Educational Studies in Mathematics (2023) 114:503–524
https://doi.org/10.1007/s10649-023-10261-w

# Study of modeling questions in a first-year university mathematics online course

Camilo Andrés Ramírez-Sánchez[1,2] · Avenilde Romo-Vázquez[3] · Rebeca Romo-Vázquez[4] · Diana Velásquez-Rojas[1]

Accepted: 25 July 2023 / Published online: 5 September 2023


**Abstract**
The training of non-specialists, particularly engineers, in mathematics requires designing specific didactic proposals that make the importance of mathematics evident. One approach to creating such proposals consists in analyzing the mathematics used in authentic contexts of engineering research and then effectuating a didactic transposition to mathematics teaching. To this end, in this research, we use elements of the anthropological theory of the didactic and the methodology of didactic engineering. Initially, we analyze the blind source separation method, a case of inverse modeling widely used in numerous engineering contexts (e.g., telecommunications, acoustics, geophysics, biosignal analysis). More specifically, we examine the algorithm based on the $A\mathbf{x} = \mathbf{b}$ matrix model and its use in a signal processing context. Based on this analysis, we designed a didactic device that focuses the study on a simulated signal mixture of pure tones and implemented it in two first-year university mathematics courses in the virtual modality. This article reports the results of the students' first approximation to mathematical modeling activity in a setting of authentic research in engineering using a blind source separation method.

**Keywords** Authentic mathematical modeling questions · BSS method · Higher education · First-year math course · Study and research path

## 1 Introduction

Mathematical modeling and technological tools are essential resources for dealing with the numerous, varied challenges and critical, break-down, or novel situations that characterize engineering workplaces (Bissell & Dillon, 2000, 2012; Gainsburg, 2007; Kent et al., 2007; Pollak, 1988). In engineering research defined as a specific workplace, mathematical modeling is used to resolve tasks both theoretical and real; for example, engineers specialized in signal processing work with doctors and use mathematical models in the blind source separation method with the data of real patients (e.g., brain signals) (Romo-Vázquez & Artigue, 2023). Engineers use mathematical models like black boxes (Velten, 2009; Williams & Wake, 2007) or, as professional modelers, like gray or white boxes (Frejd & Bergsten, 2016; Velten, 2009), as well as sophisticated software that encapsulates advanced

Extended author information available on the last page of the article

mathematics, and acquire techno-mathematical literacies during their workplace practice (van der Wal et al., 2017). These specific workplace conditions demand that future engineers receive an education that covers the aspects of mathematics relevant to enterprises of this kind, as highlighted in the ICMI 20 study "Educational interfaces between mathematics and industry" (Damlamian et al., 2013), and the curriculum framework proposed to the SEFI (European Society for Engineering Education) by Alpers et al. (2013).

Interest in integrating mathematical modeling into engineering education has increased in recent years (Schukajlow et al., 2018). In this context, such pedagogical approaches as "learning-by-doing," that motivate students to learn as they play an active role in the learning process, have been proposed. These include problem-based learning (PBL), project-based learning (PjBL), case-based learning (CBL), inquiry-based learning (IBL), model-eliciting activities, STEM integration, and problem posing. The latter involves students actively formulating their own mathematical problems and aligns with the trend of promoting student autonomy in the learning process. This student-centered instructional strategy has a history of producing solid educational results in engineering (Muhammad & Srinivasan, 2020) that could help students become more creative when learning mathematics and help them become better problem-solvers (Nedaei et al., 2019). According to Savery (2006), all these approaches are based on the same core features: (a) students must take responsibility for their learning; (b) instruction must be weakly structured to allow free inquiry; (c) learning must be embedded in an interdisciplinary framework; (d) collaborative work; and (e) real-world connections. Though suitable for enhancing engineering students' learning and developing their inquiry skills and critical thinking (Rozhkova et al., 2020), they do not necessarily include mathematical modeling activities for engineering or address real issues of enterprises. Today, few instructional practices include authentic engineering tasks in mathematics courses for engineers (Alpers, 2011; Moon et al., 2022; Vázquez, 2017; Schmidt & Winsløw, 2021; Siero González et al., 2022). Indeed, bringing an industrial or engineering task into mathematics courses entails enormous complexity because these are two entirely different worlds, even more so for first-semester math courses, where students do not have a solid background in either mathematics or engineering. Faced with this challenge, we decided to revisit blind source separation (BSS), a mathematical modeling activity used in engineering that allows mixtures to be separated. Vázquez (2017) demonstrated the didactic potential of BSS by designing instructional material for an introductory linear algebra course. Against this background, three main research questions oriented our work: (1) what kind of mathematical modeling instructional device based on BSS can be designed for a first-semester online mathematics course?; (2) what kind of didactic "milieu" would best allow employing this instructional device in online mathematics teaching with students with only an elementary background in mathematics and engineering?; and (3) what institutional conditions are required to implement such an instructional device? To address these questions, we utilized elements of the anthropological theory of the didactic (ATD) and didactic engineering as our research methodology.

## 2 Elements of the ATD

The ATD proposes an epistemological model for analyzing human activity in its institutional dimension (Chevallard, 1999, 2019). It considers that mathematics activities and mathematical modeling activities are similar because, as Barquero et al. (2019) stated, doing mathematics consists in acting on mathematical models (by producing, teaching, and using them), and this can be analyzed through praxeology. The four components of praxeology are the type of task ($T$), technique ($\tau$), technology ($\theta$), and theory ($\Theta$). “Task” refers to what is to be done; “technique” is how it is to be done; “technology” refers to a discourse that produces, justifies, and explains the “technique”; while “theory” produces, justifies, and explains the “technology.” The first two elements correspond to the praxis block $[T, \tau]$, and the latter two to the logos block $[\theta, \Theta]$. An example of praxeology is $T$, finding the distance between two points on a plane; $\tau$, using the Euclidean formula; $\theta$, the Pythagorean theorem; and $\Theta$, Euclidean geometry.

An institution is understood as a stable social organization that makes it possible to deal effectively with problematic tasks, mainly in a collective way. The people who belong to an institution are recognized as individuals subject to institutional rules who may occupy different positions, such as professors, students, and citizens, among many others. There are many kinds of institutions: for example, families, schools, and industries. With respect to education in engineering, Romo-Vázquez (2009) identified three types of institutions that differ in terms of their relation to knowledge: production or research, teaching, and using institutions. Producing institutions, such as mathematics or engineering disciplines (e.g., calculus, signal processing, control theory), create praxeologies; teaching institutions (e.g., mathematics and engineering schools) transmit those praxeologies; while using institutions (e.g., workplaces and industries) employ them. According to Muller and Young (2014, p. 132), “professional knowledge displayed a new form of organization. As it absorbed and recontextualized contextual knowledge from the sciences, usually more than one, it had perforce to attend internally to the growing theoretical knowledge structures of basic science.”

It is important to point out that praxeologies can be created, taught, and used in any institution and can be passed from one to another. However, during that process, they are, in effect, transformed. Specifically, when praxeologies produced in researching or using institutions are to be taught, they must go through a process called didactic transposition (Chevallard, 1991). This transformative process of praxeologies or knowledge (academic, scientific, using) into knowledge to be taught often involves various institutions, as Gascón and Nicolás (2022, p. 16) illustrated (Fig. 1):

Thus, introducing a mathematical modeling praxeology from engineering research, called *Pe*, into school mathematics at university entails performing a didactic transposition of *Pe* (Fig. 2).

Here, $T^e$ is an engineering type of task (e.g., separating a mixture of sound signals), $\tau^{em}$ is the modeling technique, which has mathematical and engineering elements (e.g., determining the inverse of a matrix that modeled the mixture of sound signals), the technology, $\theta^{em}$, is a mathematical model related to engineering knowledge (e.g., matrices used as mixing models in a blind source separation method), and the theory, $\Theta^e$, is taken from engineering (e.g., signal processing). The didactic transposition of *Pe* to obtain *Ps* must ensure that it remains close to *Pe* but is understandable in university math courses. To achieve this, the engineering type of task and the mathematical modeling technique must be maintained but adapted by, for example, reducing the number of variables.

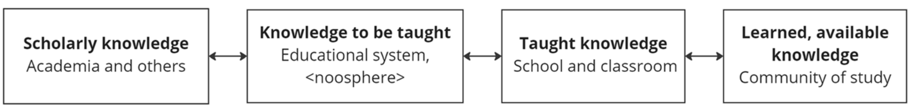


**Fig. 1** Institutions and phases involved in didactic transposition

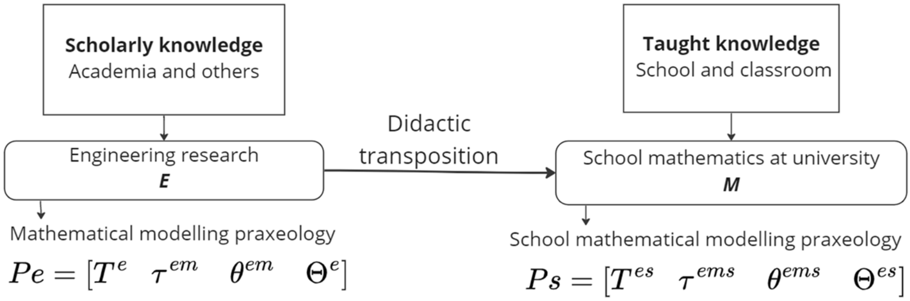


**Fig. 2** Didactic transposition process from engineering research to school mathematics

## 2.1 Study and research paths

Study and research paths (SRP) form a didactic device defined in the "questioning the world" paradigm. They focus on studying open-ended, researchable questions, $Q$, that may arise from any institution—researching or industrial, or even from everyday life—that permit inquiry in the classroom (Chevallard, 2015) and share the principles of "learning-by-doing" approaches. But they also have a theoretical model to represent and analyze the inquiry process; in this case, the Herbartian scheme: [S (X;Y;Q)] → M → A$^{\heartsuit}$, where $S$ represents a didactic system made up of $X$, the group of students, $Y$, the group of professors who conduct the study process, and $Q$, a generating question that serves as the object of study. $M$ represents the didactic milieu, made up of all the resources used to study the $Q$: the derived questions, $Q_i$, that lead to identifying "media"; that is, digital resources, experts, and sources of information, such as the works, $W_i$ (e.g., videos, websites, research articles) that allow students to recognize existing answers, called $A_i$. These elements make it possible to elaborate an answer, represented by $A^{\heartsuit}$, that is significant for a certain institution, $I$. A more developed conception of the Herbatian scheme is depicted below:

$$[S(X;Y;Q)] \to \{Q_1, Q_2, \ldots, Q_n, A^{\blacklozenge}_{n+1}, A^{\blacklozenge}_{n+2}, A^{\blacklozenge}_{n+3}, \ldots, A^{\blacklozenge}_{m}, W_{m+1}, \ldots, W_p\} \to A^{\heartsuit}]$$

The inquiry process to study $Q$ can be analyzed through various dialectics, such as question-and-answer, or elucidating the questions that emerge, the answers found, and the new questions that arise. This generates an arborescence that can be schematized in a Q–A map as proposed by Winsløw et al. (2013).

In regular courses, mathematics subjects are often presented as finished works, with little or no time devoted to analyzing their importance for, and relation to, other courses or workplaces. In the ATD, this is known as the paradigm of visiting works. When an SRP is implemented in a regular course as a project or specific device, the pre-existing relations among students, professors, and milieu are modified. This can be analyzed through three didactic functions: topogenesis, which is concerned with the subjects' interactions [$X$;$Y$], chronogenesis, which deals with time, and mesogenesis, which involves the construction of the milieu, $M$. These functions respond to the questions of who, when, and how the process of mathematical modeling is developed, and the praxeologies studied are constructed.

## 3 The study

This research constitutes an exploratory case study (Yin, 2018) shaped by the design of an SRP called "pure tone mixing" and its implementation in two first-semester online mathematics courses at a university in Colombia. A team of four researchers conducted the research: a specialist in BSS, two university math professors, and a specialist in mathematics education. Three participated in designing the SRP, and two were responsible for the math course and analyzing students' SRP during its development.

## 4 Didactic engineering

Didactic engineering is a research methodology that offers a solid route for designing and implementing didactic devices in the classroom (Artigue, 2014). It was adapted to design SRP by García et al. (2019). Its four phases are as follows: (1) preliminary analysis; (2) design and a priori analysis; (3) experimentation and in vivo analysis; and (4) implementation and a posteriori analysis.

### 4.1 Preliminary analysis

Following Barquero and Bosch (2015), Bartolomé et al. (2018), and Vázquez (2017), in this phase we analyze, first, a didactic phenomenon involving mathematics for non-specialists and, second, a mathematical modeling activity in engineering, the BSS praxeology. We then perform a didactic transposition to obtain a school BSS praxeology, and so develop an epistemological model from engineering to design, in the second phase, the SRP called "pure tone mixing."

#### 4.1.1 First-year online math course for non-specialists

In our experimental context, a teaching institution and a first-year online math course that is compulsory for all majors (e.g., economics, engineering, management) but is attended mainly by engineering students, we identified an important didactic phenomenon: the lack of modeling activities related to engineering research and workplaces. The 8-week course was developed in a Canvas online platform asynchronous and synchronous modalities. Its syllabus includes basic topics covered in high school math courses required for studying university mathematics, such as the operations and properties of real numbers, linear equation systems, and elementary functions. An interactive forum enabled student–teacher interaction. Instructional activities included *scenarios* and the *projec*t, focusing on definitions and paradigmatic examples, and assigning practice exercises. Thus, each scenario proposed through virtual learning objects (e.g., animations and presentations, audiovisual media) has four moments: entry, initiation, encounter, and closure. *Entry* focuses on studying a mathematics topic, *initiation* on solving problems to apply this topic, and the *encounter* on performing exercises. Finally, *closure* concludes the scenario by offering final considerations and summarizing the knowledge acquired. The project consists of a context-specific task to apply previously studied topics designed freely by the professor. It is worth 30% of the final grade. Both the scenario and the project fall within the paradigm of visiting works. Since the professors are free to design the project, proposing an SRP based on the BSS, a mathematical modeling activity in engineering was possible.

#### 4.1.2 The BSS praxeology in an engineering research institution

The principle of BSS is inverse problem (modeling) since it is generally used in situations where neither the sources that are mixed nor the way in which they were mixed is known (type of task). The ideal case is to have the same number of sources and captors (observations). The technique used to separate this type of mixing (e.g., sound signals) is based on the matrix model $x = As$, where $\mathbf{x}$ represents the mixed signals recorded by the sensors, $A$ is the mixing matrix, and $\mathbf{s}$ represents the sources. Blind source separation (BSS) consists in recovering the sources of origin, $\mathbf{s}$ (unknown) by means of a separation matrix, $B$, that ideally corresponds to the exact inverse of the matrix $A\left(B = A^{-1}\right)$, such that a separation model, $y = B\mathbf{x}$, is produced in which $\mathbf{y}$ represents the estimated sources, which are expected to show high similarity to the sources, $\mathbf{s}$ ($\mathbf{y} \approx \mathbf{s}$). The technology is framed by several BSS algorithms that are used to obtain the separation matrix, $B$, including FastICA (Hyvarinen, 1999) and SOBI (Belouchrani et al., 1997), among others. The theoretical block $(\theta, \Theta)$ is based on notions from linear algebra (e.g., matrices, matrix transformation, the inverse of a matrix, linearity) and signal processing.

#### 4.1.3 Transposition of BSS praxeology to the school setting

The didactic transposition of BSS praxeology considers, as Vázquez (2017) described, an audio context and pure tones (sound signals). For our study, we proposed a system consisting of two sound sources that emitted pure tones of different frequencies, placed arbitrarily in a room, with two observations (recorders) that registered the mixture of the tones. Pure tones were chosen over other sounds or voices because their signals correspond to sinusoidal functions, $y(t) = a\sin(2\pi ft)$, where $t$ is the time in seconds, and $a$ and $f$ measure amplitude and frequency, respectively. Since sinusoidal functions belong to the vector space of continuous functions in $\mathbb{R}$, addition and multiplication by scalar are defined. Therefore, two continuous functions can be combined linearly, resulting in another continuous function. In the case analyzed herein, the mixture, $o_i$, of two pure tones resulted in a tone represented by $o_i(t) = a_{i,j}\sin(2\pi f_1 t) + a_{i,j}\sin(2\pi f_2 t)$, where the amplitude, $a_{i,j}$, is the intensity of the tone, which is related to the distance between the observations, $i$, and sources, $j$. The problem is to identify the frequencies of the pure tones, $f_i$, resulting in a system of equations:

$$o_1(t) = a_{1,1}\sin(2\pi f_1 t) + a_{1,2}\sin(2\pi f_2 t)$$

$$o_2(t) = a_{2,1}\sin(2\pi f_1 t) + a_{2,2}\sin(2\pi f_2 t)$$

The matrix $A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$ is called the mixture matrix. One of the techniques applied to accomplish the task—that is, finding the sources, $s_j = \sin(2\pi f_j t)$, that produce the mixtures—is to discretise the mixtures in time intervals (e.g., 14,700 intervals per second) to obtain 14,700 equations to be solved. The problem can be solved by calculating its inverse; given that $\mathbf{o} = A\mathbf{s}$, where $\mathbf{o}$ represents the vector of the mixtures (known) and $\mathbf{s}$ the vector of the sources (unknown). If the inverse of $A$ exists, $A^{-1} = B$, then the solution is $\mathbf{y}^* = B\mathbf{o}$, where $\mathbf{y}^*$ represents the vector of the estimated sources recovered. Thus, the school BSS praxeology consists of a *type of task*, in this case modeling a simulated mixture of two pure tones arbitrarily placed in a room. The *technique* involves several steps: (a) associating the recorded sound with a mixture of pure tones; (b) determining the characteristics of the mixture and its components, since pure tones are defined as a function of time by relating amplitude or intensity and frequency, the mixture must combine these components in an unknown way; (c) studying the amplitude and frequency of the pure tones where intensity is inversely proportional to the square of the distance between the source and the observation (the shorter the distance, the greater the intensity), the frequency of the tone is constant and, when known, the pure tone is determined; (d) finding the distances involved in the system, and using them to determine the intensities of the sources and observations; and (e) expressing the relation among the pure tones, intensities, and mixtures by means of linear combinations. The mixture of pure tones can be modeled by a system of linear equations, where the unknown are the sources. With reference to the *technology*, the notions of continuous function, vector, and vector space ensure the existence of the mixture as a continuous function. The concept of the equation as equality or equivalence permits representing the mixture as a combination of pure tones, and then modeling it using a system of two equations with two unknowns (hence, it corresponds to sinusoidal functions). The existence and uniqueness of the solution are based on elements of linear algebra, the matrix model, and the notions of the inverse matrix. The existence of the mixture is based on the principles of vector spaces, linear transformations, and linear combinations. The study of audio signals, finally, is based on signal processing. Finally, *theory* is based on linear algebra and signal theory.

### 4.2 Design of the "pure tone mixing" SRP and a priori analysis

In this phase, the SRP named "pure tone mixing" was designed to build the school BSS praxeology so that we could answer our first research question. An a priori analysis was conducted to determine the milieu required to develop the SRP in a first-semester math course and to envisage the general way in which the students in this course could develop it in order to answer our second research question. The SRP was proposed in the following situation: "Suppose that two speakers (sources) and two recorders (observations) are placed randomly in a room. Each speaker emits a different pure tone, and each recorder receives the sound produced by both speakers. How are the sounds combined to produce the mixtures? $\left(\boldsymbol{Q}_0\right)$. In other words, the generating question is: how is a mixture of sounds modeled? Three stages associated with building the school BSS praxeology were conceptualized: (1) to study the first approach to sound mixing; (2) to analyze the simulated pure tone mixing and variable selection; and (3) to examine the systems of equations used to model the simulated mixture. Accordingly, three main media were proposed, one for each stage: a video entitled "The science of hearing" (youtu.be/LkGOGzpbrCk) on the brain's ability to separate different sound sources and accurately identify their nature and position, and two digital resources created by a math professor on the research team that simulate a 10 m × 10 m room with two speakers that emit pure tones, plus two recorders. The first resource only allowed the students to listen to two different mixtures with the distance between speakers and recorders fixed. In the second, they could modify both the distance and frequencies of the tones. Finally, three initial questions considering these media were proposed for each stage: $Q_1$: what does the brain do to the mixture of sounds?, $Q_2$: how were the sources combined to produce the recorded mixtures? And $Q_3$: how to obtain the mix recorded by the microphones? These stages, the use of these media, and these questions $(Q_1, Q_2, Q_3)$ generated some derivate questions (see Fig. 3).

We hypothesized that the students would build the school BSS praxeology based on the study of initial questions $(Q_1, Q_2, Q_3)$ by creating a milieu based on the use of the two digital resources, and that this would help them move forward to study the variables of frequency, distance, and amplitude, and to perform a first approach to a mathematical model

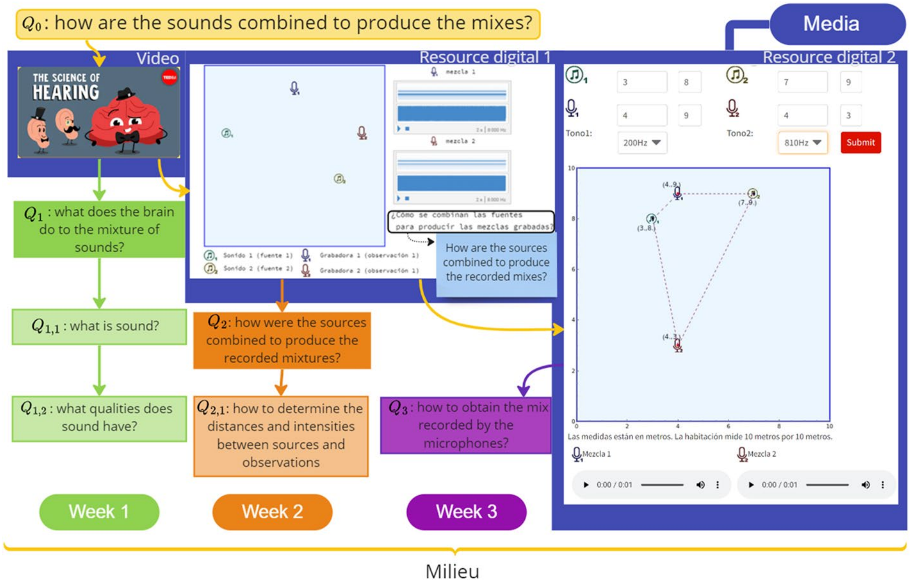


**Fig. 3** Media, derivate questions envisaged, and organization of time

for sound mixing by following this organization of the work developed by teams of students over 3 weeks. Each team could organize itself autonomously but had to hand in a weekly report on their activities. At the end of each week, a synchronous meeting was held to present the advances of some students and discuss doubts, ideas, and routes for continuing the work. This procedure is schematized in Fig. 3.

### 4.3 SRP experimentation and in vivo analysis

The SRP was implemented over 4 weeks, one more than originally considered in the a priori analysis, during two online courses in the second semester of 2019, with 232 students enrolled in each one. The students worked in teams (4–5 per team, 47–50 teams per course). Each team used a unique forum to study specific questions during the week and record their work. Synchronous meetings were held weekly in both courses (to share answers). In the final week, each team presented a report on all the questions it had studied. Two professors who were members of the research team implemented the project, reviewed the forums, conducted the synchronous meetings, and reviewed the teams' reports.

An in vivo analysis was developed parallel to the implementation of this SRP to identify the derived questions proposed by the students in their forums, the media they used, the websites they consulted, their use of the two digital resources, their dialogues with experts, and the answers they found and adapted. This analysis helped the professors prepare for the synchronous meetings in which the paths taken by the teams were compared, their feasibility questioned, and the best route to be followed determined. A general Q–A map was elaborated that includes the topics that the students considered most relevant. To illustrate this, a Q-map is shown in Fig. 4.

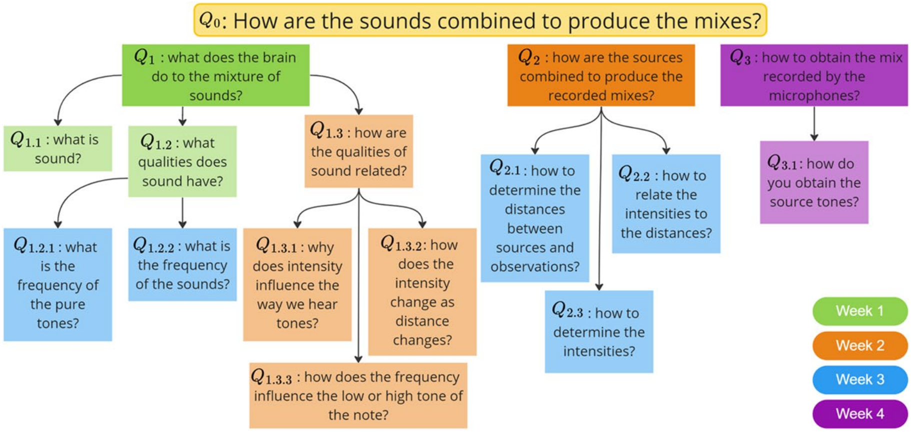


**Fig. 4** Q-map from the student's work in vivo analysis

### 4.4 A posteriori analysis

The a posteriori analysis was performed once the students had made their SRP. Focused on the inquiry processes they had carried out, it evidenced how the media suggested (video, digital resources) and those proposed by the students themselves (websites) made it possible to address both the initial study questions $\left(Q_1, Q_2, Q_3\right)$ and the later, derived questions to shape the process that determined the consensual answer, $A^\heartsuit$. Seven teams (with a total of 28 students) selected by convenience were taken as the sampling units. While each team planned their work autonomously, the plenary sessions played a significant role in having them propose, collectively, one sole way of developing the SRP. A first analysis showed that the teams addressed the same questions, consulted similar websites, and used similar techniques to study and relate the variables. But the ways in which they communicated varied widely. Some teams provided less detail on how they found responses on the websites and how they were studied. Two data types were collected: participation in the forums and the videos of the four synchronous meetings. The teams selected were 2, 3, 4, 12, 26, 30, and 44. All forum participations were coded with the student's number—1, 2, 3, 4, or 5—and the team number. For example, S1-T30 corresponds to student 1—team 30.

## 5 Student activities to address the SRP "pure tone mixing"

In this analysis, we considered three stages associated with building the school BSS praxeology: (1) study the first approach to sound mixing; (2) analyze the simulated pure tone mixing and variable selection; and (3) examine the systems of equations used to model the simulated mixture. This is illustrated below.

### 5.1 The first approach to sound mixing

The students' first approach to sound mixing involved analyzing the video "The science of hearing" (Oliver, 2018) and asking question $Q_1$: what does the brain do to the mixture of sounds?, and the derived question $Q_{1.1}$: What is sound? Though the students' answers varied widely, specific trends were observed. Students from non-science majors (e.g., psychology) focused their attention on anatomy, the functioning of the nervous system, and humans' ability to identify and interpret sounds. An example of these answers is shown in Fig. 5. Students in scientific programs (e.g., engineering), in contrast, mainly identified elements of sound, such as frequency, intensity, wave amplitude, and distance, as illustrated in Fig. 6.

The description of these variables was related to everyday contexts. For example, the sound was related to vibration: "[…] the most important characteristic of such sounds [string vibrations] is the pitch or frequency, because if a string is struck it vibrates, the more times it is struck, the louder the sound will be" (S2-T44). Distance was recognized as a variable in the model and initially associated with closeness or remoteness, which abstracts the relation that can be established between sound intensity and distance from the receiver: "[…] when an airplane passes by, or it's going to rain, you hear thunder, the sound will be louder or [softer] according to [their] proximity" (S1-T30). Reflections like these on the initial questions motivated the students to investigate the qualities/properties of sound. This corresponds to step (a) of the school praxeology technique.

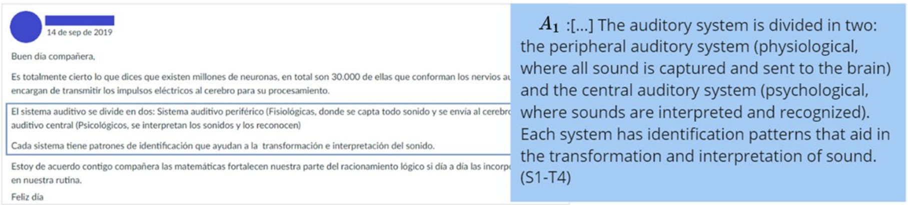


**Fig. 5** Participation in the S1-T4 forum

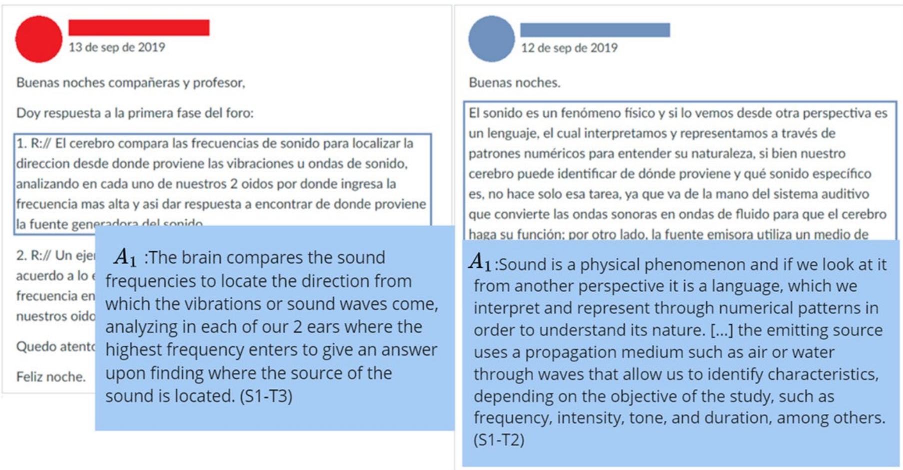


**Fig. 6** Participation in the S1-T2 forum

## 5.2 Simulated mixtures and identification of variables

In the second stage, simulated mixtures were studied using the first digital resource (Fig. 7), and a selection of variables was made: distance, frequency, and amplitude, corresponding to steps (a), (b), and (c) of the school praxeology technique. The students were thus able to explore the combination of sounds and begin to study question $Q_2$: how were the sources combined to produce the recorded mixtures?

The first digital resource simulated an audible mixture of two pure tones of different frequencies (sources 1 and 2). Two recorders (observations 1 and 2) captured the combination of the pure tones from the sources, which differed because the recorders were placed at distinct distances from sources 1 and 2. Students listened to the mixtures and began to investigate how to model them. Some groups decided, first, to do additional research on the nature of sound, its qualities, characteristics, and relations $\left(Q_{1.3}\right)$. For example, student S1-T2 first described the sound elements and then used a graphic representation of the pure tone to discuss frequency and amplitude (Fig. 8). This corresponded to steps (b) and (c) of the school BSS technique.

After identifying the first variables involved in how the mixture is combined, this student talked about distance and how the "mixture depends on the emission distances of each one" (S1-T12). This statement is supported by the representation of the two-source, two-recorder configuration of the first digital resource (Fig. 9). Each recorder captured the same sources at a constant frequency (since they are pure tones), but the mixtures were distinct, leading to the conclusion that distance is the variable that determines the relative proportions of the sources in the mixture.

Other groups that needed help in recognizing distance as an essential variable failed to perceive the sound of each recorder as a mixture of the sources because the sounds of the mixed pure tones were similar. The professor first clarified that the devices recorded mixtures of the two pure tones, before introducing the idea of intensity to guide the way in which sound was considered: "Since one is closer than the other, the intensity is higher, so the mixture is heard more loudly" (Fig. 10). The teams thus identified a relation between distance and the mixtures produced but also began to establish another, this one between intensity and distance.

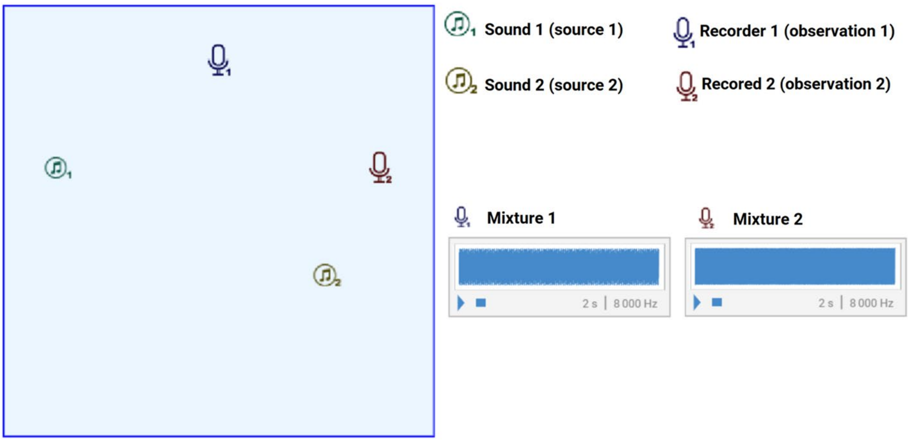


**Fig. 7** First digital resource

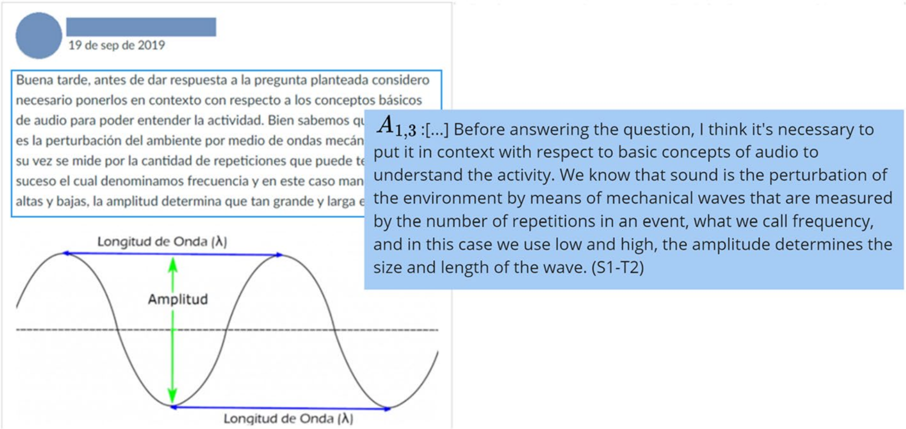


**Fig. 8** Participation by S1-T2. Research on sound

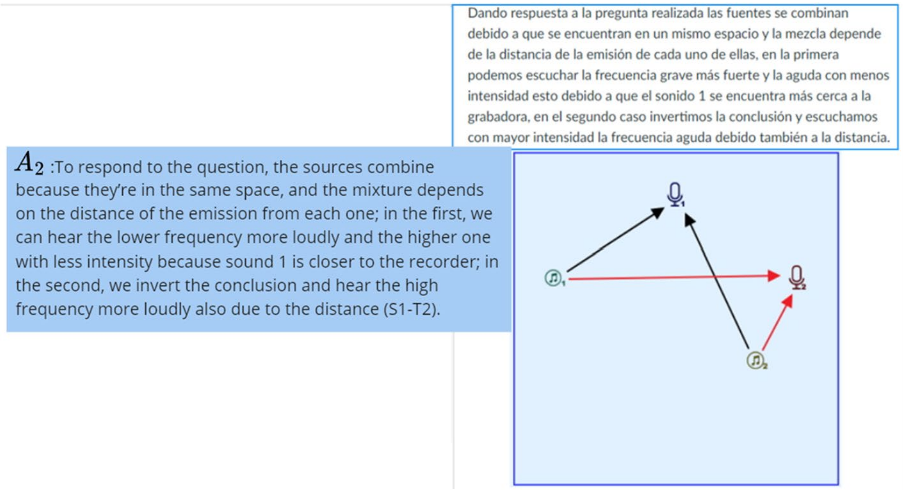


**Fig. 9** Continued participation by S1-T2

The exchanges between the students and the professor gave the course continuity. A new objective that emerged consisted in relating intensity to distance mathematically (i.e., how does the intensity change with distance?). Continuing with excerpts from group T30's forum, student S2-T30 described the physical property of intensity "[…] The larger the wave, the louder the sound". The professor took advantage of the fact that the students had moved on from simply listening to the mixtures to studying the variables involved to propose an analysis of the relations among intensity, distance, and frequency: "Intensity influences the way we hear the tones –why?– and changes as the distance changes, but how? Frequency influences how low or high the pitch is, but how?" ($Q_{1.3.1}$, $Q_{1.3.2}$, and $Q_{1.3.3}$). Students did research to attempt to answer these questions, as illustrated by student S2-T30's participation in the forum (Fig. 11).

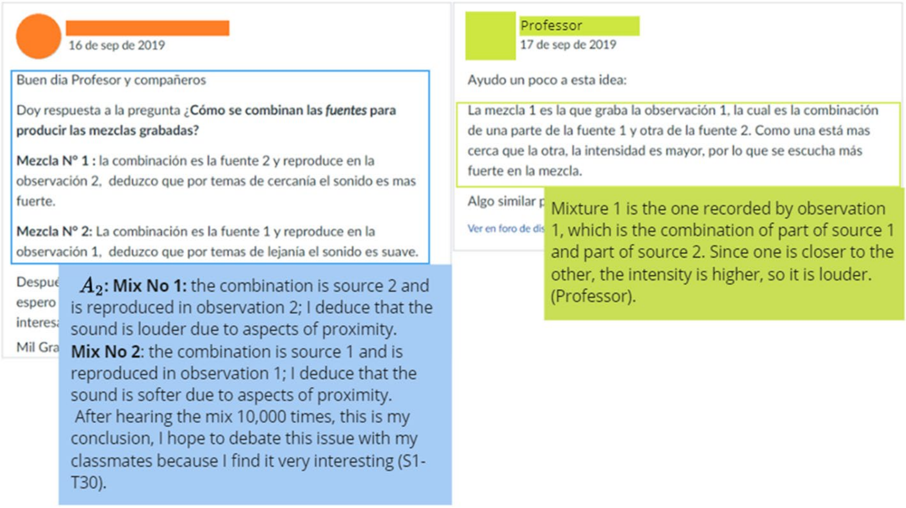


**Fig. 10** S1-T30's participation and the professor's feedback

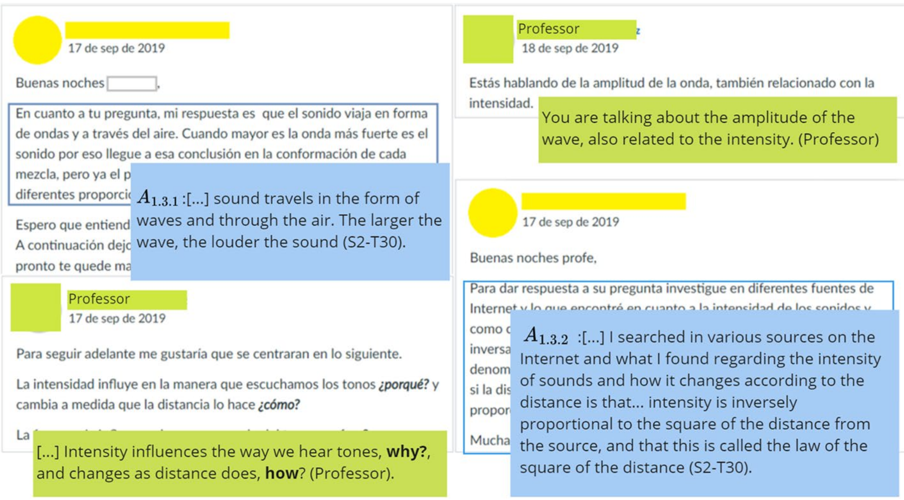


**Fig. 11** S2-T30's and the professor's comments on the relation of intensity to distance

In the synchronous encounter that closed this stage (week 2), several variables were identified in the simulated mixtures: the speed of sound, the medium in which sound is propagated, the shape of the room, the echo or direction of the sound, and its intensity, frequency, and distance (Fig. 12). Participants agreed that the mixtures heard in the recorders were higher or lower due to the type of sources that produced them and the distance between the sources and the recorders. This discussion resulted in the selection of two variables: the frequency of the sources—related to high vs. low tones—and the intensity of the tones, which depends on the distance between the sources and the recorders; as the professor pointed out:

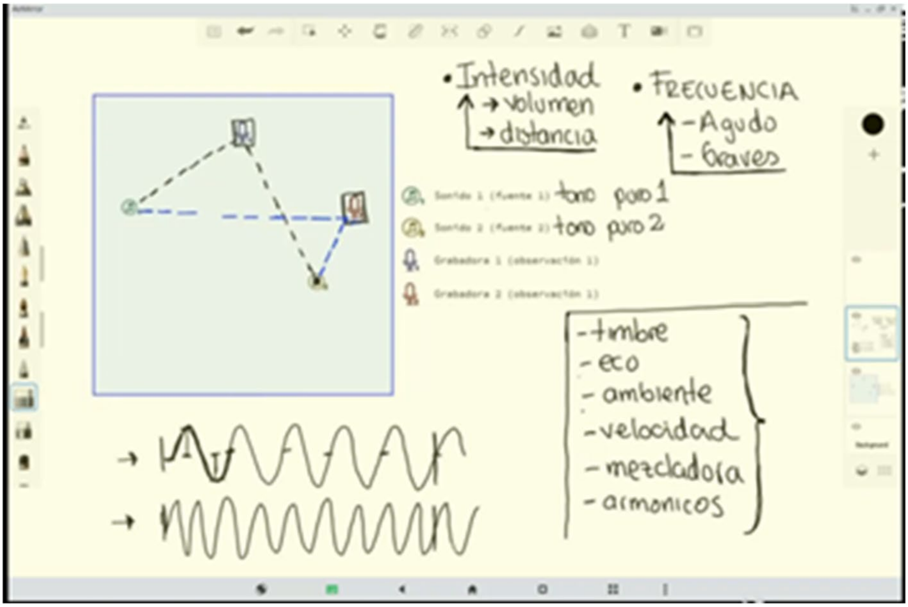


**Fig. 12** The synchronous meeting, interaction of students with the professors

> Distance is related to intensity; how close or how far the source is to the observation will tell me how intense [the sound] is, and that will have repercussions on when we hear [it] and how loudly or softly we hear it. Another variable that intervenes in this situation is frequency, which we identify with high and low tones […] (professor during the synchronous meeting).

Participants discussed and agreed that the mixtures heard in the recorders were higher or lower due to the type of sources that produced them, and that the distance between the sources and the recorders was a determining variable. The decision was taken to select three variables for further study: source-to-recorder distance, frequency, and source amplitude (or loudness). New questions arose, some related to the frequency of pure tones and mixtures. These included $Q_{1.2.1}$: what is the frequency of the pure tones?, $Q_{1.2.2}$: what is the frequency of sounds?, and $Q_3$: how to obtain the mix recorded by the microphones?

### 5.3 Systems of equations used to model a simulated mixture

In the third stage, students explored configurations between sources and recorders, and frequencies and listened to several mixtures using the second digital resource. The study of specific configurations allowed them to obtain specific frequencies and calculate the source-to-recorder distance using the Euclidean formula. It was possible, through collaborative work between students and professors during the synchronous meeting where professors initially proposed a specific notation, to reach the mathematical model for simulated mixing. They then translated a configuration studied in first equality, then in a two-equation linear system, and finally as a family of time-dependent systems as in the mathematical model of simulated mixing. That process made it possible to move ahead and work on steps (d) and (e) of the school BSS technique.

Students initially encountered certain difficulties in making sense of each variable. For example, S2-T30 began by proposing a configuration of sources and observations (Fig. 13) before identifying that distance determines that a "higher frequency" tone is heard less in the microphone which is farther away. He said, "[…] Although the frequencies of each tone are different [200 Hz and 800 Hz], tone 2 is louder and is heard with less intensity in microphone 1 because it is at a greater distance" (Fig. 13). This comment revealed that S2-T30 had confused the coordinates (2, 2) with the distance 2.2, so the professor intervened:

> […] The (2, 2) of microphone 1 is not the distance, but the coordinates, so it isn't 2.2, but (2, 2) […] that is, two meters to the right and two meters above. […] I invite the team to study and find the distances between the sources and observations [$Q_{2.1}$]. You can start with S2-T30's configuration (Fig, 13), but if someone wants to use another, that's fine.

After several interactions, S2-T30 proposed the Euclidean formula: $d(P1,P2)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ and all teams experimented with the configurations they had chosen in order to find the distances. Team 30 calculated the distances without determining that they were the coefficients of the system of linear equations. During the penultimate (week 3) synchronous meeting, the students' most important answers were presented, and, on that basis, the technique for determining the intensity of each source for each observation, and its importance in the problem, was explained; that is, the greater the distance, the lower the intensity. Specifically, intensity is inversely proportional to the square of the distance $\left(A_{2.2}\right)$. The professors (i) introduced a notation so that the students could express each observation (mixture) considering the sources $F_1$ and $F_2$: $O_1=\text{combination}(F_1,F_2)$, $O_2=\text{combination}(F_1,F_2)$; (ii) discussed the role of intensities $\left(I=\frac{1}{d^2}\right)$ and proposed $O_1=\text{combination}\left(I_{(F_1,O_1)}\cdot F_1, I_{(F_2,O_1)}\cdot F_2\right)$, and $O_2=\text{combination}\left(I_{(F_1,O_2)}\cdot F_1, I_{(F_2,O_2)}\cdot F_2\right)$; and (iii) pointed

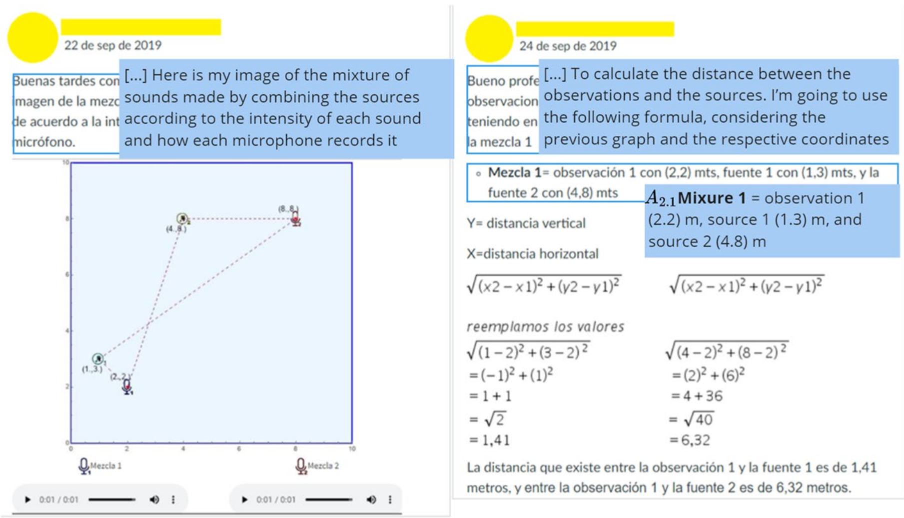


**Fig. 13** Participation of S2-T30

out that determining the mixture requires superimposing the two sources: $O_1 = I_{(F_1,O_1)} \cdot F_1 + I_{(F_2,O_1)} \cdot F_2$ and $O_2 = I_{(F_1,O_2)} \cdot F_1 + I_{(F_2,O_2)} \cdot F_2$ to obtain a system of linear equations.

As a final activity, each team was given a different configuration and asked to determine the system of equations associated with it. The teams calculated the distances between sources and observations using, primarily, the Euclidean formula to determine intensity with the formula $I = \frac{1}{d^2}$, then multiplying by the frequency, which was still handled as a black box (Williams & Wake, 2007). Through teamwork, the techniques were compared, validated, and agreed upon. In this way, all the teams arrived at the initial mathematical model of the simulated mixture (Fig. 14).

During the final synchronous meeting (week 4), the professors emphasized that the system of equations found cannot be solved yet because of the data, $O_1$ and $O_2$. The model corresponds to a family of time-dependent systems because sounds, whether pure tones or mixtures ($F_1$ and $F_2$, $O_1$ and $O_2$), can be represented as time-dependent functions. Thus, to solve a system of equations, it is necessary to determine an instant that can be measured discretely by dividing each one into small intervals, such that the sound can be represented by a number at a given instant. For example, if each second is divided into 14,700 intervals, then the first ten data obtained in the first second of the mixtures $O_1$ and $O_2$ are $\{0, 0.161, 0.320, 0.475, 0.622, 0.762, 0.891, 1.009, 1.115, 1.208\}$ and $\{0, 0.080, 0.157, 0.227, 0.287, 0.335, 0.369, 0.388, 0.391, 0.38\}$, respectively. The discretization of the mixtures made it possible to work with a set of specific values. A system of linear equations was created to model the simulated mixture at a given instant and thus obtain a solution for that instant (Fig. 15).

This third phase made it possible to construct an initial model—a system of linear equations—of the simulated mixture $(A_3)$ with which the values of the sources can be obtained for each instant. Distance was the variable that students found most understandable. The professors showed that frequency has a proportional relation to each mixture, and that superimposing the two mixtures generates a first model for the observations. However, that

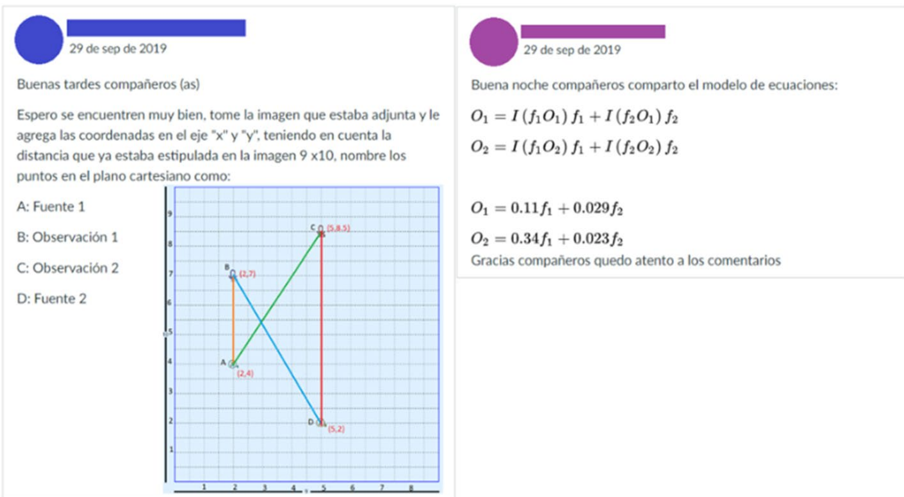


**Fig. 14** Calculation of distances and the system of equations found (S1-T26 and S2-T26)

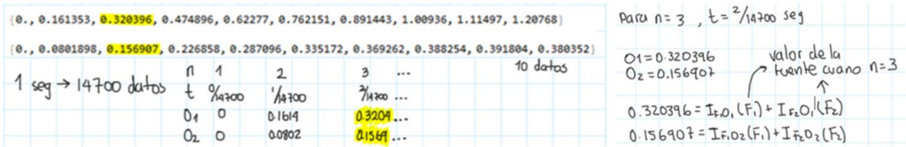


**Fig. 15** Professor's explanation of the system of equations determined

model failed to optimally exploit the constant frequency of the pure tones and their mathematical expression, $y = a\sin(2\pi ft)$.

## 6 Overview of the implementation of the SRP

Analyzing the inquiry dynamic induced by the SRP in two first-semester math courses and how this made it possible to construct the school BSS praxeology is key to illustrating both the scope and limitations of this inquiry device and the role of the institutional conditions. To this end, which also allows us to answer our third research question, we use the three didactic functions mentioned at the outset: *mesogenesis*, *topogenesis*, and *chronogenesis*.

The *mesogenesis* function makes it possible to analyze the dynamics of inquiry that the students followed to study the mixtures of sounds and their mathematical modeling. The media—video, websites, forums, and digital resources, plus the experts' and professors' explanations and students' answers—all played essential roles in constructing the milieu, the dynamics of inquiry, and the study of $Q_0$. In the first stage, the video analysis allowed students to generate a natural language discourse of sound mixtures, a key element in modeling activities (Lobry, 2003). The websites they used most often to search for information included *Wikipedia*, scientific blogs, and video sites, which they learned to use differentially. At first, their work consisted only of paragraphs copied from websites, but the students gradually began to use the information found to express their own ideas and progress in the study of $Q_0$. This constitutes a modest transition between the paradigms of visiting works and questioning the world.

The first digital resource induced the students to focus on the variables of frequency, distance, and intensity. The exploratory work with a single configuration of sources and recorders motivated them to delve more deeply into the mixture of two pure tones and the relation between intensity and distance. The second digital resource led them to consider numerous configurations, analyze the relations among the variables, and then express them through a system of linear equations.

The synchronous meetings were fundamental in several ways: for choosing a route for the research, validating proposals on the relations between variables, and proposing the system of linear equations as a model for the collaborative, progressive construction of the mathematical model. The professor's explanation of discretization gave meaning to the model developed as it represented not only a linear equation system but also a family of instantaneous systems. Knowing the value of the mixture at each instant makes it possible (with the model) to find the value of the two pure tones at that instant. In this way, these first-semester students developed a first approach to the matrix model used in the ideal case of the BSS praxeology while also opening possibilities to deepen their explorations of this topic in other courses, such as linear algebra or calculus.

The *chronogenesis* function facilitates analyzing the time organization of the SRP. Four weeks of work—instead of three—dedicating 1 week to the initial questions ($Q_1, Q_2$) and 2 weeks to $Q_3$ led the students to work with families of systems of linear equations. The quality and quantity of their participations in the forum were high: 78% vs. 60% in traditional projects in previous years. This suggests that they devoted a great deal of time to the inquiry process and to researching and structuring their contributions (use of images and mathematical writing, among other activities). The professors also devoted more time to the SRP than to traditional projects through their daily review of forums, weekly meetings of the teaching team to review students' progress, determining subsequent activities, modifying or developing the ensuing SRP activities, and the synchronous meetings that required reviewing decision-making and the progress made towards conclusions.

*Topogenesis*—the activity of the students and professors—was characterized by the inquiry. The students learned how to work efficiently as a team. At first, they did not consider the contributions presented by each one but eventually, at least in some of the groups, discussions came to revolve around key participations that allowed the activity to progress. A concrete example was determining the relation between intensity and distance. Team members helped each other find the distances of different configurations, or correct those presented by classmates to calculate the Euclidean distance. The teaching team primarily played the role of rapporteur by showing the progress of some groups, promoting reflections on the results of their research, and guiding the discussion questions. Most of the groups welcomed these dynamics; in fact, they did not always wait for the professor's explanations or validations but began to look for final answers, shared them with the team, and tried to use them to construct the answer to the generating question. In some cases, mathematical models worked as "black boxes" because the duration of the SRP was insufficient to open all the relevant boxes of this kind. However, this may spur students' interest in investigating fields where they have weaknesses or building upon the knowledge they acquired.

## 7 Conclusion

The answer to our first research question was the SRP "pure tones mixing," a didactic device for teaching mathematical modeling based on the engineering BSS method for the first-year math course that shares several principles of the "learning-by-doing" approach, such as guiding students towards interdisciplinary, collaborative, autonomous work. Moreover, the design of this didactic device, based on didactic engineering methodology, gives it a base structure for the inquiry process. An essential step was determining the school BSS praxeology and a question, $Q$: how is the simulated mixing sound modeled? That constituted the first step in studying the more complex question: how to separate mixtures? which is the engineering research question. The inquiry process to study $Q$ in a first-year math course thus consists in constructing the school BSS technique for mathematical modeling through three stages: (1) study the first approach to sound mixing; (2) analyze the simulated pure tone mixing and variable selection; and (3) examine the systems of equations used to model the simulated mixture. Performing these three stages (the SRP) required determining the didactic milieu, as posed in our second research question. To that end, the media were created and chosen based on the epistemological model composed by the engineering and school BSS praxeologies; in the case of this virtual modality: video, digital resources, forums, and online synchronous meetings. The digital resources provided allowed the students to progressively analyze three variables—intensity, frequency, and amplitude—and thus develop a base to be proposed in the virtual synchronous meetings and, under the professors' guidance, a first mathematical model for the simulated mixing of sounds.

Regarding our third research question—the institutional conditions that permitted implementing the SRP—we recognize that the opportunity to freely design a project, an instructional material, and a way to develop it in four weeks provided an optimal space for proposing the SRP. Also, it is essential to recognize that a math professor who participated in all phases of didactic engineering reached a solid background in the BSS method, allowing the math professor's team to conduct this SRP in a regular math first-year course. A question arises: how can we communicate this SRP with other math professors without this BSS background? The final conclusion we take from this study is that the approach developed led the first-year university students to experiment with a highly challenging mathematical modeling task, learn to conduct research, and explore and analyze multidisciplinary questions that, to a considerable degree, represent actual work in engineering.

**Funding** This work was supported by the Secretaría de Investigación y Posgrado, Instituto Politécnico Nacional (SIP-20211182) and the Institución Universitaria Politécnico Grancolombiano.

**Data Availability** Following our institutions' ethical guidelines, we can provide the data to everyone to ask the corresponding author.



## References

Alpers, B., Demlova, M., Fant, C.-H., Gustafsson, T., Lawson, D., Mustoe, L., Olsson-Lehtonen, B., Robinson, C., & Velichova, D. (Eds.). (2013). *A framework for mathematics curricula in engineering education*. European society for engineering education (SEFI).

Alpers, B. (2011). The mathematical expertise of mechanical engineers: Taking and processing measurements. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), I*nternational perspectives on the teaching and learning of mathematical modelling* (pp. 445–456). Springer, Dordrecht. https://doi.org/10.1007/978-94-007-0910-2_44

Artigue, M. (2014). Perspectives on design research: The case of didactic al engineering. In A. Bikner-Ahsbah, C. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education. Advances in mathematics education* (pp. 467–496). Springer, Dordrecht. https://doi.org/10.1007/978-94-017-9181-6_17

Barquero, B., Bosch, M., & Gascón, J. (2019). The unit of analysis in the formulation of research problems: The case of mathematical modelling at university level. *Research in Mathematics Education, 21*(3), 314–330. https://doi.org/10.1080/14794802.2019.1624602

Barquero, B., & Bosch, M. (2015). Didactic engineering as a research methodology: From fundamental situations to study and research paths. In A. Watson & M. Ohtani (Eds.), *Task design in mathematics education* (pp. 249–272). Springer, Cham. https://doi.org/10.1007/978-3-319-09629-2_8

Bartolomé, E., Florensa, I., Bosch, M., & Gascón, J. (2018). A 'study and research path' enriching the learning of mechanical engineering. *European Journal of Engineering Education, 44*(3), 330–346. https://doi.org/10.1080/03043797.2018.1490699

Belouchrani, A., Abed-Meraim, K., Cardoso, J.-F., & Moulines, E. (1997). A blind source separation technique using second-order statistics. *IEEE Transactions on Signal Processing, 45*(2), 434–444. https://doi.org/10.1109/78.554307

Bissell, C., & Dillon, C. (2000). Telling tales: Models, stories and meanings. *For the Learning of Mathematics, 20*(3), 3–11.

Bissell, C., & Dillon, C. (Eds.). (2012). *Ways of thinking, ways of seeing*. Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-25209-9

Chevallard, Y. (1991). *La transposition didactique [The didactic transposition]* (Rev). La pensée sauvage.

Chevallard, Y. (1999). L'analyse des pratiques enseignantes in théorie anthropologique du didactique [The analysis of teaching practice in the anthropological theory of the didactic]. *Recherches in Didactique Des Mathématiques, 19*(2), 221–266.

Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education, 12*, 71–114.

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. Cho (Ed.), *The proceedings of the 12th international congress on mathematical education* (pp. 173–187). Springer, Cham. https://doi.org/10.1007/978-3-319-12688-3_13

Damlamian, A., Rodrigues, J. F., & Sträßer, R. (Eds.). (2013). *Educational interfaces between mathematics and industry*. Springer international publishing. https://doi.org/10.1007/978-3-319-02270-3

Frejd, P., & Bergsten, C. (2016). Mathematical modelling as a professional task. *Educational Studies in Mathematics, 91*(1), 11–35. https://doi.org/10.1007/s10649-015-9654-7

Gainsburg, J. (2007). The mathematical disposition of structural engineers. *Journal for Research in Mathematics Education, 38*(5), 477–506. https://doi.org/10.2307/30034962

García, F. J., Barquero, B., Florensa, I., & Bosch, M. (2019). Diseño de tareas en el marco de la teoría antropológica de lo didáctico [Task design in the framework of the anthropological theory of the didactic]. *Avances de Investigación en Educación Matemática, 15*, 75–94. https://doi.org/10.35763/aiem.v0i15.267

Gascón, J., & Nicolás, P. (2022). ATD on relationships between research and teaching. The case of a didactic problem concerning real numbers. In Y. Chevallard, B. Barquero, M. Bosch, I. Florensa, J. Gascón, P. Nicolás, & N. Ruiz-Munzón (Eds.), *Advances in the anthropological theory of the didactic* (pp. 13–24). Birkhäuser, Cham. https://doi.org/10.1007/978-3-030-76791-4_2

Hyvarinen, A. (1999). Fast and robust fixed-point algorithms for independent component analysis. *IEEE Transactions on Neural Networks, 10*(3), 626–634. https://doi.org/10.1109/72.761722

Kent, P., Noss, R., Guile, D., Hoyles, C., & Bakker, A. (2007). Characterizing the use of mathematical knowledge in boundary-crossing situations at work. *Mind, Culture, and Activity, 14*(1–2), 64–82. https://doi.org/10.1080/10749030701307747

Lobry, C. (2003). Modèles mathématiques et modèles informatiques [Mathematical models and computer models on the theory of the model]. *Annales Des Ponts Et Chaussées, 2003*(107–108), 10–18. https://doi.org/10.1016/s0152-9668(03)80003-7

Moon, H. A., Asaki, T. J., & Snipes, M. A. (2022). Application-inspired linear algebra. *Springer*. https://doi.org/10.1007/978-3-030-86155-1

Muhammad, N., & Srinivasan, S. (2020). A problem solving based approach to learn engineering mathematics. In M. Auer, H. Hortsch, & P. Sethakul (Eds.), *The impact of the 4th industrial revolution on engineering education. ICL 2019. Advances in intelligent systems and computing* (pp. 839–848). Springer, Cham. https://doi.org/10.1007/978-3-030-40274-7_81

Muller, J., & Young, M. (2014). Disciplines, skills and the university. *Higher Education, 67*(2), 127–140. https://doi.org/10.1007/s10734-013-9646-4

Nedaei, M., Radmehr, F., & Drake, M. (2019). Exploring engineering undergraduate students' attitudes toward mathematical problem posing. *Journal of professional issues in engineering education and practice, 145*(4). https://doi.org/10.1061/(asce)ei.1943-5541.0000418

Oliver, D. L. (2018). The science of hearing [Video]. In *TED Talks*. https://www.ted.com/talks/douglas_l_oliver_the_science_of_hearing. Accessed January 2023

Pollak, H. O. (1988). Mathematics as a service subject – Why?. In A. Howson, J. Kahane, P. Lauginie, & E. Turckeim (Eds.), *Mathematics as a service subject* (28–34). Cambridge university press. https://doi.org/10.1017/cbo9781139013505.004

Romo-Vázquez, A. & Artigue, M. (2023). Challenges for research on tertiary mathematics education for non-specialists: Where are we and where are we to go? In R. Biehler, M. Liebendörfer, G. Gueudet,

C. Rasmussen. C. Winsløw. (Eds.), *Practice-oriented research in tertiary mathematics education.* (pp. 535–557). Springer. https://doi.org/10.1007/978-3-031-14175-1_26

Romo-Vázquez, A. (2009). *La formation mathématique des futurs ingénieurs* (Unpublished doctoral dissertation). Université Paris-Diderot. https://tel.archives-ouvertes.fr/tel-00470285/document

Rozhkova, S. V., Ustinova, I. G., Yanuschik, O. V., & Korytov, I. V. (2020). The use of the project-based learning in the study of the course of mathematical analysis. In M. Auer, H. Hortsch, & P. Sethakul (Eds.), *The impact of the 4th industrial revolution on engineering education. ICL 2019. Advances in intelligent systems and computing* (pp. 871–879). Springer, Cham. https://doi.org/10.1007/978-3-030-40274-7_85

Savery, J. R. (2006). Overview of problem-based learning: Definitions and distinctions. *Interdisciplinary journal of problem-based learning*, *1*(1). https://doi.org/10.7771/1541-5015.1002

Schmidt, K., & Winsløw, C. (2021). Authentic engineering problems in service mathematics assignments: Principles, processes and products from twenty years of task design. *International Journal of Research in Undergraduate Mathematics Education, 7*(2), 261–283. https://doi.org/10.1007/s40753-021-00133-0

Schukajlow, S., Kaiser, G., & Stillman, G. (2018). Empirical research on teaching and learning of mathematical modelling: A survey on the current state-of-the-art. *ZDM-Mathematics Education, 50*(1–2), 5–18. https://doi.org/10.1007/s11858-018-0933-5

Siero González, L. R., EchevarríaCepeda, L. A., Romo-Vázquez, A., & Navarro Torres, J. (2022). Design of a rehabilitation device for thrombosis: A mathematical modelling activity in the training of engineers. *Avances de Investigación en Educación Matemática, 21*, 107–134. https://doi.org/10.35763/aiem21.4258

van der Wal, N. J., Bakker, A., & Drijvers, P. (2017). Which techno-mathematical literacies are essential for future engineers? *International Journal of Science and Mathematics Education, 15*(S1), 87–104. https://doi.org/10.1007/s10763-017-9810-x

Vázquez, R. (2017). *Diseño de actividades didácticas basadas en modelización para la formación matemática de futuros ingenieros [Design of didactic activities based on modelling for the mathematical training of future engineers]* [Unpublished doctoral dissertation]. CICATA – IPN.

Velten, K. (2009). *Mathematical modeling and simulation: Introduction for scientists and engineers*. John Wiley & Sons.

Williams, J., & Wake, G. (2007). Black boxes in workplace mathematics. *Educational Studies in Mathematics, 64*(3), 317–343. https://doi.org/10.1007/s10649-006-9039-z

Winsløw, C., Matheron, Y., & Mercier, A. (2013). Study and research courses as an epistemological model for didactic. *Educational Studies in Mathematics, 83*(2), 267–284. https://doi.org/10.1007/s10649-012-9453-3

Yin, R. K. (2018). *Case study research and applications: design and methods*. SAGE Publications.

## Authors and Affiliations

**Camilo Andrés Ramírez-Sánchez**[1,2] · **Avenilde Romo-Vázquez**[3] · **Rebeca Romo-Vázquez**[4] · **Diana Velásquez-Rojas**[1]

✉ Avenilde Romo-Vázquez
avenilde.romo@cinvestav.mx

Camilo Andrés Ramírez-Sánchez
caramirezs@poligran.edu.co

Rebeca Romo-Vázquez
rebeca.romo@academicos.udg.mx

Diana Velásquez-Rojas
dvelasquez@poligran.edu.co

1 Institución Universitaria Politécnico Grancolombiano, Calle 61 # 7-69, Bogotá, Colombia

2 Instituto Politécnico Nacional, Av. Luis Enrique Erro S/N, Unidad Profesional Adolfo López Mateos, San Pedro Zacatenco, C.P. 07738 Mexico City, Mexico

3 Cinvestav, Av. Instituto Politécnico Nacional, 2508, San Pedro Zacatenco, Gustavo A. Madero, C.P. 07360 Mexico City, Mexico

4 Departamento de Bioingeniería Traslacional, CUCEI, Universidad de Guadalajara, Blvd. Marcelino García Barragán #1421, Esq. Calzada Olímpica, C.P. 44430 Guadalajara, Jalisco, Mexico