students' thinking goes one step further, from sharing and pointing out to what Correnti et al. (2015) call "uptake," which is to take up students' ideas in the collective (Staples, 2007) for discussion and to develop them. In this process, accessing and sharing play an important part, but the teacher might start by giving time to develop ideas to be shared (or allow time for students to work; see Warshauer, 2015), and the teacher does not limit her/himself to pointing out but instead continues to work on selected student ideas. Researchers have described key parts of this process as elaborating (Henning et al., 2012), encouraging reflection, reasoning, and going beyond the initial method by pushing for alternatives (Cengiz et al., 2011), extending student thinking (Boaler & Brodie, 2004; Fraivillig et al., 1999), and exploring mathematical meanings and relationships (Boaler & Brodie, 2004). Uptake means that a student's idea is the core element of the discourse, used for learning in the classroom by discussing, comparing, and exploring the idea. While it is still the teacher that decides which ideas should be worked on or taken up for discussion, uptake is clearly distinct from pointing out, where the teacher's comments are the core element. Stockero et al. (2020) studied who is publicly given the opportunity to consider students' mathematical thinking (the teacher, the same student, other students, or the whole class) and what they do or are allowed to do with that opportunity (such as clarify, develop, dismiss, evaluate, justify). In this way, it is possible to achieve a deeper understanding of who contributes in what way when uptake occurs.

While accessing and sharing clearly have value by letting students share ideas and listen to each other, access and sharing without uptake might create a discourse pattern without depth and development and instead become what Ball (2001) calls "show and tell," meaning that as many as possible must be allowed to speak with no subsequent pointing out or extension or discussion. Mercer (1995) describes a similar pattern as "cumulative talk," in which speakers respond positively but uncritically to each other. Instead, uptake can be connected to a number of seemingly productive discourse patterns, such as making students a source of mathematical ideas (Hufferd-Ackles et al., 2004), building the discussion on students' thinking (Kooloos et al., 2020), and all five targeted discussions suggested by Kazemi and Hintz (2014). The distinction between accessing and sharing and uptake is the difference between just telling and using this telling as a springboard to develop the thinking together.

A teacher who uses students' ideas to create learning positions students as owners of ideas that they are willing to share, and as being willing to engage in the joint development of their own or others' thinking. Table 5 uses the positioning theory constructs of rights and duties (positions) and communication acts to summarize the findings about a teacher who uses students' ideas to create learning and a student who owns the ideas and is willing to share them.

2.4.2 Teacher Position 4: A teacher who orchestrates

The fourth position describes where the teacher refrains from discussing the actual mathematics and instead orchestrates student discourse about the mathematics, much like the conductor of an orchestra who does not play an instrument. Refraining from discussion of the content is what separates Position 4 from Position 3, where the teachers engage in the mathematics by pointing out, suggesting, challenging, and taking up ideas for further development. Based on the literature review, there are three rather distinct communicative acts that characterize three ways to orchestrate a discourse without directly participating in the mathematical content.

The first type of orchestration is basic management, such as choosing who should speak and moderating the discourse (Drageset, 2019; Drageset & Allern, 2017). But management also includes what Drageset (2019) calls "guiding participation and norms," such as allowing questions focusing on understanding and not assessment, or telling students that it is the process that should be focused on and not the answer.

The second type of orchestration is when the teacher orchestrates with a focus on developing ideas. This can be done by giving the students time to think (or "wait time," as Chapin et al., 2013 call it), or using turn-and-talk (Kazemi & Hintz, 2014), where two students are given time to develop an idea. A third way is to request alternative methods (Drageset, 2019), and combine this with either waiting time or turn-and-talk if necessary.

The third type of orchestration is to help students focus on peer thinking. Ing et al. (2015) have described teacher responses that encourage students to engage with each other, such as asking students to respond to each other's strategies or to use another student's strategy, focusing on contrasts by asking students to discuss differences among shared strategies, and focusing on connections by asking students to make connections among ideas. In addition, following a student sharing an idea or a solution, the teacher might invite fellow students to ask questions or to evaluate the suggestion (Conner et al., 2014). A focus on peer thinking can be limited to a single evaluation, but might also create discourses of several turns between students, perhaps consisting of questions, answers, clarifications, and challenges.

A teacher who orchestrates in a classroom positions students as people who contribute, making space for students to share ideas, ask questions, suggest new approaches, and assess ideas. As the teacher does not interfere with the content (does not assess, suggest, point out), arguably the students are the actual teachers of content by asking questions, explaining, and evaluating. Table 6 uses the positioning theory constructs of rights and duties (positions) and communication acts to summarize the findings about a teacher who orchestrates and a student who contributes.

2.5 Storyline 3: Teachers as participants in learning

The 'teachers as participants in learning' storyline is based on the idea that learning can be created by participants exploring a challenge together, where no participant has the power to just state right or wrong, but must argue and explain to convince others.

2.5.1 Teacher Position 5: A teacher who participates

The fifth teacher position describes teachers that participate in the solving of problems together with students. Based on the literature review, we suggest that there are two types of this position.

The first type is seen when the teacher collaborates with the students. Staples (2007) defines collaboration as a joint building of mathematical ideas, which is distinguished from cooperation that is limited to sharing. In more detail, Mueller et al. (2012) suggest three types of collaboration. The first, the co-construction of arguments, describes a joint construction of arguments from the ground upward. The second, the integration of arguments, describes a process of integrating peers' arguments into one's own, in which reformulation (Alrø & Skovsmose, 2002, 2004; Skovsmose, 2001) will be an important part. The third, the modification of arguments, describes a process where students use what Skovsmose (Alrø & Skovsmose, 2002, 2004; Skovsmose, 2001) calls "evaluations and challenges." These three types of collaboration describe ways students either construct understanding together or enhance one another's. An important part of such learning through collaboration would be what Wegerif and Mercer (1997) call "explorative talk," while the less productive pattern of disagreeing without enhancing one another's understanding (disputational talk; see Wegerif & Mercer, 1997) would be a sign that collaboration as the joint building of mathematical ideas is not taking place. However, all research on collaboration in this review is based on students working together, with the teacher either as supporter or facilitator.

Some concepts point to a possible position where the teacher collaborates with students in cases where none of them knows how to progress. One such model is the inquiry–cooperation model (Alrø & Skovsmose, 2002, 2004), which describes eight communicative features that do not distinguish between student and teacher, indicating a situation where the power lies in the arguments and not in the position. Another model is a description of landscapes of investigation, where Skovsmose (2001) describes situations where students explore mathematics together with the teacher. In such landscapes of investigation, it seems possible for teachers to take a position as a collaborator, particularly if the teacher does not know the method or solution. It should be possible for the teacher to work together with the students in all three types of collaboration suggested by Mueller et al. (2012) (co-construction, integration, and modification), using different communicative features as a participant and not as one who knows more and leads.

The second type of teacher who participates is seen when the teacher is acting as a teacher-in-role. It might be challenging for a teacher to collaborate on equal terms with the students, as the teacher often knows more, the students have certain expectations, and the teacher has certain habits related to claiming power. Drageset and Allern (2017) describe how teachers taking a role can contribute by sidestepping these expectations and habits and thus make space for students to participate in new ways. Furthermore, the teacher can invite the students to fill this space by asking questions in response to fellow students' ideas and requesting alternative methods. While teacher-in-role (O'Neill, 1995) is a widely used method in the field of drama, Drageset and Allern (2017) develop the method, using teacher-in-role to change the mathematical discourse by deliberately inviting students into new positions and thereby creating a different discourse in which students explain, argue, and decide. This method can be used to create a teacher position that fully and equally participates with the students in solving mathematical problems, as the teacher-in-role might know as much as or less than the students. In this way, using teacher-in-role might also be a strong tool to use to position students.

A teacher who participates positions students as participators on equal terms through collaboration or role-play. Table 7 uses the positioning theory constructs of rights and duties (positions) and communication acts to summarize the findings about a teacher who participates and a student who participates.

3 Discussion

Although the positions of teacher and student can be deconstructed, they are commonly recognized positions in society, and each is defined by the other, with the teacher usually positioned as powerful and the student as subordinate (Davies & Hunt, 1994). Most people have experience of schooling that perpetuates this view of teachers' and students' rights and duties. For many, rights and duties in learning mathematics are particularly clear, perpetuating particular storylines about how mathematics teaching should be done (Xu & Clarke, 2019). Attempts to change these perceptions often center on changing classroom discourse patterns (Kazemi & Hintz, 2014; Mercer & Littleton, 2007), but this is not easily done (Heyd-Metzuyanim et al., 2016, Xu & Clarke, 2019). Therefore, this review posed the question: What storylines and associated positions can be identified in mathematics education literature on talk moves, teacher interactions, and discourse patterns, and what are the implications of these for equity and access to important mathematical ideas? The findings of five positions nested into three broad storylines has implications for equity and access to mathematical ideas, offers some explanation for why it is hard to change classroom interaction patterns, and even has implications for school mathematics itself.

3.1 Equity and access to mathematical ideas

Each of the five positions yielded from the analysis has associated rights and duties that distinguish it from the other positions, and the use of positioning theory revealed the potential consequences of these rights and duties for students, whose responses did not always feature in the reviewed articles. In each of these scenarios, the opportunity for students to learn is shaped by the position taken up by the teacher, and whether or not the students decide to take up the position they are offered by the teacher's attempts to position them. For example, if students are positioned as a contributor or collaborator who has the right to have their ideas considered, and the duty to discuss and debate ideas, they have the opportunity to learn (Hiebert & Grouws, 2007) the tools and warrants of mathematical argumentation. But the opportunity to learn is only one aspect—some students may consider that the duty of discussion and debate is too taxing and sit quietly waiting for the lesson to end. If all the students do this, it will shape the teacher's positioning, placing the teacher back in the storyline of providing insight by telling (Position 1) or supporting (Position 2).

Thus, what is made possible to learn is shaped by communication acts that ascribe positions. With multiple people in the classroom, different students can be in different storylines. To the extent that some storylines yield better access to important ideas, some students will be disadvantaged by being in a storyline that is less helpful. Storylines and positions in the classroom are intersected by broader social discourses, such as 'mathematics is not for people like me.' In this way groups of learners can be marginalised by perpetuation of harmful storylines. Speakers of languages other than the language of instruction can additionally be marginalised in classroom interaction by not recognising the position they are offered or the storyline they are in (Hunter, 2010).

However, there are equity issues that go beyond whether everyone can access the ideas and discourse in the classroom. Gutiérrez (2009) describes two axes for thinking about inequity in mathematics education: one between achievement and access, and one between identity and power. Her argument is that most discussion of inequity focuses on access and achievement, while identity and power remain under-considered. Research on classroom interaction is often focused on increasing access to ideas and raising achievement (Hiebert & Grouws 2007). Using positioning theory to group types of interaction from the literature reveals how communication acts build, and are built by, positions and storylines that bring with them ideas of identity and power. The relationship between identity and positioning is complex (Anderson, 2009; Langer-Osuna & Nasir, 2016) and beyond the scope of this discussion, but the way that students experience mathematics teaching and learning impacts the development of their mathematics learner identity (Langer-Osuna & Nasir, 2016), and the multiple identities that students bring to mathematics impact their willingness to take up positions in mathematics classes (Darragh, 2016). In the analyses presented here, the amount of power held by students increases from teacher Position 1 to teacher Position

5 in two ways. First, from being positioned as a listener in Position 1, to being an equal participant in Position 5, students have increasing space in interaction for their ideas to be taken up and genuinely considered. Second, if students resist taking up the position offered by the teacher's communication acts, then they have the power to subvert the storyline and bring positions back to a storyline in which they are comfortable. Some of this resistance may originate in student identity (Darragh, 2016). Power resides both in the opening up of space for the student voice (a handover of power from teacher to students), and in the ability of students, as participants in storylines, to control whether or not this can happen. It is important to recognize that because of intersecting marginalizing storylines, power is not evenly distributed amongst students, nor is the willingness and capability to assert power in a mathematics classroom (Langer-Osuna & Nasir, 2016). Of the five positions outlined above, the fourth and fifth provide the most potential for students to have power in relation to ideas in the mathematics classroom, to focus on the mathematics that concerns them, and to bring their language, culture, and identity to bear on the positions they accept. These may also be the hardest positions to establish and maintain because they are the furthest away from the traditional mathematics teaching and learning storyline, Position 1.

The paragraphs above consider students as one group, impacted in the same way by teachers' communication acts. In practice, students are a diverse group of people for whom teachers' communication acts will have differential impacts. Identity, language, and culture, for example, will shape how students interpret communication acts, what storylines they are prepared to participate in, and what positions they will take up. A dominant culture and confident mathematics students with positive mathematics identities will be impacted by changes in classroom interaction and teacher–student positions in different ways from students of nondominant groups, or those who do not speak the language of instruction. Teachers' attempts to share power with students by opening up space for student talk and thinking may inadvertently exacerbate inequity in mathematics classrooms. This issue has been taken up by authors, such as Hunter (2010), who use a change in storylines and position through interaction patterns to deliberately enhance the learning experience of marginalized learners.

3.2 Making change in classroom interaction patterns

Position 1, a teacher who tells and a student who listens, is a pervasive storyline in mathematics teaching and learning (Brendefur & Frykholm, 2000; Wood et al., 2006). Teachers and students seem to reproduce, in the moment, storylines about teaching and learning that are recurrent and persistent (Wagner & Herbel-Eisenmann, 2009). Teachers, students, and communities may not recognize other storylines and positions as legitimate. Changing this storyline is not easy (Ing et al., 2015). Even the most open question can be turned back by a student who is working in a 'teacher tells' storyline and is not willing to take up the position offered by a 'teacher facilitates' storyline (Bennett, 2010). For many years, researchers have outlined how changing classroom interaction patterns might improve the learning of mathematics (Alrø & Skovsmose, 2004; Chapin et al., 2013; Kazemi & Hintz, 2014; Mercer & Littleton, 2007; Stein et al., 2008). Positioning theory helps us to see this challenge as shaped by longer-term, transcendent storylines that intersect classrooms because teachers and students position one another through their communication acts. Positioning theory implies that without considering the positioning functions of discourse as well as the learning functions, meaningful shifts in discourse will be hard to establish (Anderson, 2009).

Positioning theory suggests that participants in communication have to be willing and capable of taking up the positions they are offered and have the power to do so (Davies & Harré, 1999; Huang & Wang, 2021). Students can be unwilling to take up positions offered by their teacher's communication, or not have the capability to do so; or teachers may offer a position through their communication, but not structure the situation so that students have the power to pick up on what is offered. Explicitly discussing roles and responsibilities has been shown to support change in discourse patterns (Hunter, 2010). Observation of classroom discourse (e.g., Conner et al., 2014) and interventions intended to change classroom communication (Hunter, 2010; Kazemi & Hintz, 2014) describe emergent communication patterns that our analysis suggests draw from transcendent storylines about mathematics teaching and learning (Wagner & Herbel-Eisenmann, 2009).

Considering findings about classroom interaction as describing communication acts, which function as positioning tools, gives us a productive way to understand the difficulties inherent in trying to change classroom discourse patterns. Furthermore, positioning theory with rights and duties can explain why some classrooms are stuck in one storyline and the teacher in one position while other classrooms seem to change fast between positions and storylines (Anderson, 2009). This insight also gives us the power to develop classroom teaching and learning by attending to rights and duties, by developing capabilities to take on new positions, and by facilitating students' changes of position.

3.3 The position of mathematics

As well as suggesting positions for teachers and students, each storyline from the positioning analysis suggests a position for mathematics. Mathematics is part of the teaching–learning dynamic because it brings with it disciplinary structures and norms of communication, warrants for knowledge, and ideas about what is valuable to know. In school mathematics, these ideas have been contested in research and policy (Skovsmose, 2001). Learning mathematics brings with it particular storylines about mathematics itself, for example: that it is hard to learn; that only some people can do it; that it is abstract; that it has a logical order and hierarchy of ideas; that it requires memorization and practice. In addition, the mathematics that is valued also creates a storyline. Depending on jurisdiction, school mathematics might be constructed on Western principles, ignore Indigenous knowledge, or perpetuate dominant cultural narratives (Meaney & Trinick, 2020). This becomes another way that power is held and exercised through mathematics teaching and learning.

In Tables 3, 4, 5, 6, 7, we suggest the storylines about mathematics implied by each of the teacher positions found in the review process. In each of the three storylines, we suggest mathematics has a different character, and learning mathematics a different emphasis. Using positioning theory to think about this literature on communication acts, shows that the positions taken up by teachers and students might have consequences for the mathematics available to learners and consequently their opportunities to learn (Hiebert & Grouws, 2007). Positioning theory suggests that by assigning rights and duties to participants through mathematics classroom discourse, communication acts also shape storylines about the mathematics to be studied. To the extent that some of these storylines are inclusive of all students, they are helpful for promoting equity; to the extent that they exclude groups of learners, they are risky. Dichotomized thinking about 'good' and 'bad' teaching of mathematics is prevalent in many jurisdictions (Lobato et al., 2005), but misses the moment-by-moment subtleties of positioning and the negotiation of storylines that emerge in particular contexts. Understanding that there are consequences for mathematics itself in the way teachers and learners position themselves highlights the importance of communication acts in the classroom (Cobb, 2000).

4 Conclusion

To answer the first part of our research question, we explored what storylines and associated positions can be identified in mathematics education literature on talk moves, teacher interactions, and discourse patterns. A conceptual review identified 44 articles and 94 concepts describing interactions and used these to develop five positions and three storylines (Table 2 provided an overview, Tables 3, 4, 5, 6, 7 more detail).

To answer the second part of our research question, we explored what the implications are of these storylines and positions for equity and access to important mathematical ideas. Positioning theory helps us understand moment-by-moment interactions in complex mathematics classrooms, and how these arise from, build, and maintain broader stories about what it is to teach and learn mathematics, who mathematics is for, and what mathematics is. For equity in mathematics teaching and learning, we need to recognize that classroom interaction is simultaneously of-the-moment and of-the-wider-context. Recognizing that teacher–student communication acts draw on transcendent storylines and intersect with broader narratives as participants both offer, and accept or reject, positions through communication, makes it clear why shifting classroom interaction patterns is a difficult task. Taking a view informed by positioning theory, if we are to realize the potential of different teacher and student communication acts outlined in classroom interaction studies such as those in this review, we need to explicitly deal with the forces that hold current practices in place. These forces can be characterized as storylines, and once they are seen in this way, we can think about how to change the storylines through communication acts and associated positions, being explicit with teachers and students about what is coming into play as they work together in classrooms, and working together to develop storylines where everyone can participate and succeed. The ability of this review to fully consider storylines is limited by the nature of the studies we included. As we are working from studies that were conducted in a range of theoretical frames, we can only build from the observed discourse patterns to potential storylines as implications of the patterns we saw in our analysis. A future work, conducted within a framework of positioning, could explicitly consider and chart storyline emergence and construction through discourse to further develop these ideas and to examine the impact and role of storylines on identity, power, participation, and success.

While a limitation to the study is its somewhat narrow scope (starting with two journals, three keywords, and focusing the analysis on teacher interactions), the study has illustrated the potential of using positioning theory to review the literature on classroom interactions in mathematics education and developed possible storylines, positions, and communication acts. A wider scope might identify additional or alternative storylines and positions that can be useful tools to use to deepen our understanding of how these affect equity and access, and also enable or hinder change.

Appendix 1 Overview of all storylines, positions, communication acts, interaction types, and authors

Storylines	Positions	Communication acts	Interaction concepts found in the literature review	Author(s)
Developed from the positions	Developed from the communcation acts	Developed from the interaction concepts found in the literature review	Describing talk moves, interactions, discourse patterns (see research question). These 94 concepts are data gathered through the review and the unit of analysis	For each interaction concept
(Storyline 1) Teachers are providers of insight	(Position 1) A teacher who tells	Telling to initiate student work	Informing and suggesting	da Ponte and Quaresma (2016)
			Setting the scene	Haavold and Blomhøj (2019)
			Teacher as initiator	Lobato et al. (2005)
		Telling how to do	Advising a new strategy	Drageset (2014b)
			Conventional textbook culture	Wood et al. (2006)
			Demonstration	Drageset (2014b)
			Modeling the way an expert thinks	Fukawa-Connelly (2012)
			Teacher explanation	Henning et al. (2012)
			Unidirectional communication	Brendefur and Frykholm (2000)
		Telling about connections	Connection	Rowland et al. (2005)
			Relationship to other actions	Lobato et al. (2005)
	(Position 2) A teacher who supports	Reduction of complexity	Closed progress details	Drageset (2014b)
			Conventional problem-solving culture	Wood et al. (2006)
			Cues	Henning et al. (2012)
			Direct contributions to arguments	Conner et al. (2014)
			Directed guidance	Warshauer (2015)

Storylines	Positions	Communication acts	Interaction concepts found in the literature review	Author(s)
Developed from the positions	Developed from the communcation acts	Developed from the interaction concepts found in the literature review	Describing talk moves, interactions, discourse patterns (see research question). These 94 concepts are data gathered through the review and the unit of analysis	For each interaction concept
			Funneling	Steinbring (1989) Wood (1998)
			Guided algorithmic reasoning	Lithner (2008)
			Guiding	Drageset (2014a)
			Imitative reasoning	Lithner (2008)
			Hints and highlights	Conner et al. (2014)
			Lowering of cognitive demand	Boston and Smith (2009)
			Proceduralization	Stein et al. (1996)
			Ritual student participation	Lavie et al. (2019)
			Routinizing	Boston and Smith (2009)
			Simplification	Drageset (2014a)
			Supporting and guiding	da Ponte and Quaresma (2016)
			Topaze effect	Brousseau and Balacheff (1997)
		Assessing	Confirmation	Henning et al. (2012)
			Correcting question	Drageset (2014b)
			Directing	Conner et al. (2014)
			Evaluating	Conner et al. (2014)
			Rejection	Henning et al. (2012)
		Progressing students' thinking	Open progress initiatives	Drageset (2014b)
			Probing guidance	Warshauer (2015)
			Promoting	Conner et al. (2014)
			Supporting children's conceptual understanding	Fraivillig et al. (1999)

Storylines	Positions	Communication acts	Interaction concepts found in the literature review	Author(s)
Developed from the positions	Developed from the communcation acts	Developed from the interaction concepts found in the literature review	Describing talk moves, interactions, discourse patterns (see research question). These 94 concepts are data gathered through the review and the unit of analysis	For each interaction concept
(Storyline 2) Teachers are facilitators of learning	(Position 3) A teacher who uses students' ideas to create learning	Access and share	Asking questions that elicit parts of arguments	Conner et al. (2014)
			Eliciting students' thinking	Fraivillig et al. (1999) Henning et al. (2012)
			Inviting	Da Ponte and Quaresma (2016)
			Monitoring	Stein et al. (2008)
			Probing for facts	Bennett (2010)
			Probing for understanding	Bennett (2010)
			Probing students to explain their thinking	Boaler and Brodie (2004)
			Providing students' opportunities to share their solutions	Teuscher et al. (2016)
			Questions that anticipate predictable responses	Gaspard and Gainsburg (2020)
			Questions that anticipate unpredictable responses	Gaspard and Gainsburg (2020)
			Requesting justification	Drageset (2014b)
			Selecting	Stein et al. (2008)
			Sequencing	Stein et al. (2008)
			Teacher as elicitor	Lobato et al. (2005)
		Pointing out	Clarifying	Alrø and Skovsmose (2002) Conner et al. (2014) Correnti et al. (2015)
			Focusing	Wood (1998)
			Orienting and focusing	Boaler and Brodie (2004)

Storylines	Positions	Communication acts	Interaction concepts found in the literature review	Author(s)
Developed from the positions	Developed from the communcation acts	Developed from the interaction concepts found in the literature review	Describing talk moves, interactions, discourse patterns (see research question). These 94 concepts are data gathered through the review and the unit of analysis	For each interaction concept
			Pointing to relationships among mathematical ideas	Boaler and Brodie (2004) Drageset (2021)
			Recapping	Henning et al. (2012)
			Refocusing	Conner et al. (2014)
			Reformulating	Alrø and Skovsmose (2002) Henning et al. (2012) Kooloos et al. (2020)
			Repeating	Conner et al. (2014) Henning et al. (2012)
			Revoicing	O'Connor and Michaels (1993) Chapin et al. (2013)
		Uptake	Allowing time for students to work	Warshauer (2015)
			Building the discussion on students' thinking	Kooloos et al. (2020)
			Deepening	Correnti et al. (2015)
			Elaborating	Henning et al. (2012) Correnti et al. (2015)
			Encouraging reasoning	Cengiz et al. (2011)
			Encouraging reflection	Cengiz et al. (2011)
			Exploring mathematical meanings or relationships	Boaler and Brodie (2004)
			Extending students' thinking	Boaler and Brodie (2004) Correnti et al. (2015) Fraivillig et al. (1999)

Storylines	Positions	Communication acts	Interaction concepts found in the literature review	Author(s)
Developed from the positions	Developed from the communcation acts	Developed from the interaction concepts found in the literature review	Describing talk moves, interactions, discourse patterns (see research question). These 94 concepts are data gathered through the review and the unit of analysis	For each interaction concept
			Going beyond the initial method by pushing for alternative methods	Cengiz et al. (2011)
			Making students a source of mathematical ideas	Hufferd-Ackles et al. (2004)
			Uptake	Correnti et al. (2015) Staples (2007)
	(Position 4) A teacher who orchestrates	Management	Choosing who should speak	Drageset and Allern (2017)
			Guiding participation and norms	Drageset (2019)
			Moderating	Drageset (2019)
		Developing ideas	Requesting alternative methods	Drageset (2019)
			Turn and talk	Kazemi and Hintz (2014)
			Wait time	Chapin et al. (2013)
		Focusing on peer thinking	Asking students to explain each other's strategies	Ing et al. (2015)
			Discussing differences among shared strategies	Ing et al. (2015)
			Making connections among ideas	Ing et al. (2015)
			Requesting evaluation	Conner et al. (2014)
			Responding to or using another student's strategy	Ing et al. (2015)

Storylines	Positions	Communication acts	Interaction concepts found in the literature review	Author(s)
Developed from the positions	Developed from the communcation acts	Developed from the interaction concepts found in the literature review	Describing talk moves, interactions, discourse patterns (see research question). These 94 concepts are data gathered through the review and the unit of analysis	For each interaction concept
(Storyline 3) Teachers are participants in learning	(Position 5) A teacher who participates	Collaborates	Challenging	Alrø and Skovsmose (2002) Alrø and Skovsmose (2004)
			Co-constructing arguments	Mueller et al. (2012)
			Explorative talk	Wegerif and Mercer (1997)
			Getting in contact	Alrø and Skovsmose (2002)
			Identifying	Alrø and Skovsmose (2002)
			Integrating arguments	Mueller et al. (2012)
			Locating	Alrø and Skovsmose (2002)
			Modification of arguments	Mueller et al. (2012)
			Thinking aloud	Alrø and Skovsmose (2002)
		Teacher-in-role	Facilitating students' participation in discourse	Drageset and Allern (2017)
			Open discourse space for students	Drageset and Allern (2017)
			Teacher-in-role	O'Neill (1995) Drageset and Allern (2017)

Appendix 2 Overview of the articles identified

Search in *Educational Studies of Mathematics* and *Journal of Mathematics Teacher Education* resulted in the following 15 articles after selection

- Brendefur, J., & Frykholm, J. (2000). Promoting mathematical communication in the classroom: Two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher Education*, *3*(2), 125–153. https://doi.org/10.1023/a:1009947032694
- Cengiz, N., Kline, K., & Grant, T. (2011). Extending students' mathematical thinking during whole-group discussions. *Journal of Mathematics Teacher Education*, *15*(5), 1–20. https://doi.org/10.1007/s10857-011-9179-7
- Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics*, *86*(3), 401–429. https://doi.org/10.1007/s10649-014-9532-8
- da Ponte, J. P., & Quaresma, M. (2016). Teachers' professional practice conducting mathematical discussions. *Educational Studies in Mathematics*, *93*(1), 51–66
- Drageset, O. G. (2014b). Redirecting, progressing, and focusing actions: A framework for describing how teachers use students' comments to work with mathematics. *Educational Studies in Mathematics*, *85*(2). https://doi.org/10.1007/s10649-013-9515-1
- Fukawa-Connelly, T. P. (2012). A case study of one instructor's lecture-based teaching of proof in abstract algebra: Making sense of her pedagogical moves. *Educational Studies in Mathematics*, *81*(3), 325–345. https://doi.org/10.1007/s10649-012-9407-9
- Gaspard, C., & Gainsburg, J. (2020). Abandoning questions with unpredictable answers. *Journal of Mathematics Teacher Education*, *23*(6), 555–577. https://doi.org/10.1007/s10857-019-09440-5
- Henning, J. E., McKeny, T., Foley, G. D., & Balong, M. (2012). Mathematics discussions by design: Creating opportunities for purposeful participation. *Journal of Mathematics Teacher Education*, *15*(6), 453–479. https://doi.org/10.1007/s10857-012-9224-1
- Ing, M., Webb, N. M., Franke, M. L., Turrou, A. C., Wong, J., Shin, N., & Fernandez, C. H. (2015). Student participation in elementary mathematics classrooms: The missing link between teacher practices and student achievement? *Educational Studies in Mathematics*, *90*(3), 341–356. https://doi.org/10.1007/s10649-015-9625-z
- Lavie, I., Steiner, A., & Sfard, A. (2019). Routines we live by: From ritual to exploration. *Educational Studies in Mathematics*, *101*(2), 153–176. https://doi.org/10.1007/s10649-018-9817-4
- Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, *67*(3), 255–276. https://doi.org/10.1007/s10649-007-9104-2
- Mueller, M., Yankelewitz, D., & Maher, C. (2012). A framework for analyzing the collaborative construction of arguments and its interplay with agency. *Educational Studies in Mathematics*, *80*(3), 369–387. https://doi.org/10.1007/s10649-011-9354-x
- Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*(3), 255–281. https://doi.org/10.1007/s10857-005-0853-5
- Teuscher, D., Moore, K. C., & Carlson, M. P. (2016). Decentering: A construct to analyze and explain teacher actions as they relate to student thinking. *Journal of Mathematics Teacher Education*, *19*(5), 433–456. https://doi.org/10.1007/s10857-015-9304-0
- Warshauer, H. K. (2015). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education*, *18*(4), 375–400. https://doi.org/10.1007/s10857-014-9286-3

The following 29 articles were subsequently added in two ways

1) An article from the search in ESM and JMTE referenced a concept that led to the addition of this article *and* this (or other) concepts in the article met the criteria for inclusion (see the methods section)

2) A search in Google Scholar showed that these articles had referenced one or more of the articles from the search in ESM and JMTE *and* this article offered new concepts that met the criteria for inclusion (see the methods section)

- Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. Kluwer Academic Publishers
- Alrø, H., & Skovsmose, O. (2004). Dialogic learning in collaborative investigation. *Nordic Studies in Mathematics Education*, *2*, 39–62
- Bennett, C. A. (2010). "It's hard getting kids to talk about math": Helping new teachers improve mathematical discourse. *Action in Teacher Education*, *32*(3), 79–89. https://doi.org/10.1080/01626620.2010.10463561
- Boaler, J., & Brodie, K. (2004). The importance, nature, and impact of teacher questions. In D. E. McDougall & J. A. Ross (Eds.), *Proceedings of the twenty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 774–782)
- Boston, M. D., & Smith, M. S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education*, *40*(2), 119–156. http://www.jstor.org/stable/40539329
- Brousseau, G., & Balacheff, N. (1997). *Theory of didactical situations in mathematics*. Kluwer
- Chapin, S. H., O'Connor, C., & Anderson, N. C. (2013). *Classroom discussions in math: A teacher's guide for using talk moves to support the common core and more, Grades K-6*. Math Solutions
- Correnti, R., Stein, M., Smith, M., Scherrer, J., McKeown, M., Greeno, J., & Ashley, K. (2015). *Improving teaching at scale: Design for the scientific measurement and learning of discourse practice* (pp. 315–332). https://doi.org/10.3102/978-0-935302-43-1_25
- Drageset, O. G. (2014a). Knowledge used when orchestrating mathematical discourses: Doing, guiding and requesting. *Nordic Studies in Mathematics Education*, *19*(3–4), 151–168
- Drageset, O. G., & Allern, T. H. (2017, February). Using drama to change classroom discourse. In T. Dooley & G. Gueudet, G. (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (pp. 3049–3056). European Society for Research in Mathematics Education
- Drageset, O. G. (2019). How teachers use interactions to craft different types of student participation during whole-class mathematical work. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the eleventh congress of the European Society for Research in Mathematics Education* (pp. 3622–3629). European Society for Research in Mathematics Education
- Drageset, O. G. (2021). Exploring student explanations. What types can be observed, and how do teachers initiate and respond to them? *Nordic Studies in Mathematics Education*, *26*(1), 53–72
- Fraivillig, J. L., Murphy, L. A., & Fuson, K. C. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education*, *30*(2), 148. http://search.ebscohost.com/login.aspx?direct=true&db=afh&AN=1655007&site=ehost-live
- Haavold, P. Ø., & Blomhøj, M. (2019). Coherence through inquiry-based mathematics education. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the eleventh congress of the European Society for Research in Mathematics Education* (pp. 4389–4396). European Society for Research in Mathematics Education
- Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, *35*(2), 81–116. https://doi.org/10.2307/30034933
- Kazemi, E., & Hintz, A. (2014). *Intentional talk: How to structure and lead productive mathematical discussions*. Stenhouse Publishers
- Kooloos, C., Oolbekkink-Marchand, H., Kaenders, R., & Heckman, G. (2020). Orchestrating mathematical classroom discourse about various solution methods: Case study of a teacher's development. *Journal Für Mathematik-Didaktik*, *41*(2), 357–389. https://doi.org/10.1007/s13138-019-00150-2
- Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education*, *36*(2), 101–136. https://doi.org/10.2307/30034827
- O'Connor, M. C., & Michaels, S. (1993). Aligning academic tasks and participation through revoicing: Analysis of a classroom discourse strategy. *Anthropology and Education Quarterly*, *24*(4), 318–335
- O'Neill, C. (1995). *Drama worlds: A framework for process drama*. Heinemann Drama
- Skovsmose, O. (2001). Landscapes of investigation. *ZDM—International Journal on Mathematics Education*, *33*, 123–132. https://doi.org/10.1007/BF02652747

- Staples, M. (2007). Supporting whole-class collaborative inquiry in a secondary mathematics classroom. *Cognition and Instruction*, *25*(2–3), 161–217. https://doi.org/10.1080/07370000701301125
- Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313–340
- Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, *33*(2), 455–488. https://doi.org/10.3102/00028312033002455
- Steinbring, H. (1989). Routine and meaning in the mathematics classroom. *For the Learning of Mathematics*, *9*(1), 24–33. http://www.jstor.org/stable/40247942
- Stockero, S. L., van Zoest, L. R., Freeburn, B., Peterson, B. E., & Leatham, K. R. (2020). Teachers' responses to instances of student mathematical thinking with varied potential to support student learning. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-020-00334-x
- Wegerif, R., & Mercer, N. (1997). A dialogical framework for researching peer talk. In R. Wegerif & P. Schrimshaw (Eds.), *Computers and talk in the primary classroom* (pp. 49–64). Multilingual Matters
- Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167–178). National Council of Teachers of Mathematics
- Wood, T., Williams, G., & McNeal, B. (2006). Children's mathematical thinking in different classroom cultures. *Journal for Research in Mathematics Education*, *37*(3), 222–255

Funding Open access funding provided by UiT The Arctic University of Norway (incl University Hospital of North Norway)

Data availability The data used in this article are other articles, and all articles used is included in the reference list.

Declarations

Conflict of interest The authors declare no competing interests.

References

Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. Kluwer Academic Publishers.

Alrø, H., & Skovsmose, O. (2004). Dialogic learning in collaborative investigation. *Nordic Studies in Mathematics Education, 2*, 39–62.

Anderson, K. (2009). Applying positioning theory to the analysis of classroom interactions: Mediating micro-identities, macro-kinds and ideologies of knowing. *Linguistics and Education, 20*, 291–310.

Ball, D. L. (2001). Teaching, with respect to mathematics and students. In T. L. Wood, B. S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 11–22). L. Erlbaum Associates.

Barwell, R. (2013). Discursive psychology as an alternative perspective on mathematics teacher knowledge. *ZDM-Mathematics Education*, *45*, 595–606.

Bennett, C. A. (2010). "It's hard getting kids to talk about math": Helping new teachers improve mathematical discourse. *Action in Teacher Education, 32*(3), 79–89. https://doi.org/10.1080/01626620.2010.10463561

Boaler, J., & Brodie, K. (2004). The importance, nature, and impact of teacher questions. In D. E. McDougall & J. A. Ross (Eds.), *Proceedings of the twenty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 774–782).

Boston, M. D., & Smith, M. S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education*, *40*(2), 119–156. http://www.jstor.org/stable/40539329. Accessed 23 Oct 2020.

Brendefur, J., & Frykholm, J. (2000). Promoting mathematical communication in the classroom: Two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher Education, 3*(2), 125–153. https://doi.org/10.1023/a:1009947032694

Brousseau, G., & Balacheff, N. (1997). *Theory of didactical situations in mathematics*. Kluwer.

Cazden, C. B. (1988). *Classroom discourse: The language of teaching and learning*. Heinemann.

Cengiz, N., Kline, K., & Grant, T. (2011). Extending students' mathematical thinking during whole-group discussions. *Journal of Mathematics Teacher Education, 15*(5), 1–20. https://doi.org/10.1007/s10857-011-9179-7

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2013). *Classroom discussions in math: A teacher's guide for using talk moves to support the common core and more, Grades K-6*. Math Solutions.

Cobb, P. (2000). The importance of a situated view of a learning to the design of research and instruction. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 527–548). Ablex Publishing Corporation.

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics, 86*(3), 401–429. https://doi.org/10.1007/s10649-014-9532-8

Correnti, R., Stein, M., Smith, M., Scherrer, J., McKeown, M., Greeno, J., & Ashley, K. (2015). *Improving teaching at scale: Design for the scientific measurement and learning of discourse practice* (pp. 315–332). https://doi.org/10.3102/978-0-935302-43-1_25

da Ponte, J. P., & Quaresma, M. (2016). Teachers' professional practice conducting mathematical discussions. *Educational Studies in Mathematics, 93*(1), 51–66.

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics, 93*, 19–33.

Davies, B., & Harré, R. (1990). Positioning: The discursive production of selves. *Journal for the Theory of Social Behaviour, 20*(1), 43–63.

Davies, B., & Harré, R. (1999). Positioning and personhood. In R. Harré & L. van Langenhove (Eds.), *Positioning theory: Moral contexts of intentional action* (pp. 32–52). Blackwell.

Davies, B., & Hunt, R. (1994). Classroom competencies and marginal positionings. *British Journal of Sociology of Education, 15*(3), 389–408.

Drageset, O. G. (2014a). Knowledge used when orchestrating mathematical discourses: Doing, guiding and requesting. *Nordic Studies in Mathematics Education, 19*(3–4), 151–168.

Drageset, O. G. (2014b). Redirecting, progressing, and focusing actions: A framework for describing how teachers use students' comments to work with mathematics. *Educational Studies in Mathematics*, *85*(2). https://doi.org/10.1007/s10649-013-9515-1

Drageset, O. G., & Allern, T. H. (2017). Using drama to change classroom discourse. In T. Dooley & G. Gueudet, G. (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (pp. 3049–3056). European Society for Research in Mathematics Education.

Drageset, O. G. (2019). How teachers use interactions to craft different types of student participation during whole-class mathematical work. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the eleventh congress of the European Society for Research in Mathematics Education* (pp. 3622–3629). European Society for Research in Mathematics Education.

Drageset, O. G. (2021). Exploring student explanations. What types can be observed, and how do teachers initiate and respond to them? *Nordic Studies in Mathematics Education*, *26*(1), 53–72.

Erath, K., Ingram, J., Moschkovich, J., & Prediger, S. (2021). Designing and enacting instruction that enhances language for mathematics learning: A review of the state of development and research. *ZDM-Mathematics Education, 53*, 245–262.

Fraivillig, J. L., Murphy, L. A., & Fuson, K. C. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education*, *30*(2), 148. http://

search.ebscohost.com/login.aspx?direct=true&db=afh&AN=1655007&site=ehost-live. Accessed 15 Nov 2020.

Fukawa-Connelly, T. P. (2012). A case study of one instructor's lecture-based teaching of proof in abstract algebra: Making sense of her pedagogical moves. *Educational Studies in Mathematics, 81*(3), 325–345. https://doi.org/10.1007/s10649-012-9407-9

Gaspard, C., & Gainsburg, J. (2020). Abandoning questions with unpredictable answers. *Journal of Mathematics Teacher Education, 23*(6), 555–577. https://doi.org/10.1007/s10857-019-09440-5

Gutiérrez, R. (2009). Framing equity: Helping students 'play the game' and 'change the game.' *Teaching for Excellence and Equity in Mathematics, 1*(1), 5–7.

Haavold, P. Ø., & Blomhøj, M. (2019). Coherence through inquiry-based mathematics education. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the eleventh congress of the European Society for Research in Mathematics Education* (pp. 4389–4396). European Society for Research in Mathematics Education.

Harré, R., & Moghaddam, F. (2003). *The self and others: Positioning individuals and groups in personal, political, and cultural contexts*. Greenwood Publishing Group.

Henning, J. E., McKeny, T., Foley, G. D., & Balong, M. (2012). Mathematics discussions by design: Creating opportunities for purposeful participation. *Journal of Mathematics Teacher Education, 15*(6), 453–479. https://doi.org/10.1007/s10857-012-9224-1

Herbel-Eisenmann, B. A., Wagner, D., Johnson, K. R., Suh, H., & Figueras, H. (2015). Positioning in mathematics education: Revelations on an imported theory. *Educational Studies in Mathematics, 89*(2), 185–204.

Heyd-Metzuyanim, E., Tabach, M., & Nachlieli, T. (2016). Opportunities for learning given to prospective mathematics teachers: Between ritual and explorative instruction. *Journal of Mathematics Teacher Education, 19*, 547–574.

Hiebert, J., & Grouws, D. A. (2007). The effect of classroom mathematics teaching on students' learning. *Second handbook of research on mathematics teaching and learning*, *1*(1), 371–404.

Huang, X., & Wang, C. (2021). Pre-service teachers' identity transformation: A positioning theory perspective. *Professional Development in Education*, 1–18. https://doi.org/10.1080/19415257.2021.1942143

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81–116. https://doi.org/10.2307/30034933

Hunter, R. (2010). Changing roles and identities in the construction of a community of mathematical inquiry. *Journal of Mathematics Teacher Education, 13*(5), 397–409. https://doi.org/10.1007/s10857-010-9152-x

Ing, M., Webb, N. M., Franke, M. L., Turrou, A. C., Wong, J., Shin, N., & Fernandez, C. H. (2015). Student participation in elementary mathematics classrooms: The missing link between teacher practices and student achievement? *Educational Studies in Mathematics, 90*(3), 341–356. https://doi.org/10.1007/s10649-015-9625-z

Kazemi, E., & Hintz, A. (2014). *Intentional talk: How to structure and lead productive mathematical discussions*. Stenhouse Publishers.

Kooloos, C., Oolbekkink-Marchand, H., Kaenders, R., & Heckman, G. (2020). Orchestrating mathematical classroom discourse about various solution methods: Case study of a teacher's development. *Journal Für Mathematik-Didaktik, 41*(2), 357–389. https://doi.org/10.1007/s13138-019-00150-2

Krummheuer, G. (2011). Representation of the notion 'learning as participation' in everyday situations of mathematics classes. *ZDM-Mathematics Education*, *43*, 81–90.

Langer-Osuna, J. M., & Nasir, N. I. S. (2016). Rehumanizing the "other": Race, culture, and identity in education research. *Review of Research in Education, 40*(1), 723–743.

Lavie, I., Steiner, A., & Sfard, A. (2019). Routines we live by: From ritual to exploration. *Educational Studies in Mathematics, 101*(2), 153–176. https://doi.org/10.1007/s10649-018-9817-4

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*(3), 255–276. https://doi.org/10.1007/s10649-007-9104-2

Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education, 36*(2), 101–136. https://doi.org/10.2307/30034827

Meaney, T., & Trinick, T. (2020). Indigenous students in mathematics education. In S.Lerman (Ed.) *Encyclopaedia of Mathematics Education*. Springer.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom*. Harvard University Press.

Mercer, N. (1995). *The guided construction of knowledge: Talk amongst teachers and learners.* Multilingual Matters.

Mercer, N., & Littleton, K. (2007). *Dialogue and the development of children's thinking: A sociocultural approach*. Routledge.

Mueller, M., Yankelewitz, D., & Maher, C. (2012). A framework for analyzing the collaborative construction of arguments and its interplay with agency. *Educational Studies in Mathematics, 80*(3), 369–387. https://doi.org/10.1007/s10649-011-9354-x
O'Connor, M. C., & Michaels, S. (1993). Aligning academic tasks and participation through revoicing: Analysis of a classroom discourse strategy. *Anthropology and Education Quarterly, 24*(4), 318–335.
O'Neill, C. (1995). *Drama worlds: A framework for process drama*. Heinemann Drama.
Podsakoff, P., MacKenzie, S., & Podsakoff, N. (2016). Recommendations for creating better concept definitions in the organisational, behavioural and social sciences. *Organisational Research Methods, 19*(2), 159–203.
Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255–281. https://doi.org/10.1007/s10857-005-0853-5
Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses and mathematizing*. Cambridge University Press.
Skovsmose, O. (2001). Landscapes of investigation. *ZDM-International Journal on Mathematics Education, 33*, 123–132. https://doi.org/10.1007/BF02652747
Staples, M. (2007). Supporting whole-class collaborative inquiry in a secondary mathematics classroom. *Cognition and Instruction, 25*(2–3), 161–217. https://doi.org/10.1080/07370000701301125
Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313–340.
Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488. https://doi.org/10.3102/00028312033002455
Steinbring, H. (1989). Routine and meaning in the mathematics classroom. *For the Learning of Mathematics*, *9*(1), 24–33. http://www.jstor.org/stable/40247942. Accessed 26 Oct 2020.
Stockero, S. L., van Zoest, L. R., Freeburn, B., Peterson, B. E., & Leatham, K. R. (2020). Teachers' responses to instances of student mathematical thinking with varied potential to support student learning. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-020-00334-x
Tait-McCutcheon, S., & Loveridge, J. (2016). Examining equity of opportunities for learning mathematics through positioning theory. *Mathematics Education Research Journal, 28*, 327–348.
Teuscher, D., Moore, K. C., & Carlson, M. P. (2016). Decentering: A construct to analyze and explain teacher actions as they relate to student thinking. *Journal of Mathematics Teacher Education, 19*(5), 433–456. https://doi.org/10.1007/s10857-015-9304-0
Van Langenhove, L., & Harré, R. (1993). Positioning and autobiography: Telling your life. In N. Coupland & J. F. Nussbaum (Eds.), *Discourse and lifespan identity* (pp. 81–99). Sage Publications.
Vogler, A., Prediger, S., Quasthoff, U., & Heller, V. (2018). Students' and teachers' focus of attention in classroom interaction: Subtle sources for the reproduction of social disparities. *Mathematics Education Research Journal, 30*, 299–323.
Vygotsky, L. S., Cole, M., John-Steiner, V., Scribner, S., & Souberman, E. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.
Wagner, D., & Herbel-Eisenmann, B. (2009). Re-mythologizing mathematics through attention to classroom positioning. *Educational Studies in Mathematics, 72*(1), 1–15.
Warshauer, H. K. (2015). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education, 18*(4), 375–400. https://doi.org/10.1007/s10857-014-9286-3
Wegerif, R., & Mercer, N. (1997). A dialogical framework for researching peer talk. In R. Wegerif & P. Schrimshaw (Eds.), *Computers and talk in the primary classroom* (pp. 49–64). Multilingual Matters.
Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167–178). National Council of Teachers of Mathematics.
Wood, T., Williams, G., & McNeal, B. (2006). Children's mathematical thinking in different classroom cultures. *Journal for Research in Mathematics Education, 37*(3), 222–255.
Xu, L., & Clarke, D. (2019). Speaking or not speaking as a cultural practice: Analysis of mathematics classroom discourse in Shanghai, Seoul, and Melbourne. *Educational Studies in Mathematics, 102*(1), 127–146. https://doi.org/10.1007/s10649-019-09901-x
Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*, 458–477.