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Structure sense in students' quantity comparison and repeating pattern extension tasks: an eye-tracking study with first graders

Demetra Pitta-Pantazi[1] · **Eleni Demosthenous**[1] · **Maike Schindler**[2] · **Achim J. Lilienthal**[3,4] · **Constantinos Christou**[1]

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Abstract
There is growing evidence that the ability to perceive structure is essential for students' mathematical development. Looking at students' structure sense in basic numerical and patterning tasks seems promising for understanding how these tasks set the foundation for the development of later mathematical skills. Previous studies have shown how students use structure sense in enumeration tasks. However, little is known about students' use of structure sense in other early mathematical tasks. The main aim of this study is to investigate the ways in which structure sense is manifested in first-grade students' work across tasks, in quantity comparison and repeating pattern extension tasks. We investigated students' strategies in quantity comparison and pattern extension tasks and how students employ structure sense. We conducted an eye-tracking study with 21 first-grade students, which provided novel insights into commonalities among strategies for these types of tasks. We found that for both tasks, quantity comparison and repeating pattern extension tasks, strategies can be distinguished into those employing structure sense and serial strategies.

Keywords Eye tracking · Quantity comparison · Repeating pattern extension · Structure sense · Serial strategies

1 Introduction

The term "structure sense" describes the ability to recognize how a mathematical whole consists of parts as well as the relationships between these parts (Hoch & Dreyfus, 2004; Lüken, 2012). It was first used by Linchevski and Livneh (1999), and subsequently, the idea was developed and refined by Hoch and Dreyfus (2004). Structure sense seems important

✉ Demetra Pitta-Pantazi
dpitta@ucy.ac.cy

1 University of Cyprus, Nicosia, Cyprus

2 University of Cologne, Cologne, Germany

3 Örebro University, Örebro, Sweden

4 TU München, Munich, Germany

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across various mathematical content domains, especially in algebra and arithmetic (Mulligan et al., 2006). Lüken (2012) found that structure sense at the beginning of first grade is an early predictor of arithmetical competence at the end of second grade.

So far, there is insufficient knowledge of how structure sense is manifested in students' work in early mathematics. Thus, there is a need to deepen the understanding of young students' structure sense and the processes related to the awareness of this structure. In this context, eye-tracking studies can provide important insights into young students' strategies for solving mathematical tasks (Obersteiner & Tumpek, 2016). *Eye tracking* is a technique that captures participants' eye movements using the anatomic feature of human vision that the eyes need to move so that an observer sees objects or regions of interest in high resolution. According to Radford (2010), attending to something in a certain way requires an intentional act that he calls "domestication of the eye" (p. 4), and this attention allows students to recognize things from a mathematical perspective. Hence, examining students' strategies by tracking the eye gaze as an indicator of visual attention is promising in revealing insights into how students engage with mathematical tasks in their early years.

There is a growing consensus that "eye tracking offers unique ways to understand cognitive processes in mathematics education" (Strohmaier et al., 2020, p.167). For the investigation of students' strategies in early years, this is particularly beneficial for two reasons. First, it is possible to observe the strategies without interrupting students (e.g., Weijden et al., 2018). Second, it is possible to explore strategies that may not be consciously accessible or that young students may not be able to communicate (e.g., Ott et al., 2018; Schindler & Lilienthal, 2018).

Sprenger and Benz (2020) used eye tracking and found that when five-year-old students enumerate quantities, they are aware of structures, and some of them can even use these structures to determine the cardinality of sets. Ten-year-old students also use strategies such as enumerating all the dots simultaneously or enumerating groups of dots (Schindler, Schovenberg, & Schabmann, 2020), which suggests that students can identify structures in the visual representation of quantities. In these studies (Schindler, Schovenberg, & Schabmann, 2020; Sprenger & Benz, 2020), it was the use of an eye-tracking methodology that allowed inferences about students' structure sense to enumerate quantities.

However, researchers need to understand better how structure is used in other mathematical activities. Since structure sense was found to be a significant predictor of mathematical learning (Lüken, 2012), in this paper, we study how structure sense can be identified across different tasks and how it can be assessed with eye tracking. We study students' structure sense when students compare quantities, an ability often developed after enumeration, and when students extend repeating patterns, a type of task primarily linked with structure. The main aim of our study is to investigate the ways that structure sense is manifested in first-grade students' work across tasks, in particular, in quantity comparison and repeating pattern extension tasks.

2 Literature review

In our literature review, we first focus on structure sense. Then, we present findings from students' use of structure sense in enumeration tasks, with and without eye tracking, because these findings guided the investigation of eye-tracking strategies in quantity comparison and pattern tasks. Following this, we explore findings on quantity comparison and pattern tasks. Finally, we present how the current study draws on the existing findings and state the research questions.

2.1 Structure sense

When we refer to structure, we adopt Battista's definition. In his view, "spatial structuring is the mental operation of constructing an organization or form for an object or set of objects. It determines the object's nature, shape, or composition by identifying its spatial components, relating, and combining these components, and establishing interrelationships between components and the new object" (Battista, 1999, p.418). Lüken (2012) describes early structure sense as an individual's ability to (a) identify a configuration as a familiar structure or pattern (e.g., dots on a dice), (b) break a pattern into sub-structures, (c) recognize and find connections and relationships between sub-structures (i.e., similarities and differences, detect regularity), and (d) integrate substructures to see a pattern as an entity (e.g., extend a pattern).

Many researchers relate patterning ability or even equate it with the ability to perceive and use structures (Hutchinson, 2011; Lüken, 2012). According to Lüken (2012), students may consciously and/or subconsciously use structure sense to determine a quantity or extend a pattern. It seems likely that students' structure sense might be manifested when working with other types of tasks, such as quantity comparison. Although several studies identify students' structure sense in enumeration tasks through students' actions and verbal responses (Schindler, Schovenberg, & Schabmann, 2020; Sprenger & Benz, 2020), it is not clear whether it is possible to identify structure sense through eye tracking in quantity comparison and pattern extension tasks.

So far, there is evidence that students use structure sense when enumerating quantities. The ability to enumerate quantities (i.e., to grasp sets of items and say how many there are) is crucial for children in preschool and the beginning of primary school. Research studies that investigated students' actions and verbal responses in enumeration tasks led researchers to the identification of a set of students' strategies such as counting, subitizing, and groupitizing (Schleifer & Landerl, 2011; Starkey & McCandliss, 2014). Counting is the "one-to-one mapping between a set of objects and number words" (Schleifer & Landerl, 2011, p. 280). Subitizing is the ability to enumerate small quantities fast and precisely without counting and is considered an essential requirement for arithmetic learning (Fischer et al., 2008). Finally, groupitizing involves understanding the concepts of numbers and part-whole schema (Starkey & McCandliss, 2014) and the idea of composing and decomposing (Clements, 1999). In groupitizing, children perceive sets in subsets even at a young age (Clements, 1999).

Eye-tracking studies with 10-year-old students on enumeration tasks found that students use simultaneous enumeration and enumeration through the use of structures for the canonical arrangement of 2–9 dots (Schindler, Schovenberg, & Schabmann, 2020). For random arrangements of 2–4 dots, students again used simultaneous enumeration and enumeration of groups of dots. For random arrangements of 5–9 dots, students used quasi-simultaneous enumeration and partial enumeration of groups of dots. In this paper, we classified these strategies, which draw on simultaneous or quasi-simultaneous enumeration or the use of structures, under *strategies employing structure sense*. On the contrary, we classified the enumeration of single dots (i.e., counting all dots one by one) as a *serial strategy*. Sprenger and Benz (2020) also found that 5-year-old students used structures to determine the cardinality of sets when they were asked to find how many eggs were presented in an egg carton. For example, strategies employing structure sense included that of enumerating in groups, (de)composing, and subitizing (Sprenger & Benz, 2020).

While studies on students' enumeration appear to identify students that use structure sense or serial strategies, to the best of our knowledge, there are no eye-tracking studies that investigate students' use of structure sense across tasks, in this case, across quantity comparison and patterning tasks. This is the focus of our study.

2.2 Quantity comparison

Comparing quantities and identifying equal and unequal sets is another common topic among early education curricula (e.g., Department of Education, 2013) and commonly a part of mathematics instruments for students' number development (e.g., Beltrán-Navarro et al., 2018). Human beings intuitively make perceptual judgments about the relative magnitude of quantities (Sarama & Clements, 2008).

Such judgments about the magnitude of quantities (i.e., which set of objects has more/less) are based on two non-symbolic cognitive numerical systems. According to the first system, the comparison is executed through object tracking, in which small sets (less than 4) are enumerated based on subitizing without counting serially (Trick & Pylyshyn, 1994). The second system is referred to as the approximate number system (ANS) and is used for larger numbers. The ANS supports the estimation of the magnitude of a set without relying on language or symbols; instead, it is ratio-dependent (Nieder & Dehaene, 2009). Huntley-Fenner and Cannon (2000) suggest that 3- to 5-year-old children's decisions about numerical magnitude are mediated by a similar mechanism, which does not depend on their ability to count verbally.

Huntley-Fenner and Cannon (2000) also show that comparison tasks with a 2:3 ratio in the number of dots of the two sets were more difficult than the ones with a 1:2 ratio. It was also observed that error rates and response times for number comparison increased when the ratio of the smaller over the larger number increases (Moyer & Landauer, 1967). In non-symbolic comparison tasks with numerosity of 5–22 dots, 7- to 9-year-old students showed that those with high ANS acuity tended to have high achievement scores (Inglis et al., 2011).

It seems that preschoolers do not use counting often to compare sets of items (Clements & Sarama, 2007). Children who are approximately 3.5 years old can match homogenous visual sets; when they are approximately 4.5 years old, they can match equivalent collections of heterogeneous objects (Sarama & Clements, 2008). Initially, Piaget and Szeminska (1952) and later Fuson (1988) corroborated that children at the age of 4 to 5 years, when comparing sets, focus on misleading length cues and do not use counting. Even though children may count two sets to compare them, they tend to still decide which set is bigger based on appearance and extension (Piaget & Szeminska, 1952). For example, they may count the sets, recognize that the number is the same, but still mention that one set has more items based on the spatial extension of the item arrangement. All in all, several studies indicate that only over time do children come to trust the results of the counting process to compare the magnitude of sets.

Only a few studies studied quantity comparison strategies using eye-tracking methodology (e.g., Fuson, 1988). These studies examined how adults respond to non-symbolic comparison when varying the ratio effect (Huntley-Fenner & Cannon, 2000) and the cumulative area (Odic & Halberda, 2015).

2.3 Patterning

Patterning is the ability to discover regularities among ordered sets of units (Clements & Sarama, 2007). To foster their patterning skills, students are often taught (a) repeating patterns, which contain a discernible unit (Threlfall, 1999) generated by the alteration of a smaller part based on objects (e.g., numbers, letters, shapes) and/or their characteristics (e.g., color, size) (Liljedahl, 2004; Papic, 2015); and (b) growing patterns which increase or decrease systematically. In this study we focus on repeating patterns.

Papic et al. (2011) investigated students' strategies when dealing with repeating patterns in an interview setting. They found that 4- to 5-year-old students use one of the following strategies: random arrangement of the pattern elements without attention to the pattern, direct comparison by matching one element at a time, alternation by focusing on successive elements, and identification of the repeating unit. In extending patterns, 5- to 6-year-old students may only notice the changes between object characteristics, for example, "the colour yellow comes after green and green comes after yellow," thus identifying only the element that follows. A shift in their understanding occurs when they are able to identify the repeating unit, for example, "yellow, green" (Economopoulos, 1998).

According to Lüken (2012), first-grade students could perceive the succession of colours in repeating patterns. However, not all of them were able to relate the figure to the mathematical aspects in ways that connect the spatial structure of the pattern with its numerical one (e.g., two red and two yellow). Lüken (2012) based her findings on students' use of specific vocabulary or actions that indicated an awareness or lack of awareness of the repeating units during interviews. Hutchinson (2011) found that pre-primary school students may apply structure sense consciously and/or subconsciously. Hence, students may be in a position to successfully complete pattern tasks but may not be able to communicate how they have reached their answer (Lüken, 2012; Van Nes, 2009). Such difficulties in communicating structure sense or not being consciously aware of it create methodological obstacles in the interpretation of findings. This is one of the reasons why eye tracking can provide complementary or otherwise unobtainable insights into students' thought processes. Yilmaz's (2019) eye-tracking study on repeating patterns showed that 4- to 5-year-old students had extended unfocused gazes on the overall given patterns (AB, ABB, ABC) while they primarily focused on the last repeating unit of the pattern. The findings indicate that students may be implicitly aware of the repeating unit (Yilmaz, 2019), while other studies have suggested that students compare one-to-one the middle elements of the patterns and the elements at the beginning of the pattern (Collins & Laski, 2015; Threlfall, 1999). It is worth mentioning that orientation of eye movements could be overt (can be observed through eye tracking) or covert (cannot be observed) (Posner, 1980). In the case of covert orientation, information can be perceived using peripheral vision based on extra-foveal processes (Posner, 1980; Shvarts et al., 2019), which may play a role when students identify the repeating unit without focusing on it visually.

2.4 Research questions

The present study set the stage for the framework of the *Digital identification and support of under-achieving students* project (DIDUNAS). The DIDUNAS project, which was conducted from 2020–2023, addressed the identification of under-achieving students in mathematics in Grade 1 (see www.didunas.eu). In previous publications based on DIDUNAS studies, we investigated the types of strategies students use in pattern tasks (Baumanns et al., 2022; Baumanns et al., 2023; Demosthenous et al., 2022). In this paper, we investigate for the first time—with the use of eye tracking—students' strategies and the use of structure sense across tasks. Specifically, we explore students' strategies and use of structure across quantity comparison and patterning tasks. Therefore, the two research questions are:

(1) What strategies do first-grade students employ to respond to quantity comparison and repeating pattern extension tasks, and in what ways is structure sense manifested in students' strategies?
(2) Is there a relationship between students' correct answers and the strategies they use?

3 Method

3.1 Participants

All thirty first-grade students at a primary school in Cyprus were invited to participate in the study, and 21 students (mean age: 6.5 years) agreed to participate. All students were proficient in Greek and, according to their teachers, performing well in mathematics. The research took place during two consecutive days in the first trimester of the first grade. This specific period was chosen to investigate the strategies students use at the beginning of primary school. All students were taught the same mathematics curriculum, and none of them received any supplementary mathematics instruction.

Before the study, the students' parents, teachers, and the school principal were informed about the study and the interview procedure. Parent's written consent was necessary for their child to participate and the eye-tracking videos to be published anonymously. All parents were also informed that they and their children could withdraw from the research study at any point without any consequences. Additionally, specific steps were taken to ensure the anonymity and confidentiality of all participants.

3.2 Setting

To record students' eye movements, we used a remote eye tracker, Tobii × 3–120, with a sampling rate of 120 Hz (infrared, binocular, 9-point calibration). The eye tracker was connected to a 22'' Full HD computer monitor. The eye-tracking arrangement was free from distractions and permitted head movements, which allowed the young students to express natural behaviors. The eye-tracking accuracy in our study was 0.51° on average (SD 0.17°), with a minimum of 0.28° and a maximum of 0.88°. The computer screen was placed approximately 70 cm from the eyes of the students, which means the imprecision on the screen amounted to around 0.62 cm on average (max. 1.08 cm). We accounted for this imprecision by designing the tasks accordingly (e.g., the dots in the quantity comparison tasks had a distance of more than 1 cm from one another).

In the data collection, the individual students sat in a comfortable chair, and its height could be adapted to accommodate the different heights of the students. A researcher gave the instructions and asked the questions. The students responded to the tasks while looking at the screen monitor. Additionally, the utterances were recorded by an audio recording device.

3.3 Tasks

Students worked on quantity comparison tasks and repeating pattern extension tasks, as shown in Fig. 1. All tasks were given to all students in the same order. Before each task, all students were asked to first look at a star on the screen to ensure that all students' eyes were fixated on the same point before attempting the task.

3.3.1 Quantity comparison tasks

Students worked on six quantity comparison tasks, and in each task, there were two sets, one with green dots and one with yellow dots. In three tasks, the two sets were in columns, while the other three were in rows. In four tasks, the groups were equal, and in the other two, there was a difference of one dot. There were (a) two equal groups of the same length, (b) two unequal groups of the same length and (c) two equal groups of different length (see Fig. 1). The students were asked to say which set had more dots (the yellow or the green) or whether the sets were equal and to answer as fast as possible. In the comparison tasks, dots were arranged in this manner based on previous research (Piaget & Szeminska, 1952; von Aster et al., 2006) and on the appearance of these types of tasks in the mathematics textbooks of this age group.

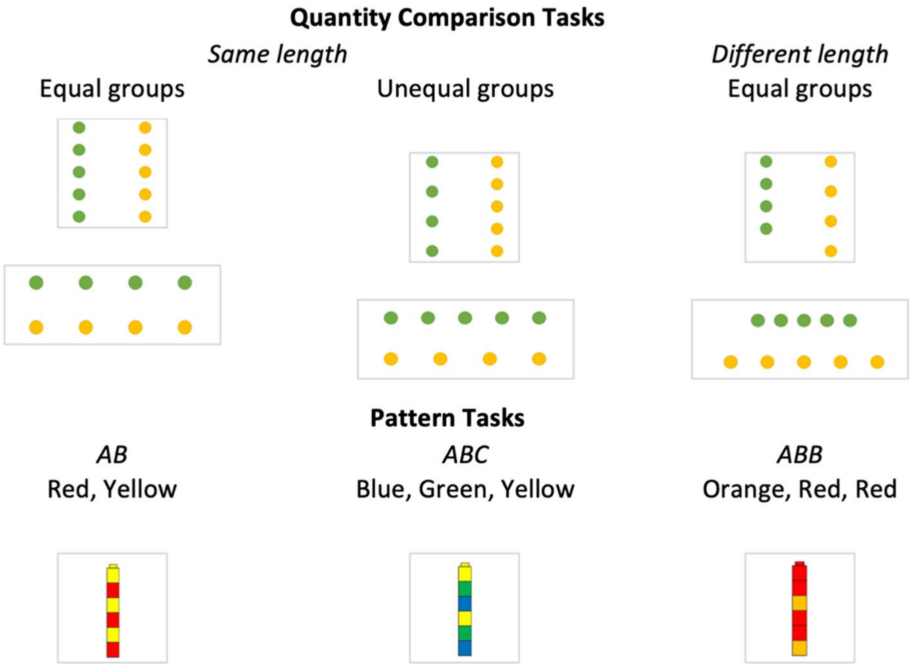


Fig. 1 Types of tasks

3.3.2 Repeating pattern extension tasks

Students were presented with a picture of a tower of 6 unifix blocks (Fig. 1). The colors of the first tower were red, yellow, red, yellow, red, yellow; the second: blue, green, yellow, blue, green, yellow; and the third: orange, red, red, orange, red, red. Students were asked to say which block comes next as the tower is built.

3.4 Analysis

We collected eye-tracking data and transcribed the recordings of students' verbal responses. For the eye-tracking video analysis, we produced gaze-overlaid videos. Although more time-consuming and tedious for researchers, gaze-overlaid videos allowed observations that would not have been possible in the investigation of students' strategies if we recorded only the number of eye fixations.

To answer the first research question, our analysis followed the four stages described by Schindler et al. (2019). In the first stage, we watched the gaze-overlaid videos and assigned initial categories according to the strategy used for each task. These initial categories were labeled according to the respective common strategy. In the second stage, through a constant comparative method, we reached saturation and finalized the description of the categories by preparing a codebook with a description and gaze plot of each strategy. In the third stage, all gaze-overlaid videos were coded as correct or wrong based on the audio recordings of students' verbal responses. In the fourth stage, 25% of all the data were coded by two raters independently (Mayring, 2014) to establish inter-rater reliability. For the coding of students' strategies, the inter-rater reliability was calculated using Cohen's kappa (Cohen, 1988) and found to be 0.96 for the comparison tasks and 0.83 for the pattern extension tasks, which is considered an almost perfect and substantial agreement, respectively.

To answer the second research question, a chi-square (X^2) statistic test was applied to investigate whether there was a relationship between students' strategy use and the correctness of their solutions to the tasks.

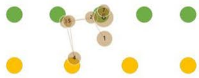
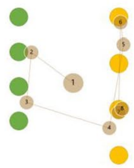
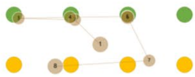
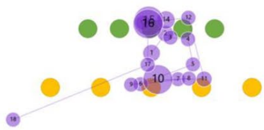
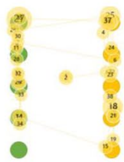
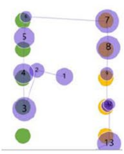
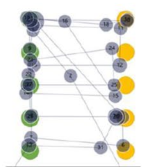
4 Results

In the following section, we respond to the two research questions of the study. First, we respond to the research question by presenting the strategies that first-grade students used in quantity comparison and repeating pattern extension tasks. Then we refer to the relationship between students' correct answers and the strategies they use.

4.1 Students' strategies in quantity comparison and repeating patterns

For each type of task, we describe the strategies and illustrate them with an exemplary scanpath. While the term scanpath can be defined "as the route of oculomotor events through space within a certain timespan" (Holmqvist et al., 2011, p. 254), we analyzed dynamic scanpath visualizations (gaze-overlaid videos), but for visualization purposes in this paper use static visualizations (gaze plots). We respond to the first research question by grouping students' strategies into those that employ structure sense and those that are serial. To the best of our knowledge, no other research study has identified categories of student strategies for quantity comparison tasks based on scanpath analysis in particular.

Table 1 Students' strategies in comparison tasks

	Strategy	Description	Gazeplot (Example)	Frequency
Strategies employing structure sense	(1a) Simultaneous comparison of the two sets	The gaze moves from one dot or two dots of one set (e.g., green) to another dot in the other set (e.g., yellow).		9
	(1b) Partial comparison of groups of dots of the two sets	The gaze goes (i) to parts of dots (indicating use of groups of dots) (Example i) or (ii) to individual dots of subset(s), of one set (green dots) (Example ii). Then, the gaze goes to parts of dots or in-between spaces (indicating the use of groups) of the other set (e.g., yellow dots). The gaze makes one transition between the two sets.	 Example i  Example ii	37
	(1c) Extended partial comparison of dots of the two sets	The gaze goes to subset(s) of dots of one set (e.g., green dots). Then, the gaze goes to subset(s) of dots of the other set (e.g., yellow dots). The gaze goes back and forth several times between dots of the same set or between the two sets of dots.		52
Serial strategies	(2a) Attending to/ comparing all	The gaze goes to every dot or to all dots but one in each set and in some cases moves back-and-forth between the two sets. The gaze may follow a sequential order (Example ii) or not (Example i).	 Example i  Example ii	33
	(2b) One-to-one correspondence	The gaze moves between pairs of dots (e.g., one from the green group and one from the yellow group of dots), in a back-and-forth movement. The gaze goes to every pair of dots.		1

The scanpaths that we identified, with indicative examples and the frequency of appearance among students' responses, are presented in Table 1. Examples of all the strategies presented in Table 1 are illustrated with videos that can be reached through the URL in the respective references. For example, the URL for Strategy 1(a) can be found in the reference Pitta-Pantazi et al. (2023a).

4.1.1 Students' strategies in quantity comparison tasks

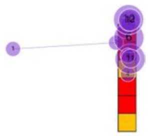
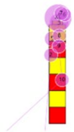
In Strategy 1(a), students compare the two sets at once, either identify a difference or match dots between the two sets, and then decide which set has more dots (Pitta-Pantazi et al., 2023a). In Strategy 1(b), students make partial comparisons of groups of dots of the two sets. They identify groups of dots or individual dots in one set and then move to the other set and look again, either at groups of dots or at individual dots (Pitta-Pantazi et al., 2023b, c). In Strategy 1(c) (Pitta-Pantazi et al., 2023d), students make extended partial comparisons of the two sets. The gaze goes to subsets of dots of one of the sets and then back and forth to a subset of dots of the other set. For Strategies 1(a), 1(b), and 1(c), students relied on selected dots or parts of dots. Therefore, we grouped these strategies and labeled them *strategies employing structure sense*. In Strategy 2(a), students' gazes go to each dot of each set, either in a sequential or non-sequential order, implying that the student was enumerating the dots one by one (Pitta-Pantazi et al., 2023e, f). In Strategy 2(b) (Pitta-Pantazi et al., 2023g), the gaze focuses on one-to-one correspondences between the dots of the two sets. We grouped Strategies 2(a) and 2(b) and labeled them *serial strategies* since students focused on all the dots when comparing the two sets. The gaze moves from one dot in one of the sets to a corresponding dot in the other set.

Both strategies, employing structure sense or serial strategies, could lead to correct or erroneous responses. For example, a student's gaze may indicate structure sense in a comparison task (e.g., 1b) since it indicates a partial comparison between subsets of the green and yellow dots. If the respective student is focusing on the length of the two sets of dots, this may lead to an erroneous response (if there are two unequal sets but the dots are spread to the same length). An error with enumeration may occur if, for example, a student double counts or misses a dot.

4.1.2 Students' strategies in pattern extension tasks

Students' strategies, indicative examples, and the frequency of the strategies identified in the pattern tasks are presented in Table 2. Among students' responses, we found evidence suggesting that some students focused on the last repeating unit of the pattern (Yilmaz, 2019) while other students appeared to look at each element of the pattern one by one (Collins & Laski, 2015; Threlfall, 1999). Specifically, Strategy 1(a) (Pitta-Pantazi et al., 2023h) involved directly identifying the repeating unit, while in Strategy 1(b) (Pitta-Pantazi et al., 2023i), students identified the repeating unit, and then their gazes jumped to (an)other repeating unit(s), implying a comparison between the repeating unit and another block or group of blocks. When applying Strategies 1(a) or 1(b), the students seemed to rely on selected elements of the pattern to decide how the pattern continued by identifying the repeating unit. It appears that they identified the repeating unit at once and stopped their gaze as soon as the next repeating unit started (1a), or they identified the repeating unit, and then their gaze jumped to another repeating unit (not necessarily the next one) to make a comparison in order to reach their answer (1b). We grouped these strategies and labeled

Table 2 Students' strategies in pattern extension tasks

	Name	Description	Gazeplot (Example)	Frequency
Strategies employing structure sense	(1a) Simultaneous identification of the repeating unit	The gaze starts from the top part of the pattern. The gaze (a) goes only to one repeating unit (e.g., red, red, yellow) or (b) goes to one repeating unit plus one adjacent block from the next repeating unit (e.g., red, red, yellow, red).		6
	(1b) Identification of the repeating unit and comparison with other elements or groups of elements (repeating unit)	The gaze goes to each block of one repeating unit. Then, the gaze jumps to the bottom or middle part of another group of blocks (repeating unit), without gazing at individual blocks.		26
Serial strategies	(2) Sequential, continuous "attending to all"	The gaze goes to all the blocks of the pattern or goes to all blocks but one. This gaze might repeat, more than once, in different directions (e.g., top to bottom, bottom to top, middle to bottom).		31

them *strategies employing structure sense* since gazes focused on repeating units. In Strategy 2 (Pitta-Pantazi et al., 2023j), students gazed at each block of the pattern before finding how to extend the pattern; we labeled it *serial strategy*.

4.1.3 The ways in which structure sense and serial strategies are manifested in students' strategies when working on quantity comparison and repeating pattern extension tasks

In the comparison tasks, strategies employing structure sense appeared more frequently than serial strategies (Table 3). On one hand, it may be inferred that counting the dots, one by one, sometimes led to erroneous solutions due to miscounting. On the other hand, students who looked at the structure of the dot arrangements may sometimes have been misled by the fact that unequal sets of dots were arranged at the same distance, or equal sets of dots were not evenly distributed. The highest frequency of errors ($f=9$ and $f=6$) was observed in the comparison of equal groups of dots, which were distributed in different lengths.

Table 3 Frequencies of strategies employing structure sense and serial strategies in the comparison tasks

	Strategies employing structure sense			Serial strategies		
	All	Correct	Wrong	All	Correct	Wrong
Total (Percentage)	93 (74%)	68 (54%)	25 (20%)	33 (26%)	29 (23%)	4 (3%)

All strategies employing structure sense and serial strategies (74% + 26%) sum up to 100%. Correct and wrong answers sum up to 100% (54% + 20% + 23% + 3%)

Table 4 Frequencies of strategies employing structure sense and serial strategies in the pattern tasks

	Strategies employing structure sense			Serial strategies		
	All	Correct	Wrong	All	Correct	Wrong
Total (Percentage)	32 (51%)	30 (48%)	2 (3%)	31 (49%)	28 (44%)	3 (5%)

All strategies employing structure sense and serial strategies (51% + 49%) sum up to 100%. Correct and wrong answers sum up to 100% (48% + 3% + 44% + 5%)

In the pattern tasks, strategies employing structure sense were used with a frequency of 51% compared to the serial strategies with 49% (Table 4). In one of the pattern tasks (AB), strategies employing structure sense were more often used (f = 18 vs. f = 3), whereas, in the other two pattern tasks (ABC and ABB), the frequency of the strategies employing structure sense was lower than the frequency of serial strategies (f = 7 vs. f = 14). In the pattern tasks, the number of erroneous answers was generally low and almost equal between those who employed structure sense and those who used serial strategies (3% and 5%, respectively).

We observe that in the quantity comparison and pattern tasks, students applied either strategies employing structure sense or serial strategies.

4.2 Relationship between students' correct answers and the strategies they used

To further investigate what strategies the students employed, we explored the frequency of the strategies employing structure sense and serial strategies in each type of task. Through the chi-square test of independence, we examined whether the strategies employed by students were likely to be related to correct or erroneous answers to the tasks. According to the values of the chi-square test, there was no significant correlation between the strategies the students used and their correct or erroneous solutions for the quantity comparison tasks X^2 (25, N = 21) = 16.42, p = 0.902 and the pattern tasks X^2 (6, N = 21) = 10.55, p = 0.10.

5 Discussion

In our study, we used eye tracking to inquire into young learners' use of structure sense in different mathematical tasks. More concisely, we investigated first-graders' use of structure sense across tasks, in particular, across repeating pattern tasks and quantity comparison tasks. Our study contributes to the research landscape in mathematics education (a) through its insights into how young learners' structure sense is employed across two domains and (b) through—methodologically—demonstrating the potential of eye tracking to gain insights into the distinction between strategies employing structure sense and serial strategies (Hunting, 2003; Lüken & Sauzet, 2021; Schöner & Benz, 2017).

How did the students employ structure sense? In the quantity comparison tasks, structure sense was identified when students compared quantities simultaneously or compared sub-groups of dots from each set and made connections. On the other hand, we identified serial strategies when students enumerated dots sequentially or performed one-to-one correspondence comparisons between the two sets. Erroneous answers were more likely given when students miscounted the number of dots. When employing structure sense strategies, it is possible that erroneous answers were given when students were misled by the dots' spatial arrangements. It seems that some students erroneously decided that an equal length/width corresponded to the same number of dots or that a shorter length/width implied a smaller number of dots. Our findings provide further insight into how the cumulative area effect (i.e., the effect of varying the area occupied by a set of objects) (Odic & Halberda, 2015) appears when using the analogue number system (ANS) to estimate the magnitude of two sets.

For structure sense in students' work with pattern tasks, we also grouped students' strategies into those employing structure sense and those using serial strategies. Students who appeared to identify the repeating unit at once (or identified the repeating unit and made comparisons between units) demonstrated structure sense. In contrast, students who attended to all the pattern elements sequentially, one by one, demonstrated a serial strategy. When investigating students' strategies in repeating pattern tasks, Yilmaz (2019) found that children focused on the last repeating unit of the pattern but also had extended unfocused gazes. In our study, we did not notice such unfocused gazes, but we found students attending to the last repeating unit of the pattern. Another study using a revised set of pattern tasks with a different group of students led to four categories (Baumanns et al., 2022). The first three respective categories were: identifying one repeating unit of the pattern, identifying one repeating unit and validating/applying it, and looking at each element. An additional category was the unsystematic jumping over the pattern.

In this study, we present findings from an exploration of students' structure sense, highlighting strategies employed across different domains (quantity comparison and pattern tasks). What we observed across the two domains investigated in our study — both of which involved visual tasks requiring perception and processing of given information — was that the first-graders exhibited two predominant approaches: They either visually attended to all given elements (serial strategies) or attended to parts of the given information, utilizing structures to infer the information sought to determine the color of the next element or to identify the larger quantity (using structure sense). It is noteworthy that although the two domains under investigation, namely patterns (early algebra) and quantity comparison (arithmetic), were inherently different, we could observe apparent commonalities in students' strategies. We also found that structure sense was utilized quite frequently, occurring in more than half of the cases in both domains. It is interesting that although the participants of the study were first-graders in the first months of primary school, they could already rely on structure sense to a great extent. However, the use of structures did not necessarily coincide with correct answers in either of the two domains. This was particularly evident in the quantity comparison tasks, where an emphasis on spatial distribution and associated inferences sometimes may have led to incorrect answers. Making use of structure sense, thus, was not related to success in terms of correctness in these mathematical tasks.

Furthermore, our eye-tracking investigation provided further insights into the strategies identified by previous researchers, which were conducted without the use of eye tracking, such as matching one element at a time, focusing on successive elements, and identifying the repeating unit (Collins & Laski, 2015; Lüken, 2012; Papic et al., 2011). We found that the identification of the repeating unit does not necessarily result from attending to each element of the whole pattern. Instead, some students can capture the repeating unit directly, while other students compare a repeating unit with elements of another repeating unit. In addition, in our study, we did not find evidence to indicate that students matched one element of one repeating unit to the respective element of the following repeating unit, as suggested by Yilmaz (2019).

Students who used strategies employing structure sense in the pattern tasks could directly spot the repeating unit, while in the comparison tasks, they could identify the equality and inequality of the two sets without going through and attending to each element (dots in the comparison tasks or blocks in the pattern tasks). This relates to Chumachemko et al.'s (2014) findings that experts tend to make fewer saccades when dealing with coordinate systems and rely on peripheral vision, for example, to perceive prototypical geometric figures using the entire perceptual field (Shvarts et al., 2019). Referring to peripheral vision, the data of this study also indicate that students who used strategies employing structure sense did not fixate serially on individual dots and seemed to rely on the extrafoveal perception of the display. For example, it can be assumed that some students who only scanned the top of the pattern in the pattern task used peripheral vision to perceive the rest of the pattern. This relates to the findings of Shvarts et al. (2019) that extrafoveal processes were involved in the identification of squares and rectangles.

The findings of this study suggest that it is possible through the investigation of dynamic scanpath visualizations, namely gaze-overlaid videos, to identify students who employ structure sense and students who employ serial strategies in quantity comparison and pattern tasks. We noticed that there were students who looked at the configuration and searched for a structure with more efficient and flexible eye movements. For example, in the comparison tasks, students simultaneously compared the two sets and identified the equality or inequality of the two sets, while in the repeating pattern task, they found the repeating unit all at once. In contrast, students with serial strategies used the more sequential and elaborate process of focusing on each dot in the comparison tasks or on each colored block in the pattern tasks. Students who showed structure sense were also able to divide the dots or patterns into substructures when looking at groups of dots in the comparison tasks, while in the pattern tasks, their eye movements jumped between the repeating units. Furthermore, in the comparison tasks, they recognized and established connections between the two sets of dots when they compared them, either through comparison at once or partial comparison. In the repeating pattern tasks, they identified the repeating unit at once or first identified the repeating unit and then compared it with other elements of the group. They seemed to have been able to do the comparison and extend the repeating pattern by viewing the groups of dots and the pattern as an entity either simultaneously or partially. All these actions appeared to be more efficient and flexible. In contrast, the serial strategies, where students attended to each element one-by-one, were more sequential and elaborate (Lüken, 2012).

The findings of this study suggest that it is possible through eye tracking to identify what Radford (2010) calls "the domestication of the eye" (p. 4). By "domestication of the eye," Radford (2010) means the lengthy procedure by which the eye recognizes things, in our cases, dots or colored blocks, from a mathematical perspective. However, both Radford (2010) and Lüken (2012) identified this "domestication" through students' actions and verbal responses, whereas in our study, we found that eye tracking allows this distinction as well. With the explosion of the use of eye tracking in mathematics educational research, it appears that in a few years, it may be possible not only to identify students' strategies but to be able to interpret them and analyze students' thinking processes in more detail.

In this study, we analyzed the eye-tracking data manually, which is time-consuming and requires experts who have domain knowledge and are experienced in eye tracking. For further research (especially with larger sample sizes) and practical applications, eye-tracking data can also be examined using Artificial Intelligence (AI). Qualitative analyses, as presented in this article, can be supported by AI approaches, for example, by using unsupervised machine learning (Schindler, Schaffernicht, & Lilienthal, 2020, 2022; Simon et al., 2023). Supervised machine learning approaches can be used to automate the analysis of eye-tracking data, as demonstrated, for example, by Schindler et al. (2019).

In this study, we did not find significant differences in students' achievement in the quantity comparison and pattern tasks based on the strategies used, whether structure sense or serial. It may be possible that future studies could examine if this changes with a different age group or different tasks.

In the future, studies with larger sample sizes (including students at the lower end of the performance level) may allow the examination of clusters of students (different ages or abilities) to investigate whether students tend to use the same strategy across different types of tasks. Furthermore, future studies could also investigate the relationship between students' use of strategies and mathematical performance. Researchers could examine which kinds of tasks may best allow the early identification of students at risk or students with exceptional abilities based on students' strategies.

A limitation of our study is that, due to relying only on eye tracking and no other source of information, identifying structure sense may not automatically reveal students' reasoning, as in the case of the comparison tasks. For instance, instead of seeing the structure of the numerosity of objects, students could have concentrated on the space taken by the objects. Another limitation of our study is that we did not use example tasks, and this may have resulted in differences in students' responses, especially to the first task. Lastly, it would have been helpful to include more pattern tasks and patterns in a horizontal arrangement to resemble the number of tasks, as well as the two orientations of dots appearing in the comparison tasks.

6 Conclusion

Even though the exploration of young students' strategies with eye tracking is still at an early stage, our study illustrated its potential. Through eye tracking, we inquired into students' use of structure sense and we observed that young students exhibited a variety of strategies. Students seemed to either utilize structure sense or follow a serial process across two types of tasks. The ability to shift attention from single elements to groups of elements has been identified as fundamental for the development of students' understanding of numbers, arithmetic operations, part-whole relationship, multiplication, and patterns (Hunting, 2003; Lüken, 2012; Schöner & Benz, 2017). The results of this study suggest that it is possible through eye tracking, and specifically through the analysis of students' scan paths (here through video analysis), to identify students' abilities to move their attention away from single elements and towards a sense of structure. The eye-tracking method can be seen as a tool to identify and assess students' structure sense across tasks. Thus, it is possible to get a better idea of students' work in these mathematical activities. This possibility opens the door to numerous applications for eye tracking in various activities. Further studies and evidence that draw from various methodological approaches would contribute to solidifying the theoretical basis for a comprehensive understanding of how students work across tasks during the early years of education, what the involved processes are, how strategy use relates to mathematical performance, and in what ways teaching tailored to the individual strategy use profiles could enhance students' learning.

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Declarations

Competing interests None.

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