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How narratives about the secondary-tertiary transition shape undergraduate tutors' sense-making of their teaching

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Abstract

Drawing on the commognitive framework, we construe the secondary-tertiary transition (STT) as a distinctive element in the pedagogical discourses of various communities. Our interest rests with university tutors in light of the emergent recognition of their impact on undergraduates' mathematics learning in many tertiary contexts worldwide. We aim to understand the roles of STT communication in tutors' reflections on incidents that took place in their tutorials. Our participants were undergraduate students in the advanced stages of their mathematics degrees in a large New Zealand university and who were enrolled in a mathematics education course. Throughout the semester, the participants led tutorial sessions for first-year students and wrote reflections on classroom incidents that drew their attention. Our data corpus consisted of 58 reflections from 38 tutors collected over four semesters. The analysis revealed that STT communication featured in tutors' descriptions of classroom incidents, assisted them in making sense of unexpected events, positioned their instructional actions as replications of what was familiar to them from their own STT experience, and contributed toward generating new pedagogical narratives. We situate these findings in the literature concerning undergraduate tutoring and teachers' perspectives on STT.

Keyword Commognitive framework · Pedagogical discourse · Secondary-tertiary transition · Teaching assistants · Undergraduate tutoring

An earlier version of this study was shared with the RUME community (Kontorovich & Ovadiya, 2022).

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1 An introductory snippet

Annie, a soon-to-be mathematics major, is a novice tutor (or "teaching assistant") in a first-semester mathematics service course at a New Zealand university.[1] In one of the sessions, the students were given $f(x) = x^2 - 2$ and asked to find $f(2 - x)$. This is how Annie reflected on an incident that drew her attention in that tutorial:

> During the tutorial, I had more than three students asking me how to solve this question. I tried to explain it by telling them that function is like a factory. $2 - x$ is the input and $x^2 - 2$ is like a machine. This is what my maths teacher used as an example when he taught us the definition of functions in school. But they told me they didn't understand it at all. So I added more content to my explanation saying this is a factory to make apple pie, whatever is in the brackets is the apple we need to put in the machine to make apple pie. So to solve the question we just use $2 - x$ to replace x. They did that but I am not sure they really understood why. [...] After the tutorial I remembered the first time my maths teacher used the factory example and at that time I also didn't understand it. But things are way more paced in school, I can't imagine how hard it must be for first-year students to meet this idea for the first time.

Putting the accuracy of Annie's description of the incident aside, notice all the places where she referred to the secondary-tertiary transition (*STT* hereafter). On the face of it, it seems only reasonable for her to bring up STT. Indeed, Annie was aware that this was the first tertiary mathematics course that her students were taking, and this explains her referring to her tutees as "first-year students." It was also not that long ago that Annie was in the same shoes as her students, which explains why she recalled her first encounter with "the factory example." However, while Annie worked with this same group of students throughout the semester, she did not acknowledge their STT in all of her reflections. This leads to the proposal that weaving STT into this particular account was not an unavoidable necessity, but a deliberate discursive move that Annie made as part of her reflection on her teaching.

We are interested in how the STT finds its way into the pedagogical discourses of university teachers, the characteristics of teachers' communication about STT, and its affordances in terms of teachers' sense-making of their own teaching. In this paper, we pursue this interest in the case of novice tutors. The literature on the transition from school to university mathematics is prolific. Yet, it has rarely focused on undergraduate tutors as agentive actors at the interface between faculty and students in transition (John & Burks, 2022). By working with a cohort that was experienced in mathematics learning and was making its first steps in teaching, we hoped to gain access to detailed STT stories and understand their roles in tutors' emerging pedagogies.

[1] Some readers may be surprised by the fact that an undergraduate student tutors undergraduate students. In "The study" section, we elaborate on the specific context that afforded this arrangement.

2 Undergraduate tutoring and university teachers' perspectives on STT

The mathematics education community has extensively explored STT (e.g., Gueudet, 2008; Hochmuth et al., 2021; Thomas et al., 2015). We locate this investigation in the intersection of research on undergraduate tutoring (2.1) and on university teachers' perspectives on STT (2.2). The first body of research shows that university teachers occasionally raise issues of STT, but these are often considered part of broader pedagogical views (e.g., Jaworski, 2002; Nardi et al., 2005). Other studies focus on university teachers' perspectives on STT (e.g., Hong et al., 2009; Klymchuk et al., 2011). These studies beg the question of whether and how these perspectives play out in teachers' interactions with students in transition. Next, we provide a short overview of the key findings from each body of research that shaped our study.

2.1 Undergraduate tutoring

Over the last two decades, research into university mathematics education has become interested in undergraduate tutoring. Mostly coming from the USA and Europe, research has associated this endeavor with graduate students who are employed by mathematics departments to contribute to the instruction of specific courses (for a review, see Speer et al., 2005). The studies show that the scope of tutors' responsibilities varies from one country to another (e.g., Jaworski, 2002; Lawson & Croft, 2021; Püschl, 2017; Yee et al., 2022), ranging from working in drop-in mathematics support centers, through leading regular tutorial sessions, to independently teaching entire courses.[2] In many tertiary contexts, tutor–student interactions constitute a significant course component, meaning that the way in which the former teaches can have a considerable impact on how the latter learns (e.g., Speer et al., 2005). In New Zealand, tutors are often in charge of the planning and instruction of weekly sessions that the course students are expected to attend regularly throughout the semester. These tutorials bring together sub-sets of the course students (usually less than 25) and engage them with questions related to topics discussed earlier in the whole-course lectures.

Research into undergraduate tutors has explored a range of aspects, including preparation programs (e.g., Speer et al., 2005) and mentoring (e.g., Yee et al., 2022). Given our interest in tutors' sense-making of their teaching, we acknowledge the Undergraduate Mathematics Teaching Project (UMTP) (Jaworski, 2002; Nardi et al., 2005). It aimed to explore the complexities of tutors' epistemologies, pedagogies, and craft knowledge through reflective interviews. To avoid discussions on general and dis-embedded levels, the project engaged its participants – six tutors in Oxford, each of whom held a doctorate in mathematics – in reflections on specific incidents that took place during their tutorials. Mostly, the incidents were selected by the researchers, but at the end of each interview, the tutors were asked to reflect on an event or idea from the tutorial that they perceived as significant. On a general note, it is worth mentioning that the use of incidents or cases is

[2] This variety can explain the lack of universal terminology for referring to this cohort (e.g., see Speer et al., 2005, for "teaching assistants"; Yee et al., 2022, for "graduate student instructors"; and Jaworski, 2002 for "tutors"). Previous research in the New Zealand context has used the term "tutors" (e.g., Oates et al., 2005), and we continue this tradition.

common in mathematics teacher education (for a comprehensive review, see Markovits & Smith, 2008). Shulman (1992) explains that "[b]ecause they are contextual, local, and situated – as are all narratives – cases integrate what otherwise remains separated" (p. 28).

One of the outcomes of the UMTP was a spectrum of pedagogical awareness (Nardi et al., 2005). The spectrum encompassed three strands: tutors' conceptualizations of their first-year students' difficulties, tutors' descriptions of what strategies they used to help overcome these difficulties, and tutors' self-reflective accounts of their teaching practices. The spectrum for each strand spreads across four levels: naïve and dismissive, intuitive and questioning, reflective and analytic, and confident and articulate. In another study, Jaworski (2002) explored tutors' sensitivity to students and the mathematical challenge, which constitute two elements in her previously developed "teaching triad" model. In it, "sensitivity to students" pertains to teachers' knowledge of students' thinking, needs, and well-being; "mathematical challenge" relates to the activity a teacher initiates to engender students' mathematical thinking.

The UMTP illustrates how context-specific and example-centered reflections can be mobilized to explore broader issues in tutors' pedagogies. We adapted this approach to study tutors' sense-making of their teaching (see "Data collection and analysis").

2.2 University teachers' perspectives on STT

Several studies examined university teachers' perspectives on STT. Klymchuk et al. (2011) developed an open survey that was answered by 63 university teachers from 24 countries. One of the survey questions asked teachers to propose reasons for the gap between school and university mathematics. The researchers grouped teachers' responses into categories, the most popular of which was "higher level of thinking at university mathematics" (p. 109). Klymchuk et al. illustrate this category with colorful quotes, in which the respondents criticize school mathematics instruction and emphasize its inferiority (e.g., "High school math is very mechanical and situational […] We [the university] expect more out of the students", Klymchuk et al., 2011, p. 111).

Such views go beyond individuals and recent times. Back at the Third International Congress on Mathematics Education in 1976, a study group brought together delegates from 15 countries to discuss STT. In their report, STT is described as a "problem," the three major aspects of which are the unavailability of "topics supposedly covered in the secondary curriculum […] when needed in later study" (Fey, 1977, p. 406); the inability of "[m]any students […] to see the relations between specific ideas" (Fey, 1977, p. 406); and "students leaving secondary school [with] a narrow and formal approach to mathematics" (Fey, 1977, p. 407). Similar perspectives (put in less disparaging terms) can be found in the report of the London Mathematical Society (1995):

> Recent changes in school mathematics may well have had advantages for some pupils but they have not laid the necessary foundations to maintain the quantity and quality of mathematically competent school leavers and have greatly disadvantaged those who need to continue their mathematical training beyond school level. (p. 3).

Note the connections between tertiary educators' perspectives on STT and the actions taken to address it. For instance, remedial courses are often presented as a "solution" to the "problem" of school graduates' under-preparedness (e.g., Fey, 1977; Klymchuk et al., 2011). Alternatively, Sfard (2014) argues that school and university mathematics constitute almost distinct disciplines. Endorsing Sfard's perspective, Pinto (2019) discusses pedagogies that university teachers can implement to support their students to make this transition.

3 A commognitive lens for studying STT

The literature on STT and undergraduate tutoring led us to three observations. First, teachers' perspectives on STT are inseparable from their broader views on epistemology, mathematics, and didactics of mathematics. Second, while the literature reports on the perspectives of individual teachers, these are often consistent with how STT has been discussed in broader communities. Third, the perspectives on students' challenges with STT may shape teachers' instruction of students in transition. These observations convinced us that the socio-cultural perspective may be a useful one to undertake this study. Within this perspective, "the human mind is seen as constituted discursively, through practices, and in particular through language that carries the specificities of social contexts and practices and regulates human functioning" (Lerman, 1998, p. 334). Specifically, this perspective affords accounting for various aspects of teacher activity (including its discursive and practical components) and viewing STT as a collectively constituted construct. The latter aspect appears especially relevant for novice teachers: Having a limited teaching experience so far, it seems more reasonable to construe their take on undergraduate teaching with references to a social plane that they internalized as participants in various educational systems (e.g., mathematics students), rather than purely individual constructions.

The socio-cultural perspective encompasses various theoretical frameworks (e.g., Lerman, 1998). Our choice rests with commognition (Sfard, 2008) for two main reasons. First, it has been acknowledged that commognition offers a coherent set of conceptual and analytical tools to investigate human thinking, learning, and development in relation to mathematics in general (e.g., Morgan, 2020) and in the tertiary context in particular (Nardi et al., 2014). This aspect makes the framework relevant to our study, which is with novice tutors. Second, commognition has been previously applied to investigate teaching in first-year courses (e.g., Viirman, 2021), including studies that focused on STT (e.g., Kontorovich & Locke, 2022; Kontorovich et al., 2019; Pinto, 2019; Thoma & Nardi, 2018). In the school setting, commognition has been utilized not only to scrutinize teachers' practices but also to gain a deeper understanding of their underlying assumptions about learning and instruction (e.g., Heyd-Metzuyanim & Shabtay, 2019). In the context of an advanced mathematics course, Kontorovich (2021) used the framework to dissect the interplay between one mathematician's assessment practices and her broader pedagogical perspective. These studies encouraged us to adhere to commognition to explore tutors' undergraduate teaching of students in transition.

Commognition assumes that *discourses* underlie all aspects of human activity (both communicational and practical), dividing society into partially overlapping communities (Sfard, 2008). In the context of teaching, Heyd-Metzuyanim and Shabtay (2019) define *pedagogical discourse* as something that shapes and orients teachers "towards *what* to teach students, *how* to teach them, *why* certain teaching actions are more effective than others and, often not talked about but still very important, *who* can learn (or not learn)" (p. 543, italics in the original).[3] The construct has been explored in relation to school teaching (e.g., Heyd-Metzuyanim et al., 2016), but we see no reason to confine it to a particular educational setting. Indeed, Viirman (2015, 2021) uses the same term and with a similar

[3] Bernstein and Solomon (1999) adhere to the same term, while emphasizing the institutional dimensions of a discourse. In turn, Heyd-Metzuyanim and Shabtay (2019) use "pedagogical discourse" to capture issues of content and its teaching.

meaning to investigate the instructional practices of university mathematics teachers. Given that "the membership in the wider community of discourse is won through participation in communicational activities" (Sfard, 2008, p. 91), we perceive tutors as a community that takes part in the pedagogical discourse on university mathematics.

In this investigation, we focus on tutors' sense-making of their teaching rather than on their actual instruction. Thus, we use Sfard's (2020) approach to discourse as a special type of communication that "has been constructed along history as a toolbox for constructing potentially useful accounts of different segments of reality" (p. 90). This description is not very far from Bruner's (1991) approach to narrative as "a conventional form, transmitted culturally and constrained by each individual's level of mastery and by [their] conglomerate of prosthetic devices, colleagues, and mentors" (p. 4).

Discourses offer conventional building blocks (specifically, keywords, narratives, and routines) to construct such accounts, but they rarely determine individual choices. This is because the selection of what to bring into the discursive existence and how to do so rests with the individual. This is especially relevant when people capture segments of classroom reality in words. Such *pedagogical accounts* belong to the communicational sphere. This makes their objects *discursive*, i.e., arising through keywords and narratives. Some of these keywords point to perceptually accessible entities that exist independently of human discourse. But even then, the choice of words lies with the narrator. For instance, a narrator needs to decide how to refer to humans: by name, their assumed role (students, learners, mathematicians), gender, etc. The discursive choices become especially critical when one comes to construct accounts that refer to purely abstract objects (e.g., "knowledge," "understanding").

A recurrent finding on how teachers account for their teaching pertains to *deficit* types of discourses (e.g., Anthony et al., 2018). Adiredja and Louie (2020) explain that deficit discourses focus on students' academic and intellectual shortcomings, locating them "in students themselves, their families, or their culture" (p. 42). In this way, such discourses are silent about a broader social and historical context in which the specific students' activity unfolds. The lecturers' perspectives in Klymchuk et al. (2011) illustrate that deficit narratives can feature in teachers' discourses in relation to STT. These findings raised our sensitivity to deficit narratives that feature in tutors' reflections on their teaching.

4 The study

Consistently with the previous section, we conceptualize STT as an element of a pedagogical discourse – a line of communication distinguishable through keywords and narratives that points to students' transition from the secondary to tertiary educational context. For instance, in Annie's reflection in the "An introductory snippet" section, we find STT in the sentences about her "maths teacher," her not understanding "at that time," "things […] in school," and the reference to "a first-year student." To emphasize the discursive character of this conceptualization, we shall dub it as *STT communication*.

Our central research question is "what roles does STT communication play in tutors' sense-making of the incidents that took place in their tutorials when participating in pedagogical discourse?" We aim to offer analytically informed interpretations for tutors' initiation of STT communication. The term "sense-making" refers to tutors' accounting for "different segments of reality" (Sfard, 2020, p. 90) and reflection – an "active, persistent and careful consideration" (Dewey, 1910, p. 6). The question is informed by case-based approaches to study tutors' pedagogies (see "Undergraduate tutoring") and previous research on university teachers' perspectives on STT (see "University teachers' perspectives on STT").

4.1 Context and participants

Our data come from an undergraduate course in mathematics education (*MathEd* hereafter) offered in the mathematics department at a large New Zealand university.[4] The course was not required by a particular program, and it mostly attracted students in the last stages of their mathematics majors who were interested in educational issues. In a collaborative and student-centered environment, the students engaged with various aspects of university mathematics education (for additional course details, see Oates et al., 2005). Tutoring first-year mathematics students was the central activity of the course.

It may seem unconventional for a mathematics department to let undergraduates tutor other undergraduates. This arrangement should be contextualized in the rather intricate school qualification system in New Zealand. Due to space limitations, we do not delve into its details here (see Locke et al., 2020, for a short overview). It suffices to say though, that high-school mathematics is structured in modules and considerable freedom is given to schools and teachers regarding what modules to offer to students. Consequently, high-school graduates vary significantly in the mathematics they studied. Being aware of this issue, the syllabi of first-semester courses for non-mathematics majors include concepts and methods that are not very different from those covered in high school but are new to those students who did not study the relevant modules. The situation is different with undergraduates majoring in mathematics who have already been exposed to a broad range of mathematics as part of their degree.

All the students in the MathEd course tutored in one of three mathematics courses: We refer to them as *preparation*, *service-I*, and *service-II*. The first course is intended for students who do not meet the standards necessary to succeed in first-year mathematics. The course covers topics in basic algebra, trigonometry, functions, differentiation, and integration. Completing this course does not provide its students with credits toward a university degree. Service-I is a general entry course for non-mathematics majors, usually students of commerce, life sciences, and social sciences. Its syllabus contains standard topics in calculus (e.g., differentiation, integration) and linear algebra (e.g., linear equations, matrices). Service-II continues service-I and covers series, calculus of two variables, algebra of vector spaces, and differential equations.

The novice tutors[5] are allocated to specific groups for the whole semester and are expected to co-lead, in pairs, ten 1-hour tutorial sessions. Nearly a week before each tutorial, the course lecturers publish sets of questions for the tutorial. In preparation for it, tutors are expected to examine the questions, consider issues that might emerge, and develop strategies to address them. Overall, the expected role of a tutor can be described in the words of Moore (1968):

[4] The analysis was conducted after the students completed the course. The necessary approval was obtained from the university ethics committee. It ensured participants' consent, confidentiality, and the right to withdraw.

[5] From here onwards, we use "tutors" for students in the MathEd course in relation to their teaching. Those who attended the tutorial sessions as part of their first-year studies are "students.".

> The tutor is not a teacher in the usual sense: it is not [their] job to convey information. […] The teacher [sic] acts as a constructive critic, helping [students] to sort it out, to *try* it out sometimes, in the sense of exploring a possible avenue, rejecting one approach in favour of another. (p. 18, italics in the original).

In the MathEd course, the tutors engaged with the mathematics education literature and discussed their experiences as "seasoned" university learners and beginning teachers. The course teachers supported the tutors by providing opportunities for reflection and encouraging experimentation in their tutorials. For instance, at the beginning of the course, the tutors were requested to select specific aspects that they would like to trace in their classes throughout the semester and design a format for their tutorial sessions to advance these aspects (these could be revised later). This is to illustrate that the course provided a space for tutors to pursue their pedagogical interests without expecting them to implement specific course-driven agendas. This is consistent with our initial positioning of undergraduate tutors as agentive actors, in the sense of distinct teachers who co-author social interactions and communal practices for their students in transition (cf. Vygotsky, 1978).

4.2 Data collection and analysis

As part of the individual MathEd coursework, the tutors were expected to submit a coherent piece of text in which they reflect on a specific incident that drew their attention in their tutorial session that week (for specific guidelines, see the Appendix). The guidelines were inspired by the design of the UMTP (Nardi et al., 2005): They allowed tutors to choose an incident they conceived to be significant, made room for their elaborations, and encouraged critical questioning of their teaching. Consistent with Mason (2002), the guidelines directed tutors to distinguish between the incident and its interpretation. To enhance the potential usefulness of reflection writing, tutors were also asked to formulate "take-outs" for their further teaching. Selected submissions were read and discussed in the MathEd course to support the authors and the cohort as a whole in their first teaching steps. These interactions typically started with the authors re-telling the stories of the incidents and continued to a whole-class discussion.

Overall, we collected 363 reflections from 42 tutors over four semesters. To construct the data corpus, we used AtlasTi software to scrutinize the reflections in search of the STT narratives. We drew on Sfard's (2008) definition of a narrative as a "sequence of utterances framed as a description of objects, of relations between objects, or of processes with or by objects" (p. 134). As part of the search, we included smaller textual units, such as words, phrases, and comments. This process resulted in 14 keywords that we associated with STT communication (e.g., "first-year," "first-semester," "school," "previous studies," "university," "teacher," "lecturer"). In the next stage, we re-examined each reflection to decide whether the STT communication in it was substantial enough to include the reflection in the data corpus. We decided based on the number of STT instances and their perceived significance. Eventually, our corpus consisted of 58 reflections generated by 38 tutors. We conceptualized each reflection as a snapshot of the tutors' pedagogical discourses where the tutors were the ones to initiate STT communication.

The data analysis started with the commognitive distinction between "mathematizing" – narrating about mathematical objects – and "subjectifying" – narrating about participants of mathematical discourse (Sfard, 2008). The STT instances were categorized based on their main objects: mathematics (e.g., "The topic of this week's tutorial, differentiation, is something that most students have encountered before"), the course the participants were tutoring (e.g., "This is a crash course of high school knowledge"), and people. The last category was especially diverse, and for this, we drew on Heyd-Metzuyanim and Sfard's (2012) three levels of generality of subjectifying narratives. The levels distinguish between one's performance of a particular action, a person's typical or routine performance, and inherent properties that identify a person. This classification led us to distinguish between tutors' reference to someone's *actions* (e.g., "I tried to explain it by telling them …"), *general narratives* identifying people or "things in the world" (e.g., "When coming to university many students struggle"), and *descriptions of routines* (e.g., "This is how our lecturer taught us"). The level of detail that the participants provided in their reflections varied, and then we use "descriptions of routine" to refer to tutors' narratives describing frequently repeated actions.

To characterize the roles of STT communication in tutors' reflections, we approached each instance with such questions as "what is its added value to the tutor's story?" and "how will this story change if the instance is omitted?" To better understand the relations between STT instances and the surrounding story, we were mindful of the logical connectors the tutors used (e.g., "because" – reason, "also" – similarity, "but" – contrast).

5 Four roles of STT communication

This section is structured around the four roles of STT communication that we identified in tutors' reflections. They are weaving STT into the descriptions of tutorial incidents, making sense of unexpected incidents, presenting their (tutors') instructional actions as replications of their own STT experiences, and generating new pedagogical narratives. These roles are neither strictly distinct nor exclusive, as it was rare that any reflections contained STT instances associated with a single role only. We exemplify these roles with excerpts chosen based on their clarity and the potential to illustrate the gist of each category.

5.1 Weaving STT into the descriptions of tutorial incidents

As mentioned previously, tutors had agency in choosing the aspects they captured in their pedagogical accounts. The guidelines for reflection asked the tutors to separate the description of a tutorial incident (i.e. "facts") from its interpretation (see Appendix). Yet, STT communication permeated the descriptions in nearly a quarter of tutors' reflections, at least once for each tutor. We illustrate this by showing how the tutors identified their students and referred to mathematics as the focus of the incidents.

Let us consider an excerpt from a reflection made by Betty^{Preparation}[6]:

> After visiting a few groups, it surprised me when I found out that there actually were a lot of [Preparation] students who were struggling with fractions. Most of them had some idea of what fractions are, but they had many misconceptions that they had brought from school. For example, they did not know how to multiply or add fractions, or find common factors. […] Some students with richer background math knowledge did finish all the tutorial questions.

[6] All tutors' names are pseudonyms; a superscript indicates the course they tutored.

Betty writes about her “Preparation students,” whom she identifies as “having some idea” and “many misconceptions.” She points to “school” as the source of both. However, students who completed the tutorial questions are identified through “richer background math knowledge.” Betty’s association of her students’ “math knowledge” with their school studies cannot be taken for granted. To recall, students come to the tutorials less than a week after the relevant mathematics has been discussed in the lectures. Then, the preparation course appears as an alternative point of reference for addressing students’ successes and challenges. This is what we found in other accounts, where tutors identified their students with the beginning stages of university studies (e.g., “first-year students,” “students who are new to uni”).

Some students referred to “mathematics,” “topic,” or specific questions around which the incidents revolved with comments linking them to school or university mathematics. For instance, “to remind them of *this concept from school math*, I said […]” and “the students used a *notation from high school* […].” In his reflection on the fourth tutorial, one tutor wrote: “this tutorial was on vectors, *which is the first university math in this course*.” This instance implies that the mathematics covered in the first three sessions does not deserve the label of “university math,” even though it was discussed in a university course.

5.2 Making sense of unexpected incidents in STT terms: “communizing” and “empathizing”

Around eighty percent of reflections contained descriptions of incidents where the students acted differently from how the tutors expected. This made room to interpret and rationalize the unexpected, which nearly all tutors did with STT communication. Specifically, we identified two discursive moves through which the tutors made their peace with such cases: *commonizing unexpected incidents* and *empathizing with the students*. We illustrate both moves with Annie’s excerpt from the “An introductory snippet” section.

Annie wrote that after explaining that “function is like a factory,” she was not content with the students telling her that they “didn’t understand it all.” Even after “they did” what she suggested to find $f(2-x)$, she was still “not sure they really understood why.” However, this “not understanding” appears differently if considered under the assumption that “a first-year student […] meet[s] this idea for the first time.” Through the lens of this general STT narrative, what looked special and unexpected becomes a logical derivation of a broader pedagogical “truth.” Indeed, if “the factory example” is “hard,” there seems to be little surprise in the fact that “more than three students” raised questions and repeatedly declared their “not understanding.” We use “commonization” to highlight that the tutors used general STT narratives, within which students’ actions appeared less unexpected and could be rationalized.

Table 1 presents additional examples of unexpected incidents and general STT narratives that commonized them. Some revolved around the challenge of university mathematics and the diverse range of experiences that students bring to first-year courses. Others, however, were deficit and stressed the innate lack of students’ interest and abilities. Notably, the deficit narratives criticized school mathematics and its instruction, but we found no instances of tutors condemning these issues in the university context.

The other sense-making move is empathizing with the students, which led to personal (on the part of the tutor) STT narratives. For instance, Annie wrote that the first time she encountered “the factory example” (in school), she “also didn’t understand it.” Thus,

Table 1 Examples of communizing unexpected incidents through general STT narratives

Description of unexpected incidents	*General STT narrative*	*The gist of commonizing*
"Students learned more on the topic of functions in basic algebra, the exponential function, and the natural logarithmic function. It was difficult for students to solve questions with $ln(a)$. Before the tutorial, I gave them some hints. But they still couldn't find the way and couldn't understand why I did what I did."	"It is difficult for [students] to find the way themselves and know why they need to do it in this way. […] They are not very good in solving questions which combine the old knowledge they learned before and new knowledge together. […] Many students have weak background of mathematical knowledge. It is a common situation for many students that when they learn new knowledge, they forget the old knowledge."	The students engaged in extensive studies of algebra and functions, and received hints from the tutor. Nevertheless, they struggled to solve questions and understand tutors' solutions. This can be communized by a general STT narrative, suggesting that students' mathematical background is weak, and they are especially not successful in questions that combine multiple topics. Within this interpretation, a particular struggle with $ln(a)$ becomes an instance of a broader pattern when learning new material comes at the expense of the old
The tutor describes students not reacting to his explanation of a question on series: "I let the class answer questions that were simple to solve like 'What's the pattern that's occurring as I keep adding more terms to the partial sums?' and 'What's the limit of the right hand side.' A couple of students answered these questions, but there were no questions and no reactions from the rest of the class."	"Service-II is the endpoint for a lot of students not intending to further study mathematics or any other subject that requires rigorous mathematics – subjects like commerce and biology. […] To them, maths is a tool, not something they want to engage with at a deeper level. They would be more excited about this if they were real maths students."	Despite the tutor's invitations to contribute to the solution of a problem and react to his explanations, the majority of the students did not engage. This can be communized by a general STT narrative, suggesting that non-mathematics majors take the course because their programs require them to do so, opposed to mathematics majors who would be excited about the content. Service-II is the last mathematics course for most non-mathematics majors, and their particular disengagement can be explained with the general lack of interest in the subject

drawing parallels between her own and her students' experiences provided Annie with access to what appears as a similar reaction in the same situation. These parallels explain Annie's ability to empathize with her students on a personal level. Indeed, we interpret her "I can't imagine how hard it must be" as an exaggerated version of "I *can* understand how hard it must be, because I was in a similar situation." Through the lens of empathizing, "not understanding" emerges not as an attribute of the particular students but as a legitimate reaction to an intricate mathematical topic.

Table 2 presents additional examples of tutors empathizing with their students by drawing on their own personal STT narratives. Notably, after sharing their narratives, the tutors often wrote that they "associate," "connect," and "understand what students are going through." Furthermore, a careful reading of the reflections suggested that this was often a result of the tutors recognizing mathematical nuances that they had been taking for granted

Table 2 Examples of empathizing with students through personal STT narratives

Description of an unexpected incident	*Personal STT narrative*
"This week the students were introduced to functions, which included function notation, how to find the inverse of a function, and how to transform function graphs. One episode that caught my attention was one student who was trying to use all this maths to solve the exercises in the tutorial. […] The student looked rather confused about how to apply the proof that was taught in class to the questions given in the tutorial."	"I did understand why the proof may have seemed confusing. […] I thought back to the beginning of my studies when I had been introduced to a whole new palette of mathematical notations that represented crazy things in my own maths class. […] Comparing my maths journey to the student's, I felt a strong sense of sympathy for him."
The tutor wrote about a student being overwhelmed with the amount of content she was expected to know when coming to a tutorial or a test	"When I was in my first year, I really struggled with not being attuned to the standard of studying in university and the adjustment I needed to go through wasn't smooth"

(e.g., the abundance of mathematical notation). We see these realizations as tutors growing appreciation for and sensitivity to the mathematical intricacies of STT.

5.3 Tutors' instructional actions as replications of their own STT experience

Every tutor elaborated on the instructional actions that they undertook in the specific incidents at least in one of their reflections. Most of these reflections contained descriptions of routines that the tutors experienced as students in STT. Accordingly, in this role, STT communication acted as a bridge between the tutors' teaching and their own experiences in transition, tacitly accounting for their actions in the classroom.

Consider an excerpt from the reflection of Connor$^{Service-I}$. He wrote,

> The interesting thing I found in this week's tutorial was that there are two different ways to do chain rule to solve questions: "inside function/outside function" and "define a new function *u*." Only the second way is written in their [students'] coursebook, but I personally always used the first way and my high school maths teacher didn't mention "define new function *u*" at all. At that time, I felt that it's hard to realize or notice there are two functions here. So I did lots of exercises to make myself get used to this. […] So when students came and asked me how to use chain rule, this is what I told them: look for "inside/outside function" and practice, practice, practice.

In his reflection, Connor distinguished between "two different ways" (or routines, in our terms) to apply the chain rule. For instance, to derive $(2x-1)^2$ according to the first routine, one needs to recognize $2x-1$ as an "inside function" and x^2 as the "outside function," and then derive with respect to x. In the second routine, one introduces u to be $2x-1$ and derives with respect to u before rewriting the result in terms of x in the last step. The "inside/outside function" emerges as a characteristic routine of Connor's mathematical discourse; a routine that he "always used." Connor traces his first encounter with it back to his days in school and his "high school math teacher," who introduced this routine without

Table 3 Examples of instructional actions described as implementations of familiar routines

Description of instructional actions	*Description of familiar routines from tutors' STT*
Concerning a question asking to find $\sum_{k=1}^{\infty}\left(\frac{1}{k+2}-\frac{1}{k+1}\right)$: "This was exactly what I demonstrated on the board: writing out partial sums for the first few values of k, pointing out that the middle terms cancel out, and taking care to point out that $\sum_{k=1}^{\infty}$ really means $\lim_{n\to\infty}\sum_{k=1}^{n}$."	"I had merely reproduced what I observed from [name of another tutor] tutoring in the first few weeks. His way of tutoring has worked for him for three years and seems similar to how most tutors do their jobs."
Toward the end of a tutorial, a student asked the tutor how to solve a particular question. The tutor wrote in her reflection, "I referred her to the subsequent coursebook notes that applied to this type of questions as well as some external links and YouTube channels."	"I had used them [the links] when I was at her stage and I found they helped me. […] When I was in my first year, I really appreciated all the little tips that various peers who were older and more experienced gave me."
A student insisted that the derivative of $f(x) = x + a$ is 0, and the tutor engaged in explaining the confusion. She wrote, "I showed her that according to $f'(x) = nx^{n-1}$, the expression becomes $1 \bullet x^0$ which is 1."	"It's amazing how I remembered this explanation on the spot. Mr Chang, my math teacher, taught us that many years ago and I still remember it."

mentioning an alternative. Connor's decision to teach this routine to his students is not obvious since he conceives it as different from what "is written in their [students'] coursebook." Furthermore, Connor remembers his struggle with the initial implementation of this routine. Nevertheless, he got "used to this," and this is the advice he gave to his students.

Table 3 offers additional examples where tutors described their actions in the tutorial as replications of routines with which they were familiar as students. The tutors traced their initial exposure to these routines back to their school teachers, course tutors, and peers.

5.4 Updating initial STT narratives

This category emerged from just above thirty percent of reflections where the tutors capitalized on unexpected incidents to update their initial narratives on STT. Let us consider the reflection of Ely$^{Service-I}$ as an example. Its focal incident revolved around the question asking students to find the maximum number of zeros in a 3×3 matrix such that its determinant is not zero. The incident involved "Student1," whom Ely described as "being confident with his answer of 2 zeroes." Ely wrote,

> Before I could ask him to justify his answer, a student next to him (Student2), who was having trouble with the same question, asked me for help. My first instinct was to start answering his question myself in my usual way, which is to give the first step and maybe ask a few prompting questions to get Student2 thinking. […] However, this time I took a different approach and asked Student1 to explain his answer to Student2.

In his interpretation, Ely explained:

> I had never considered this approach before, as I thought that a first-year student's understanding of a topic would be too shaky to help another student. Neither student understood how to solve this question before this encounter, however both stu-

Table 4 Examples of new pedagogical narratives

Abbreviation of the incident	*Pre-incident STT narrative*	*New pedagogical narrative*
A tutor asked the students to solve $\sin x = 0$ without the calculator and the students "just gave up."	"It came to me as a surprise that after all these years in school some students couldn't draw out the graph for $\sin x$ or $\cos x$."	"I now think we need to consider the fact that some students may have learnt about trigonometry a long time ago. […] It would be helpful if we revise the basic features and properties (namely the function graphs) at the beginning of each tutorial in trigonometry."
A student said that she did not know how to simplify $\left(2-\sqrt{5}\right)\left(3-\sqrt{5}\right)$. The tutor suggested to replace $\sqrt{5}$ with x and expand the brackets like in a parametric expression	"It would not have occurred to me personally that simplifying an equation with surds would be more difficult to a university student than simplifying an equation with x. After some thought, it made a lot of sense because in school, students got used to equations with variables rather than equations that had square roots."	"From this episode, I learned that what *I* believe is easier and simpler might not be easier or simpler for others. […] Furthermore, I now have new insight […] that there is no 'best' way to solve a problem. If a student does not understand the first way you explain/solve a problem, it might be a good choice to try to explain it to them using another interpretation."
A student asked for help with multiplying 2 by $\sqrt{3}$. In her explanation, the tutor invited the student to use a calculator for a decimal representation of $2\sqrt{3}$. The tutor then explained, "by cutting off an irrational number, it is technically not the same number since it is less precise. Therefore, we must keep it as $2\sqrt{3}$." The tutor also wrote that "it was difficult to explain on the spot why we don't multiply the two together."	"This episode really drew my attention because the multiplication of rational and irrational numbers just seemed so simple to me. I learned it in year ten and never questioned why we don't make them one real number."	"After this episode, I learned that it is important to ask why we do things. […] Before the next tutorial, I want to work on more examples and examine why we apply certain rules and how they would violate mathematical laws if they were done in a different way."

> dents left with a better understanding, with next to no involvement from me. Since my first maths course, I have always preferred to work alone.

In these excerpts, we see Ely describing his "usual way," that is, his routine of interaction with a student who asks for mathematical help, "to give the first step and maybe ask a few prompting questions." In what appears as an attempt to explain why involving additional students never crossed his mind, Ely elaborates that his "usual way" aligns with his personal STT narrative as a student who "always preferred to work alone" and with a general STT narrative on "first-year student's understanding of a topic would be too shaky to help another student." However, Ely deviated from the usual and invited "Student1" to share his solution with "Student2."

"Student1" agreed to step in, which yielded an interaction that Ely described in very positive terms. This interaction impressed him:

> This episode drew my attention as it gave me an alternative way to help a student, and one that did not even require me to say anything. From this encounter, I can see that working in groups offers its own advantages. […] I believe that this method of learning math is far more effective than a student listening to me talk about the topic. Since the answers and work came from them, it is obviously a better way to help solidify their understanding. They think about the question on their own, which means they also develop and enhance their own problem-solving skills, which can be applied to any future maths problems that they may come across.

In this excerpt, we see Ely generalizing his instructional action into a "method" and describing an alternative routine "to help a student." The description is not very detailed, but it suggests that in the new routine, a help-seeking student would not be "listening to [Ely] about the topic," but students would be "working in groups" and "think about the question on their own." He also elaborates on the advantages of this routine in comparison to his "usual way."[7] Ely's "method" and arguments for it are consistent with what has been known in the literature as dialogic learning (e.g., Resnick et al., 2015).

Table 4 presents additional illustrations of pedagogical narratives and descriptions of routines, all highlighting their novelty to the tutors. Similar to Ely's example, these narratives appear as generalized versions of particular aspects of the incident, and they differ from the STT narratives that the tutors associated with pre-incident times. Accordingly, in these reflections, STT narratives act as a benchmark against which the tutors juxtapose their incident-driven generalizations; the contrast emphasizes their novelty.

6 Summary and discussion

Mathematics education research has considered STT through various theoretical lenses to study what changes the transition to the new tertiary realm entails (e.g., Gueudet, 2008; Hochmuth et al., 2021; Thomas et al., 2015). Using the commognitive framework, we introduced STT communication as an element of pedagogical discourse practiced by various teaching communities. In this way, we highlight that working with students in transition requires teachers to construct accounts of their pedagogical reality, which makes the

[7] Off note, Ely seems to overlook the fact that a 3×3 matrix can have up to six zeroes without having the zero determinant.

processes of this discursive construction matter. We explored STT communication with first-year tutors in light of the growing acknowledgement of the tutors' impact on undergraduates' learning (e.g., Jaworski, 2002; Johns & Burks, 2022; Speer et al., 2005; Yee et al., 2022). Accordingly, this investigation contributes to a small number of studies that view tutors as key actors in students' STT (e.g., Lawson & Croft, 2021).

Our findings emerged from tutors' written reflections on their teaching. The reflections came from an elective MathEd course that the tutors – students themselves, took toward a mathematics major. On the one hand, the particularity of this setting requires caution and moderation in regard to the empirical generalizability of the findings (Hammersley, 2012). Indeed, these may have emerged from wishful reflections or were representative neither of other tutors nor of the same tutors in other circumstances. But even if so, the findings open the door for what Hammersley (2012) terms as theoretical inferences – ideas and hypotheses that can be generated via careful consideration. In "On the four roles of STT communication," we structure these inferences around the four identified roles of STT communication. Beforehand, let us make two general comments in relation to the areas in which we grounded this study in the "Undergraduate tutoring and university teachers' perspectives on STT" section.

6.1 STT communication and undergraduate tutoring

From the commognitive perspective, STT communication has the power to shape the thinking and actions of those who participate in the pedagogical discourse. The evidence for this theory-driven corollary can be shown by the fact that we identified instances of STT communication in at least one reflection generated by most tutor participants. Indeed, STT issues were discussed in the MathEd course, but the tutors were neither guided nor expected to build on these discussions in their reflections. Accordingly, the tutors' decision to do so suggests that STT communication was useful for them to make sense of their own teaching, at least in some cases (see "On the four roles of STT communication" for elaboration).

Is STT communication unique to our tutor participants? Our preliminary attempt to identify STT communication in some previous studies cannot be declared utterly successful. But this is not unexpected. Jaworski (2002) and Nardi et al. (2005) present many data excerpts from tutorials in rather advanced courses (e.g., real analysis, group theory), while other studies do not share tutors' quotes (e.g., Lawson & Croft, 2021). On the other hand, while not referring to tutoring specifically, the mathematicians in Nardi (2008, p. 93–101) consistently refer to STT issues. Furthermore, in our experience, mathematicians often converse about teaching first-year students as a craft that is distinct from teaching other student cohorts. Accordingly, future research may be interested in exploring teachers' pedagogical discourses and tutoring practices with special attention to what makes them distinct in the case of students in transition.

6.2 STT communication and teachers' perspectives on STT

This study joins a rather thin line of research on university teachers' perspectives on STT. Specifically, the findings in "Making sense of unexpected incidents in STT terms: 'communizing' and 'empathizing'" present a range of general STT narratives that the tutors shared in their reflections. Many of these narratives revolve around the inadequacy of students' previous mathematics studies in school, not unlike how these issues have been described in the literature (e.g., Gueudet, 2008; Nardi et al., 2005; Thomas et al., 2015).

What is notable to us is how similar our tutors' de-evaluative narratives are to those generated by the university lecturers in Hong et al. (2009) and Klymchuk et al. (2011). For instance, some of the narratives were consistent with the view that STT is a problem and school is its main source (e.g., Fey, 1977; London Mathematical Society, 1995). We find this consistency fascinating: Even if such narratives are as global and widespread as typically assumed, how do they become part of the pedagogical discourses of beginning university teachers? After all, it is hard to believe that someone "sat our tutors down" and purposefully exposed them to these narratives. This question draws attention to salient processes through which collectively constituted perspectives on STT are internalized by tutors and novice teachers in general. Further research in that direction seems paramount to better understand STT as a discursive and social phenomenon.

That said, our tutors also generated narratives acknowledging the difficulty of tertiary mathematics and transitioning into it. These narratives echo Nardi et al.'s (2005) findings on Oxford tutors, whose higher levels of pedagogical sensitivity to students' difficulties were linked with an awareness that a large part of mathematics studied in school needs to be re-grounded on university foundations. Further research can tap into the apparent tension between the two kinds of narratives and the teachers' sense-making of their teaching in one way rather than another.

6.3 On the four roles of STT communication

The main finding of our study pertains to the four roles of STT communication. Specifically, we found that it (i) featured in tutors' descriptions of tutorial incidents; (ii) assisted tutors to make sense of unexpected incidents; (iii) positioned tutors' instructional actions as replications of their own STT experience; and (iv) contributed to tutors' generation of new pedagogical narratives. We discuss each of these roles next.

Mason (2002) acknowledged that it takes time and effort to learn to write teaching accounts that are free from evaluation and judgment. An effort to separate between incidents and their interpretation was visible in many reflections that we analyzed. Yet, the first role of STT communication demonstrates that maintaining the separation can be hard. For instance, in some reflections, the tutors identified their tutees as "first-year students" (as opposed to using pseudonyms or referring to them as "just students") and categorized the mathematics that they worked on in the tutorials as "school" or "university" (as opposed to only naming the topics, for example). This allows proposing that STT constituted a segment of tutors' classroom reality, a segment that they could not leave behind in their pedagogical accounts.

The second role pertains to tutors accounting for incidents where students' actions deviated from tutors' expectations. This is where general STT narratives became handy as they turned students' actions from attention-drawing oddities into instantiations of broader patterns. This commonization often drew on deficit narratives about students, their knowledge, and their abilities. In Nardi et al.'s (2005) terms, these narratives can be seen as naïve and dismissive since they largely ignored students' difficulties and left little room to consider how students can be supported. Let us recall that these narratives featured in tutors' coursework that they submitted for assessment. This may suggest that our tutors did not consider deficiency-based interpretations to be an issue. These interpretations illustrate the gap between novices to pedagogical discourse and the growing anti-deficit movement in the mathematics education community (e.g., Adiredja & Louie, 2020). In the school context, the phenomenon of teachers viewing students' learning through deficit spectacles has been well documented, and professional development appears as a conventional countermeasure (e.g., Anthony et al., 2018). We are not familiar with any systematic efforts of this sort in the tertiary context in Australasia. Thus, we appreciate the colleges and universities in the USA, where the design, implementation, and systematic scrutiny of professional development programs for tutors have been institutionalized (e.g., Speer et al., 2005; Yee et al., 2022).

Empathizing with students was another discursive move our tutors implemented to make sense of students' actions. As part of it, the tutors harked back to their time as students in transition and shared their experiences through personal STT narratives. In their reflections, they recalled situations that were similar to the tutorial incidents and, having referred to their own personal struggles, the tutors "related," "connected," and "associated" with their students. This move echoes Jaworski's (2002) findings on tutors expressing affective sensitivity to students: drawing on personal STT narratives may be a trigger of this sensitivity.

Nearly half of the analyzed reflections contained tutors' personal STT narratives. We believe that this is not accidental: Our participants – undergraduates themselves – were in their students' shoes not so long ago, which provided them with access to narratives from their own STT. The accessibility to a wide range of relevant personal stories may be out of reach to other tutoring cohorts, such as doctoral students and faculty (e.g., Yee et al., 2022). Thus, we draw attention to what may be a characteristic of tutors who completed their transition relatively recently. We also invite future research to view personal STT narratives as a resource and explore how they can be leveraged in tutor training and undergraduate teaching in general.

The third role concerns tutors presenting their instructional actions as replications of routines from their STT. If we take tutors' reflections at face value, these replications are somewhat expected. Indeed, at the time of data collection, the tutors' teaching experience was limited, which explains why they turned to their STTs for precedents. Specifically, they drew on the actions of their former teachers and tutors – what Vygotsky (1978) dubbed knowledgeable others.

If we were to compare rigorously between tutors' learning then and contemporary teaching now, it might turn out that the connections between the two are shakier than our tutors described. Nevertheless, the study shows that these connections can be tight in tutors' pedagogical discourses. This finding adds to the STT literature, which has typically been concerned with the impact of students' transition "here and now." Our study adds that the transition can also have a deferred effect that comes into play years later, when some current students become university tutors and lecturers. Practically speaking, these findings suggest that assisting students who are in STT today may have long-term gains in how they will support their future students to make the transition.

The fourth role of STT communication puts the third role in perspective by showing that tutors' instruction can go beyond replicating the familiar and that their STT experience is not the only resource they can turn to in their teaching. Within this role, our tutors engaged in a critical reflection on incidents from their tutorials and generated narratives and descriptions of routines that they presented as new to them. Commognitively speaking, these are accounts of *learning* since they lead to tutors' enriching their pedagogical discourses with new insights (Sfard, 2008).

We appreciate our tutors' attention to incidents, critical reflection on them, and insight generation. We argue that tutors' pedagogical and mathematical growth through teaching and reflection deserves further research. Take Ely's reflection as an example. It is fascinating to see how one classroom incident appears sufficient for him to de-evaluate a teaching routine that he described as instinctive, familiar, and experience-based. Note how he reframes the instructional actions that he took in a particular situation into a "method" that now "can be applied to any future maths problems that [the students] may come across." In a similar vein, many of our tutors wrote about how their teaching made them appreciative of the inherent complexity of mathematics, something that they considered as "basic" beforehand. Exploring the processes of tutors' growth seems especially relevant when it comes to designing effective professional development and mentoring frameworks.

Sfard (2014) notes that a "discursive vision offered by the commognitive framework impacts our understanding of the learning and teaching of university mathematics" – while not necessarily offering "innovative idea[s] about how [the] deeply entrenched [tertiary] practices could be changed" (p. 202). We find this acknowledgment relevant to our study, in which commognition helped us notice and take the first step towards considering the roles of STT-communication in tutors' pedagogical discourses. With an eye to change, let us recall that from the socio-cultural standpoint, tutors' STT communication does not exist in a vacuum; it constitutes an individualized version of actions and talk practiced in broader communities. This is to say that with university teaching practices being entrenched so deeply, it seems unlikely that a sustainable change in students' experiences of STT can come only from focusing on tutors. Nevertheless, our study suggests that tutors can present their teaching as a substantial modification of the familiar that they made to support their students. Further research is necessary to explore whether and how these narratives of change materialize in mathematics tutorials.

Appendix

Guidelines for post-tutorial report

Format: 500-word document that is produced individually by each student. The document should contain a consistent and self-contained narrative in a PDF format. Please type the document and make sure that mathematical formulas, if there are any, properly appear.

Writing: You are requested to write an entry after each tutorial that you deliver and to bring the entry to the following lesson. Selected entries will be shared and discussed.

1. **What.** Describe *one episode* from the tutorial that drew your attention: Who did the episode involve? Around which mathematics did it emerge? What was said and done? If you were involved in the episode, you may choose to describe your in-the-moment feelings and thoughts. Generally, try to avoid interpretations as much as possible in this part to enable the readers to develop their own impressions.
2. **So what.** Offer an interpretation of the episode: Why did it draw your attention? What sense can you make of it? What alternative interpretations can you offer? Which interpretation is more appealing to you and why?
3. **Now what**. What did you learn from the episode and from your reflection on it? Your answer can be formulated as an insight or a question that seems valuable to you. Your answer can also contain some practical actions that you may want to pursue in your future teaching.

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Declarations

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References

Adiredja, A. P., & Louie, N. (2020). Untangling the web of deficit discourses in mathematics education. *For the Learning of Mathematics, 40*, 42–46.

Anthony, G., Hunter, R., & Hunter, J. (2018). Challenging teachers' perceptions of student capability through professional development: A telling case. *Professional Development in Education, 44*(5), 650–662. https://doi.org/10.1080/19415257.2017.1387868

Bernstein, B., & Solomon, J. (1999). "Pedagogy, identity and the construction of a theory of symbolic control": Basil Bernstein questioned by Joseph Solomon. *British Journal of Sociology of Education, 20*(2), 265–279. https://doi.org/10.1080/01425699995443

Bruner, J. (1991). The narrative construction of reality. *Critical Inquiry, 18*(1), 1–21. https://doi.org/10.1086/448619

Clark, M., & Lovric, M. (2008). Suggestion for a theoretical model for secondary-tertiary transition in mathematics. *Mathematics Education Research Journal, 20*(2), 25–37. https://doi.org/10.1007/bf03217475

Dewey, J. (1910). How we think. *Heath & Co*. https://doi.org/10.1037/10903-000

Fey, J. T. (1977). Report of study group D: Minimal competencies in mathematics. *The Arithmetic Teacher, 24*(5), 405–407. https://doi.org/10.5951/at.24.5.0405

Gueudet, G. (2008). Investigating the secondary-tertiary transition. *Educational Studies in Mathematics, 67*, 237–254. https://doi.org/10.1007/s10649-007-9100-6

Hammersley, M. (2012). Troubling theory in case study research. *Higher Education Research & Development, 31*(3), 393–405. https://doi.org/10.1080/07294360.2011.631517

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: On learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research, 51*, 128–145. https://doi.org/10.1016/j.ijer.2011.12.015

Heyd-Metzuyanim, E., & Shabtay, G. (2019). Narratives of 'good' instruction: Teachers' identities as drawing on exploration vs. acquisition pedagogical discourses. *ZDM-Mathematics Education, 51*, 541–554. https://doi.org/10.1007/s11858-018-01019-3

Heyd-Metzuyanim, E., Tabach, M., & Nachlieli, T. (2016). Opportunities for learning given to prospective mathematics teachers: Between ritual and explorative instruction. *Journal of Mathematics Teacher Education, 19*, 547–574. https://doi.org/10.1007/s10857-015-9311-1

Hochmuth, R., Broley, L., & Nardi, E. (2021). Transitions to, across and beyond university. In V. Durand-Guerrier, R. Hochmuth, E. Nardi, & C. Winsløw (Eds.), *Research and development in university mathematics education* (pp. 193–215). Routledge.

Hong, Y. Y., Kerr, S., Klymchuk, S., McHardy, J., Murphy, P., Spencer, S., Thomas, M. O. J., & Watson, P. (2009). A comparison of teacher and lecturer perspectives on the transition from secondary to tertiary mathematics education. *International Journal of Mathematics Education in Science and Technology, 40*(7), 877–889. https://doi.org/10.1080/00207390903223754

Jaworski, B. (2002). Sensitivity and challenge in university mathematics tutorial sessions. *Educational Studies in Mathematics, 51*(1/2), 71–94. https://doi.org/10.1023/A:1022491404298

Johns, C. A., & Burks, L. C. (2022). A Framework for mathematical knowledge for undergraduate mathematics tutors. *International Journal of Research in Undergraduate Mathematics Education,* 1–30. https://doi.org/10.1007/s40753-022-00165-0

Klymchuk, S., Gruenwald, N., & Jovanovski, Z. (2011). University lecturers' views on the transition from secondary to tertiary education in mathematics: An international survey. *Mathematics Teaching-Research Journal Online, 5*(1), 101–128.

Kontorovich, I. Herbert, R., & Yoon, C. (2019). Students resolve a commognitive conflict between colloquial and calculus discourses on steepness. In J. Monaghan, E. Nardi, & T. Dreyfus (Eds.), *Calculus in upper secondary and beginning university mathematics - Conference proceedings* (pp. 119–122). MatRIC. https://matric-calculus.sciencesconf.org/data/pages/CalcConf2019_Papers_190910.pdf.

Kontorovich, I. (2021). Minding mathematicians' discourses in investigations of their feedback on students' proofs: A case study. *Educational Studies in Mathematics, 107*(2), 213–234. https://doi.org/10.1007/s10649-021-10035-2

Kontorovich, I., & Locke, K. (2022). The area enclosed by a function is not always the definite integral: Re-learning through transitioning within learning-support systems. *Digital Experiences in Mathematics Education*. https://doi.org/10.1007/s40751-022-00116-z

Kontorovich, I., & Ovadiya, T. (2022). Secondary-tertiary transition and undergraduate tutoring: Novice tutors make sense of their teaching of first-year courses. In S. S. Karunakaran & A. Higgins (Eds.), *Proceedings of the 24th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 314–322). RUME.

Lavie, I., Steiner, A., & Sfard, A. (2019). Routines we live by: From ritual to exploration. *Educational Studies in Mathematics, 101*(2), 153–176. https://doi.org/10.1007/s10649-018-9817-4

Lawson, D., & Croft, T. (2021). Lessons for mathematics higher education from 25 years of mathematics support. In In V. Durand-Guerrier, R. Hochmuth, E. Nardi, and C. Winsløw (Eds.), *Research and development in university mathematics education* (pp. 22–40). Routledge.

Lerman, S. (1998). Research on socio-cultural perspectives of mathematics teaching and learning. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity* (pp. 333–350). Kluwer Academic Press.

Locke, K., Frankcom-Burgess, R., Passmore, R., & Kontorovich, I. (2020). Calculus in secondary school and in teacher education in New Zealand. ResearchGate https://www.researchgate.net/publication/353257699_Calculus_in_secondary_school_and_in_teacher_education_in_New_Zealand

London Mathematical Society. (1995). *Tackling the mathematics problem*. https://dokumen.tips/documents/tackling-the-mathematics-problem-mei-tackling-the-mathematics-problem-c-the-london.html

Markovits, Z., & Smith, M. S. (2008). Cases as tools in mathematics teacher education. In D. Tirosh, & T. Wood (Eds.), *The international handbook of mathematics teacher education, tools and processes in mathematics teacher education* (vol. 2, pp. 39–65). Sense Publishers.

Mason, J. (2002). Researching your own practice: The discipline of noticing. *Routledge*. https://doi.org/10.4324/9780203471876

Moore, W. G. (1968). *The tutorial system and its future*. Pergamon Press.

Morgan, C. (2020). Discourse analytic approaches in mathematics education. In S. Lerman (Ed.), *Encyclopaedia of mathematics education* (pp. 223–227). Springer.

Nardi, E., Jaworski, B., & Hegedus, S. (2005). A spectrum of pedagogical awareness for undergraduate mathematics: From "tricks" to "techniques." *Journal for Research in Mathematics Education, 36*(4), 284–316. https://doi.org/10.2307/30035042

Nardi, E., Ryve, A., Stadler, E., & Viirman, O. (2014). Commognitive analyses of the learning and teaching of mathematics at university level: The case of discursive shifts in the study of Calculus. *Research in Mathematics Education, 16*(2), 182–198. https://doi.org/10.1080/14794802.2014.918338

Nardi, E. (2008). *Amongst mathematicians: Teaching and learning mathematics at university level*. Springer.

Oates, G., Paterson, J., Reilly, I., & Statham, M. (2005). Effective tutorial programmes in tertiary mathematics. *International Journal of Mathematics Education in Science and Technology, 36*(7), 731–739. https://doi.org/10.1080/00207390500271461

Pinto, A. (2019). Towards transition-oriented pedagogies in university calculus courses. In J. Monaghan, E. Nardi, & T. Dreyfus (Eds.), *Calculus in upper secondary and beginning university mathematics - Conference proceedings* (139–142). MatRIC. https://matric-calculus.sciencesconf.org/data/pages/CalcConf2019_Papers_190910.pdf

Püschl, J. (2017). Identifying discussion patterns of teaching assistants in mathematical tutorials in Germany. In T. Dooley, & G. Gueudet (*Eds.*), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (pp. 2225–2232). DCU Institute of Education and ERME.

Resnick, L., Asterhan C., & Clarke, S. N. (Eds.). (2015). *Socializing intelligence through academic talk and dialogue*. AERA.

Rowland, T., Turner, F., & Thwaites, A. (2014). Research into teacher knowledge: A stimulus for development in mathematics teacher education practice. *ZDM-Mathematics Education 46*, 317–328. https://doi.org/10.1007/s11858-013-0564-9

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher, 27*(2), 4–13. https://doi.org/10.3102/0013189x027002004

Sfard, A. (2008). Thinking as communicating: Human development, the growth of discourses, and mathematizing. *Cambridge University Press*. https://doi.org/10.1017/CBO9780511499944

Sfard, A. (2014). University mathematics as a discourse – Why, how, and what for? *Research in Mathematics Education, 16*(2), 199–203. https://doi.org/10.1080/14794802.2014.918339

Sfard, A. (2020). Learning, discursive fault lines, and dialogic engagement. In N. Mercer, R. Wegerif, & L. Major (Eds.), *The Routledge international handbook of research on dialogic education* (pp. 89–99). Routledge.

Shulman, L. S. (1992). Towards a pedagogy of cases. In J. H. Shulman (Ed.), *Case methods in teacher education* (pp. 1–29). Teachers College Press.

Speer, N., Gutman, T., & Murphy, T. J. (2005). Mathematics teaching assistant preparation and development. *College Teaching, 53*(2), 75–80. https://doi.org/10.3200/ctch.53.2.75-80

Thoma, A., & Nardi, E. (2018). Transition from school to university mathematics: Manifestations of unresolves commognitive conflict in first year students' examination scripts. *International Journal of Research in Undergraduate Mathematics Education, 4*, 161–180. https://doi.org/10.1007/s40753-017-0064-3

Thomas, M. O. J., de Freitas Druck, I., Huillet, D., Ju, M. K., Nardi, E., Rasmussen, C., & Xie, J. (2015). Survey team 4: Key mathematical concepts in the transition from secondary to university. In Cho, S. (Ed), *The Proceedings of the 12th International Congress on Mathematical Education.* Springer, Cham. https://doi.org/10.1007/978-3-319-12688-3_18

Viirman, O. (2015). Explanation, motivation and question posing routines in university mathematics teachers' pedagogical discourse: A commognitive analysis. *International Journal of Mathematical Education in Science and Technology, 46*(8), 1165–1181. https://doi.org/10.1080/0020739x.2015.1034206

Viirman, O. (2021). University mathematics lecturing as modelling mathematical discourse. *International Journal of Research in Undergraduate Mathematics Education, 7I*, 466–489. https://doi.org/10.1007/s40753-021-00137-w

Vygotsky, L. (1978). Mind in society: The development of higher psychological processes. *Harvard University Press*. https://doi.org/10.2307/j.ctvjf9vz4

Yee, S., Deshler, J., Cervello Rogers, K., Petrulis, R., Potvin, C. D., & Sweeney, J. (2022). Bridging the gap between observation protocols and formative feedback. *Journal of Mathematics Teacher Education, 25*, 217–245. https://doi.org/10.1007/s10857-020-09485-x