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Stories of devoted university students: the mathematical experience as a form of ascesis

Francesco Beccuti[1,2] · **Paola Valero**[3] · **Ornella Robutti**[1]

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Abstract
Drawing on autobiographical essays written by master's students in mathematics preparing to become teachers, we investigate what mathematical identity these students articulate and how. By means of a discursive thematic analysis centered on the notion of ascesis, we show that the participants' identity revolves around a characterization of mathematics as a challenging, useful, and comforting activity or knowledge, which is however regarded negatively by others. Indeed, mathematics is described as a uniquely challenging activity which requires an increasingly demanding self-discipline. Moreover, mathematics is depicted as a variously useful form of knowledge which is additionally capable to offer comfort to those who engage with it. However, the participants often remark that other people regard mathematics negatively, a fact explained by stressing others' inability or unwillingness to understand or appreciate mathematics' inherent positive features. This sets the boundary of an ideal club of math enthusiasts whose elitist membership is regulated in terms of acceptance or refusal of its constitutive values. Belonging to the club as well as proselytizing in order to recruit new members appears to be central to the participants' mathematical identity.

Keywords University students · Prospective teachers · Identity · Subjectification · Ascesis

1 Introduction

Engagement with mathematics is not solely an intellectual matter but relates more broadly to the identities that individuals and groups develop with respect to the discipline and to the world. In other words, coming to know mathematics goes hand in hand with becoming a type of person (Radford, 2008, 2018). Research in mathematics education has employed the notion of *identity* to investigate students' relationships with mathematics predominantly with reference to failure, disengagement, and exclusion (cf. Darragh, 2016; Lutovac & Kaasila, 2018; Radovic et al., 2018). In particular, research on identity within tertiary

✉ Francesco Beccuti
francesco.beccuti@unito.it

[1] Department of Mathematics, University of Turin, Turin, Italy

[2] Department of Mathematics and Computer Science, University of Cagliari, Cagliari, Italy

[3] Department of Teaching and Learning, Stockholm University, Stockholm, Sweden

mathematics education mainly focused on students who experienced drop-out (e.g., Hernandez-Martinez, 2016) and disengagement (e.g., Solomon & Croft, 2016). Also, researchers explored how, while being academically successful, some students are nevertheless at risk of being marginalized or excluded, for instance in connection to markers expressing race (e.g., McGee, 2015; Stinson, 2013) or gender (e.g., Hall & Suurtamm, 2018; Solomon et al., 2015). Identity research has thus primarily illuminated lack of success and/or exclusion. What about the identity of successful non-marginalized university students in mathematics?

Wood et al. (2012) addressed the broad issue of the development of a mathematical identity of university students and graduates. They understood this as a years-long process of transformation and analyzed it in connection with their participants' views on mathematics, mathematical learning, and the use of mathematics in professional working life. Bartholomew et al. (2011) investigated the way in which university students' identity incorporates their relationship with mathematics. They argued that the choice of studying (or not) mathematics "is closely bound up with an individual's sense of self – the kind of person they see themselves as being and present to the world" (Bartholomew et al., 2011, p. 916). They concluded by metaphorically describing the dominant discourses on "being mathematical" in terms of zealous adherence to the values of an ideal club of math enthusiasts.

These two studies can be advanced in at least three directions. First, the inclusion in these studies of data from non-mathematics majors renders it difficult to tie the notion of identity to a specific mathematical experience. Second, both these studies were carried out in English-speaking countries. Thus, an exploration of the identity of university students having a completely mathematical background as well as coming from other geographical areas seems to be important for advancing research on identity. Third, Bartholomew et al. (2011) strictly focused on their participants' mathematical identity as learners, while Wood et al. (2012) studied the professional identity of mathematics' graduates in various areas with no particular reference to the field of teaching. As Graven and Heyd-Metzuyanim (2019) and Lutovac and Kaasila (2018) argued, there is a need to investigate learners' and teachers' identity simultaneously. Indeed, explorations of the development of the self of university students in mathematics while they are preparing to become teachers seem to have been limited in the literature, hence constituting an open terrain for investigation. If "being successful in school mathematics requires students to develop a strong mathematical identity" (Valero, 2015, p. 25), then the same should be true of those students who, after having been successful in school, enrolled in a bachelor's program entirely devoted to mathematics. The same, and even more so, should apply to students who chose to specialize with a master's program in mathematics and moreover, within it, decided to follow courses in mathematics education to become mathematics teachers.

The present study brings further these unexplored directions by addressing the following question: What mathematical identity do master's students in mathematics preparing to become teachers develop in relation to their lifelong mathematical experience in school and university?

2 Alice's story

To approach this question, we investigated master's level students in mathematics enrolled in their first course in mathematics education in Italy. At the beginning of the course, these students were asked to write an essay entitled "What mathematics is to me." Contrary to

our expectation that they would solely write about their view of mathematics, the students wrote stories about their personal relationship to mathematics and about its role and significance in their life. Alice [pseudonym] is one of these students. She told a particularly touching story:

> When Alex [pseudonym] died […] my world was broken […] My head was spinning without relief, my heart was crying and I was absent. The only moment in which I had relief from pain was when I sat down doing mathematical exercises. […] in those moments I could breathe. As the years passed by, many things changed, and even mathematics, from a simple, natural, safe and comforting subject, became to me cause of insecurity and misunderstandings, from mechanical, clear and fast to obscure sometimes but also capable to arouse in me questions and make me curious to find their answers, in an active process at the end of which, step by step, I was finding myself. […] To me mathematics is difficult because it is difficult to talk about it, to share it […] with friends [who do not study mathematics] […] But in the end, mathematics was my salvation [la mia salvezza], when I was feeling lost, and I wish that it shall be the same for many others too.[1]

In this passage, Alice's relationship with mathematics appears to be strongly tied to crucial lifetime experiences. Doing mathematical exercises, she tells, comforted and helped her through a traumatic mourning. Advancing in her studies, the comforting side of mathematics turned into a more challenging side, sometimes provoking uneasiness and insecurity. These difficulties are complemented with the difficulty of communicating and sharing mathematics with others. Despite the various worries she is now experiencing, Alice describes mathematics as a practice capable of allowing her to discover herself. Furthermore, she wishes that others too shall be rescued, as she herself was once rescued, by means of this peculiar form of salvation.

In the essays, the students consistently conveyed ideas that resonate with Alice' story. Can we thus disentangle the relationship between, for instance, the challenging dimension and the comforting dimension of mathematics, which together are seemingly of help in finding one's self? What about the non-mathematician friends with whom it is difficult to interact? And what of this idea of saving others through mathematics?

3 Mathematical identity, subjectification, and ascesis

The notion of identity has been defined by researchers in different ways depending on their adopted theoretical frameworks (Darragh, 2016). Drawing from Gee (2000), we start by understanding identity as the discursive positioning of someone as a "kind of person" belonging to a group. As Radovic et al. (2017) wrote, "Identities develop in relation to others, […] during a process in which individuals position themselves and are positioned by others as certain kinds of people" (p. 437). This apparently static definition of identity does not contradict current thinking in mathematics education research which has emphasized the fluid, changeable, and precarious nature of identities (cf. Andersson et al.,

[1] As reported in Section. 4 below, the present study was entirely conducted on the original data written in Italian. This and the other passages quoted in the present article were translated into English for the purpose of presenting our results to an international audience. The translations were performed by the first author and checked for meaning-preservation by the other authors.

2015; Chronaki, 2016; Stentoft & Valero, 2009). We do not deny that in general identities are malleable and fluid. Nonetheless, positionings of belonging can become crystallized across the space of the individuals' possible self-positionings and across the positionings of the communities they are part of. Indeed, while identity can be thought of as a property or feature which pertains to some particular person (and which defines him/her as a person), it is also, by its very nature, a feature which is shared within some community (and which defines it as a community) either generally or with respect to, say, some topic, interest or discipline.

But what shall we take to be mathematical identity more specifically? Gee further argued that post-modern neoliberal societies came to increasingly stress *affinity identities*: forms of identities related to the practices and values of the groups towards which identification is directed. However, in investigations relying mainly on linguistic data, a positive expression of affinity identity may be missing or difficult to capture in explicit operational terms. In the second part of their study, Bartholomew et al. (2011) thus employed a psychoanalytic lens which allowed them to attend to the hidden meanings lurking behind the surface of their data, which they linked to the metaphoric notion of the *math club*: an ideal club of math enthusiasts whose values are tied to the expression of their participants' mathematical identities. This notion resonates with the understanding of identity expressed by Mendick (2005) who discussed the case study of a high school student in mathematics whose identity work she described as a bond to an "imagined community of like-minded individuals" (p. 171). Mendick conceptualized mathematical identity as a discursive construct built on oppositions (e.g., rational/emotional, objective/subjective, masculine/feminine). We add to Mendick's theorization that the oppositional character of identity is in general unavoidable, in the sense that identity is always constituted in opposition to some other (often dominant) identity, as Connolly (2002, p. xiv, p. 64) argued.

This observation allows us to remain within the metaphor employed by Bartholomew and collaborators by expanding Gee's definition: the existence of the math club is tied to the exclusion of some individuals from it and, concomitantly, to the exclusion of members of the math club from other communities. This is because any identification or inclusion within a group occurs concomitantly (albeit possibly implicitly) to an exclusion or aversion from the same group, and vice versa. In the case of an individual, for instance, it is crucial to her identification with some group not only the positive positioning of her inside the group, but also the concomitant negative positioning of others outside the group. An affinity group is therefore always also an aversion group (i.e., it can be defined by both positive and negative acts of positioning) and thus can be conceptualized as an *affinity/aversion group*. Therefore, we understand *mathematical identity* as being discursively positioned within an affinity/aversion group in relation to mathematics.

Notice that our aim here is not to conceptualize identity as a feature existing in discourse independently from people's and groups' life trajectories. On the contrary, our aim will be to frame our participants' mathematical identity as both the results of a past institutional and extra-institutional lifelong mathematical experience and, likewise, as possibly informing their future relationship with the society and the institutions that they will contribute to shape. In the case of Alice's story, for instance, we may recognize that her engagement with mathematics relates to a process of identity building connected to her distant personal past. Furthermore, a multi-faceted form of identification is concomitantly at work in the present as well as projected into the future. On the one hand, the recognition of a type of people outside of the math club (with whom communicating about mathematics is difficult) reinforces her present identification with the math club itself. On the other hand, such identity extends towards the future through Alice's wish to increase the number

of subscriptions to the math club: i.e., towards the possible development of others' mathematical identity.

Hence, following the need to attend to the institutionalized time/space dimension of the individuals' positions and belongings, we further link mathematical identity to broader processes of *subjectification* which occur in educational experiences: the processes of being posited and simultaneously positing oneself in the world. According to Radford, 2008, 2018), if we assume a socio-cultural perspective on learning, then processes of knowledge-acquisition of concepts or techniques are inextricably intertwined with processes of subjectification, or subject-constitution (cf. Presmeg et al., 2018; Valero & García, 2014). We choose here to employ the term "subjectification" in a Foucauldian sense (cf. Walshaw, 2007), in order to stress the relationship between the (mostly passive) process of subjection which is implied in any educational experience and the (potentially active) agency which may be the result of it. In the case of Alice's story above, her mathematical experience across the years in school and university, with its joys and pains, has molded her through a disciplinary process (the process of subjecting to mathematical discipline) resulting in the self-revelation of a particular (mathematical) identity, which she appears to interpret as an essential property of her character.

Foucault (1995) argued that in modern educational institutions, power is primarily effected through disciplinary technologies. In his later work (e.g., Foucault, 2006), he conceptualized these mainly as technologies for the government of the self, where individuals conduct their own conducts through self-reflection and self-regulation in turn associated to the gradual acquisition of moral, behavioral, and cognitive attributes. Foucault further characterized these technologies of the self within educational institutions as forms of *ascesis*:

> We find the mould, the first model of the pedagogical colonisation of youth, in this practice of the individual's exercise on himself, this attempt to transform the individual, this search for a progressive development of the individual up to the point of salvation, in this ascetic work of the individual on himself for his own salvation. (Foucault, cited in Ball, 2017, pp. 21–22)

Various authors explicitly relying on Foucault's philosophy further detailed how mathematics education brings about a form of shaping of the subjectivities of the students involved in it (e.g., Andrade-Molina & Valero, 2015, 2017; Diaz, 2014). In particular, Kollosche (2014) argued that mathematics education functions as a mechanism of selection producing forms of ascetic behaviors which favor the reproduction of power in society. Furthermore, Kollosche (2017) provided a preliminary explorative socio-political analysis of how the subjectivities of students enrolled in German secondary schools develop through their involvement in mathematics education which he again described as a particular form of ascesis.

But what is ascesis? The term "ascesis" is a direct borrowing of the Latin *ascesis* derived from the ancient Greek ἄσκησις (from the verb ἀσκέω), originally meaning "training" or simply "exercise." The term—progressively through the centuries—has taken the meaning of defining the general process of both material and mental discipline one imposes on oneself as a (usually fundamental) component of the path to attaining a special spiritual status in reference to some transcendental ideal (*Merriam-Webster's Encyclopedia of World Religions*, 1999, p. 80; Antonaccio, 1998, p. 7). Nowadays, the term is mostly used to refer to the spiritual experiences of individuals connected to religious organizations. However, there is a sociological and philosophical tradition which has used the notion of ascesis to characterize axiological behaviors (and discourses) of secular lay-men (and women), their

secular motifs, and their secular/immanent (yet somewhat always transcendent) ideals (cf. Abbruzzese, 2001; Sloterdijk, 2013). Thus, in an extended sense, we employ here the term "ascesis" and the adjective "ascetic" to refer to experiences or trainings involving the following features:

1. A complex system of punishments and rewards, ideally self-administered (*discipline*).
2. Such disciplined way of life is meant to bring (closer) towards a higher (state of) being (*transcendence*).
3. An ideal line of separation between the ascetic and the non-ascetic, suggesting a higher or more desirable position of the former (*elitism*).
4. The disciplined way of life (perhaps in a mitigated form) has to be propagated to other people (*proselytism*).

Discipline and transcendence are features common to any ascetic experience as well as elitism (Fuchs, 2005), while proselytism only relates to some particular ascetic experiences (e.g., proselytism of at least a moderate ascetic ideal is encouraged within most understandings of Christianity). These four features appear to be generally shared between forms of religious practice, military training, professional athleticism, and some types of corporate career climbing, as well as devoted participation to the schooling system (Sloterdijk, 2013).

Indeed, as mentioned above, one of Foucault's seminal theses is that modern education contains elements which derive from religious or ascetic education and institutions (cf. Deacon, 2002). These elements had been transformed through new educational practices where knowledge and science had become new organizing principles to fabricate modern forms of subjectivities (cf. Ball, 2017; Tröhler, 2011). If this is the case for education in general, we may assume the same to hold (in specific forms) for mathematics education. As Kollosche, 2017, p. 186) remarked, "[…] this ascesis is unique to mathematics and therefore has a unique function in the process of the students' construction of a mathematical individuality." Thus, the notion of ascesis appears to be appropriate for detailing the sort of experience people in general go through (and the sort of identity they develop) when they engage with mathematics within educational institutions from the lowest to the highest levels, and significantly more so, as we will see, in the case of university mathematics education.

4 Context and method

As anticipated in Section. 2, we carried out an empirical study with students enrolled in the master's program in mathematics at the University of Turin, Italy, in order to research their mathematical identity. Access to this two-year program focusing on pure and applied mathematics requires completing a bachelor's in mathematics with good proficiency. The program attracts students who graduated from this and other recognized Italian universities. In Italy, holding a master's degree in mathematics is the primary path for accessing the state examination leading to mathematics teaching positions in upper-secondary schools. The third author lectures the first course in mathematics education in this master's program, which is offered in spring and is typically followed by the students in the second semester of their first year. In the very first day of the course, before being exposed to any material or discussion, she asks the students to write a short essay entitled "What mathematics is to me" [Che cos'è per me

la matematica],[2] which they compose in the class by writing individually in Italian for about 40 min. No further instructions are provided except the title. A total of 103 essays in three subsequent years—2018, 2019 and 2020[3]—are the primary data for this study.

In order to collect information about the participants, a form is also administered right after the students finish writing the essays. Using the information collected in this form, the group of participants can be characterized as follows. Overall, 60.6% of the students were females and the rest males, while their median age was 23 years.[4] 97.2% of the students had already obtained their bachelor's degree,[5] with a median final grade of 96 (out of 110). Finally, 71.4% of the participants stated to consider teaching mathematics their primary career goal. Almost all the participants were native speakers of Italian. Only one participant had completed her previous education abroad.

All the students provided consent to allow their essays and information to be used as research material in accordance with the university's regulations and with the Italian and European law. The essays, of average length about 300 words, were anonymized and digitalized in a unique text file. The essays were then analyzed by a discursive thematic analysis: a two-stage procedure combining a thematic analysis with a Foucault-inspired discourse analysis on the already thematically analyzed data (cf. Clarke and Braun, 2014, p. 1948). Overall, in the analysis, we chose to treat the texts in their *exteriority*; that is, we attended to what they say and do not search for hidden or causal explanations. This choice implies that connecting information about particular students to what is said in their essays was not relevant for our analysis. The latter was conducted solely on the essays and, with respect to these, our interest was in finding recurrences and resonances of expressions as related to how ideas about being in/with mathematics are put forward in order to give an account of a shared mathematical identity prevalently expressed by the participants.

In the first stage of the analysis, we thus asked ourselves: What is being said in the essays? What do the students express? Following Braun and Clarke (2006) and Clarke and Braun (2014), after a preliminary reading and collective discussion among the authors, codes for different types of experiences or perceptions of mathematics were created and assigned to sentences or expressions. The coding was conducted on the original untranslated data by the first author. The codes were then checked against the data and refined in discussion with the second and the third author. In this phase, all textual expressions were evaluated literally without consideration of latent meanings, but when ambiguous or metaphoric expressions were found, their meaning was evaluated by context.[6]

The codes are presented with representative expressions in Table 1. The table also shows the occurrences of the codes in the data, i.e., for each code, we chose to count all individual essays which contain at least one instance of expression belonging to such code. Given that only the (quite general) title of the essay was prompted to the students (as opposed to, say,

[2] This title was inspired by the title "Me and mathematics" of the essays employed by Di Martino and Zan (2010, 2011) to study compulsory school students' attitudes towards mathematics.

[3] Due to the COVID-19 pandemic, in the year 2020 the students wrote the essays in their homes under the same time constraints and modalities during the first lecture of the course (delivered online in synchronous form). We did not observe substantial differences in content or form between the 2020 cohort and the previous cohorts.

[4] Notice that secondary education in Italy is one year longer than in most Western countries.

[5] Some students attend courses of the master's while being in the process of completing the last steps of their bachelor's program.

[6] For example, in expressions such as "when you climb a mountain […]", the meaning of the intended metaphor (mathematics is like climbing a mountain) was understood in the context of the whole essay.

Table 1 Preliminary codes and their occurrences in the data

Code	Description	Examples of sentences or expressions (translation)	Occurrences over 103 essays
A	Mathematics is an easy game-like activity	"[mathematics] is an easy game"; "The easiest among the subjects […]"; "It is similar to solving a puzzle"; "It is fun"; "[it is] like doing a crossword"	24
B	Mathematics is difficult	"It is objectively difficult"; "[you need] dedication and strength"; "[it is] hard"; "[it is] a continuous challenge"; "[it is like] when you climb a mountain […]"	34
C	Mathematics is arduous	"It is frustrating"; "often I think that it is too much for me"; "[it is] painstaking"; "I had to struggle"; "[it requires] sacrifice"	41
D	Mathematics is satisfying	"When I arrived at the solution, I was happy"; "[it brings you] an injection of pride"; "the feeling of satisfaction in seeing that the result was correct"; "a perception of competence that I felt when I owned the concepts"; "It makes me joyful"	18
E	Mathematics is a refuge from worldly preoccupations	"I used mathematics as a way to let off steam"; "very reassuring"; "It was a refuge"; "It was a friend upon which I could count"; "[you can use it to] evade from the world in which we live"; "[it is] a safe harbor"	30
F	Mathematics is omnipresent or everything is mathematizable	"Mathematics is everywhere"; "in every place you look there is mathematics"; "the language which God has written […] in nature"; "[mathematics allows] to grasp the true essence of things"; "you can […] retrace it in […] all facts and objects"	33
G	Mathematics is useful in everyday life	"It is an instrument of everyday life"; "you can use it each day"; "[it is] behind many aspects of life"; "it simplifies everyday affairs"; "you find it […] in everyday choices"	48
H	Mathematics helps to reason rigorously	"[It leads to] a more abstract and rational intellectual approach"; "it shapes the mind"; "it opens the mind"; it leads to a method of reasoning"; "[through it] you learn a way of making connections and seeing reality"	24
I	Mathematics provides with job-security	"[studying it opens] job possibilities"; "you can apply it to very different working environments"; "[it will secure me] a future job"; "[it has] many corporate fields of application"; "it gives me a [professional] future"	22
J	Mathematics is a tool in science	"[It is] an instrument with which […] to model reality"; "it is the language through which [sciences] are studied"; "[it serves] to make sense of phenomena"; "it explains things"; "it is a rigorous and short way […] to describe the world"	26
K	Mathematics is experienced negatively by many people	"People hate it"; "they are terrified by mathematics"; "mathematics was [for others] a torture"; "not many […] like this subject"; "[…] others' difficulty with it"	44
L	Mathematics has to be transmitted to others	"My goal would be to […] convey my passion for mathematics"; "I wish to help others understand it"; "I want to communicate its essence"; "make other understand how fascinating it is"; "turn people to mathematics"	35

interviews or questionnaires which typically prompt students to reflect on items chosen in advance by the researchers and thus possibly not relevant for them), we chose to consider one occurrence of a concept sufficient for deeming it an aspect that the writer judged relevant to his or her own mathematical experience (cf. Di Martino & Zan, 2010, pp. 32–33). Moreover, we chose to not count repetitions of expressions belonging to the same code within each single essay. This is because the repetition of a concept within one essay does not by itself mean that its writer assigned more importance to it, as opposed to other concepts which are mentioned fewer times (since within one essay, it is often the case that the same concept is repeated at different locations for mere argumentative or formal reasons).

After re-reading the original data, the first author identified candidate themes which were then refined together in discussion with the second and third author by developing, merging, and discarding across the codes.

- Codes A, B, and C were merged into a single theme referring to different aspects of the challenging nature of mathematics (the theme of *Challenge*).
- Similarly, codes D and E were merged into a single theme referring to the satisfying or comforting experience of mathematics (the theme of *Comfort*).
- Codes G, H, I, and J in turn were merged into a theme referring to the manifold utility of mathematics (the theme of *Utility*)
- Code K was developed into a new code centered on the expression of difference to others, in view of the fact that the negative experience of others was almost always contrasted with the experience of the writer. This was later promoted to a theme (the theme of *Difference to others*).
- We deemed code F to be interesting, but we decided to set it aside for the moment, as it referred to a problematic and almost mystical characterization of mathematics (frequently articulated by means of terms directly belonging to the religious or quasi-religious semantic field, e.g., "God has written […] in nature," "the true essence of things").
- Code L was also deemed interesting but problematic. On the one hand, it represented an important focus of many essays. On the other hand, it was not really relevant to experience with mathematics per se, as it referred to the future and not to the past. We left this code aside for the moment.

In selecting the four final themes (displayed in Table 2), we followed a criterion of prevalence and of straightforwardness with respect to the whole corpus. Indeed, the final themes are by far the most prevalent in comparison to other possible candidate themes of comparable simplicity that we could conceive; i.e., they all appear in more than one third of the essays.

Table 2 Final themes and their occurrences in the data

Theme	Description	Originating codes	Occurrences over 103 essays
Challenge	Mathematics is challenging, stimulating, demanding, exigent, difficult, hard	A, B, C	51
Comfort	Mathematics is comforting, pleasurable, consoling, alleviating	D, E	40
Utility	Mathematics is useful in everyday life, in developing skills, in science and technology, in finding jobs	G, H, I, J	61
Difference to others	Mathematics is viewed differently by other people	K (developed)	52

At this point, we were able to identify a story involving the selected themes referring to the participants' mathematical experience. Simply put, mathematics is viewed as challenging and useful and as generally providing pleasurable or comforting feelings while being regarded negatively by others. However, this simple story did not fully capture the depth and complexity of the data both in general and in relation to our research question.

Therefore, in the second stage of the analysis, we asked ourselves: How are the themes related to each other and to the participants' process of subjectification? In trying to understand this, the connections between the themes were unfolded by pondering the recurrences in the material across individuals and cohorts of students, searching for resonances among the expressions that articulate statements of a mathematical identity. Here a Foucauldian-inspired analytical move started. Arribas-Ayllon and Walkerdine (2017) suggest that (Foucauldian) discourse analysis explores ways of accessing the continuities (and discontinuities) of experiences over time and of understanding subjectification as an ascetic practice on the self. In autobiographical material, such as the texts in our data, problematizing some domain knowledge, in our case mathematics, narrated by those who can be considered expert in this knowledge, in our case master's students in mathematics, discursive analysis provides ways for nuancing the process in which selves are constituted *through* this knowledge (Arribas-Ayllon & Walkerdine, 2017, p. 116). Furthermore, it allows to trace "how individuals problematize and regulate their own conduct" (*ibid.*) and others' conducts "in order to attain a certain state of happiness, purity, wisdom, perfection, or immortality" (Foucault cited *ivi*, p. 117).

Further collective reflection and discussion on the resonances of expressions in the data led us to progressively recognize a relationship between the four features of ascesis introduced above (discipline, transcendence, elitism, proselytism) and the discursive connections between the themes. Indeed, in the data, a disciplinary characterization of mathematics is articulated foremostly in parallel to the themes of Challenge, Comfort, and Utility. A transcendental characterization of mathematics is implied by statements belonging to code F and is voiced in correspondence with enunciations of the theme of Comfort as well as Utility (in particular with reference to the everyday use of mathematics). An elitist characterization is encompassed in the theme of Difference to others, since difference is expressed together with a privileged position of the mathematics-literate or enthusiasts, in turn explained in terms of the challenging, comforting or utilitarian aspects of mathematics. The latter aspects are also voiced in parallel to explanations of why mathematics should be transmitted (the previously considered code L).

The four components of ascesis were hence employed to articulate the previous themes in order to construct a story on how a shared mathematical identity was formed through the practices of mathematics education. This final phase of the analysis was again carried out primarily by the first author on the original data in periodic consultation and discussion with the other authors. In this last step, particular attention was paid to the temporal structuring of the texts by identifying time-dependent points of rupture signaling a change in the significance of the mathematical experience for the students. In the following section, we present the resulting account of the discourses pervading the data with reference to the mathematical experience of the participants as they build an identity with respect to mathematics. This aims at making evident the general mathematical and extra-mathematical *ethos* which permeates the community of students selected (cf. Walshaw, 2016, p. 61) and which both affects as well as is retroactively made visible in the texts that constitute our data.

5 Findings

One of the most recurrent topics of many essays is the depiction of the mathematical experience as an increasingly demanding challenge. Mathematics was a natural and easy game-like activity in the students' earlier school years while its challenging yet stimulating aspects became more visible to them during high school. The challenge of mathematics however transformed into a struggle for many in the later years of university, when the content of several courses became too difficult or even obscure. Indeed, difficulty is described as a feature belonging to the mathematical endeavor per se*:* learning mathematics is inherently hard and painstaking. The experienced difficulties with mathematics in the past and present are narrated as confirming the students' determination to succeed. Moreover, difference between the participants and others is articulated at the level of individual motivation or will to accept the mathematical challenge. What discourages most people is that they lack the discipline which is necessary to be proficient in mathematics.

> It never was an easy relationship, it [mathematics] is a form of knowledge which is sometimes very difficult and obscure and this is what, I believe, in general discourages people to study it in depth. At a human level, what mathematics taught me in all these years is how to not give up and how to continue to insist on long and difficult proofs, on exercises which sometimes seemed really too much for me.

Difficulty thus renders mathematics a type of teacher: a "teacher of life" or a "gym of life," because it is a useful disciplining practice whose inherent difficulty mirrors the difficulties of life itself. Furthermore, people who are profane to mathematics seem incapable of understanding what sort of totalizing struggle the students have been and still are going through in their studies. This results in frequent incomprehension between them and mathematicians.

> […] and I have lost friends who could not believe that I was studying so much and I have lost much hair too.

Nonetheless, any ascetic practice is not only constituted by mere disciplining or renunciatory components but also integrates forms of spiritual satisfaction which are inherently part of the discipline which constitutes the ascesis itself. Indeed, mathematics is also an activity that brings relief from anxieties and a source of confidence in face of the uncertainty and messiness of the outside world in which often the students have felt to be at discomfort: a transcendental safe-haven built on certainties and truths.

> When I am sad or worried mathematics to me is a refuge. It is certainty when I am in doubt.

The comforting dimension of the mathematical experience is linked by the participants to the rule-following procedures characteristic of arithmetic and early algebra and, perhaps even more fundamentally, to the more challenging further steps of the curriculum. As a matter of fact, many essays describe how students are motivated to submit themselves to the discipline required for solving mathematical problems by the sense of accomplishment that each problem rewards its solver with: a sudden sense of pleasure or satisfaction which constitutes another intrinsic use of engagement with mathematics. The harder the problem, the harder the effort and the greater the associated satisfaction or confidence.

> And when you are able alone to solve a problem that you thought to be impossible, you feel a great joy and a sense of confidence.

Furthermore, mathematics is not only described as being useful for its disciplinary and comforting features, but also in relation to its use in offering models for the real world, and for its value in providing job-security. As to the use of mathematics in modelling the real, a repeated claim of many essays is that all aspects of the world or of life are mathematizable or prone to be understood mathematically. Indeed, frequent statements are that "mathematics is everywhere" or that it "explains everything." This corroborates the transcendental characterization of mathematics linked to its comforting dimension, as seen above. Mathematics is thus depicted as a higher form of knowledge or activity because it founds, it explains, or simply it is in everything. As a consequence, those who engage in mathematics can be considered to be closer to a higher state of knowing/being.

> Since all things are mathematical […] doing mathematics makes me capable to grasp the true essence of things.

This belief in the "transcendental immanence" of mathematics is also explicitly argued in many essays to be a specific product of mathematical studies. More generally, the epistemological disposition towards the world described above (i.e., numbers and mathematics are seen in everything and conversely everything is seen mathematically or quantitatively) is sometimes narrated as a pre-existing attitude progressively unveiled to the student through his or her lifelong engagement with mathematics as connected to the discovery of his or her self (cf. Alice's story above).

> […] because [mathematics] is in everything, it is in me, and step by step I see it everywhere […]

> […] mathematics is an entity in which I can find myself because it reflects my being, my disposition, my tendency to classify things as right or wrong.

On the contrary, other people do not see mathematics in the same way. Often the essays contrast the views and general incomprehension between the students and their acquaintances on the topic of mathematics. The majority of people consider mathematics to be "boring," "difficult," "too complex," "abstract and distant," "ugly," "useful to nothing," "a jumble of numbers, functions, and other nonsense," "source of boredom, fear, and frustration," etc. The sort of public disgrace with which mathematics is regarded is described as providing a line of separation between our students and others. This difference appears as the starting or motivating question of many essays to which the whole text attempts at building an answer. Furthermore, such difference sometimes takes the form of a conflict, whose oppositional nature contributes to the students' mathematical identity (i.e., to their membership to the math club understood as an affinity/aversion group).

> I often find myself defending mathematics when I discuss with others […] Many people have difficulties in understanding mathematics, I on my part have difficulties in understanding many people.

This fracture between the participants and others (i.e., the inability of others to enjoy mathematics) is explained in many essays as stemming from other people's failure to see mathematics' inherent positive qualities. Indeed, recognizing the overall utility of mathematics, accepting the positive role of its challenging dimension, and recognizing its comforting side are seen as prerequisites which make it possible to commit to the mathematical endeavor. Remarkably, our participants see themselves as endowed with the role of the agents who are (or will be) able to dispel the "prejudices" attached to mathematics among their peers or among their future students. This often appears at the core of the participants'

expressed desire to become teachers and becomes literally a mission (sometimes characterized as similar to that of a religious or a humanitarian organization) which points at transforming others into passionate math lovers as they themselves have come to be.

> If someday I will become a teacher, I will try to convey all that mathematics is […], which, to me, is comparable to the help offered by CARITAS[7] or WWF,[8] a good humanitarian deed.

> My biggest aspiration is to come to see myself and my love for mathematics in the eyes of my future students.

6 Discussion and conclusion

Figure 1 schematically condenses the interaction between the themes and the features of ascesis which we now summarize.

Overcoming the challenges posed by mathematics is described by the participants as requiring an increasing amount of self-discipline. The latter in turn renders mathematics a character-shaping activity, a feature contributing to its overall utility. Furthermore, the satisfaction experienced in solving each mathematical problem relates to a comforting and pleasurable dimension associated with doing mathematics. Additionally, the multifaceted utility of mathematics, depicted as variously stemming from its transcendental characterization, is related to a general feeling of reassurance and comfort when dealing with mathematics. Moreover, an ideal divide from mathematicians and others is articulated at the level of the views these two groups fail to share on mathematics. Similarly, the proselytistic mission of bringing mathematics to others is tied to the possibility or opportunity of changing others' views of mathematics. First, the challenging nature of mathematics is narrated as a positive feature which must be accepted and embraced by others for their own (mathematical) benefit. Second, the utility of mathematics must be recognized by others if they want to be proficient in mathematics. Third, others will also benefit in this respect by recognizing the comfort or satisfaction which doing mathematics brings about.

The interplay between the themes thus offers a window to understand the participants' process of subjectification resulting in the development of a particular mathematical identity. Such identity revolves around the polarizing character of mathematics: the participants' and others' recognition of exceptionality related to engagement with mathematics is what sets the boundaries of a particular math club. As said, other people are often framed as different (and concomitantly frame the writers of the essays as different, according to them) not so much because they do not engage with the mathematical activity per se, but most crucially because they do not share the same views on mathematics: they fail to see (and enjoy) the challenging, comforting and useful features of mathematics. Thus, insiders and outsiders of the math club are separated mainly by the ways in which they view mathematics, i.e., by their *opinions* on the topic of mathematics, i.e., their mathematical identity is that of the mathematics-enthusiast (those capable or willing to see the qualities of mathematics) rather than that of the mathematics-skilled or literate. Even when articulating the theme of Challenge, particular emphasis is put on motivation or dedication, rather than ability. To corroborate this conclusion, we have identified almost no expressions related to

[7] A charitable organization active in Italy and connected to the Catholic Church.
[8] An international organization specialized in the protection of the wildlife.

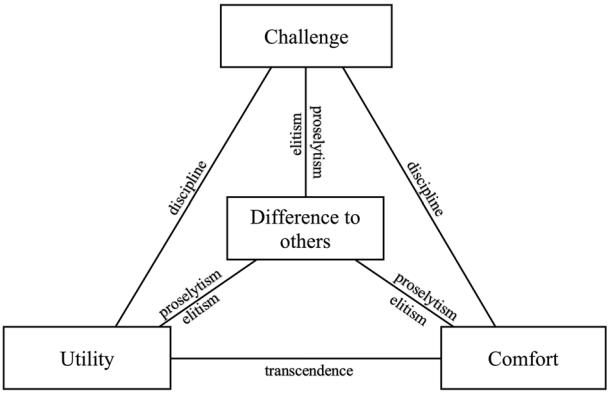

Fig. 1 Diagrammatic summary of the findings

the innateness of mathematical ability or to the idea that some students are "mathematical whizzes" while others are irredeemably doomed to fail (as it is instead reported by, e.g., participants in the studies of Kollosche, 2017 as well as Bartholomew et al., 2011).

Furthermore, no essay articulates explicitly the transmission of specific content knowledge, while many essays refer either to a generic willingness to transmit mathematics, or to the willingness to change people's views on mathematics. These are narrated as reformable inclinations which the participants themselves will be able to mold in their future professional activity. Therefore, the mathematical identity articulated by the participants appears to be tied not only to the feeling of belonging to a group of mathematics-enthusiasts but also to the eagerness to transform others into enthusiasts: the participants see themselves simultaneously as product and agents of the math club, i.e., both as satisfied members and eager recruiters (cf. Montecino & Valero, 2017). Thus, the mathematical subjectification the participants experienced in the past appears to have crystallized in a present mathematical identity which is also projected towards the future through the will to proselytize for the mathematical cause, i.e., to contribute to others' mathematical subjectification.

This identity, as said above, is also related to a specific outlook to the world in which mathematics, perhaps unsurprisingly, has more than just a significant place. As we have seen, adherence to the math club means, for instance, agreement on the beneficial and useful nature of mathematics (e.g., viewing even its hardships as necessary or useful/beneficial). These views are incorporated in the more general creed of mathematics' "transcendental immanence" connected to the idea that everything is mathematizable or that mathematics can be employed in all domains. Similar views were reported in the literature on students across the globe (e.g., Wood et al., 2012; Kollosche, 2017) and are consistent with the prevailing narrative concerning mathematics and mathematics education (cf. Beccuti & Robutti, 2022; Skovsmose, 2020). This metaphysical stance, we may conjecture, is not based on personal experience and can well be interpreted as a myth, "as a power–knowledge relation in the Foucauldian sense, […] which might be effective in providing a technique for students and teachers, with which they can make sense of their own involvement in mathematics education, eventually reproducing that myth themselves" (Kollosche, 2017, p. 186). More generally, the overall ascetic conduct that we described may function as a multifaceted technology serving both to endow mathematics with various meanings (e.g., that it

is inevitable, useful, and beneficial) within the life trajectories of the participants as well as, concomitantly, to frame their interpretation of others' life trajectories.

In conclusion, we advanced the understanding of identity and subjectification by concentrating on a disciplinary-homogeneous group of university master's students in mathematics in Italy who prepare to become teachers. Further research would be needed to investigate the relationship of our findings with the role of mathematics education in society in general and with the problem of the transmission of mathematical knowledge in particular. It is reasonable to expect that, for instance, the mathematical-ascetic posture that we depicted will be passed on to (or will at least affect) new generations of math club members within educational institutions or elsewhere, if these master's students will become journalists, popularizers, academics or, most likely, parents and teachers. Indeed, the choice of young students of the twenty-first century to dedicate (a significant portion of) their life to mathematics is not merely the fulfillment of an intellectual inclination. It is instead the realization of and the endurance in a totalizing and quasi-religious disciplinary endeavor affecting epistemological as well as axiological views. The incessant supply of people devoted to these appears to be vital to the existence of the math club, i.e., to the self-perpetuation of the institutional practice and culture of mathematics education.

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Data availability The data analyzed in the current study are not publicly available as they constitute personal and possibly sensitive data in accordance with the European Union Regulation 2016/679 on the protection of natural persons: http://data.europa.eu/eli/reg/2016/679/2016-05-04.

Declarations

Conflict of interest The authors declare no competing interests.

Authorship statement The theoretical framework of the present study was developed by the first author in consultation with the second author. The first author also conducted preliminary research on the relevant literature. The original design of the experiment was conceived by the third author, who also conducted data collection with help from the first author. The method of data analysis was developed by the first author in collaboration with the second author. Data analysis was conducted primarily by the first author in periodic discussion with the second and third author. The first and subsequent drafts of the manuscript were written by the first author and all authors commented on each version of the manuscript. The first author also revised the manuscript and produced its final version. All authors read and approved the final manuscript.

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