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Learning from English and Kuwaiti children's transcoding errors: how might number names be temporarily adapted to assist learning of place value?

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Abstract
This study identifies language specific errors made with transcoding tasks to inform possible future pedagogic decisions regarding the language used when teaching early number. We compared children aged 5–7 years from Kuwait and England. The spoken Arabic language of Kuwait gave the opportunity to compare not only languages where the tens and units digits are said in a different order, but also where the direction of writing is different. We asked 396 children from Kuwait and 256 children from England to write down 2-, and 3-digit numbers which were spoken to them. We found that the direction of the language did not affect the nature of errors made, but that other aspects of the two languages could account for some of the differences we found. As well as supporting previous studies regarding the significance of the order in which the tens and units are said, we found significance in the role the word *and* can play in marking the number of digits involved. We also noted that the way the numbers 20, 100 and 200 are said in Arabic can set up particular symbolic associations which could account for other differences we found. Having identified language-specific errors, we discuss possible pedagogic decisions to temporarily use more regular language for the number names in each of the languages and propose the order in which number names are taught might be different to their mathematical order of magnitude.

Keywords Transcoding · Number names · Language · Mathematics education

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1 Background

Much of mathematics learning is strongly grounded in number. A lack of understanding of, and confidence with, the structure of number can mean that subsequent work using number also becomes problematic. Addition, subtraction, multiplication, division, estimation, fraction work and even other mathematics areas such as angle, area and perimeter, use numbers. Even students as old as 14 years can still have significant issues regarding the meaning they give to numbers expressed in symbolic form (Hewitt & Brown, 1998). Numbers are sometimes said and sometimes written. Knudsen et al. (2015) found that mapping heard number names onto quantities was in advance of being able to do that with number symbols. What is of interest is to bring the spoken number names and number symbols closer together so that there is equal confidence with both forms. This places an educational importance to the learning of writing numbers in symbolic form beyond that aspect of the curriculum. The dynamic between what is said and what is written can either assist or obfuscate a sense of the underlying mathematical structure of number, such as place value. Some languages are very regular in the way numbers are said, and this can support the underlying mathematical structure of written numbers. For example, many East Asian languages, such as Chinese, Japanese and Korean, have regular number names which include words which explicitly assist the understanding of the value of each numeral, such as 'ten-two' for 12 and 'three-ten-four' for 34. In contrast, many Western European languages have far less regularity, particularly with the *-teen* numbers. This can affect not only gaining a sense of the underlying place value structure of number, and hence the relative size of numbers, but also the carrying out of arithmetical tasks as well. For example, the language of 'one-ten-two' add 'four-ten-six' assists gaining the answer of 'five-ten-eight', whereas the language of 'twelve' add 'forty-six' does not. The significance of language in the teaching of early number has an impact way beyond just the learning of number names. Ng and Rao (2010) found that the Chinese number naming appeared to help children learn about place value, counting and carrying out various arithmetical tasks. Children from Japan and Korea tended to represent two-digit numbers using appropriate ten and unit blocks whereas children from America, Sweden and France tended to use only single unit blocks, indicating a weaker sense of place value (Miura & Okamoto, 2003; Miura et al., 1993). Transparency of mathematical structure within the Chinese language has led to strategies being offered within textbooks to 'make-a-ten', whereas American textbooks do not, as the language does not support this so much (Ng, 2012). Göbel et al. (2011) highlighted that number words, in particular, have an influence on number representations. There are also factors other than regularity which might assist children in their learning of number. The Chinese names for the digits tend to be short, compared with English. This can ease load on working memory when the children are having to memorise the number names (Ng & Rao, 2010). There are also other linguistic differences, such as the way in which numbers are part of daily life, with weekdays in China being referred to as weekday 1, weekday 2, etc., and family members being known by their order of birth in the family (Zhang & Zhou, 2003). It is difficult to disentangle these cultural differences with linguistic differences. However, in a comparison between 8 and 10 year-old Welsh-speaking children and English-speaking children, where there was less contrast in cultural and educational matters than in other cross-national studies, Welsh-speaking children found it easier to read and compare two-digit numbers (where they have regular number names) than the English-speaking children (Dowker et al., 2008). There were even gains made by children whose first language was English but who were taught through the medium of Welsh, compared with those taught through English. This suggests that it was the linguistic differences which were significant in gaining this stronger sense of place value, and gives weight to the significance of the language through which numbers were taught over a child's first language. This leads to the awareness that the dynamic between what is said and what is written in numerals is important within children's mathematics education. There have been many studies with a focus on counting, subitising and ordinality (for example, Askew & Venkat, 2020; Bruce & Threlfall, 2004) and these important aspects of a child's mathematical development ultimately depend upon their understanding of, and confidence with, spoken and written number names. Identifying particular linguistic issues which may lie at the heart of children's errors can inform ways in which language of number names might be temporarily adapted to better assist children's learning of place value notation. For example, Magargee and Beauford (2016) carried out a study where some English and Spanish children were given an intervention in pre-kindergarten using explicit, regular number names when learning to count. They continued to benefit throughout a 6-year period, in terms of an accelerated acquisition of place value recognition and concepts of numeracy, compared with those children who had been taught using their non-regular natural language names for numbers.

The connection between the language of number names and the symbolic ways of writing those numbers has created much interest in transcoding — the process of converting a number said in words to its symbolic representation (e.g. Moeller et al., 2015; Zuber et al., 2009). Byrge et al. (2014) highlighted that some errors appeared to come from attempts to directly map words into written symbols. In which case, the way in which numbers were said in different languages becomes particularly significant. There have been many studies comparing children's early number learning in languages which differ in terms of their regularity (e.g. Clayton, et al., 2020; Moeller, et al., 2015), but as far as we are aware there have not been any which have analysed transcoding tasks comparing languages which have different reading directions. This may be of some significance as Bergeron and Herscovics (1990) found that kindergarten children sometimes focused on the order of writing digits by writing 21 for twelve when asked to write out the numbers they know from right-to-left (i.e. ….., 5, 4, 3, 2, 1). Bergeron and Herscovics called this a chronological level of understanding positional notation. The issue of writing direction was one reason why we have focused on comparing children's ability to transcode spoken number names into written symbolic form between English and Kuwaiti children, as the languages of English and Arabic have different writing directions. We seek, firstly, to examine relative differences in errors and, secondly, to consider educational implications for the teaching of early number within school settings. Decisions for teaching approaches do not only depend upon the mathematical content being taught, but also upon the language associated with that content. So, we want to find out ways in which language might affect the children's relative success to inform possible future pedagogical decisions over the associated language used when teaching early number. The languages of Arabic and English will both have their affordances and hindrances regarding learning the structure of how number names are written in their respective languages. We seek to explore how numbers are spoken can support or obfuscate children's writing of numbers in conventional place value form. We start by reviewing some of the already established difficulties that children experience. Children will write numbers in different ways on their journey towards coming to know what is accepted as the socially agreed conventional way we write numbers. What a student writes, is their way of expressing the meaning they have at that moment in time. The meaning could be sound mathematically, even if they have not written it in the socially agreed conventional way. We use the word 'error' throughout this paper to mean only that what was written was not conventional.

1.1 Inversion

Inversion is where "the order of tens and units in number words as compared to digital notation is inverted" (Helmreich et al., 2011, p. 600). This happens in German where 24 is said *four and twenty* rather than *twenty four*. Clayton et al. (2020) found that German speaking children made more inversion errors compared with English speaking children. Where the two languages both had inversion, in the *-teen* numbers, both German and English children had a similar proportion of inversion errors. Results from other studies, which have looked at transcoding, have also shown that children in countries with language inversion have more inversion errors than those from countries without inversion in their language (Imbo et al., 2014; Moeller et al., 2015). Pixner et al. (2011) looked at the Czech language where two systems exist for saying two-digit numbers, one inverted and the other not inverted. Children had to perform a transcoding task in both systems. They found that about half the errors were inversion related with the inverted number-word system, whereas hardly any such errors were found in the non-inverted system. The inversion effect has been seen to be present even though children might be expected to have 'mastered' two-digit numbers according to the curriculum (Krinzinger et al., 2011; van der Ven et al., 2017; Van Rinsveld & Schiltz, 2016) and can even persist in 7- to 9-year-olds arithmetic work (Göbel et al., 2014). Ganayim et al. (2020) found that the error of swapping tens and units digits in transcoding tasks remained present for many adult university students. These students were bilingual with Arabic as their first language (with two-digit numbers said with the units digit first) and Hebrew as their second (with two-digit numbers said with the tens digit first). Ganayim et al. (2020) noted that this type of error was found mostly in non-decade numbers and non-*teen* numbers, which were the numbers where the order of the naming structure is particularly important. Interestingly, error rates were lower when given transcoding tasks in Arabic compared with Hebrew. This could indicate that confidence with the first language may be more significant than the language structure for number names with regard to this type of error. However, it should be noted that both languages are written right-to-left and as such the two-digit XX numbers are written in the same direction with Arabic as opposed to Hebrew. Hence, we argue that it is Hebrew, rather than Arabic, which has inversion.

1.2 Other types or errors

Byrge et al. (2014) found that 4- to 6-year-olds added extra zeros or '100' when being asked to write three-digit numbers. They said that the "expanded productions seem to preserve the structure in the heard form at the expense of position or place in the written form" (p. 442). This was an example of what Zuber et al. (2009) called syntactic errors, where there are correct digits involved but the overall magnitude was incorrect. They subdivided syntactic errors into those which had additive composition errors, such as 10023 for *one hundred and twenty three*; those which were multiplicative composition errors, such as 4100 for *four hundred*; and those which were inversion errors.

1.3 Reading and writing direction

Dehaene (1992) showed that participants tended to have an internal number line where smaller numbers are positioned to the left and larger numbers to the right. This is known as the SNARC effect (Spatial-Numerical Association of Response Codes). The orientation of imagined numbers within this number line was also seen to relate to the direction of writing within the main language of the student, with the effect being reversed for Iranian participants, whose script is written right-to-left (Dehaene et al., 1993). Such a reversal was also found by Zebian (2005) with Lebanese participants, and by Shaki et al. (2009) with Palestinians, where both speak Arabic, which has a right-to-left script. This may, of course, not just be about the direction of writing but also about associated cultural ways in which numbers are shown, for example, on rulers or public signs (Dehaene et al., 1993), or indeed different curricular effects (Krinzinger et al., 2011). However, in a different study, Shaki and Fischer (2008) did show that reading activity was causally involved with the way in which numbers were spatially imagined.

There is a gap in these studies about the possible differences which may occur with transcoding errors when comparing children from countries where the language is read left-to-right and right-to-left. Nuerk et al. (2011) pointed this out and expressed the view that "we believe that it is well conceivable that writing words from right-to-left, but digits in multi-digit Arabic numbers from left-to-right (e.g. in Hebrew) may produce additional culture-specific interference in multi-digit number processing" (p. 17). It is for this reason that we decided to compare children transcoding from the two countries of England and Kuwait. In England, text is written left-to-right and numbers are said and written left-to-right. Arabic is spoken in Kuwait, this being written right-to-left while numbers are said in a way similar to German: left-to-right but with the tens and units digits inverted. Thus, we might expect some similar effects to those studies which have included German participants, but there might be other factors due to the different direction when writing text. Specifically, we wanted to answer these questions: What type of errors do these 5- to 7-year-old children make when transcoding one, two and three-digit numbers? What differences are there between English and Kuwaiti children's errors? How might these errors be affected by the way in which numbers are said and the direction of writing text? What consequences might there be for how numbers are said during the teaching of early number work?

2 The Arabic and English number naming systems

In Arabic, most two-digit numbers are said, and invariably written, with the units digit first, followed by the tens digit. The exceptions are the multiples of ten, which are written left-to-right. So, *ushroon* (*twenty*) would be written with 2 first and then 0, whereas *wahed wa ushroon* (*twenty-one*) would be written as 21 but with the 1 first and then the 2. With three digits numbers, the hundred digit would be said and written first with the rest of the number following the two-digit conventions. The spoken language associated with the written numbers has different words for the numbers 1 to 10 (see Table 1).

The *-teen* numbers are said with the units digit word followed by a variation of *ashrah* (*ten*). For example, 13 would be said *thalathata-ashar* (*three ten*). Other two-digit numbers than multiples of ten, are said in a similar way but with the inclusion of *wa* (*and*) between the digits. For example, 57 (٥٧) would be said as *saba'ah wa khamsoon* (*seven and fifty*). For the numbers 30 (٣٠), 40 (٤٠), 50 (٥٠),… 90 (٩٠), the beginning of the name is similar to that for 3 (٣), 4 (٤), 5 (٥), …, 9 (٩) but has an *-oon* sound ending which is different to the name for 10 (١٠), whose name is *ashrah*. The beginning of the word for 20 (٢٠) has a sound derived from that for 10 (١٠), rather than the name for 2 (٢), before finishing with the *-oon*

Table 1 Number names for the Arabic numerals. Please note that the spelling of these names are approximations to the sounds of the spoken names, since the Arabic written script is quite different

Numeral		Spoken sound	Numeral		Spoken sound	Numeral		Spoken sound	Numeral		Spoken sound
English	Arabic		English	Arabic		English	Arabic		English	Arabic	
1	١	Wahed	11	١١	Ahda-ashar	10	١٠	Ashrah	100	١٠٠	Me'ah
2	٢	Ethnan	12	١٢	Ethna-ashar	20	٢٠	Ushroon	200	٢٠٠	Me'atan
3	٣	Thalathah	13	١٣	Thalathta-ashar	30	٣٠	Thalathoon	300	٣٠٠	Thalatho-me'ah
4	٤	Arba'ah	14	١٤	Arba'ta- ashar	40	٤٠	Arba'oon	400	٤٠٠	Araba'o- me'ah
5	٥	Khamsah	15	١٥	Khamsta- ashar	50	٥٠	Khamsoon	500	٥٠٠	Khamso- me'ah
6	٦	Sittah	16	١٦	Sittata- ashar	60	٦٠	Sittoon	600	٦٠٠	Sitto- me'ah
7	٧	Saba'ah	17	١٧	Sab'ata- ashar	70	٧٠	Saboon	700	٧٠٠	Sabo- me'ah
8	٨	Thamaniah	18	١٨	Thamaniata- ashar	80	٨٠	Thamanoon	800	٨٠٠	Thamano-me'ah
9	٩	Tes'ah	19	١٩	Tes'ata- ashar	90	٩٠	Tesoon	900	٩٠٠	Tesoo- me'ah
									35	٣٥	Khamsah wa thalathoon
									683	٦٨٣	Sitto- me'ah wa thala-tha wa thamanoon

sound. The difference between 20 (٢٠) and the other multiples of ten is found within the names for the hundreds as well. 100 (١٠٠) itself is just a single word *me'ah* (equivalent to *hundred* rather than *one hundred*). 200 (٢٠٠) takes on the 'pair' form of the name for 100 (١٠٠), *me'atan*, which has an additional *-an* sound added at the end. The rest of the multiples of a hundred take the form equivalent to *five hundred* for 500 (٥٠٠).

With the exception of multiples of ten, two-digit numbers are written in Arabic with the unit digit written first, followed by the tens digit. This makes the issue of inversion particularly interesting since the orders of both speaking and writing these numbers are the same. Thus, the language of these number names is not an inverted language if inversion is defined as the comparison of the order of the spoken number names and the written notation. This also applies to the multiples of ten. Even though the digits are written tens first and the zero after, the number names also reflect this (see Table 1).

The English numbers have separate names for 1 to 12, followed by the *-teen* numbers. These mostly have regularity with the unit digit followed by the sound *-teen*. Numbers from 20 onwards are written and spoken from left-to-right, with the multiples of ten following the rule of saying the name for the appropriate digit followed by *-ty* only from 60 to 90 (e.g. *six-ty*). The start of the names for 20, 30, 40 and 50 all have some exceptions. In contrast to the USA, in the UK the word *and* is said after the number of hundreds with three-digit numbers.

3 Frameworks

We use two frameworks which have their roots within a Vygotskian perspective. The first is based upon the notion that language and thought are interconnected. Indeed Vygotsky (1992) claimed that "thought is not merely expressed in words; it comes into existence through them" (p. 218). The meaning children develop for the symbolic notation of number will be partly a consequence of the language used in relation to that notation. Whether the notation 62 is expressed as *sixty-two* or *two and sixty* matters. Vygotsky (1992) talks about whether associated signs have the property of being reversed. For example, the symbol 62 may become associated for a German child with the equivalent words of *two and sixty*; however it is another matter whether the words *two and sixty* are associated for them with 62 rather than 26 or 260 or something else. If a child has not developed a meaning for certain number words then we are mindful that "a word without meaning is an empty sound" (Vygotsky, 1992, p. 6). In such a situation, a child might respond to *two and sixty* using symbols other than 2, 6 or 0, or they might not write anything at all. The words and notation associated with numbers are part of a mathematical structure, in this case place value. The desired educational result is that children gain meaning for these words and symbols which align closely with this mathematical structure. By 'meaning' we do not consider this in terms of cardinality at this stage. We use the word 'meaning' to indicate a sense of underlying rules which govern the way in which number names are expressed in symbols. We see meaning, words, notation and underlying mathematical structure being linked as indicted in Fig. 1. Vygotsky (1992) said that "Speech itself is based on the relation between a sign and a structure of higher intellectual operations, rather than purely associative connections" (p. 109) and as such we feel the direct link between words and notation for a child is only established through the sense of a structure which forms a temporal meaning accompanying these. Temporal in the sense that a sequence of approximate structures may be formed on the way to establishing a sense of structure which is consistent with our

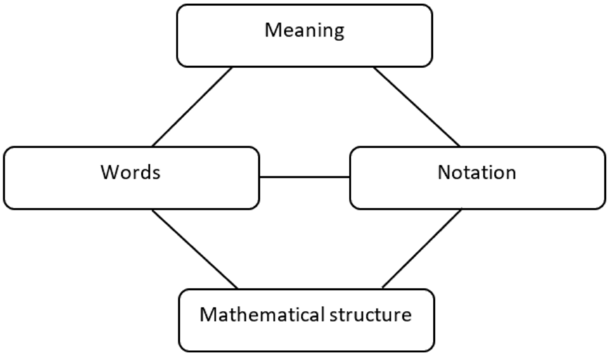


Fig. 1 A generalised association between words and notation is established through the meaning which someone develops in relation to them. There is a mathematical structure which lies behind the words and notation, and the educational aim is for the child's meaning to align with this structure

place value system. We consider it is not possible for the underlying structure to be 'given' directly to learners; this they need to do for themselves through trying to abstract rules for what gets written when certain words are said, and vice-versa. So the link between spoken words and written symbols is of considerable educational significance. What is educationally desirable is not just associating an arbitrary word with an arbitrary sign, but gaining a sense of a system of words and signs based upon the underlying mathematical structure of place value. This is the case with number, where the association is generalised beyond a particular collection of examples which have previously been met. For example, your ability to say out loud the number 7289 is due to the generalised meaning you developed, as it is likely that you have never been asked to say this number before in your life.

The second framework comes from Vygotsky's development of Piaget's notion of spontaneous and non-spontaneous, or scientific, concepts. Spontaneous concepts originate from a child's personal reflections on perceptual attributes whereas scientific concepts are "culturally formulated and transmitted" from external sources such as family and school (Alves, 2014, p. 25). Vygotsky (1992) argued that these two concepts interact and unite into a total system. As children are being introduced to the 'scientific' notion of how numbers are written within a place value system, they are also developing their own personal notion of what might be the rules behind such a system. Being told by a teacher how to write numbers does not always coincide with the developing spontaneous notions a child is developing about this system. Otherwise, 'mistakes' of how numbers are written would not happen. As Brizuela and Cayton (2008) say, "notations being constructed have an impact on the concepts and meanings being constructed, and vice versa" (p. 212). We consider that when scientific concepts are introduced, it is not as simple as children just taking these on board. Instead, there is a dynamic with the spontaneous concepts they bring to these new ideas. As Vygotsky (1992) said, "the child, while assimilating adult concepts, stamps them with characteristics of his [sic] own mentality" (p. 154). Actions taken by a child may begin to be more than just memorised attempts to reproduce what they have been told. Instead, there can be a shift from memorising rules given to them by a teacher to something more related to an awareness based upon a personally abstracted rule from their experiences with these ideas. This effectively shifts their actions from being informed 'top-down' as a scientific concept to a 'bottom-up' spontaneous concept. The inverse shift can also take place, with a child acting upon their personal spontaneous ideas only to find that they are at odds with what a teacher is saying is correct. Here, there can be an abandonment (at least temporarily) of their original spontaneous way of thinking and a return to trying to take this on board as a scientific concept from the teacher. Thus, the learning of a new concept happens over a period of time, involving a dynamic between spontaneous and scientific concepts until there is a meeting of the two. Vygotsky (1992) articulated this by saying "The child's and the adult's meanings of a word often "meet," as it were, in the same concrete object, and this suffices to ensure mutual understanding" (p. 111). However, these respective 'meanings' may meet within a certain range of variation, say for two-digit numbers, but find that this is not the case when numbers are extended outside that range, for three-digit numbers for example.

4 Methods

We asked children from Kuwait and the UK to answer three types of questions: write down numbers which were spoken to them; write in numerical form numbers which were written in words; and say numbers which were written in numerical form. This paper will concentrate on the first of these. Children from Kuwait came from two elementary schools, one for boys and one for girls. A total of 191 children from the first grade (5–6 year olds) participated, and 205 children from the second grade. The children were all Arab speakers. Children from the UK came from four different primary schools with 142 from year 1 (5–6 year olds), 114 from year 2 and 87 from year 3. This paper will focus on years 1 and 2 only from the UK along with all the children from Kuwait. There were also 12 other children from the UK whose answers were discounted due to either consent forms not being signed or due to incomplete data following illness and absence on the relevant days. All the data from Kuwait were used. We did not have data on the first language of the UK children, but extrapolating from the proportion of children within each of the schools whose first language was not English, we estimate that about 11.5% of the children in our data set may not have English as their first language. We note that Dowker et al. (2008) showed that the language of instruction may be of more significance than the first language of the children in the comparison of two-digit numbers.

We asked the children to write and say 28 numbers, which were a mix of six one-digit, 13 two-digit and nine three-digit numbers. In this paper, we concentrate on 2- and 3-digit numbers. The numbers were chosen to take into consideration getting *-teen* and *-ty* numbers mixed up (1X and X0 numbers), having non-*teen* XX two-digit numbers, and having a selection of X00, XX0, X0X and XXX three-digit numbers. We kept the overall number of questions as small as possible, whilst accommodating the above, so as not to make these tests too long for the children.

Data were collected in the UK by the class teachers. They read out a series of numbers and the children were asked to write the number in boxes on an answer sheet. Questions were given with a short break after every four numbers, with there also being a visual gap in the answer boxes on the question sheet. This was to prevent children being unsure of which box they were to write their current answer in. In Kuwait, a similar process was carried out but with one of the researchers reading out the numbers.

Incorrect answers were coded, initially based upon the literature and our own previous knowledge as teachers, educators and researchers. This included 'inversion' errors, writing digits reversed as in a mirror image, and additive and multiplicative composition errors (Zuber et al., 2009). We also interviewed some elementary teachers in Kuwait to gather their thoughts about possible errors children might make. We were open to the possibility of finding several occurrences of errors which we had not considered beforehand. In this way there was an element of thematic analysis which also took place. We classified scribbles which did not visually resemble digits, or drawings, the same as questions not answered. Many answers were given more than one code as there could be different types of errors appearing within the same answer. For example, when *four hundred and eighty* was written as 40018, this was given a code for writing the *four hundred* out fully with two zeros, and a code for mixing up *eighty* with the *-teen* number 18. Steiner et al. (2021) also found a high frequency of combination errors. In addition, we highlighted answers which did not fit into common errors or had additional, idiosyncratic, aspects to their answers. There was then a lengthy process of the two researchers reviewing codes to ensure consistency of coding across the UK and Kuwait data sets. We collected the total numbers of occurrences of the codes along with a percentage of these, compared with the number of opportunities students had to make those types of errors given the numbers they were asked to write down. We then reflected upon the nature of language-specific errors made in the respective languages in two ways: how specific errors might be connected with the language used in saying numbers; and to gain a sense across the year group of the alignment between the apparent spontaneous notions of how numbers were written with the socially agreed scientific conventions for writing numbers in a place value system. Lastly, we considered the educational implications for assisting children with gaining a sense of the underlying place value structure of number.

5 Results

5.1 Two-digit numbers

The swapping of tens and units digits was by far the most common error amongst the Kuwaiti children, with 10.2% of the written answers having this mistake in year 1 and still 7.1% in year 2. This is not technically an inversion error, since the order of the spoken number name matches the order in which the children are taught to write the digits. This was, therefore, a surprise to us. It mirrored errors found in other languages where inversion was part of the language and also how such errors could persist beyond a time when the curriculum might suggest children would be confident with two-digit numbers (Krinzinger et al., 2011). A possible reason behind this could be that there is little consistency in the order in which the digits are written in Arabic. With XX numbers, the digit on the right is written first; but with X0 numbers, the digit on the left is written first. This lack of consistency means that developing a more spontaneous sense of how to write these numbers is more difficult and so the children are left to try to memorise what might appear to be rather arbitrary rules, as a scientific concept. Furthermore, as will be discussed in the next section, three-digit XXX numbers involve the left-most digit written first, then the right-most digit and finally the digit in the middle. It is maybe not surprising then that Kuwaiti children had difficulty in deciding whether the left-most or right-most digit was written first. Although less frequently, we did also find swapped-digit errors with X0 numbers with the Kuwaiti children. This was despite the fact that 02 (٠٢), for example, might look a bit unusual, and with the words being said left-to-right. This supports the notion that this error is not so much about inversion, but about the lack of consistency with the order in which the number is said compared with how it is written.

The greater consistency with the XX numbers in English can help those children develop a spontaneous concept for these numbers which is consistent with the socially accepted convention for how they are written. However, such spontaneous concepts which work for nearly all the XX numbers, do not fit with the 1X *-teen* numbers. The English

-teen numbers are the only numbers where there is inversion within the language of the number names. It was therefore not surprising that it was with these numbers that we found inversion errors taking place with the English children.

The effect of the word *wa* (*and*) can be seen within the Kuwait data with two-digit numbers. The *-teen* numbers are said in Arabic as the unit digit followed by a variation of the word for 10. For example, 18 (١٨) is said *thamaniata-ashar* (*eight ten*). Non-*teen* XX numbers are said with the word *wa* (*and*) added in between the name for the unit digit and the name for the appropriate number of tens. For example, 48 (٤٨) is said *thamaniah wa arba'oon* (*eight and forty*). The inclusion of the word *wa* means that the words for each of the two digits were separated and increased the chance that they were viewed and written separately as 408 (٤٠٨) rather than 48 (٤٨) compared with the *-teen* numbers. This is borne out in Table 2 where this error rarely occurred with the *-teen* numbers but occurred more than ten times as often with non-*teen* two-digit numbers. This error hardly occurred at all with the English children (on just five occasions). In English, the word *and* is not said for two-digit numbers and so was not present as a marker for the tens and units words to be treated separately. This indicates for us the role that the word *wa* can play in establishing rules behind developing spontaneous concepts which clash with the socially accepted place value conventions.

This error was particularly prevalent for the Kuwaiti children with the numbers 25 (٢٥) and 26 (٢٦) (14.3% and 16.4% in year 1 respectively) and far less so with the other XX numbers (the next highest was 52 (٥٢) with 5.2% in year 1). With the two-digit numbers above the twenties, the Arabic word for the tens digit is based upon the sound of the single-digit number with the additional ending of the *-oon* sound (see Table 1). For example, 5 (٥) is said *khamsah* and 50 (٥٠) is said *khamsoon*. However 20 (٢٠) is different, with 2 (٢) being said *ethnan* and 20 (٢٠) being said *ushroon*. This can result in a weaker association being formed between *ushroon* and the digit 2 (٢), compared with *khamsoon* and the digit 5 (٥). Instead, there is a stronger association of *ushroon* with 20 (٢٠) per se, and hence the greater likelihood of 20 (٢٠) being included in the writing of 25 (٢٥) or 26 (٢٦) (i.e. ٢٠٥ or ٢٠٦). A similar issue arose with the three-digit numbers, which will be discussed in the next section.

The only other significantly occurring error made by the English children, was to confuse the *-teen* and *-ty* endings of number names. Table 3 shows the percentages of given answers with such errors. It seems as if this type of error quickly goes away as the year 2 children rarely made this error. We have included 12 within the table to indicate that it was more of an issue with the *-teen* and *-ty* endings than the common starting sound of *twe-*. This linguistic issue of *-teen* and *-ty* was not present for the Kuwaiti children and there were very few errors of mixing up 13 and 30 for example. There is a clear linguistic difference in the endings of 1X and X0 number names in Arabic. This shows that the sound of the words is significant as well as the composition of the words involved in a number name.

Table 2 Occurrences of error of writing out the tens digit of a two-digit number as a separate number in Kuwaiti year 1 children (e.g. 48 (٤٨) written as the equivalent of 408 (٤٠٨))

	Number of times error occurred	Number of attempts	Percentage of error
Non-teen	77	1011	7.62%
Teen	6	863	0.70%

Table 3 Percentage of errors made mixing the *-teens* with the *-ty* numbers (e.g. writing 30 for *thirteen*)

Number said	England		Kuwait	
	Year 1	Year 2	Year 1	Year 2
12	0%	0%	0.6%	0%
13	6.4%	1.8%	0.6%	1.0%
17	2.9%	0%	0.6%	0.5%
19	5.0%	1.8%	2.5%	0%
20	0.7%	0%	3.4%	0%
30	3.6%	0%	8.4%	0.5%

5.2 Three-digit numbers

5.2.1 Placement of the hundreds digit

With three digits, there are extra directional complications with Arabic. Unlike German, the natural reading of text is right-to-left and this fits in well with writing and reading two-digit XX numbers, where the unit digit is said and written first, followed by the tens digit. However, with a three-digit number, the practice of teaching is that the hundreds digit is said first, followed by the units and then the tens. This means that, coming from a natural right-to-left perspective, there is a jump needed to go to the left digit, then come back to the right digit and then deal with the middle digit. With this issue of changing direction, we found that the most common mistake with the Kuwaiti year 1 children was starting to write the three-digit numbers from the right, resulting with the hundreds digit in the units place. An example would be writing *thalatho-me'ah wa khamsta-ashar* (*three hundred and fifteen*) as 153 (١٥٣). There are two factors involved with this; the first being that the written script is usually from the right anyway, and the second is that this is what they had become used to doing when writing two-digit numbers. We consider that the spontaneous concepts built up from experience with two-digit numbers have been transferred by many children to the three-digit number situation. This mistake occurred in over 50% of all the year 1 attempts at writing three-digit numbers. This continued through to year 2, with over 16% of all attempts having this error. This was despite three-digit numbers being explicitly taught during that year. As Vygotsky (1992) points out, "the curve of development does not coincide with the curve of the school instruction" (p. 185). This type of error was extremely rare with the English children, with only five occurrences in total across years 1 and 2. In the English situation, the writing of three-digit numbers follows what they did for two-digit numbers and also continues the natural writing from left to right.

5.2.2 Swapping tens and units digits

In year 2, when the Kuwaiti children were explicitly taught three-digit numbers, there was an error which became more prominent than with year 1. This was the swapping tens and units digits error with three-digit numbers (13.3% of attempts in year 2 compared with 9.4% in year 1) despite a decrease in doing so with two-digit numbers. This seemed to be a consequence of trying to learn what to do with three-digit numbers. If they remembered to start with the hundreds digit on the left then to write the number correctly, they also had to remember to leave a gap before writing the units digit, and then fill the gap with the tens

Fig. 2 Changes in direction needed in Arabic to write the three-digit number 259 (٢٥٩) correctly

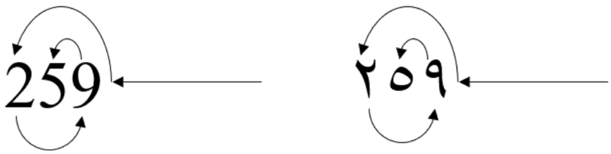
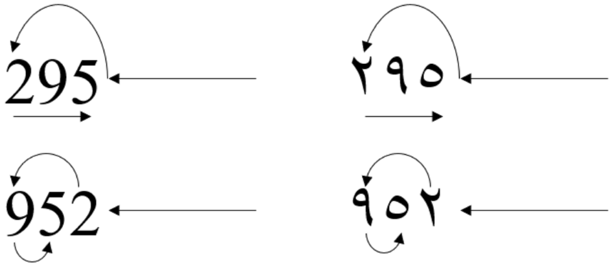


Fig. 3 Error in writing 259 (٢٥٩) by forgetting one of the two new rules of **a** leaving a gap, and **b** starting at the left. Both of these produce a swapping of tens and units error



digit. This requires a new scientific concept to be taken on board. There were effectively two rules to be remembered (see Fig. 2 for the changes in direction necessary). Forgetting just one of these rules would result in the tens and units digits being inverted, albeit with those two digits ending up in different positions in the final three-digit number (see Fig. 3).

When the children did apply just one of these rules, they usually continued in one direction. For example, with a number such as 259 (٢٥٩), the children would hear the digits in the order 2 (٢), then 9 (٩), then 5 (٥). If they put the hundred digit on the left, then writing it as 295 (٢٩٥) would keep the direction of going to the right (Fig. 3a), rather than continually changing direction to write it correctly (as in Fig. 2).

It was rare that the swapping error was accompanied by also writing the hundreds at the right with XXX numbers, as in Fig. 3 b. This may be because starting with the hundreds digit on the right already indicated an inclination to stay with what they learned with two-digit numbers, and so they just continued going to the left as they did with two-digit numbers. So, this resulted with avoiding this particular error.

5.2.3 Writing 100 or 00 after the hundreds digit

The way in which the hundreds part of a number is said is generally through two words: the first is the digit word (e.g. *four*) and the second is the value word (e.g. *hundred*). This can result in children writing the digit followed by writing something for the value word, such as 100. So, *eight hundred and thirty* could be written as 810030. This was a common error with year 1 Kuwaiti children with nearly a third of attempts having this error. It was less common amongst the English year 1 children, although still occurring in nearly a tenth of the attempts. For both countries, this mistake rarely occurred in year 2, so it seemed short-lived. What was of particular interest was that language seemed to play a significant role in this type of error. With the Kuwaiti year 1 children, although it was a common mistake with other three-digit numbers, it only happened on one occasion with 259 (٢٥٩). In Arabic, 200 (٢٠٠) is said not as *two hundred* — a digit word followed by a value word — but

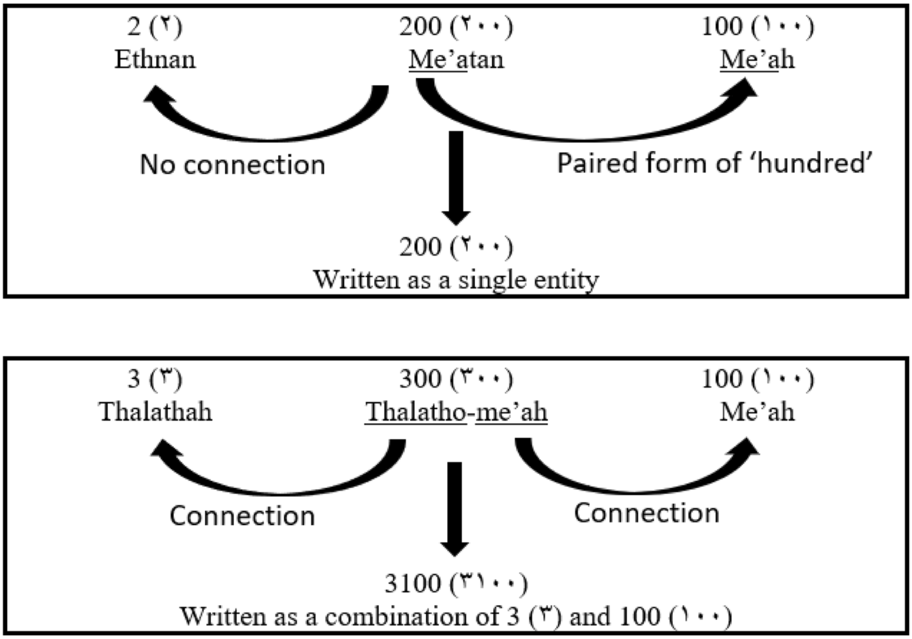


Fig. 4 Arabic sound associations which can result in different ways of writing 200 (٢٠٠) numbers compared with other hundred numbers (300 (٣٠٠) is taken here as an example)

as the 'paired' form *me'atan* of the word *me'ah* (*hundred*). As such, it is a single word and so does not encourage the writing down of two numbers, one for the 'digit' word and another for the 'value' word (see Fig. 4). The error of writing 259 (٢٥٩) as 210059 (٢١٠٠٥٩) was therefore extremely rare, happening only once, whereas it was a common error for the other hundreds. This also relates to the similar linguistic issue discussed above with 20 (٢٠). We felt this indicated the way in which children's developing spontaneous concepts impacted upon the way in which they wrote the number names. The fact that there were distinct errors made by many of the Kuwaiti children for 259 (٢٥٩) compared with other three-digit numbers indicated the fact that they were not just memorising a scientific concept given by their teacher, but developing more spontaneous concepts based upon the language of those particular number names. In English there is, of course, the word *two* as well as the word *hundred*. As a consequence, the English children made the error of breaking up the hundreds into the number of hundreds followed by 100 just as often with 259 as with the other hundreds.

We looked further into the way the hundreds digit was written. In both English and Arabic, the inclusion of the word *and* helps to separate the *four hundred* from the *eighty* and thus increased the chance that these might be written separately, with 400 first, followed by the 80. Interestingly, this type of error was made more than twice as often with the English children than with the Kuwaiti children; this was in contrast to the error of including 100 after the hundreds digit, which happened more than three times as often with Kuwaiti children than the English (see Table 4). This links with what we discussed previously concerning the likelihood of all but the 200 numbers being associated with the value digit followed by 100, rather than 00 (Fig. 4). This difference could also be affected by the different ways in which 100 is said in English

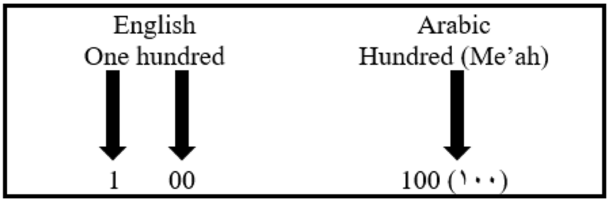
Table 4 Percentage of attempts with the error of writing 00 (٠٠) or 100 (١٠٠) after the hundred digit (year 1)

	Kuwaiti children	English children
Writing 100 (١٠٠) after hundred digit	32.5%	9.6%
Writing 00 (٠٠) after hundred digit	20.1%	43.4%

and Arabic. In our study, with the English children it was said as *one hundred* – a digit word (*one*) followed by a value word (*hundred*) — whereas in Arabic it was always said just as *hundred* (*me'ah*). This meant that in Arabic the single word *hundred* (*me'ah*) could carry with it the notation 100 (١٠٠). In English the two words could each carry with them a part of the notation 100, with *one* being associated with 1, and *hundred* being associated with 00 (see Fig. 5).

A consequence of the Arabic situation is that a teacher might feel that the linking *me'ah* (*hundred*) with the notation 100 (١٠٠) is helping establish the underlying mathematical place value structure. However, the child may be forming linguistic connections which result in the name for 300 (٣٠٠) being associated with 3100 (٣١٠٠) rather than 300 (٣٠٠) (Fig. 6) and becoming detached from the desired mathematical structure of the notation. This highlights a complex pedagogical dilemma for a teacher. What may appear to assist with developing a sound meaning for the structure of place value, may, upon more detailed analysis, be creating a potential misalignment with the conventional way of writing number names.

Fig. 5 Different possible associations with the way in which 100 (١٠٠) is said



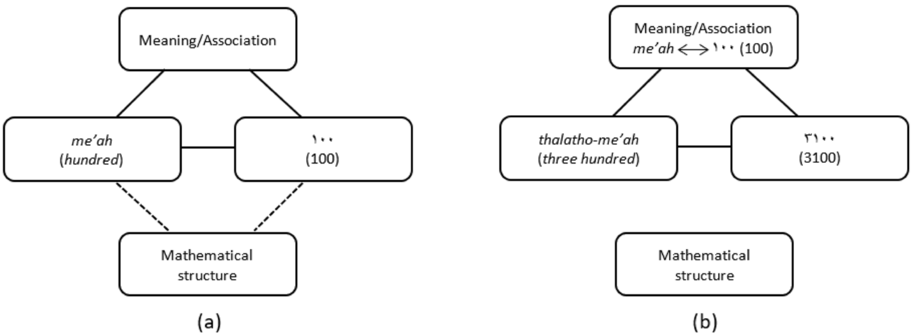


Fig. 6 **a**) A child associates the word *me'ah* with the notation 100 (١٠٠). The dotted lines indicate a link a teacher might have with these to the underlying mathematical structure, but this may not be necessarily so for the child; **b**) the learnt association is used with *thalatho-me'ah* and reveals a disconnect with the mathematical structure

This means that there is an increased possibility that a Kuwaiti child might associate *hundred* with 100 (١٠٠) whereas an English child might associate it with 00. This can account for the difference between the English and Kuwaiti children as seen in Table 4.

5.2.4 Leaving out a zero

Another significant difference between the Kuwaiti and English children concerned the possibility of leaving out zeros when being asked to write down a number said to them. This can happen when a number such as *five hundred and four* is written as 54 (٥٤). This did not happen at all with any of the English children, yet it was quite a common error with the Kuwaiti children (21.4% of attempted questions involving three-digit numbers which had a zero in them). This dramatic difference was a surprise. The English children were more inclined to add extra zeros rather than leave any out. The way a number is spoken does not say anything about the zero element. 504 is said *five hundred and four* and not *five hundred, no tens and four*. The zero can only be identified by the absence of something which might have been expected. As a listener, I have to expect to hear something about the tens and then note its absence by writing a zero. If I do not expect to hear something, then I will not note its absence and so not write a zero. The difference in the way in which English and Arabic is written, means that this decision moment, of whether to write a zero or not, occurs at different time points. For an English speaker, it is usual to hear a three-digit number, such as 564, said as *five hundred and sixty four* — hundreds then tens then units. With 504, noticing the absence of the tens occurs midway through a spoken three-digit number name. Hence the zero is likely to be written before writing the final 4. In Arabic, 564 (٥٦٤) is said *khamso- me'ah wa arba'ah wa sittoon* (*five hundred and four and sixty*) with two occurrences of *wa* (*and*) marking each of the shifts from one place value to another. With 504 (٥٠٤), noticing the absence of the tens occurs at the end, after the 5 (٥) and 4 (٤) have already been written. So two things happen here for the Kuwaiti children. Firstly, the writing of a zero can become an afterthought, something which has to be done after all the other digits have already been written. Secondly, there is the absence of a second *wa*, which marks the fact that there is another place value involved. In English, there is only one *and* whether or not the tens digit is zero. As such, it does not play such a role in marking the number of digits involved in the number.

With XX0 three-digit numbers, the decision moment of noting the absence of a digit, happens the opposite way round for the two languages. This time, for English speakers, the hundreds and tens digits are already written before there is an absence of a units digit being said. Whereas for Arabic speakers it happens in the middle, between the hundreds digit being said and the tens digit. Thus, we expected within the Kuwaiti data that there might be less errors of this kind with XX0 numbers than X0X numbers. Indeed this proved to be the case, with 50% more proportional errors per attempt for the X0X numbers than for the XX0 numbers. Even for the XX0 numbers, this remained a common mistake for the Kuwaiti children (at around 20% of attempts), which lead us to feel that the role of the words *wa/and* remained significant.

6 Conclusions

The errors children made when transcoding numbers were similar to those found in previous studies and these were swapping tens-and-units digits — visually similar to inversion errors in some other languages (Imbo et al., 2014; Moeller et al., 2015), writing out more fully the tens and hundreds digits, such as 205 for *twenty-five* (Clayton et al., 2020), leaving out zeros or adding in extra ones, and also adding in additional digits which did not appear in the number which was spoken.

We did find significant differences across the two cultural settings and many of these can be accounted for by the differences in the language of how numbers were said. Regarding the digit swapping of two-digit numbers, the Kuwaiti children made far more of these errors than the English children, with the English children mainly making those errors with the numbers said in an inverted form, the *-teen* numbers. The swapping of tens-and-units digits remained a common error for year 2 children in Kuwait, whilst it become a rare error with year 2 English children. We found that these errors were also appearing with three-digit numbers for the Kuwaiti children, particularly in year 2. The swapping of tens and units digits has common features found in other languages, such as German, where inversion errors occur (for example, Moeller et al, 2015). However, with Arabic being written right-to-left, we felt other factors may be involved. Nuerk et al. (2011) posit that some additional interference dealing with multi-digit numbers might be involved for children where the language is written right-to-left. The need for the Kuwaiti children to jump from right-to-left, then back to the right again, and finally left, brings a complexity to writing three-digit numbers which the English children did not have. Ignoring one of the two rules of (a) leaving a gap, or (b) starting at the left, would result in a swapping error. The explicit teaching of three-digit numbers in year 2 meant more children made attempts at these questions than with year 1, but showed they were not consistently following both the two rules to avoid such an error.

The similarity within the English language of *-teen* and *-ty* seemed to result in English children confusing these numbers and writing, for example, 30 for *thirteen*. Such errors did not occur with the Kuwaiti children as those linguistic similarities are not present in Arabic.

An interesting feature of the Arabic language is that when a list of items is being spoken, such as perhaps what someone had for breakfast, the word *wa* (*and*) would be used in-between each item. In English the word *and* is only said before the last item in the list. Related to this is something we have not found reported elsewhere; that is the potential role of the word *and* as a marker for the number of digits present within a spoken number. Arabic uses the word *wa* between the hundreds, units and tens digits with most of their numbers. Thus, this word marks the finishing of the name for one of the place value digits and the start of the next. This can assist each part being thought about separately, such as with *araba'o- me'ah wa thamaniah wa thalathoon* for 438 (٤٣٨). However, this is not the case with the numbers from 11 (١١) to 19 (١٩) which do not have *wa* appearing. This lack of separation can help to see the number as one entity rather being made of two recognisably separate numbers. This could be a factor in the ten-fold increase in the occurrence of writing the tens digit fully, such as writing 408 (٤٠٨) instead of 48 (٤٨), with XX numbers rather than 1X numbers with the Kuwaiti children. The word *wa* is also significant within three-digit numbers with Arabic having two inclusions of *wa* with XXX numbers, whereas there is only one in English. Again, *wa* acts as a marker between each of the three digits in Arabic. When a three-digit number is either XX0 or X0X, one of these *wa* sounds is missing, whereas in English it remains the same. This lack of the extra *wa* can signal that there are now only two digits involved, rather than three. This can account for the significant difference between leaving out the zero being a common error with Kuwaiti children (e.g. 504 (٥٠٤) written as 54 (٥٤)) and not happening at all with the English children.

The Arabic language treats 20 (٢٠) and 200 (٢٠٠) differently to the other multiples of 10 and 100, and this can affect the associations set up between the number name and the symbols. The name for 300 (٣٠٠) uses the name for 3 (٣) and the name for 100 (١٠٠) and we found occurrences of 3100 (٣١٠٠), for example, in the three-digit numbers, except for the 200 number we used, 259 (٢٥٩). With *two hundred* (*me'atan*), the 'paired' form of *hundred* (*me'ah*) is used in the number name rather than explicitly using the name for 2 (see Table 1). This results in there not being such a strong association with the digit 2 (٢) followed by the digits 100 (١٠٠). It becomes more of an entity in its own right, as 200 (٢٠٠) rather than 2 (٢) lots of 100 (١٠٠). Consequently, we found that the hundred part of an XXX number was written more as 200 (٢٠٠) rather than 2100 (٢١٠٠). We also found a similar association with two-digit numbers, where *twenty* (*ushroon*) takes the 'paired' form and does not have the sound for *two* (*ethnan*) within it and so becomes more of a single entity than with the other multiples of ten. We saw a greater rate of errors in writing the two-digit numbers of 25 (٢٥) and 26 (٢٦) as 205 (٢٠٥) and 206 (٢٠٦), than with other XX numbers which were not in the twenties. This aligned with what Byrge et al. (2014) found that the structure of what is heard can take precedence over the conventional position of digits in the written form. Thus, the way in which children write numbers is heavily influenced by the language of the number names.

The issue of associations between words and symbols is also present in the fact that 100 is said in Arabic as *hundred* and not *one hundred*. In English this is commonly said either as *one hundred* or perhaps *a hundred*. Within our study, it was said to the English children as *one hundred* and thus there can be an association of *one* with 1 and *hundred* with 00. This was not the case for the Kuwaiti children, who may associate *me'ah* (*hundred*) with 100 (١٠٠). We found that Kuwaiti children were more likely to write 100 (١٠٠) after the hundred digit whereas the English children were far more likely to write 00 after it. This can link together these two types of errors, which Zuber et al. (2009) had labelled separately as additive and multiplicative errors.

7 Educational implications

The language of the numbers names does affect the types of errors that children develop (Xenidou-Dervou et al., 2015). These errors reflect the difficulty children experience in gaining appropriate meanings for place value with both the number names and the written notation. If the language of the numbers is regular, this means that the underlying mathematical structure of place value appears within the language of the number names. Since language is integral to the development of meaning (Vygotsky, 1992), this helps children develop meanings which are more likely to align with the mathematical structure of place value. This meaning is then brought to the written notation when this is seen and in the writing of that notation (see Fig. 7). This results in a close connection between the words and the notation, with that connection being based upon the mathematical structure.

If the language of number names is not so clearly based upon the underlying mathematical structure, then children can develop spontaneous concepts which result in alternative meanings/associations placed in both the words and the notation, with these not necessarily being connected. Furthermore, neither may relate to the underlying mathematical structure (Fig. 8).

Children do tend to sort out place value over time, despite the language of number names being irregular. However, this is not so for all children and some carry significant misunderstandings of place value into their secondary education (Hewitt & Brown, 1998). Also, educational advantage seems to occur over many years for those whose language for number names is regular (Krinzinger et al., 2011). The question arises as to whether this

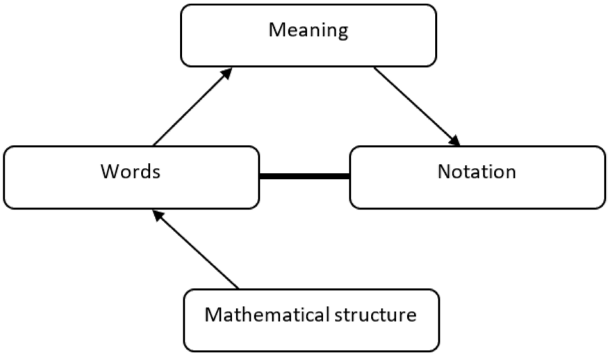


Fig. 7 The significance of language being based upon mathematical structure and how this assists appropriate mathematical meanings being brought to the written notation

is just accepted as a cultural phenomenon, that those whose language is irregular for number names will have this educational disadvantage, or whether there might be a pedagogic decision to assist the learning of place value through temporary use of an adapted language which is more regular. Piaget (2000), in his discussion of Vygotsky's points concerning spontaneous and scientific concepts, talked about a more productive form of instruction where schools "create situations that, while not "spontaneous" in themselves, evoke spontaneous elaboration on the part of the child" (p. 252). We believe in our analysis that children do bring spontaneous ideas to their tasks and that these are built upon the experiences they have gained up to that point in time. What we consider to be important educationally, is that teaching approaches might be adapted so that those educational experiences might increase the development of spontaneous concepts which align more with the way in which numbers are written in place value form. Göbel et al. (2014) concluded that "the structure of the language of instruction is an important factor in children's numerical development not only in basic numerical tasks such as transcoding and magnitude comparison but also in more complex arithmetic" (p. 25). Magargee and Beauford (2016) have shown that a pedagogic decision to use language regularity in teaching early number can have benefits. Also, Hayek, et al. (2019) found that children performed better with transcoding tasks in Arabic when numbers were presented in the order of hundreds, tens, units, rather than the traditional hundreds, units, tens. Adapted language which is more regular can help students become confident with place value and with how numbers are written, and then, once confidence is gained, can gradually shift into the use of the irregular words within the natural language. The alternative is that the irregular natural language is used, and the children

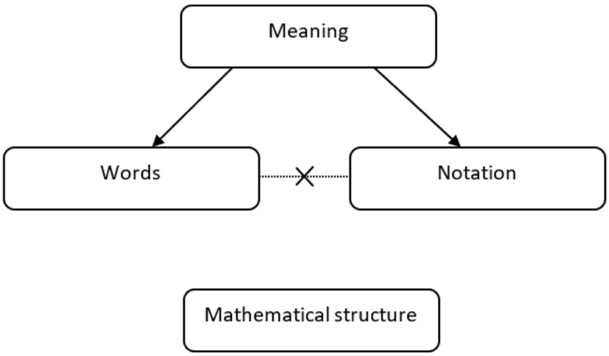


Fig. 8 The lack of mathematical structure within the words leaves children to place alternative, and possibly different, meanings/associations into the number name words and the written notation. This can result in a disconnect between the words and the notation, neither being particularly based on the underlying mathematical structure

struggle to gain a sound sense of place value and the value of written numbers. This can go on to affect their future learning of mathematics where numbers are involved. We argue that the former is worth serious consideration. Our study has shown that, for English, consideration might be given in the early stages of learning numbers to:

- consistently use *-ty* with *one-ty, one-ty one, one-ty two,...* for 10, 11, 12,...; *two-ty and three-ty and five-ty* for 20, 30 and 50.

The idea here is to reduce how many numbers are necessarily non-spontaneous, in that they are arbitrary names given to symbols (Hewitt, 1999). The digits 1 to 9 need to be given names and are necessarily non-spontaneous concepts. However, the language used with a combination of these symbols, such as 14, can allow children to develop spontaneous concepts which are more likely to align with the conventional way in which larger numbers are said and written. As Göbel et al. (2011) state, "a An explicit and regular structure might lead to a better and earlier understanding of the structure of the number system" (p. 556).

For Kuwaiti children, consideration might be given to:

- have a consistent direction in which the place value names are said, and the digits are written. Ganayim and Dowker (2021) found that native Arabic speakers performed better at transcoding tasks when three-digit numbers were presented in hundreds, tens, units order rather than the usual hundreds, units, tens order. Hayek et al. (2019) had also found similar results. However, we feel consideration should be given to an older style of saying numbers in Arabic which is used in literary academic circles where they pronounce numbers in the order of units, *wa*, tens, *wa*, hundreds, etc., even if it is not currently used outside of these circles. This order of units, tens and hundreds would fit in more with the order in which Arabic text is read and indeed used in some old periodic drama in Arabic countries. Having said this, the particular direction is not the main issue, it is that the order of what is said with what is written is matched.
- consistently write the numerals from right to left; this being in keeping with the above recommendation.
- We have noted an interesting effect with the paired form of *twenty* (*ushroon*) and two hundred (*me'atan*) used in Arabic, where the error of writing 100 (١٠٠) after the hundreds digit was not found in the two hundred number we used, whereas it was in other hundred numbers. Yet we also found that the two-digit numbers in the twenties had more errors (e.g. 25 (٢٥) written as 205 (٢٠٥)) than the other XX non-teen numbers. So, we feel that further research could be done to explore whether a temporary pedagogic decision to use explicitly the word for two in 200 and 20, rather than to use the paired form, would be beneficial.

Within the Arabic language, it is conventional to always include the word *wa* in-between any two items in a list. So, with four items, the word *wa* will be said three times. What happens with the Arabic number names is completely consistent with this. However, we have found that with X0X and XX0 numbers, the lack of a second *wa* seems to have resulted in children only writing two digits. Only one *wa* and so only two items. This error also appeared, albeit to a lesser extent, with X0 numbers, with only one digit being written. So perhaps consideration might be given to include explicit reference to a zero so that the children are still aware that three digits are involved, one being a zero. So, with 408, this might be said *thamaniah wa la shai'e* [*nothing*] *wa araba'o- me'ah*, this being consistent with our first recommendation above, saying the number right-to-left. A visual support for seeing the presence or absence of a particular place value can be the Gattegno Tens Chart (Gattegno, 1974), where the components of a particular number are

100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Fig. 9 Three rows of the Gattegno Tens Chart with the number 408 indicated

indicated through tapping, or highlighting, them on the chart (see Fig. 9 for the number 408). This also supports possible use of regularity within the number names, with a digit word for each column and a value word for each row. This can complement the use of practical materials which would focus on the cardinal sense of number (e.g. Ladel & Kortenkamp, 2016).

We suggest that in order for children to learn the place value structure, it is necessary for children to engage with numbers which may be much higher than those for which they have already gained a sense of cardinality. A meaning for place value cannot be obtained from just the numbers 1 to 20, for example. Nataraj and Thomas (2009) raised the question of whether it is "possible to enhance students' understanding of positional notation by exposing them to large numbers" (p. 103). Their study showed that students responded well when extended beyond what was stated within the curriculum for their age. Zazkis (2001) found using big numbers was helpful in an algebraic context to force consideration of structure over the urge to calculate. We suggest that exposing young children to larger numbers, along with pedagogic choices as to what language is said alongside those numbers, can also force consideration of the structure which underpins the place value system. We suggest that only when such structure is established, might the irregular exceptions for numbers such as *eleven*, *twelve*, etc., be introduced. Alternatively, the *-teen* numbers might be avoided whilst working with the tens chart until after children seem confident with the general structure. This would mean that the teaching approach would enhance the likelihood that the spontaneous concepts being developed by children would align with the mathematical structure underpinning the writing of numbers (Fig. 10).

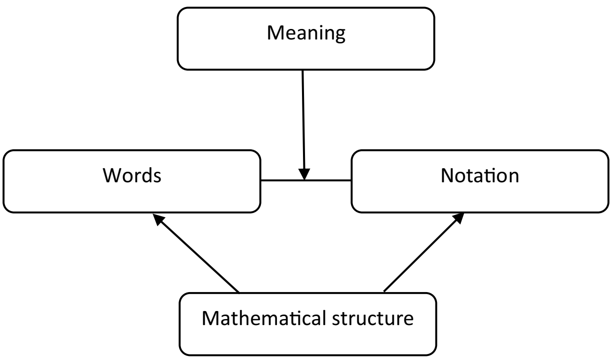


Fig. 10 When the number name words are consistent and reflect the mathematical structure of the notation, they both support the child's developing meaning of the underlying structure

Overall, we feel that further research into the temporary use of regular language during the early teaching of number would be a fruitful avenue to pursue. We suggest that there is a general principle of *regular first* (to establish the underlying structure) and *exceptions later*. This guides not only the language used but also the order in which numbers might be worked on. We suggest that the mathematical order of numbers (smallest first and then gradually up to 10, then 20, then 100) might not be the best order to teach children about the structure of number names and the writing of numbers in symbolic form. The *pedagogical order* might be different to the *mathematical order*. Pedagogic principles of regularising language could also be applied to other languages where there is irregular naming of numbers, following research into the particular errors children experience with those languages.

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Data availability The datasets generated and analysed during the current study, along with codes used, are available in the Loughborough University repository, https://doi.org/10.17028/rd.lboro.18003398.

Declarations

Competing interests The authors declare no competing interests.

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References

Alves, P. F. (2014). Vygotsky and Piaget: Scientific concepts. *Psychology in Russia: State of the Art, 7*(3), 24–34. https://doi.org/10.11621/pir.2014.0303

Askew, M., & Venkat, H. (2020). Deconstructing South African grade 1 learners' awareness of number in terms of cardinality, ordinality and relational understandings. *ZDM-Mathematics Education, 52*(4), 793–804. https://doi.org/10.1007/s11858-020-01132-2

Bergeron, J. C., & Herscovics, N. (1990).The kindergartners' knowledge of numerals. In G. Booker, P. Cobb & T. N. de Mendicuti (Eds.), *Proceedings of the 14th* Conference *of the International Group for the Psychology of Mathematics Education, 3*, 191–198 PME

Brizuela, B. M., & Cayton, G. A. (2008). The roles of punctuation marks while learning about written numbers. *Educational Studies in Mathematics, 68*(3), 209–225. https://doi.org/10.1007/s10649-007-9109-x

Bruce, R. A., & Threlfall, J. (2004). One, two, three and counting. *Educational Studies in Mathematics, 55*(1), 3–26. https://doi.org/10.1023/B:EDUC.0000017676.79430.dc

Byrge, L., Smith, L. B., & Mix, K. S. (2014). Beginnings of place value: How preschoolers write three-digit numbers. *Child Development, 85*(2), 437–443. https://doi.org/10.1111/cdev.12162

Clayton, F. J., Copper, C., Steiner, A. F., Banfi, C., Finke, S., Landerl, K., & Göbel, S. M. (2020). Two-digit number writing and arithmetic in year 1 children: Does number word inversion matter? *Cognitive Development, 56*, 100967. https://doi.org/10.1016/j.cogdev.2020.100967

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*(1), 1–42. https://doi.org/10.1016/0010-0277(92)90049-n

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General, 122*(3), 371–396. https://doi.org/10.1037/0096-3445.122.3.371

Dowker, A., Bala, S., & Lloyd, D. (2008). Linguistic influences on mathematical development: How important is transparency of the counting system? *Philosophical Psychology, 21*(4), 523–538. https://doi.org/10.1080/09515080802285511

Ganayim, D., & Dowker, A. (2021). Writing units or decades first in two digit numbers dictation tasks: The case of Arabic—An inverted language. *Brain Sciences, 11*(11), 1518.

Ganayim, D., Ganayim, S., Dowker, A., & Olkun, S. (2020). Linguistic effects on the processing of two-digit numbers. *Open Journal of Modern Linguistics, 10*(1), 49–69.

Gattegno, C. (1974). *The common sense of teaching mathematics*. Educational Solutions.

Göbel, S. M., Moeller, K., Pixner, S., Kaufmann, L., & Nuerk, H.-C. (2014). Language affects symbolic arithmetic in children: The case of number word inversion. *Journal of Experimental Child Psychology, 119*, 17–25. https://doi.org/10.1016/j.jecp.2013.10.001

Göbel, S. M., Shaki, S., & Fischer, M. H. (2011). The cultural number line: A review of cultural and linguistic influences on the development of number processing. *Journal of Cross-Cultural Psychology, 42*(4), 543–565. https://doi.org/10.1177/0022022111406251

Hayek, M., Karni, A., & Eviatar, Z. (2019). Transcoding number words by bilingual speakers of Arabic: Writing multi-digit numbers in a units-decades inverting language. *Writing Systems Research, 11*(2), 188–202. https://doi.org/10.1080/17586801.2020.1787298

Helmreich, I., Zuber, J., Pixner, S., Kaufmann, L., Nuerk, H.-C., & Moeller, K. (2011). Language effects on children's nonverbal number line estimations. *Journal of Cross-Cultural Psychology, 42*(4), 598–613. https://doi.org/10.1177/0022022111406026

Hewitt, D. (1999). Arbitrary and necessary: Part 1 a Way of Viewing the Mathematics Curriculum. *For the Learning of Mathematics, 19*(3), 2–9.

Hewitt, D., & Brown, E. (1998).On teaching early number through language. In A. Olivier, & K. Newstead (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education, 3*, 41–48. Stellenbosch, South Africa, PME

Imbo, I., VandenBulcke, C., De Brauwer, J., & Fias, W. (2014). Sixty-four or four-and-sixty? The influence of language and working memory on children's number transcoding. *Frontiers in Psychology, 5*, 313. https://doi.org/10.3389/fpsyg.2014.00313

Knudsen, B., Fischer, M. H., Henning, A., & Aschersleben, G. (2015). The development of Arabic digit knowledge in 4- to 7-year-old children. *Journal of Numerical Cognition, 1*(1), 21–37. https://doi.org/10.5964/jnc.v1i1.4

Krinzinger, H., Gregoire, J., Desoete, A., Kaufmann, L., Nuerk, H.-C., & Willmes, K. (2011). Differential language effects on numerical skills in second grade. *Journal of Cross-Cultural Psychology, 42*(4), 614–629. https://doi.org/10.1177/0022022111406252

Ladel, S., & Kortenkamp, U. (2016). Development of a flexible understanding of place value. In T. Meaney, O. Helenius, M. L. Johansson, T. Lange, & A. Wernberg (Eds.), *Mathematics Education in the Early Years* (pp. 289–309). Springer.

Magargee, S. D., & Beauford, J. E. (2016). Do explicit number names accelerate pre-kindergarteners' numeracy and place value acquisition? *Educational Studies in Mathematics, 92*(2), 179–192. https://doi.org/10.1007/s10649-016-9682-y

Miura, I. T., & Okamoto, Y. (2003). Language supports for mathematics understanding and performance. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 229–242). Lawrence Erlbaum.

Miura, I. T., Okamoto, Y., Kim, C. C., Steere, M., & Fayol, M. (1993). First graders' cognitive representation of number and understanding of place value: Cross-national comparisons: France, Japan, Korea, Sweden, and the United States. *Journal of Educational Psychology, 85*(1), 24–30. https://doi.org/10.1037/0022-0663.85.1.24

Moeller, K., Zuber, J., Olsen, N., Nuerk, H.-C., & Willmes, K. (2015). Intransparent German number words complicate transcoding – A translingual comparison with Japanese. *Frontiers in Psychology, 6*, 740. https://doi.org/10.3389/fpsyg.2015.00740

Nataraj, M. S., & Thomas, M. O. J. (2009). Developing understanding of number system structure from the history of mathematics. *Mathematics Education Research Journal, 21*(2), 96–115. https://doi.org/10.1007/BF03217547

Ng, S. S. N. (2012). The Chinese number naming system and its impact on the arithmetic performance of pre-schoolers in Hong Kong. *Mathematics Education Research Journal, 24*(2), 189–213. https://doi.org/10.1007/s13394-012-0037-4

Ng, S. S. N., & Rao, N. (2010). Chinese number words, culture, and mathematics learning. *Review of Educational Research, 80*(2), 180–206. https://doi.org/10.3102/0034654310364764

Nuerk, H.-C., Moeller, K., Klein, E., Willmes, K., & Fischer, M. H. (2011). Extending the mental number line: A review of multi-digit number processing. *Zeitschrift Für Psychologie/journal of Psychology, 219*(1), 3–22. https://doi.org/10.1027/2151-2604/a000041

Piaget, J. (2000). Commentary on Vygotsky's criticisms of Language and thought of the child and Judgement and reasoning in the child. *New Ideas in Psychology, 18*(2–3), 241–259.

Pixner, S., Zuber, J., Heřmanová, V., Kaufmann, L., Nuerk, H. C., & Moeller, K. (2011). One language, two number-word systems and many problems: Numerical cognition in the Czech language. *Research in Developmental Disabilities, 32*(6), 2683–2689. https://doi.org/10.1016/j.ridd.2011.06.004

Shaki, S., & Fischer, M. H. (2008). Reading space into numbers – A cross-linguistic comparison of the SNARC effect. *Cognition, 108*(2), 590–599. https://doi.org/10.1016/j.cognition.2008.04.001

Shaki, S., Fischer, M. H., & Petrusic, W. M. (2009). Reading habits for both words and numbers contribute to the SNARC effect. *Psychonomic Bulletin & Review, 16*(2), 328–331. https://doi.org/10.3758/PBR.16.2.328

Steiner, A. F., Finke, S., Clayton, F. J., Banfi, C., Kemény, F., Göbel, S. M., & Landerl, K. (2021). Language effects in early development of number writing and reading. *Journal of Numerical Cognition, 7*(3), 368–387. https://doi.org/10.5964/jnc.6929

van der Ven, S. H. G., Klaiber, J. D., & van der Maas, H. L. J. (2017). Four and twenty blackbirds: How transcoding ability mediates the relationship between visuospatial working memory and math in a language with inversion. *Educational Psychology, 37*(4), 487–505. https://doi.org/10.1080/01443410.2016.1150421

Van Rinsveld, A., & Schiltz, C. (2016). Sixty-twelve = Seventy-two? A cross-linguistic comparison of children's number transcoding. *British Journal of Developmental Psychology, 34*(3), 461–468. https://doi.org/10.1111/bjdp.12151

Vygotsky, L. S. (1992). *Thought and language (A. Kozulin, Trans.)*. M.I.T. Press (Original work publish 1934).

Xenidou-Dervou, I., Gilmore, C., van der Schoot, M., & van Lieshout, E. C. D. M. (2015). The developmental onset of symbolic approximation: Beyond nonsymbolic representations, the language of numbers matters. *Frontiers in Psychology, 6*, 487. https://doi.org/10.3389/fpsyg.2015.00487

Zazkis, R. (2001). From arithmetic to algebra via big numbers. In C. Helen, K. Stacey, J. Vincent & J. Vincent (Eds.), Proceedings of the 12th ICMI study conference: The future of the teaching and learning of algebra, Melbourne, Australia, (Vol. 2, pp. 676–681). Melbourne, Australia: The University of Melbourne

Zebian, S. (2005). Linkages between number concepts, spatial thinking, and directionality of writing: The SNARC effect and the REVERSE SNARC Effect in English and Arabic monoliterates, biliterates, and illiterate Arabic speakers. *Journal of Cognition and Culture, 5*(1–2), 165–190. https://doi.org/10.1163/1568537054068660

Zhang, H., & Zhou, Y. (2003). The teaching of mathematics in Chinese elementary schools. *International Journal of Psychology, 38*(5), 286–298. https://doi.org/10.1080/00207590344000097

Zuber, J., Pixner, S., Moeller, K., & Nuerk, H.-C. (2009). On the language specificity of basic number processing: Transcoding in a language with inversion and its relation to working memory capacity. *Journal of Experimental Child Psychology, 102*(1), 60–77. https://doi.org/10.1016/j.jecp.2008.04.003

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