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Secondary mathematics teachers' descriptions of student engagement

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Abstract

There is a need for a more robust conceptualization of engagement in mathematics education research. Investigating how teachers describe engagement can provide insight into relationships between purposes of engagement and dimensions of engagement. In this exploratory study, we examined how 28 secondary mathematics teachers in two states in the USA talked about their students' engagement. During interviews, we asked teachers to provide their definitions for engagement, describe their teaching strategies for engaging students, and describe their observations of engagement during a video clip from their own classroom. We interpreted teachers' talk to identify how they described the nature of mathematics engagement (dimensions such as behavioral, cognitive, affective, and/or social engagement) and purposes of engagement (engagement in learning or in schooling [Harris, 2011]). When teachers described the purpose of engagement as engagement in learning, they also tended to describe the nature of engagement with cognitive and social dimensions and with multiple dimensions of engagement.

Keywords Motivation · Engagement · Secondary mathematics teachers · Teacher voice

Engagement is a complex phenomenon, which presents challenges for both mathematics teachers (Pedler et al., 2020) and mathematics education researchers. It can be challenging for mathematics teachers to understand how to support students' engagement because engagement is a complex phenomenon comprised of cognitive, behavioral, affective, and social dimensions (Fredricks et al., 2016). Researchers tend to conceptualize engagement in terms of either its dimensions or its purposes, such as engagement in learning or engagement in schooling (e.g., Harris, 2011), but a more robust conceptualization of engagement would include both dimensions and purposes. Listening to high school mathematics teachers' voices about engagement can provide insights, including whether and how multi-dimensional descriptions of engagement may align with particular purposes of engagement.

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Understanding secondary teachers' descriptions of engagement is important because students' mathematics engagement decreases over time as students move into high school (e.g., Collie et al., 2019). Engagement is malleable, socially situated, and influenced by teachers' instructional practices in the moment and by the classroom climate (Anderson et al., 2004; Shernoff et al., 2016, 2017). In this study, we investigated the nature and purpose of engagement that high school teachers intended to promote among their students. We illustrate how attention to various dimensions of the nature of engagement could align with different purposes of engagement by studying teachers' descriptions of engagement.

1 Mathematics engagement

Engagement is "...the in-the-moment relationship between someone and her immediate environment, including the tasks, internal states, and others with whom she interacts. Engagement occurs during activity, including both observable behavior and mental activity involving attention, effort, cognition, and emotion" (Middleton et al., 2017, p. 667). Engagement manifests when a person "participates in an activity with cognitive and affective investment" (Jansen, 2020, p. 273). Mathematics engagement occurs when a student's activity is directed toward doing mathematics, learning mathematics, completing a mathematics task, or otherwise participating in school mathematics.

1.1 Nature of engagement: dimensions

Engagement is a complex, multi-faceted, meta-construct that is typically described as simultaneously accounting for *cognitive*, *affective*, *and behavioral* dimensions (Fredricks et al., 2004). *Cognitive* engagement includes self-regulation used to direct attention toward mathematics and the process of making sense of content (Peterson et al., 1984). *Behavioral* engagement aligns with observable actions, including effort and time on-task (Peterson & Janicki, 1979). *Affective* engagement encompasses students' emotional responses and interest levels (Kong et al., 2003). Engagement manifests in a student's expression of affect, observable behaviors, and cognitive effort (Jimerson et al., 2003). When students experience interest and enjoyment coupled with concentration and effort, they have an optimally engaged flow experience (Csikszentmihalyi, 1997).

Following the work of Fredricks and colleagues (2016), we incorporate a fourth dimension of engagement in our analysis: *social engagement*. Fredricks et al. (2016) incorporated social engagement when they developed a survey to study students' engagement in learning mathematics and science. They interviewed 106 students (grades 6–12) and 34 teachers (middle grades and high school grades). Interviewees described cognitive engagement (students trying to understand for themselves and figuring out mistakes), emotional / affective engagement (caring about learning, enjoying mathematics, not being bored), and behavioral engagement (effort, task completion, focus). However, students also spoke about engagement as building on each other's ideas, trying to understand ideas of others, and helping peers. Thus, *social engagement* is how students take up opportunities to interact with their teacher and their peers about mathematics (Jansen, 2020) or "how learners interact with peers and their instructor in ways that lead to learning" (Xie, 2021, p. 82). Social engagement has been of particular interest in recent times; Roman et al. (2021) found that

secondary mathematics teachers increasingly attended to supporting students' affective and social engagement over time in their instruction as the COVID-19 pandemic unfolded.

Students benefit when teachers foster engaging mathematics learning opportunities, because learning and engagement are intertwined (Middleton et al., 2017). Skillings and colleagues (2016) found that students with different histories with achievement varied in how they engaged in learning mathematics as they moved from elementary to middle school. Higher achieving students who became more engaged over time reported positive affective engagement (enjoyment and interest in learning and doing mathematics) and productive behavioral engagement (high effort, proactive actions). Lower achieving students who became less engaged over time reported lower affective engagement (frustration) and a desire for more opportunities to collaborate with peers (sought more social engagement).

2 Teachers' descriptions of engagement

Teachers may need support to focus on a range of dimensions of engagement. In prior research, teachers spoke about mathematics engagement primarily as behavioral and less as affective, social, instrumental, and cognitive (e.g., Turner et al., 2009). In Skilling and colleagues' (2016) study of 31 secondary mathematics teachers in Australia, during interviews, teachers attended more closely to students' behaviors and emotions when describing engagement and reported less about cognitive engagement. Thus, research suggests that teachers tend to describe behavioral engagement.

According to Skilling and colleagues (2021), it is productive for teachers to hold multi-faceted views of engagement. For instance, if teachers only attend to affective engagement but not cognitive engagement, they may focus on fostering students' emotional experiences over facilitating productive learning opportunities. Teachers can learn to develop multi-faceted views of engagement. In an investigation of three teachers (grades five and six) during a 10-week professional development experience (Bobis et al., 2016), teachers learned to view engagement as multi-faceted, including moving away from a sole focus on behavioral engagement toward considering cognitive, affective, and social dimensions of engagement. Turner and colleagues (2011, 2014) found that, through professional development, teachers could learn to understand and enact multiple engagement principles: autonomy, competence, relatedness, and meaningfulness.

When teachers develop understandings about students' motivation to learn mathematics, their students' engagement improves. Stipek and colleagues (1998) supported elementary teachers with learning to understand their students' motivation to learn mathematics over time, including inviting their students to reflect on their learning experiences and supporting teachers with understanding their students' reflections. These teachers' students were observed to be more enthusiastic during mathematics class (affective engagement) and more likely to be discussing mathematics (social engagement) compared to students of teachers without this support.

2.1 Purposes of engagement: engaging in learning or in schooling

Harris (2011) illustrated secondary English teachers' descriptions of student engagement in Australia as aligning either with engagement in learning or in schooling. *Engagement in learning* was indicated by teachers' talk about engagement as cognitive (metacognition, thinking, how students occupied their minds), when students saw purpose in their

learning by trying to achieve meaningful goals, and when students felt a sense of ownership and value for their learning. *Engagement in schooling* sounded like teachers' talk about students' behavior (doing what teachers asked them to do and exhibiting "appropriate," on-task actions) and students' enjoyment of school (interest in what was being taught or in what was happening in school, not exhibiting boredom). When teachers spoke about engagement in schooling, the focus was on compliance with teachers' expectations. According to Harris (2011), "While learning may take place as a result of activities focused on engaging students in schooling, the nature and purpose of that learning certainly merits scrutiny" (p. 384). Harris characterized engagement in learning as more productive than engagement in schooling because engagement in schooling was for school's own sake, but engagement in learning aligned with supporting students' personal development and deeper understanding.

It is problematic to view engagement as compliance with school. The work of students is more productive when they focus on learning and personal development. Fenstermacher (1986, 1997) indicated that the work of a student could be called "studenting," in parallel to the use of "teaching" to characterize the work of teachers. In his earlier work, Fenstermacher (1986) characterized studenting as what students do to learn. However, as cited in Goldin (2010), in Fenstermacher's later work (1997), he focused on students' efforts to navigate or get around the system of school, which students might prioritize over learning. Engagement in schooling occurs when students are focused on navigating the system of school over learning.

Studies of studenting in mathematics in secondary mathematics classrooms have revealed engagement in schooling, but students benefit from opportunities to engage in learning. In mathematics lessons when teachers enacted direct instruction in Canada, Liljedahl and Allan (2013) identified studenting behaviors aligned with pseudoengagement, or appearing to comply with school expectations, such as faking or mimicking rather than engaging in reasoning. In Webel's (2013) study of high school students' goals for working together on mathematics in the USA, students reported some goals associated with engagement in schooling (completing the task, avoiding boredom, avoiding appearing incompetent) and other goals associated with engagement in learning (seeking explanations that would help them understand, collaborating to make sense, wanting to be convinced). High school students in the USA who reported higher levels of engagement in learning experienced a match between the challenge of a task and their perceived capabilities as well as learning activities that supported their autonomy (Shernoff et al., 2014). High school students' reports of high-quality mathematics experiences have been found to be closely connected with their reports of interest (Schiefele & Csikszentmihalyi, 1995).

2.2 Teachers' voices

We value listening to the voices of teachers to understand the work of teaching and the nature of the construct of engagement. Understanding how teachers talk about their practice can support closing a research-practice gap (Miretzky, 2007). Teachers' own words about teaching and learning appears to relate, to some degree, to what they accomplish in their practice compared to their use of technical terminology in their talk (Horn & Kane, 2019). Teachers' talk can reveal how they make sense of concepts related to teaching and learning in their own contexts (Demulder & Rigsby, 2003), including the concept of engagement.

The research question we sought to answer was: *What are secondary mathematics teachers' descriptions of students' engagement?* Our findings elaborate upon Harris's (2011) distinctions in teachers' descriptions of purposes of engagement—engagement in schooling or in learning—to illustrate what these purposes sound like in secondary mathematics teachers' voices in two states in the USA. We also investigated how teachers' talk about the nature of engagement reflected dimensions of engagement (cognitive, behavioral, affective, or social). We investigated how purposes of engagement aligned with dimensions of engagement in teachers' voices to capture the robust nature of the concept of engagement.

3 Methods

We employed a phenomenographic approach to studying teachers' descriptions of engagement. According to Marton (1986), "phenomenography investigates the qualitatively different ways in which people experience or think about various phenomena" (p 31). The aim of phenomenological research is to take a second-order perspective, or to describe the world as the participant perceives it (Gibbs et al., 1982). Following Harris (2011), the purpose of our analysis was to represent ways that our participants described engagement at the time of our interviews.

> Phenomenography differs from many qualitative approaches as it focuses on the collective understanding of groups. It does not attempt to assert that individual participants 'hold' specific conceptions, but instead gathers evidence to illustrate the range of conceptions present within the population under study. (Harris, 2011, p. 378)

Thus, we do not intend to claim that these descriptions of engagement are a psychological property of the individual teacher, nor do we claim that these participants' views of engagement are held at a level of stability over time.

3.1 Participants and context

This exploratory study was conducted in the context of a larger project in which we sought to investigate engagement in high school mathematics classrooms: the Secondary Mathematics in-the-moment Longitudinal Engagement Study (c.f., Jansen et al., 2019). Project team members interviewed 28 teachers in two states in the USA (one in the Southwestern region and one in the Mid-Atlantic region). We chose these locations to capture a range of learning opportunities. Each state tended to adopt different curricular approaches: integrated mathematics (Mid-Atlantic) and topics-based courses (Southwest). Mid-Atlantic courses were titled Integrated Math 1, 2, or 3. Southwest courses were Algebra I or Geometry. The Mid-Atlantic schools implemented a block schedule with approximately 90-min class periods, and courses lasted one semester. In the Southwest, the class periods were approximately 50 min long, and courses lasted for a full academic year.

We gathered data from six schools (three from each state). In the Mid-Atlantic, the schools' student demographics ranged from 9 to 30% low income, 24 to 57% White, 27 to 46% Black, 7 to 24% Latinx, and 5% or fewer Asian-American, Native American, or mixed-race students. In the Southwest, the schools' student demographics ranged from 85 to 94% low income, 2 to 5% white, 1 to 15% Black, 74 to 96% Latinx, and 5% or fewer Asian-American, Native American, or mixed-race students.

We recruited teachers for this study by soliciting nominations from district curriculum supervisors and mathematics coaches. The 28 participating teachers averaged 10.55 years of teaching experience (range: 1–27 years). Seventeen teachers had earned a Master's degree. They self-identified their races as follows: Two as Black or African-American, one Asian-American, one Hispanic/Latinx, one multi-racial (Black and Hispanic), and the rest identified as white. They self-identified their genders: 20 identified as female and eight as male. (Names of teachers presented in the results are pseudonyms.)

3.2 Data collection

We collected data to investigate how teachers described engagement, strategies they reported using to engage students, and their observations of engagement in a video clip from one of their lessons. Our two data sources were an open-ended survey prompt and an interview. Each teacher completed a baseline survey online at the start of each course. The baseline survey item relevant to this study was: *In your own words, what does "engaging students with mathematics" mean?*

Interviews lasted between approximately 35 and 75 min. The interviews were conducted by researchers including research assistants (doctoral students) and two mathematics education professors (principal investigators of the study). All interviewers had prior experience teaching mathematics at the secondary level and/or experience as a mathematics teacher educator. (Data analysts were a subset of those who conducted the interviews.)

During the interview, we asked teachers to elaborate on their definition for engagement that they wrote on their baseline survey. We also asked questions to capture teaching strategies: *What are some of your favorite strategies you use to engage students? Why do you use these? Can you tell me about a time when you have successfully engaged students with mathematics?*

We also asked the teacher to interpret student engagement in a video clip of one of their lessons. We asked questions such as: *Overall, would you say that your students in each class period were engaged or not during this activity? Why? How did you (or could you) respond to the students' engagement or lack of engagement? What surprised you in the video (related to student engagement)? When you notice whether students are engaged, what are you looking for?* and *When you think about some of your favorite ways to engage students, did the topic for this lesson allow you to incorporate any of these strategies? Why or how?* Video clips were chosen from an activity during the lesson that the teacher had nominated to be a potentially engaging activity. The research team selected video clips that included students who gave assent to be video recorded and those students whose parents provided consent to be video recorded, and they excluded clips that included students whose parents did not provide consent and those of students who did not give assent to being video recorded.

3.3 Data analysis

To investigate teachers' descriptions of engagement, we had two analytic goals: (1) determine the dimensions of the nature of engagement described by a teacher and (2) interpret whether the teacher talked about the purpose of engagement as engaging in schooling or in learning. In the context of examining the purpose of engagement, we also investigated whether teachers who talked about different purposes of engagement also described similar or different dimensions of engagement. We divided each interview transcript into three

sections, as determined by the content of interview questions discussed in each section: (a) teachers' definitions for mathematics engagement, (b) teachers' strategies for engaging students, and (c) teachers' reflections on students' engagement during a classroom video episode.

3.3.1 Dimensions of the nature of engagement

For each interview, we used two levels of analysis to determine dimensions of engagement. We applied descriptive coding techniques (Saldaña, 2013) to the unit of a turn of talk by a teacher; we identified teachers' words in their turns when they appeared to align with a dimension of mathematics engagement: affective, behavioral, cognitive, and social. See Table 1 for indicators of codes.

For the first level of analysis, we investigated whether a dimension of engagement was *present* in each of the three sections of a teacher's interview. Each interview was coded by two researchers, and each of these researchers had conducted a subset of the interviews. Researchers compared their analyses during resolution meetings and resolved all disagreements. Each researcher identified the evidence they used to assign a code in an analysis table. Once they agreed upon a common set of evidence, researchers resolved which dimension of engagement was reflected in the teachers' talk.

For the second level of analysis, we interpreted whether a dimension of engagement was *central* in a teacher's interview. The dimension was determined to be *central* in the teacher's interview if it was determined to be a *primary* dimension in two out of three sections of the interview. To interpret whether a dimension was a primary dimension in one section of an interview, our analytic framework followed prior research by the first author (Jansen, 2006; Jansen et al., 2012) which paralleled Harris's (2011) process. We examined the repetition, emphasis, and detail that a teacher used to talk about engagement. If at least two out of the following three criteria were present in a teacher's description of an engagement dimension, we determined that this dimension was primary in that interview's section: (a) repetition throughout a section of an interview (whether a teacher mentioned the dimension multiple times), (b) degree of emphasis (whether a teacher used emphasis words such as "a lot," "very," "much more," "definitely," and/or "big" to amplify his or her description of an engagement dimension), and (c) the level of detail (substantial descriptive richness in the teacher's report of an engagement dimension). During resolution meetings, researchers agreed on primary dimensions of engagement for each of the three sections of an interview. Then, the first author of this paper examined the documentation from resolution meetings to identify central dimensions of engagement, or whether the dimension was primary in two out of three sections of the interview.

Table 1 Descriptions of dimensions of engagement

Dimensions of engagement	
Affective	Emotional responses, interest, attitudes, and expressions of values
Behavioral	Observable actions of students, including whether or not they were on task
Cognitive	Process of coming to understand, learn, and make sense of mathematics
Social	Interactions with others for the purpose of learning mathematics

3.3.2 Purpose of engagement: engagement in schooling or engagement in learning?

We incorporated a third level of analysis to identify the purpose of engagement that a teacher sought to achieve, as reflected in their descriptions of dimensions of engagement: engagement in learning or in schooling. Our criteria were informed by Harris (2011). *Engagement in schooling* was indicated by talk about engaging students so that they comply with school expectations (doing what teachers asked them to do, exhibiting "appropriate" behavior or on-task actions, completing school tasks). *Engagement in learning* was indicated by talk about engagement to evoke students' thinking (including how students occupied their minds, their metacognition, meaningful learning, understanding), to encourage students to see purpose in their learning, or to help students feel a sense of ownership or value for their learning. We identified the purpose with which the interview most aligned.

We sought reliability for interpreting purposes of engagement by having two researchers code six out of the 28 interviews (21%) of the data. The researchers independently agreed on purposes of engagement for five out of the six interviews (83.3%). They met to resolve the interpretation of the sixth interview. Then they coded the remaining interviews for purpose of engagement, as informed by the resolution conversation.

4 Results

Below, we illustrate how secondary mathematics teachers described engagement during interviews. Their descriptions reflected which dimensions of engagement (behavioral, affective, cognitive, and social) aligned with purposes of engagement in schooling or in learning. Most multi-dimensional descriptions of engagement aligned with engagement in learning.

4.1 Engagement in schooling

These teachers spoke about engagement in schooling in two ways: they talked about behavioral engagement in terms of compliance and talked about using affect as a scaffold to draw students into working on school mathematics.

4.1.1 Behavioral engagement in schooling

Teachers described behavioral engagement as students' compliance, such as following along and being on task. Two teachers talked about behavioral engagement as the sole central dimension of engagement in their descriptions. For instance, Julien defined engagement on the survey as, "Garnering their attention such that their attention and focus is on the task at hand." When asked to explain why an activity would be engaging for students, Julien said, "That they [students] were following along, that they were doing the work, that they were either getting correct, or asking questions they need to ask about. That would be it." Thus, some teachers described the behavioral dimension of engagement as aligned with the purpose of engagement in schooling.

4.1.2 Cognitive-behavioral engagement in schooling

Other teachers described engagement in its cognitive-behavioral dimensions and as aligned with engagement in schooling. Two teachers expressed cognitive and behavioral dimensions as central in their descriptions; they spoke about managing or training students to actively work on or think about mathematics. Peter reported on his survey that engaging students with mathematics means, "all students are working on what they are supposed to. All students are actively participating in their own learning, with no exceptions. Engaged means 'doing.'" Behavioral engagement was indicated by "working on what they are supposed to," and cognitive engagement was "actively participating in their own learning." He elaborated upon how he structured his teaching to engage students:

> What we do is we give them straight lecture and then we have interactive notes. They're engaged through interactive notes and we have PowerPoints that go with them… So they're engaged with closed activities inside the notes and interactive parts within the notes where they have to perform. They are engaged during the regular part of our lecture. But what we do is then we use the grouping in order for them to do practice… So it's guided practice within the group that helps them master what they're doing.

Peter spoke about guiding the students through what they needed to do (behavioral engagement) to master the material (cognitive engagement) and perform well on school tasks.

Nicole spoke about engaging students through cold calling on them by randomly choosing among popsicle sticks with students' names on them.

> I kind of force them to be a little bit more engaged for the Popsicle sticks. And then also, if they didn't know the answer, they had to listen to somebody else, and then they had to repeat it back. Like, [student] didn't know what to do, so somebody else gave the answer. And then, I made [student] repeat it so that he was listening, at least. I don't know if that's considered engagement, because to me, he's just listening, and he's just repeating. But, at least it's trying to get them to think.

Nicole used behavioral management strategies (cold calling on students) to begin to evoke students' cognitive engagement (repeating other students' words to begin to evoke thinking). These teachers appeared to describe engagement as behavioral compliance to support students' engagement with schooling.

4.1.3 Affective engagement in schooling

Four teachers described the affective dimension of engagement as aligned with engagement in schooling. These teachers spoke about targeting students' interest or enjoyment of mathematics to motivate students. Anne described her efforts to use affect to engage students as an effort to "kind of trick them into doing math."

> I like to try and do random things with them that they don't necessarily think is math. Like, for example, the other day I had Estimation 180, I don't know if you've ever heard of it, but it's like a website and they just have a bunch of things where students are estimating, but they don't like really realize that it's math. So, the other day I had on the board like a screenshot of iTunes and it was about halfway through, and it was the amount of time. And so I'm like 'How

> long do you think that song is?' And in their head they're like 'Oh, you double it and then it's that amount of time.' They're doing math, they don't realize it. They're like, 'Oh, we're just guessing how long a song is.' So I think things like that where you kind of trick them into doing math and they don't really realize it, I like doing that. I think that helps… I've noticed those are the things that get them excited to do it because a lot of kids hate math and they think it's hard and any time they know they have to do math, they're automatically 'Oh, it's math, I'm not going to be able to do it.' Whereas if you trick them into or ease them into with other strategies, I think it helps them and they're not as afraid.

This "trick" had an affective approach of freeing students from fears of mathematics so they would complete a school mathematics task. Teachers described the affective dimension as aligned with engagement in schooling when teachers spoke about using affect to hook students to complete work during mathematics class.

4.1.4 Cognitive-affective engagement in either schooling or learning

Five teachers spoke about cognitive and affective dimensions centrally. Some of these teachers spoke about affect as a scaffold to lead to compliance with completing school tasks (engagement in schooling). Other teachers spoke about how positive affect resulted from opportunities to understand mathematics (engagement in learning).

Corinne's description of cognitive-affective engagement aligned with engagement in schooling. On the survey, Corinne defined engaging students with mathematics as, "creating activities that students will enjoy and learn by doing." Enjoyment referred to affect and "learn by doing" referred to cognitive engagement. Corinne elaborated upon this definition in her interview by saying, "it's important to have more fun activities because if students are more interested in it, I think they'll actually take more away from it." This way of describing cognitive-affective engagement focused on affect as a hook to learn from school tasks.

However, when teachers spoke about cognitive-affective engagement like Addie, their description aligned with engagement in learning. Addie explained why an activity was engaging for students by saying, "it's harder for the students and it's challenging, but it engages the students because they're like, 'Oh, I really want to know how to do this because it's my creation.'" From Addie's perspective, students developed a stronger affective experience (wanting to know) from wrestling with a challenge cognitively. When teachers spoke about cognitive-affective engagement as engaging with learning, they described students as having a more personally meaningful connection with content or valuing learning due to opportunities to enact autonomy.

4.2 Engagement in learning

When teachers spoke about engagement in learning, they spoke about cognitive, social, and/or affective dimensions toward engagement. Most descriptions of engagement in learning included multiple dimensions of engagement. An exception was that a subset of the teachers who described engagement in learning only described the cognitive dimension of engagement.

4.2.1 Cognitive engagement in learning

Six teachers reported the cognitive dimension as their only central dimension in their descriptions, and they described engagement in learning. They described the cognitive dimension of engagement by emphasizing students' intellectual autonomy. On Jacob's survey, he defined engagement as, "Allowing students to grapple with mathematics and arrive at their own conclusions instead of being spoon fed procedures." During his interview, he described an engaging activity in mathematics classes as when students "are really diving in and looking at the math and thinking about it." Rachel also spoke similarly about engagement during her interview,

> It means giving students the opportunity to grapple with important mathematical concepts on their own… giving students the opportunity to have those revelations of like, 'I can explain this, I can make sense of it, I can do these problems and feel like I have ownership over my work.' I feel like that would be the ultimate goal of engagement.

These teachers described cognitive engagement in learning through promoting students' opportunities to do the thinking about mathematics.

4.2.2 Cognitive-social engagement in learning

Three teachers reported cognitive and social dimensions as central and spoke about engagement in learning. These teachers spoke about developing students' mathematical thinking through communication. On the survey, Julie reported that engaging students with mathematics means, "finding ways for students to think about and discuss mathematics in a way that deepens their understanding." Julie expanded on this definition in her interview by saying:

> I like to, from time to time, after we've done a concept, to kind of pose a question that forces them to really, first of all, think on their own. Can they generate their own thought? But then to have those discussions with their peers to see, 'Well, what do you think about that? I didn't think about it.' How can we maybe expand on each other's ideas to see different ways of viewing the same kind of problem?

She wanted students to develop their own understandings (cognitive engagement) through social engagement of interacting with peers about mathematics. Jimena spoke about an engaging activity with the dimensions of cognitive and social engagement.

> …them arguing and debating the answers. That to me, it's big, it's beautiful, makes me cry. When they start arguing about math… That they will correct each other, that's, to me, is the best, when the less I have to talk, the better the engagement and the lesson goes, and they remember it. They will take that from short-term to long-term.

These descriptions aligned with engagement in learning because students generated their own thoughts through discourse, debating, relying on each other to make sense.

4.2.3 Cognitive-social-affective engagement in learning

Two teachers described engagement with cognitive-social-affective dimensions aligning with engagement in learning. Although they expressed an affective dimension, they did not describe using affect solely as a hook to extrinsically motivate students in schooling. Rather, these teachers spoke about the affect students would experience as an outcome if they engaged in discourse (social engagement) to develop mathematical understanding (cognitive engagement in learning). For example, Kimorah spoke about how she would engage her students:

> …build some rich activities so that students could feel comfortable, confident, be able to attack a problem, just good problem solvers. …They answer each other's questions, they're constantly having little debates, which I like.

Kimorah also said, "When they find their own mistakes and correct them, it's better than when I'm telling them what their mistake is… just getting kids to communicate is my biggest thing, to feel confident." She used affect (increasing students' comfort) to scaffold students' engagement, but she also reported engagement in terms of the affective experience of students feeling comfortable and confident as an outcome of cognitive-social engagement. She emphasized students' thinking about mathematics for themselves (cognitive engagement in learning) through peer debates (social engagement in learning).

4.3 Distinguishing explicitly between engagement in schooling and engagement in learning

Some teachers' descriptions reflected an awareness of engagement in schooling, but they promoted engagement in learning. When behavioral engagement appeared in a multi-dimensional description of engagement, teachers identified behavioral engagement in schooling as insufficient engagement. Engagement in schooling may have been viewed as a step toward engagement in learning.

4.3.1 Cognitive-affective engagement in learning over behavioral engagement in schooling

Two teachers reported cognitive, affective, and behavioral dimensions in their descriptions. They reported wanting to support students' cognitive engagement in learning over behavioral engagement in schooling. Chloe said,

> I think there's two different kinds of engagement. There's engagement, and, yes, the kids are all working on something, but then there's an engagement of, yes, they're actually personally invested in something. So, I would rather like to see it shift more towards the second definition. The difference would be the excitement level, that's what I would gauge it by. Because if they're engaged in working, like when you come by with your administrators and they're doing their evaluation, 'Oh, there's 80% engagement,' which basically translates to, 'You had 80% of your classes that were working on whatever they were working on,' whether it be an activity or paper. … If they were working on an activity just because they're working on activity, yes, that's

> engagement. But if they're really interested in the outcome of the activity, that's a personal engagement. And so there will be a different energy level between the two.

A distinction between engagement in schooling and engagement in learning appeared in her explicit contrast of two different types of engagement: "working" as behavioral engagement in schooling and "personal investment" as cognitive-affective engagement in learning. Chloe described an engaging activity as follows:

> …when you can actually give them [students] a challenge that, one, makes sense to them, and that they have a good entry point to get into so that they can get into the math to get to that end-point, and they're interested in what the output is going to be.

Chloe, like other teachers who reported cognitive-affective-behavioral dimensions as central in their descriptions, recognized behavioral engagement in schooling but preferred cognitive-effective engagement in learning.

4.3.2 Cognitive-affective-social engagement in learning over behavioral engagement in schooling

Similarly, the two teachers who reported all four dimensions centrally in their descriptions were aware that behavioral engagement was a form of engagement, but they wanted more than engagement in schooling. Elise reported promoting engagement in learning:

> I see it as not just doing math. I see it as questioning math. 'Why does this work? Why doesn't this work? Why does this make sense? If what I did doesn't make sense, then how can I make it better?' That's how I want my students to be. I want them to not just say, 'I don't understand this.' I want them to say, 'What am I doing that's not making sense?'

She stated that engagement was more than engagement in schooling and that she wanted engagement in learning. She observed a range of engagement:

> You have different levels of engagement. You have the kid that hasn't even attempted to pick up a pencil, or hasn't even ... the kid that looks like he's listening or she's listening, but hasn't even read the question or hasn't tried to understand the directions and the task. And then you have the kids like, 'Oh, I got an answer. I'm done.' Then you have the kids that it starts trickling down. 'Oh, I have this answer. I'm not sure of the answer,' but they don't know how to figure out what they're missing. And then you have the kids like, 'I got this. What do you want me to do now?' It's such a big spectrum of engagement and lack of engagement that you try to address every day.

Elise spoke about engagement in terms of engagement in schooling through descriptions of complying with behavioral expectations (picking up a pencil), pseudoengagement (appearing to listen but not listening), task completion, and being motivated to do more. She talked about engagement in learning by describing an engaging activity as one that evoked positive affect and provided an opportunity for social engagement.

> Technology this year has been an amazing tool… It's been a lot of inquiry. Like, 'Okay, let's graph this. What are we looking at? What does this mean? What if I plot this point? What if I move this over here?' Whenever we've done lessons like that, the kids have been really positive about it… And the kids really enjoy,

> it takes away that anxiety about, 'I don't know how to calculate this. This process is difficult.' It takes that away, and then, okay, 'This is my understanding of what's happening.' That has been really successful... the technology, using their peers to engage in the math.

When she used her technological tools to help students make sense of math (cognitive engagement in learning), it increased positive affect (enjoyment) and decreased anxiety, as well as provided an opportunity for discourse with peers (social engagement). Cognitive-affective-social engagement in learning was important to teachers like Elise. They were aware of behavioral engagement in schooling, but they were not satisfied with it.

5 Discussion

A contribution of this paper is our illustration of how dimensions of engagement aligned with purposes of engagement in secondary mathematics teachers' voices. Characterizing variations in the structure of engagement, as revealed by teachers' voices, is relevant for researchers who seek to understand the nature of constructs from participants' viewpoints. In prior research studies, either dimensions of engagement (Skilling et al., 2016; Turner et al., 2009) or purposes of engagement (Harris, 2011) were used to analyze teachers' thinking about engagement. When we studied purposes and dimensions of engagement concurrently, our findings supported the work of Fredricks and colleagues (2016) who argued for the inclusion of a social dimension for engagement, as social engagement was reported by teachers who expressed learning as the purpose of engagement.

Affective engagement aligned with either engagement in schooling or learning for these teachers. When affect aligned with engagement in schooling, teachers would speak about using affective experiences to draw students into completing school tasks. Teachers' talk about affect aligned with engagement in learning when teachers spoke about increases in positive affect as an outcome of engaging in learning.

Teachers in this sample who expressed more than two dimensions of engagement reported promoting engagement in learning, not engagement in schooling. Most descriptions with one or two dimensions of engagement (e.g., behavioral, affective, cognitive-behavioral, and some cognitive-affective descriptions) aligned with engagement in schooling. These results suggest that engagement in learning is richer and more complex than engagement in schooling.

Most teachers in this study did not describe the purpose of engagement as compliance with schooling or behavioral compliance, which contrasts with teachers' voices in previous research (e.g., Skilling et al., 2016; Turner et al., 2009). The teachers in this study may have had opportunities to develop more robust knowledge of students' motivation and engagement compared to the participants in previous studies. Additionally, it is possible that teachers who reported that the purpose of engagement was engagement in learning may have had higher expectations for their students compared to teachers who describe the purpose of engagement as engagement in schooling. Our findings suggest that, to motivate and engage students productively in learning, teachers could benefit from developing knowledge beyond mathematics knowledge for teaching, including knowledge of multiple dimensions of engagement.

5.1 Limitations

One limitation of this study is that we investigated teachers' efforts to engage students through self-reports during interviews. We acknowledge that teachers may enact teaching practices that do not consistently align with their self-reports. However, these descriptions do provide insights into these teachers' instructional visions (Jansen et al., 2020; Munter, 2014), which indicate how teachers ideally prefer to teach.

We also did not present evidence of the degree to which students' engagement aligned with these teachers' descriptions of engagement. Researchers could investigate the degree to which students experience engagement in ways that were intended by the teacher. We examined ways that students' talk about engagement aligned (or not) with their teachers' talk about engagement elsewhere (Mohammad Mirzaei et al., in press).

5.2 Future research

More research is needed to understand how teachers develop robust and productive perspectives on engagement. Perhaps the teachers in this study had professional learning opportunities to develop their knowledge of students' motivation and engagement, which would explain why a subset of these teachers described multi-dimensional views of engagement aligned with engagement in learning. Maybe the teachers who described engagement as behavioral compliance with schooling did not have such learning opportunities.

Future research could address whether teachers' instructional practices vary depending on their descriptions of mathematics engagement. For example, how might teachers who described cognitive-behavioral engagement in schooling enact teaching differently than teachers who expressed cognitive-affective-social engagement in learning? In what ways? Although teachers' self-reports in this sample reported different teaching practices depending upon their descriptions, the enactments of teachers' instructional practices could be observed.

Additionally, researchers could investigate students' experiences with teachers who described engagement differently. Do students report different levels of motivation and engagement with teachers who express different descriptions of engagement toward mathematics? Do students' motivation and engagement improve over time with teachers who describe engagement in one way over another?

5.3 Implications for teacher education

Teachers could benefit from learning about dimensions of the nature of engagement and the two purposes of engagement (learning or schooling). They could consider what they might be promoting, intentionally or not, among their students, and whether and how changes in their teaching could impact students' engagement. We propose two goals for teachers' learning about mathematics engagement: (1) Teachers can learn to promote additional dimensions of engagement or alternative purposes of engagement. (2) Teachers can strive toward fostering their idealized descriptions of students' engagement more fully in their practice.

6 Conclusion

We hope that these findings address the call of Miretzky (2007) to listen to teachers' voices as a step toward closing a gap between research and practice. Some of these teachers reported an understanding of differences between engagement in learning and engagement in schooling. Teachers who described learning as the purpose of engagement expressed richer, more multi-dimensional descriptions of the nature of engagement.

To disrupt trends of decreases in students' mathematics engagement as they move through school (e.g.,Chouinard & Roy, 2008; Plenty & Heubeck, 2013), it is important to understand and support secondary teachers with developing their thinking about engagement. Our presentation of these teachers' descriptions of engagement offers structure to understand the complex phenomenon of mathematics engagement. To increase students' engagement with mathematics, teachers could adopt perspectives on engagement aligned with engaging in learning and attend to multiple dimensions of engagement.

Declarations

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