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Narrative characteristics of captivating secondary mathematics lessons

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Abstract
Why do some mathematics lessons captivate high school students and others not? This study explores this question by comparing how the content unfolds in the lessons that students rated highest with respect to their aesthetic affordances (e.g., using terms like "intriguing," "surprising") with those the same students rated lowest with respect to their aesthetic affordances (e.g., "just ok," "dull"). Using a framework that interprets the unfolding content across a lesson as a *mathematical story*, we examine how some lessons can provoke curiosity or enable surprise. We identify eight characteristics that distinguish captivating lessons and show how some, such as the average number of questions under consideration at any point in the lesson, are strongly related to student aesthetic experiences. In addition, the lessons that students described as more interesting included more instances of misdirection, such as when students' false assumptions provide opportunities for surprising results. These findings point to the characteristics of future lesson designs that could enable more students to experience curiosity and wonder in secondary mathematics classrooms.

Keywords Mathematics curriculum · Narrative · Aesthetic · Mathematical story

1 Introduction

There is persistent evidence of widespread boredom in mathematics in different areas of the world, such as the USA (e.g., Middleton et al., 2019), Mexico (e.g., Baños et al., 2019), and Germany (e.g., Daschmann et al., 2011). Unfortunately, negative experiences with mathematics have been connected with a range of undesirable student outcomes (Middleton et al., 2016, 2019), such as poor mathematical dispositions and identity formation (Luo et al., 2014), mathematics anxiety (e.g., Foley et al., 2017; Ma, 1999), and lack of persistence in problem-solving (Tulis & Fulmer, 2013). These ill-effects are compounded by the increasing calls for students to develop perseverance (National Governors Association

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(NGO) Center for Best Practices and Council of Chief State School Officers (CCSSO), 2010) as they engage in problem solving and complex reasoning (Stein et al., 2007; Trafton et al., 2001). Thus, teachers are increasingly expected to teach lessons that not only engage students but also keep them engaged throughout complexity and cognitive struggle. Stimulating students' desires to engage in complex mathematical activity can not only avoid these negative outcomes but also can benefit student learning (Csikszentmihalyi, 1990; Dewey, 1913; Durik & Harackiewicz, 2007; Guthrie et al., 2005; Wong, 2007), since once a student's interest is stimulated (to "catch" the student's attention (Durik & Harackiewicz, 2007, p. 598)), they are more likely to engage with the material and look forward to more (thus, "holding" the student's attention and interest (Durik & Harackiewicz, 2007, p. 598)).

So, what can be done to catch and hold students' interest in mathematics lessons within this demanding curricular environment? Many efforts have focused on augmenting mathematical content with non-mathematical elements such as typography and photographs (e.g., Durik & Harackiewicz, 2007), electronic educational games (e.g., Conati & Zhao, 2004), real-world contexts (e.g., Renninger et al., 2002), or humor (e.g., Matarazzo et al., 2010). However, Sinclair (2001) argues that relying on non-mathematical approaches to improve student mathematical experiences can convey a message to learners that mathematics is a "sterile domain" that, in itself, cannot be aesthetically pleasing or interesting (p. 25). Mathematics is inherently aesthetic in nature (Burton, 1999; Netz, 2005; Sierpinska, 2002; Sinclair, 2001), by which we mean that it can move or compel a student to act, such as asking a question, exclaiming "Woo!," or even quitting (Dewey, 1934; Dietiker et al., 2016, 2016a; Dietiker et al., 2016b; Dietiker, 2015a; Sinclair, 2001; Wong, 2007). Sinclair (2001) describes learning experiences that "enable children to wonder, to notice, to imagine alternatives, to appreciate contingencies, and to experience pleasure and pride" (p. 26) as *aesthetically rich*. Yet the ways students experience any particular lesson vary moment-to-moment and the qualities of a mathematics lesson (e.g., how it withholds information to catch students' attention) that appeals to one student may repel another. Therefore, we are interested to learn about the characteristics of aesthetically rich lessons that hold broad appeal to students. Just as some literary stories have mass appeal and interest large audiences in comparison to others, even when they appear very similar on the surface (e.g., origin stories of superheroes), some mathematics lessons excite or intrigue many students while others leave students bored from the start (Dietiker, 2016a; Richman et al., 2019).

Previously, in this journal, Dietiker (2015b) introduced a framework to describe how mathematical content can aesthetically draw a student into mathematical inquiry (i.e., catch) and support their desire to advance (i.e., hold) by interpreting mathematics lessons as *mathematical stories*.[1] Researchers have used the metaphor of mathematics-as-narrative to describe the aesthetic dimensions of mathematical learning experiences (e.g., Borasi & Brown, 1985; Gadanidis & Hoogland, 2003; Sinclair, 2005; Zazkis & Liljedahl, 2009). By using this metaphor to interpret enacted lessons with heightened student aesthetic reactions (e.g., gasps, visible excitement), some studies have provided rich analyses of how the mathematical content of these mathematical stories unfolds across a lesson (e.g., Dietiker, 2016a; Richman et al., 2019). These studies have proposed that the heightened student aesthetic reactions could be explained by potential narrative characteristics, such as overarching questions and misdirection. However, none of these studies were designed to learn if specific

[1] Although the term "mathematical story" can refer to stories used within mathematics curriculum (i.e., story problems), we are instead referring to the narrative created by the unfolding mathematical concepts and elements. This will be discussed more thoroughly in the theoretical framework section of this paper.

narrative characteristics were associated with student aesthetic reports. By only studying those with extremely positive student reactions, it was not possible to learn whether other lessons that were not interesting would have some or all of these same narrative characteristics. And, because they only relied on visible aesthetic reactions of some students at particular points of lessons, it is not clear whether these lessons also held broad appeal.

Therefore, in this study, we begin to answer the question: *When enacted high school mathematics lessons are interpreted as mathematical stories, what narrative characteristics, if any, distinguish the lessons that hold broad aesthetic appeal from those that do not?* Our analysis demonstrates how analyzing lessons for their narrative characteristics can enable a new way of understanding mathematical learning experiences and their impacts on students.

2 Theoretical framework

To connect students' experiences with how the mathematical ideas emerge and change throughout the lesson,[2] we interpret a lesson with a sequence of mathematical events (e.g., tasks, discussions, lectures) that incrementally shifts what is known as a *mathematical story* (Dietiker, 2013, 2015b). This curricular metaphor is particularly useful because a story integrates both logical (i.e., sense-making) and aesthetic dimensions (Egan, 1988). That is, the aesthetic dimensions of a story, such as the way it enables anticipation or elicits surprise, can compel a reader to keep reading and work at making sense of the story (Nodelman & Reimer, 2003). Moreover, sensemaking offers potential aesthetic benefits and consequences; when stories make sense, putting pieces of the story together can be pleasurable. Yet when stories do not make sense, readers may lose interest and quit reading.

Similar to literary analysis, which compares stories by analyzing their characteristics (e.g., characters, action, setting), mathematical stories can be recognized and distinguished by their *mathematical characters* (i.e., mathematical objects, such as a linear function), *mathematical actions* (i.e., processes that transform these characters, such as symbolic manipulation or transformation), and *mathematical settings* (i.e., the representation(s) in which the mathematical story takes place, such as a coordinate plane) (Dietiker, 2015b). In addition, mathematical stories can be compared for their potential aesthetic dimensions for students. Specifically, the *mathematical plot* describes how a story can emotionally impact a member of its audience[3] by offering revelations and withholding information, potentially compelling them to predict where the story is headed (or not). It does this by dynamically shifting the tension between what is already known and what is desired to be known by the students as the story progresses. For example, when a mathematical story offers information (i.e., progress) that hints of a future revelation, it can spur the formulation and pursuit of new questions ("Which functions are similar?"), similar to how a reader of a literary story might generate questions about how the story will progress (i.e., "Will the villain be caught?").

[2] Note that sets of lessons, such as units or courses, can also be interpreted as mathematical stories. However, the grain size in which we focus in this study is at the lesson level. Thus, all references to a mathematical story will refer to the interpretation of a lesson.

[3] Although both teachers and students experience mathematical stories enacted in classrooms, for the purposes of our study, the primary audience toward which we direct our attention is the students.

Barthes (1974) proposes that the aesthetic effects of literary stories can be understood by analyzing the ways that what a reader knows shifts as the story unfolds: creating mystery (i.e., enabling a reader to recognize that they do not know something), progress (i.e., information that advances a reader toward an answer), misdirection (i.e., information that leads a reader away from an answer), disruption (i.e., when the story halts progress), and disclosure (i.e., support for a reader to answer the question). By integrating the logical and aesthetic aspects of a story, Barthes' approach represents the story's plot (Bal, 1986). Thus, a mathematical plot represents how the aesthetic dimensions of a mathematical story support questions to be raised and answered by students as the story unfolds (Dietiker, 2015b; Richman et al., 2019). For example, a mathematical story can provide false or misleading information (i.e., misdirection, such as allowing students to assume that functions are different when, in fact, they are not) which can enable surprise and further questions to emerge when an unexpected result is later revealed. We refer to the way in which a question transitions from being asked to being answered as a *story arc*; once a question is raised, a story arc remains open until an answer is ascertained either by explicit revelation (such as by the teacher or curriculum materials) or by the sensemaking of the audience. Some story arcs can span an entire mathematical story, while others are brief mysteries. Since a mathematical story can involve answering multiple questions at any point along a sequence, story arcs can overlap. Theoretically, a lesson that nurtures prolonged curiosity will enable multiple story arcs that overlap as they remain unanswered.

3 Methods

This study is part of a larger project focused on learning how the design of high school mathematics lessons can potentially impact student experiences. Because aesthetically rich high school mathematics lessons are not common, making them difficult to study, the *Mathematically Captivating Learning Experiences* Project has worked with a group of high school mathematics teachers to design and test specialized lessons ("MCLEs") with potentially aesthetically rich experiences for students (e.g., surprise, suspense) to increase their interest in the mathematical content. For each class of students, we observed multiple lessons, both MCLEs and non-specialized lessons ("everyday lessons"), after which we measured (via surveys) student perceptions of their aesthetic experiences. By comparing lessons with aesthetic extremes, we can identify the characteristics of the way the content unfolds that can be associated with the student aesthetic reports.

In this section, we begin by describing the teachers, schools, and process to design MCLEs. We then describe how we collected the data. Next, we explain how we analyzed the mathematical plots of the lessons and formed comparison groups based on the student surveys: the lesson per teacher with the most aesthetic value (i.e., the lesson that students rated highest with respect to its aesthetic affordances) and the lesson per teacher with the least aesthetic value (i.e., the lesson that students rated lowest with respect to its aesthetic affordances). Lastly, we describe how we identified distinguishing narrative characteristics of the two groups of lessons.

3.1 Teachers, schools, and the MCLE design process

Six teachers, each with at least 4 years of teaching experience, were recruited from three high schools in the Northeastern region of the USA. One school is a small, private charter high school in the city with a predominantly Latinx student population, while a second is a large, urban public high school with multiple racial and ethnic groups. The third school is a large, suburban public high school with a predominantly white student population. In addition to differences in size and demographics, these schools reflect different curricular contexts as well, in that, the use of written curriculum varied. In all six classes we report on in this study, teachers employed group work and problem solving as a regular part of everyday instruction. Three classes were designated as college level (e.g., Advanced Placement) or honor level, one at each high school.

To create the MCLEs, teachers and researchers designed lessons by thinking about how the content would unfold in potentially aesthetic ways (e.g., surprise, suspense). The teachers drew on their knowledge of their local context and their students to make design decisions and were free to draw from their course curriculum materials as inspiration for their designs. All lessons were designed to be enacted within the teachers' curricular contexts. For example, a lesson designed to introduce logarithms was intended to be taught at the start of a unit on logarithms. To avoid competing or conflicting explanations for heightened student aesthetic experiences with non-mathematical elements, such as a fun computer game or a compelling worldly context, the MCLEs were designed without features like these. The everyday lessons had no design restrictions (i.e., they could include real-world contexts, games).

As the MCLEs were designed, teachers (with researchers) considered a set of questions, including (1) How do we predict students will be thinking throughout the lesson? (2) How might rethinking the sequence potentially offer new opportunities (aesthetic, conceptual)? (3) Is the lesson making the best use of chosen mathematical characters and/or setting? Should a "recast" be considered? (4) Is there a literary story analog for your MCLE? (Examples: romantic comedy, murder mysteries, mistaken identity), and (5) What is the moral of the story? Teachers mapped out the sequence of activities and were given wide latitude in how to attend to different aspects of lesson design (e.g., deciding what questions to ask). To plan out how the events of the lesson would unfold, teachers created representations for the unfolding events of the lesson, such as storyboards (see an example in Fig. 1). Note that teachers were not given explicit instructions to attend to the nature of their questions, the way they would respond to student ideas, or whether to have students collaborate in groups or work individually.

To support the design of MCLEs, the teachers attended a 2-week professional development during the prior summer where they learned about mathematical stories through practitioner articles (i.e., Dietiker, 2016b; Ryan & Dietiker, 2018) and studied the designs of textbook lessons for how the content unfolds and enables or limits aesthetic opportunities (such as having plot twists or a growing sense of mystery). In addition, to further advance the teachers' understanding of mathematical stories and how they can impact students in the classroom, the teachers and researchers collectively analyzed the mathematical plot of one videotaped mathematics lesson. The lesson selected to be analyzed with the teachers had not been designed using the mathematical story framework but contained evident student aesthetic reactions. The analysis presented an opportunity for teachers to study how the unfolding content was potentially related to the students' reactions.

We acknowledge that the introduction of teachers to both the metaphor of mathematical story and Barthes' framework likely influenced the mathematical structure of the teachers' lessons in this study. Note, however, that this understanding had the potential to influence all the teachers' lessons, not only their specialized lessons. In addition, this study had no assumption that the lessons with the most aesthetic value would necessarily be MCLEs. Instead, the MCLEs were created to enrich the aesthetic opportunities of the overall set of

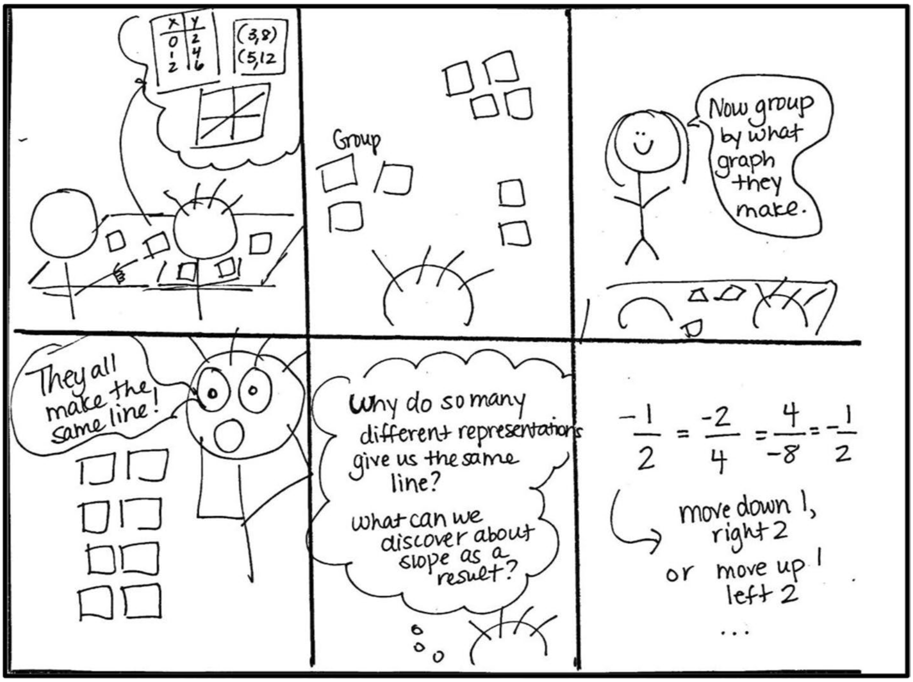

Fig. 1 A storyboard created for an MCLE on linear functions by Ms. Spruce

lessons, just as everyday lessons were also observed in order to increase the set of aesthetic opportunities and limitations that might not be present in MCLEs.

3.2 Data collection

All lessons were observed by multiple researchers using the same protocol so that students would not be able to infer whether a lesson was special or not. Multiple video cameras and audio recorders were placed strategically around the classroom to capture teacher and student interactions, students' gestures, and the progress of a focus group of students during group problem solving. Immediately after each lesson, all participating students took a Lesson Experience Survey ("LES") individually using Qualtrics (see Online Resource 1). To measure students' aesthetic experiences during a lesson, the survey asked students to rate their overall interest in the lesson on a Likert scale of 1 ("very bored") to 4 ("very interested") and to select three adjectives to describe their experience from 16 randomly arranged descriptors. A wide range of adjectives were provided to enable students to describe a variety of aesthetic experiences, especially those perceived to be most interesting. When designing the survey, the adjectives were initially identified by asking high school students from a variety of school contexts to provide their own terms for different types of experiences (e.g., "What word or short phrase would you use to describe how you felt during a day in math class when you were really curious to find something out?"). We then tested the survey after a variety of lessons, comparing student ratings of interest with the adjective selections, which enabled us to categorize the descriptors as "positive" (suspenseful, amazing, fascinating, fun, funny, enjoyable, satisfying, thought-provoking, surprising, intriguing), "neutral" (fine, just ok, and frustrating), and "negative" (dull, boring, and not special). We included more positive descriptors so that students could provide insight into different kinds of aesthetically rich experiences, since lessons that are intriguing are not necessarily suspenseful and vice versa. Since students were prompted to select three descriptors, we included at least three descriptors at each level to enable students to answer the survey without the use of positive descriptors. Any impact of the large number of positive descriptors on their selection would impact measures for all lessons consistently. This survey was tested with students and shown to distinguish between student aesthetic experiences (Riling et al., 2019).

Finally, we recognize that the student aesthetic reports for MCLEs may have been influenced by teachers' affective behavior (e.g., perhaps teachers were more enthusiastic during MCLE enactments), since teachers knew of their specialized nature. If this influence alone explains the student's improved experiences, then the mathematical structure of MCLEs and everyday lessons should be similar and no distinct patterns of how the content unfolded across the lessons should emerge between the two groups. Comparing the lessons with the greatest and least aesthetic value across multiple teachers from multiple schools limits the potential for an aspect of any particular lesson to be a distinguishing characteristic. That is, if one lesson's inclusion of a manipulative or another teacher's use of humor is the key determining characteristic influencing an increase in student positive aesthetic reports for a lesson, then the mathematical plots across the entire set of lessons should not differ in consistent ways. Therefore, any significant differences between the mathematical plots of lessons can be related to the characteristic that distinguished the two groups: namely, the student aesthetic reports.

3.3 Data analysis

This subsection describes how we coded each lesson for the mathematical plot, identified the two groups of lessons, and compared the plots of these groups to identify structural characteristics that distinguished them.

3.3.1 Coding the mathematical plots

Coding a lesson for its mathematical plot requires three passes through its transcript. For each pass, the research team broke into two groups to code separately and then came together to resolve differences. On the first pass, the research team analyzed transcripts to identify the acts of the story by identifying when the focus of the mathematical story changed. This was done by noting shifts in which mathematical characters, actions, and settings were in focus throughout the transcript. For example, when a teacher shifted from talking about the y-intercept of a set of linear functions to the slope of the same set of linear functions, this shift would start a new act, since the y-intercepts and slopes are different mathematical characters. The portion of the lesson between shifts represents an *act*[4] of the mathematical story.

[4] Note that similar to how acts operate in a literary play (e.g., *Romeo and Juliet*), an act in a mathematical story is not associated with any length of time. That is, acts represent major portions of the story and can be brief or can span a long period of time.

Table 1 Mathematical story codes adapted from Barthes (1974)

Code	Description	Examples
a. Proposal	A hint of mystery that allows for anticipation, foreshadowing	A student speculates that natural logs may be involved when finding the derivative of an exponential function, yet the importance of this observation is not recognized until later in the exploration
b. Question by teacher or environment	An indication that a mystery is raised explicitly or implicitly by teachers, students, or the environment	The teacher poses a problem
c. Question by student		A student asks a question, or a posted objective inspires a question about the meaning of a new term in the objective, such as "*How do I multiply 5 by* $-x$*?*"
d. Promise	An explicit indication that a question will be answered later	The teacher states that she will help the students learn an easier way to solve a problem that they are having trouble solving with existing methods
e. Progress by teacher or environment	An increase in what is known about a question	The teacher demonstrates the first step in a procedure (teacher) or a passage is read from the text that helps clarify a question (environment)
f. Progress by student		Students work on a problem and make progress toward an answer (student)
g. Equivocation	A misdirection by encouraging a faulty assumption which is not recognized until later	The teacher prompts the students to group cards which have different representations of a linear function. Students sort cards in different groups, assuming that not all cards represent the same linear function. Students are later surprised to learn all cards represent the same function
h. Snare	A misdirection through explicit lie or error that is not corrected until later	A student is working on a task and makes an error that is corrected much later in the lesson
i. Jamming	A disruption to progress that misleads the audience by suggesting that a question will never be answered	The teacher interrupts student work on a task
j. Suspension	A shift of focus that disrupts progress on a question	A task prompts students to consider a question about the nature of complex numbers, eliminating opportunities to make progress on existing questions about equilateral triangles
k. Disclosure	An endorsement of an answer	The teacher confirms that a student's answer to a task is correct

On the second pass, the research team identified all the mathematical questions raised, considered, and addressed throughout the lesson. Although many questions identified were explicitly posed by a teacher, student, or some type of curriculum material, some were suggested implicitly through statements. For example, if a task prompts students to "graph $y < -\frac{2}{3}x + 4$," then the questions "What is the graph of $y < -\frac{2}{3}x + 4$?" and "How do I graph $y < -\frac{2}{3}x + 4$?" are implicitly raised. If the research team found no evidence a question was ever discussed or addressed (i.e., no form of progress was made on answering the question), then it was not included in the analysis.

On the final coding pass, the research team coded each question for changes in what was known about the question across the acts. The researchers used codes adapted from Barthes' (1974) narrative theory (see Table 1). Because the questions asked by students indicate curiosity, and because the progress toward an answer is indicative of someone doing the mathematical work of the problem, we have separated questions and progress by teacher and student. For example, contributions by a teacher or environment (e.g., a textbook) that increase what is known about a question were coded "e," whereas student contributions were coded "f."

These coding passes result in a comprehensive mapping of how participants within each lesson are moved to raise and answer questions. Namely, as this coding coordinates the dynamically changing tensions between what is unknown (i.e., the emergence of new questions) and known (i.e., increased progress on questions or disclosure of answers) as acts unfold, this mapping represents the mathematical plot of the lesson.

3.3.2 Selecting the comparison groups of lessons

In all, we observed 32 lessons across the 2018–2019 year: 18 specialized and 14 not. To recognize and identify lesson narrative characteristics that are related to broad student aesthetic appeal, the research team compared the mathematical plots of the lessons with the greatest aesthetic value ("captivating") with those with the least aesthetic value ("non-captivating") for each class of students. To determine the captivating[5] group, we first eliminated lessons that did not have at least 10 LES student surveys completed or other complicating factors, such as the survey being administered after an emotional school announcement. Of the remaining 29 lessons, we identified which lesson had the highest average interest measure on the LES survey for each teacher. In the case of ties, we selected the lesson with the highest average number of positive descriptors selected by students. Once a teacher's lesson was selected for the captivating group, then the lesson for that same class with the lowest average student interest level was selected for the non-captivating group. In the case of ties, we selected the lesson for which students, on average, selected the highest number of negative descriptors for the non-captivating group. Further confirming these groups of lessons, paired *t*-tests of students' LES measures revealed significant differences in interest and/or positive descriptors for all teachers except Mr. Ash. The selected lessons, along with their measures, lesson content,[6] course, and student grade levels are listed in Table 2. Although the group selection was not based on whether the lessons were MCLEs or not, all lessons in the captivating group were MCLEs and all lessons in the non-captivating group were not.

[5] Note that our use of the term "captivating" does not refer to any particular type of aesthetic experience but instead connotes a mixed collection of positive aesthetic experiences (e.g., suspense, intrigue, surprise).

[6] The variety in topics is a benefit to our analysis. Had the topics been strongly related, the students' aesthetic experiences in the second of the two lessons would likely have deteriorated since the mathematical content was not new, thus eliminating the potential for aesthetic moments such as surprise, intrigue, and the like.

Table 2 Lesson topics and interest measures for captivating and non-captivating lessons per teacher

			Non-captivating lessons			Captivating lessons		
Teacher	Course	Grades	Topic	Avg. interest	Avg. positive descriptor	Topic	Avg. interest	Avg. positive descriptor
Mr. Ash	Alg 2	11	Properties of logs	2.69	1.38	Intro to imaginary numbers	2.80	1.50
Ms. Cherry	Alg 2H	10	Systems of ineqs	2.71	1.12	Extraneous solutions	2.89	1.61*
Ms. Elm	Math 3H	10, 11	Percent change	2.65	0.96	Rational root theorem	3.19*	2.05*
Mr. Palm	AP Calc	12	Product rule	2.60	1.33	Deriv. of exp. functions	2.82	2.00*
Ms. Spruce	Math 1	9	Graphing lin ineqs	2.86	1.79	Equivalent linear functions	3.69*	2.62
Ms. Willow	Alg 2	10	Inverse functions	2.36	1.09	Logarithmic identities	2.90*	1.27*

All names are pseudonyms

*Significantly different using 2-tailed paired t-test with $p<0.05$

3.3.3 Comparing the mathematical plots of lesson groups

Across all the lessons, we qualitatively compared the mathematical plots and identified characteristics, both theoretically and visually, that appeared to distinguish them. For example, since theory suggests that lessons in the captivating group may provide overlapping questions that remain open, we analyzed the number of questions in each act that were not disclosed and were still open in the subsequent act (what we refer to as *degree of inquiry*). Since lessons in both groups were coded for their mathematical plots, we compared the overall structural characteristics (i.e., number of acts, number of questions opened throughout the lesson), as well as the characteristics of questions (e.g., the average number of acts a story arc remains open, the percentage of questions open for more than one act) and the characteristics of acts (e.g., how many questions are open in each act on average, the degree of inquiry).

To determine if a characteristic was significantly different for lessons in the two groups, a paired-sample *t*-test was conducted. When the paired-sample *t*-test indicated a relationship, a simple linear regression analysis was performed to learn whether the characteristic predicts students' levels of interest. We used Ferguson's (2009) guidelines, which suggest that $0.04 < R^2 < 0.25$ represents a weak association, $0.25 < R^2 < 0.64$ represents a moderate association, and $R^2 \geq 0.64$ represents a strong association.

We also compared the two groups of lessons for their aesthetic opportunities by comparing the presence or absence of misdirection: equivocations, snares, and jamming. For this, we compared the number of instances in a lesson in which any of these forms of misdirection occurred (as opposed to the number of questions affected by the teachers' misdirection). For example, if the same statement led to a misleading assumption in multiple questions, we counted it as one instance of equivocation.

4 Findings

As a group, there are multiple ways the structure of the mathematical plots of the lessons in the two groups differ. To illustrate these differences qualitatively, we start by describing in detail the captivating and non-captivating lessons for one class in order to highlight narrative characteristics that distinguish how the content unfolded across the two lessons. We then present how these same narrative characteristics quantitatively differed (or not) across all lessons in both groups and identify which are associated with students' interest.

4.1 An illustrative example of contrasting mathematical plots

This pair of lessons was observed in a 9th-grade Integrated Math 1 course taught by Ms. Spruce. There were 19 students present for each lesson, 16 of whom participated in the study. We begin with a description of Ms. Spruce's captivating lesson, about linear equations, and then summarize her non-captivating lesson, about linear inequalities. We then highlight some key characteristics of the mathematical plots of the lessons and note how they differ.

4.1.1 Ms. Spruce's captivating lesson

Ms. Spruce opens the lesson (Act 1) with a question reviewing the slope–intercept form of linear equations, indicating that this question is not related to the rest of the lesson. Students, seated in groups, work mostly independently, occasionally asking each other questions. The teacher then prompts students for what m and b represent in the slope–intercept form of a linear function (Act 2) and introduces the activity that will take up the remainder of the lesson (Act 3). In this activity, Ms. Spruce distributes 14 cards, each with a representation (i.e., equation, graph, sets of points, verbal description, or table) of a linear function, to each group of three or four students (see Fig. 2). The teacher prompts students

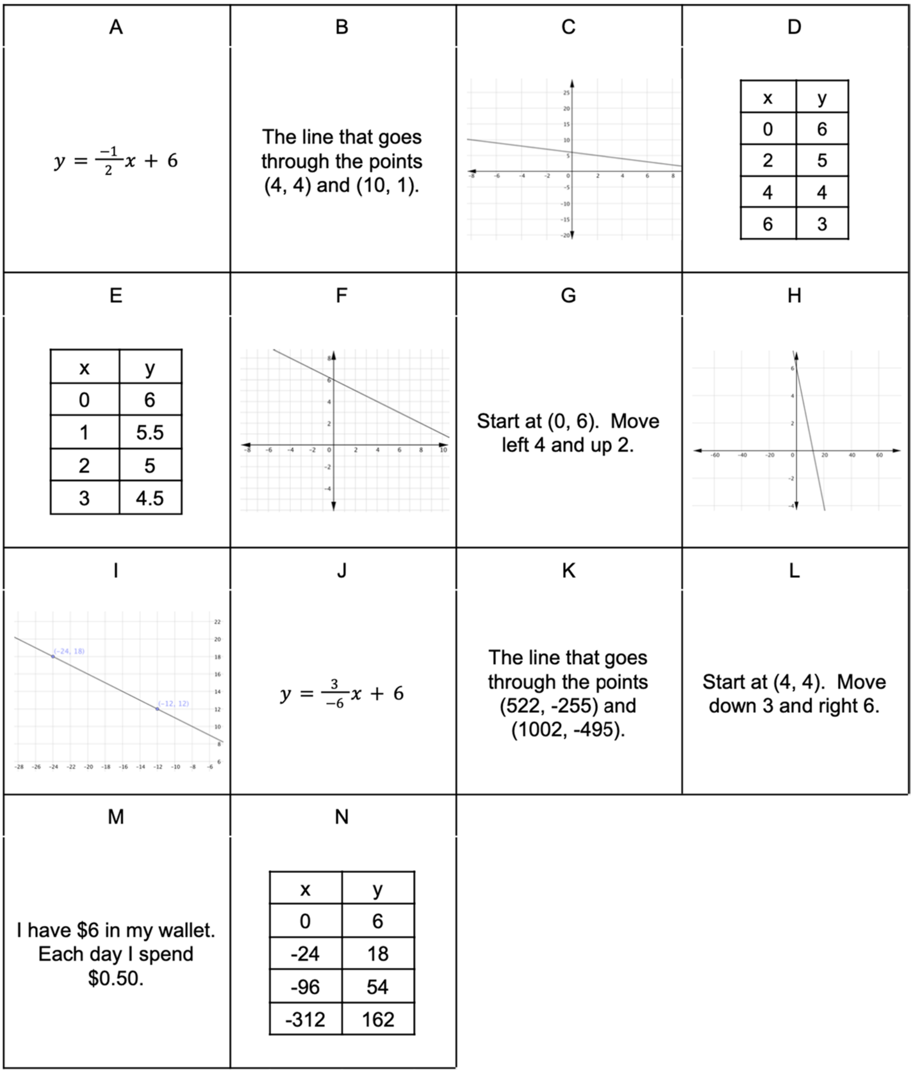

Fig. 2 Task cards for Ms. Spruce's captivating lesson

to group the cards without specifying criteria. For about 5 min, students work together to compare pairs of functions for common attributes (Act 4). For example, in our focal group, as Ashley examines Card K that says, "The line that goes through the points $(522, -255)$ and $(1002, -495)$," she and her group members recognize how one of the points makes the equation on a different card (Card A) true:

Ashley	So, what's like, this [the slope of the line on Card A] is, in decimal, this is like, negative point five, right? (Brandon nods.) And you times five twentytwo, plus six
Brandon	(Sounding impressed) What! Where did you get that?
Ashley	(smiling) Exactly!
Colin	What?
Brandon	Alright, so that's a pair
Ashley	Yeah, that's a pair bro!

Next, the teacher pulls together the class and Eugene explains that cards E, M, and C are a match (Act 5). Then (Act 6), a disagreement emerges as Kevin suggests that C and F are a pair instead. At this suggestion, Maria protests, claiming that F matches with D. Ashley agrees, explaining that they both have the same rate. The class spiritedly debates which cards belong together.

Then, in Act 7, Brandon suggests that four cards (E, F, D, C) belong together and the class erupts with disagreement. Now that it has been suggested that groups of functions may have more than 3 cards, new students enthusiastically volunteer more and more cards that potentially represent the same function, vigorously calling out to agree or disagree. Ms. Spruce plays along, challenging some matches: "You think A goes with K? That's crazy!".

Twelve minutes into this discussion (the start of Act 8), EJ claims, "All of them go together," which sparks many reactions (e.g., "Woah!"). Ms. Spruce asks, "You actually think that?" Other students indicate that they agree and explain a few additional matches. Ms. Spruce asks groups to figure out how to prove that the final few cards match. During this time, in Act 9, students excitedly call out their findings across the room ("Yo! N and I go together!"). As Ms. Spruce brings the class together one final time, for Act 10, students ask to present their work. Once Devon points out that all the representations should have a y-intercept of 6, they are able to explain the remaining connections. In Act 11, students debate whether all the graphs have the same slope. Ms. Spruce indicates that they will learn more about slope in a later lesson.

4.1.2 Ms. Spruce's non-captivating lesson

At the start of a lesson on graphing linear inequalities[7] (Act 1), a warm-up prompts students to graph $y < -\frac{2}{3}x + 4$ and explain whether $(-2, -5)$ is a solution. Some students work silently, but others discuss the tasks as they work. Zaya displays her graph on the document camera (Act 2) and Ms. Spruce leads a discussion about which points on the graph are solutions and whether the line should be solid or dashed (Act 3). Then (Act 4),

[7] This lesson occurred 12 weeks after Ms. Spruce's captivating lesson discussed in Sect. 4.1.1.

the teacher shifts the focus, reviewing how to express linear inequalities in standard form. Students then work in groups to find the x- and y-intercepts of $2x + 3y > 12$ (Acts 5 and 6). Following this, the teacher leads a discussion on how to graph the inequality (Act 7). A student expresses surprise that the line of this inequality is the same as that of Act 1. In Act 8, the teacher asks Jalila to pick “your favorite point” that should be part of the solution and another that is not, and to justify their conclusions. Finally, in Act 9, students are prompted to graph another inequality, $4x + 5y \leq 20$.

4.1.3 Contrasting characteristics of Ms. Spruce’s mathematical plots

The ways in which mathematical ideas unfolded across Ms. Spruce’s two lessons differ in several ways, especially in terms of how many acts questions remained open, how the questions overlapped and interconnected (or not) with one another, and the number of questions that were under consideration during each act. These characteristics are visible in the mathematical plots shown in Fig. 3 (for Ms. Spruce’s captivating lesson) and Fig. 4 (for her non-captivating lesson). Each row represents a question raised and addressed in the mathematical plot, in the order they appeared in the lesson. The numbered columns represent the acts of the mathematical plot. The shaded portions represent the story arcs where questions were open and under consideration.

The mathematical plot of Ms. Spruce’s captivating lesson shows that the acts are deeply interconnected, with many questions open across many acts. The central mathematical question of this lesson (Question 9: “Why do certain cards go together?”) is one of six questions that spans Acts 3 through 11, or 82% of the story (i.e., 9 of 11 acts). In contrast, the plot of her non-captivating lesson is remarkably sparse, with only a few questions connecting distinct parts of the mathematical story. Only one question (Question 5: “How do you know whether to shade above or below the line when graphing an inequality?”) spans more than half of this story (78%), joining parts of a lesson seemingly devoted to different topics (i.e., graphing linear inequalities of different forms).

Furthermore, the story arcs in the captivating lesson are considerably longer, suggesting that the questions offered students the opportunity for extended consideration. The average story arc length for this lesson is 3.7 acts long, or 34% of the story, which is approximately double that of the non-captivating lesson (1.5 acts on average, or 17% of the story). In fact, most of the story arcs in the non-captivating lesson (71%) are only one act long, whereas only 34% of story arcs in the captivating lesson have this quality. The story arcs in the captivating lesson are also more likely to contain acts in which there is no change in what is known about the question, which provided students an opportunity to wonder about questions that were not being addressed, while potentially communicating that not all questions would be answered immediately. Almost half of the story arcs of the captivating lesson (27 of 58) are open for at least one act without change (i.e., no codes). In contrast, only 3 of the 35 story arcs in the non-captivating lesson contain at least one uncoded act.

Since increasing the opportunities to wonder about the questions generates potential to sense growing mystery, we also note that there is a stark difference in the degree of inquiry as the lessons unfold (i.e., the number of questions per act that are not disclosed and are still open in the subsequent act). For example, the non-captivating lesson begins with its greatest degree of inquiry (7 questions in Act 1 remain open in Act 2) and then immediately drops to 3 in Act 2, where it remains steady (between 1 to 3 questions) until the end of the lesson. In contrast, throughout most of the captivating lesson, the degree of inquiry increases from 3 questions in Act 1 to an incredible 38 questions by Act 10. Almost all

#	Question	1	2	3	4	5	6	7	8	9	10	11
1	Do Now question.	bfed										
2	What are some mathematical ideas that could be represented by an equation?	ce										
3	What can the parts of a linear equation tell us about a real-life situation?	bfe						fe				
4	What is a linear function?		bf									
5	What does the equation of a linear function look like?		bfk									
6	What does m stand for in Y=mx+b?		bfk									
7	What does b stand for in Y=mx+b?		bfk									
8	Which groups of cards are similar?			bgfe	f	f	hf	f	fe	fe	f	k
9	Why do certain cards go together?			bfe	f	f	f	f	e		fe	f
10	Why are Cards N and I a match?			bfe	f				f		f	k
11	How do you know that two cards aren't a match?			bfe	h							i
12	How does considering (0,6) help us find matches?			bf	f		f				f	
13	What is the 'starting point' of a linear function?			bf	f			f				
14	How does considering travel behavior help us find matches?			bf	f	f		f				f
15	What might Card G be a match with?			cf					f	f	f	k
16	What could be negative about a graph?			cf	f							
17	How does a rate help find the y-intercept when the graph doesn't include it?			bf								
18	If two cards have one point in common, does that mean they go together?			bf			f				f	
19	How many cards can be in a group?				b	f		f	f			k
20	What pattern do the y-values follow in Table N?				cf							
21	How do the patterns in the y-values help find a match?				cf							
22	How do you calculate the rate between two points?				b		f					e
23	Will finding the rate of the graph give us matches?				b	f	f	f				f
24	How does Card A match with Card K?				cf				f		f	k
25	If the values on two tables are different, could they match?				ch		f		f			k
26	Why are Cards E, M, and C in the same group?					bf	h		f			k
27	Why are Cards E, M, and F a better match than Cards E, M, and C?						bfh					
28	Why are Cards F and D a group?						bfh	f	f			k
29	Do all points need to match up for two cards to be similar?						bf	f				f
30	Can one card match with two other cards?						bf		e			k
31	Why does Card F go with Cards D and E?						bhf	f	f			k
32	Why are Cards D and C a group?							bf	f			k
33	Why are Cards E, F, D, and C one large group?							cf	f			k
34	Can there be different slopes with the same points?							b				gfd
35	Why are Cards M, E, and A a group?							bfe	f			k
36	What is the equation for Card M?							bef				
37	Where do we see 50 cents in the equation y=-1/2x+6?							bf				
38	Why are Cards F and B a group?							bf	f		f	k
39	Why do all of the cards go together?								cfe	f	f	f
40	Why are Cards L and F a group?								bf		f	k
41	How can we use the transitive property with these cards?								bef	e		k
42	Why are Cards L, F, and B a group?								bef		f	k
43	Why do Cards J and H go together?								b		f	k
44	How do Cards N, G, I, H, and J connect with the other cards?									bf	f	k
45	Why do Cards G and N go together?									bf	f	k
46	Why do Cards G and I match?									cf	f	k
47	Why do Cards G, N, and I all belong together?									bf		k
48	Why does Card F belong with Cards G, I, and N?									bf		k
49	Why are Cards G and F a match?									bf		k
50	How can we show that all of the cards have a y-intercept of 6?										cfk	
51	Does Card B have a y-intercept of (0, 6)?										bfk	
52	How do you know what the y-intercept is if you can't see it?										bf	
53	Why are Cards E, B, and F a group?										bk	
54	What is the pattern that all of the cards have?											cfd
55	Do they all have the same slope?											cfgfd
56	What is the slope of each of the cards?											bf
57	What is slope?											bed
58	Do two lines have the same slope even if their fractions are different?											bfd

Fig. 3 The mathematical plot of Ms. Spruce's captivating lesson. Note that the letters in the cells refer to the mathematical plot codes in Table 1

story arcs close in Act 10, when the students became convinced that all cards represent the same function, resulting in a rapid decline of open questions.

Lastly, Ms. Spruce's captivating lesson contained several opportunities for misdirection in comparison to her non-captivating lesson. The first equivocation occurred when the teacher introduced the card sorting activity and instructed students to "make groups that are similar" from the cards. Though the teacher never explicitly misled the students, she allowed them to assume that there would be multiple groups. This expectation was later broken in a surprising manner when students realized that every card represented the same function. The second equivocation occurred when the teacher complicated the discussion of whether two cards represented the same function by questioning whether the

#	Question	1	2	3	4	5	6	7	8	9
1	What is the graph of y<-(2/3)x+4?	bf	fk							
2	Is (-2 -5) a solution to y<(-2/3)x+4?	bfe		fk						
3	Do you shade below the line when you graph y<(-2/3)x+4?	cf	k							
4	How do I use algebra to test if (-2,-5) is a solution to y<(-2/3)x+4?	ce		efk						
5	How do you decide where to shade for the solution to a linear inequality?	cf	cfe					f		
6	How do I know when a linear inequality will have a dashed or solid line?	cf	cfk							
7	Is the line dashed for y<(-2/3)x+4?	cf	fk							
8	Do I have to make table for y<(-2/3)x+4?		ck							
9	What do we call the type of line that is not solid in an inequality?		bfk							
10	Can you go about graphing y<(-2/3)x+4 in more than one way?		cefk							
11	How do you know if (-2,-5) is a solution to y<(-2/3)x+4?			bfk						
12	Is (-2,-5) in the shaded region of y<(-2/3)x+4?			bfk						
13	Does having a graph make testing solutions of inequalities easier?			bk						
14	How do you algebraically test whether (-2,-5) is a solution of y<(-2/3)+4?			bfk						
15	What is the right side of -5<(-2/3)(-2)+4 equal to?			bfk						
16	What are the different types of forms that linear inequalities could be in?				bfk					
17	How do you graph an equation in standard form, such as 2x+3y=12?				bfe					
18	What are we finding if we plug in zero for x and y in a linear inequality?				bfk					
19	How do you graph 2x+3y>12?					beef	f	f		
20	What are the x- and y-intercepts of 2x+3y>12?					bef	fk			
21	Does the sign (< vs ≤) have any impact on the inequality's graph?					ck				
22	If I plug in zero for x into 2x+3y>12, what would y need to be for it to equal 12?					bf	fk			
23	How did S10 get (0,4) and (6,0) as the intercepts for 2x+3y>12?						bfk			
24	How do you decide whether the line in 2x+3y>12 is solid or dashed?							bfk		
25	Do you shade above or below the line in 2x+3y>12?							bf		
26	What is a point that is a solution to 2x+3y>12?								bf	
27	How did S14 know (4,6) is a solution to 2x+3y>12 by looking at the graph?								bfk	
28	How do I algebraically prove that (4,6) is a solution to 2x+3y>12?								bfe	
29	What is a point that is not a solution to 2x+3y>12?								bfe	
30	Is (-2,-2) a solution to 2x+3y>12?								cf	
31	How do I graph 4x+5y≤20?									bf
32	What are the x- and y-intercepts of 4x+5y≤20?									bf
33	When graphing 4x+5y≤20, is the line solid or dashed?									bf
34	When graphing 4x+5y≤20, do you shade above or below the line?									bf
35	What is a point that is a solution of 4x+5y≤20?									bf

Fig. 4 The mathematical plot of Ms. Spruce's non-captivating lesson. Note that the letters in the cells refer to the mathematical plot codes in Table 1

cards could have the same slope: "I'm a little confused because I thought slope was change in *y* over change in *x* and some of these had different changes in *y* over changes in *x*'s." Though she did not tell the students that their conclusion of identical slopes was incorrect, she suggested that it might be. This equivocation raised a question about equivalence that offered students an opportunity to further consider the relationships between the slopes of the lines. In contrast, Ms. Spruce's non-captivating lesson had no instances of misdirection.

4.2 Narrative characteristics that distinguish captivating and non-captivating lessons

Across all 12 lessons,[8] many of the characteristics that were shown to distinguish Ms. Spruce's lessons similarly distinguish the entire group of captivating lessons from the entire group of non-captivating lessons. We start this section by comparing the narrative characteristics of both groups of lessons and describing how they are associated with students' perceptions of interest. We then compare how the opportunities for misdirection differ for the two groups of lessons.

[8] For the interested reader, the mathematical plot diagrams of all 12 lessons are provided in Online Resource 2.

4.2.1 Comparison of structural narrative characteristics between both groups of lessons

Many of the differences and similarities identified in Ms. Spruces' lessons were also found when comparing the captivating and non-captivating lessons for all six classes. The characteristics of the mathematical plots of both groups of lessons (i.e., six captivating and six non-captivating lessons) are presented in Table 3. Structurally, although captivating mathematical plots tend to have slightly more acts and formulated questions than non-captivating lessons, these differences are not statistically significant. However, there are notable differences in the story arcs: on average, a question raised in a captivating lesson tends to remain unanswered for more acts, and span more of the story, than a question in a non-captivating lesson. In addition, the longest story arc of each of the captivating lessons spans nearly the entire lesson (approximately 89%), whereas the longest story arc of a non-captivating lesson spans only 57% of the story on average. The captivating lessons also have fewer questions that are only open for one act. Approximately half of their story arcs extend multiple acts, as compared with only about a third of the story arcs in non-captivating lessons. However, neither the proportion of disclosed questions nor the proportion of coded acts within story arcs were significantly different.

The differences between the mathematical plots of captivating and non-captivating groups of lessons are also considerable when considering the characteristics of their acts. For example, as seen with Ms. Spruce's lessons, the average number of questions that a student is invited to consider per act, including both those questions that are asked in that act and those that were asked previously but remain under consideration, is higher for captivating lessons. This "thickening" of the plot within the captivating group of lessons is almost twice that of non-captivating lessons. Since this difference could be impacted by a large number of 1-act questions, it is important to also note that the average degree of inquiry, which does not include 1-act questions, shows an even greater difference;

Table 3 Narrative characteristics of the captivating and non-captivating groups of lessons

	Captivating group mean (SD)	Non-captivating group mean (SD)
Overall structure of the mathematical plot		
Number of acts	13.67 (4.37)	13.00 (2.90)
Number of formulated questions	52.17 (6.65)	46.83 (10.76)
Characteristics of the formulated questions		
Mean arc length (in acts)*	3.35 (0.56)	2.05 (0.83)
Mean arc length as proportion of story*	0.26 (0.05)	0.16 (0.03)
Max arc length as proportion of story*	0.89 (0.13)	0.57 (0.24)
Proportion of extended story arcs*	0.52 (0.09)	0.35 (0.09)
Percent of story arcs with at least one uncoded act*	0.34 (0.10)	0.16 (0.10)
Percent of formulated questions that were disclosed	0.70 (0.16)	0.69 (0.13)
Characteristics of the acts		
Mean number of questions open per act*	13.33 (3.46)	7.12 (1.51)
Mean degree of inquiry per act*	9.73 (2.92)	3.50 (1.62)
Percent of acts in story arcs with codes	0.85 (0.05)	0.92 (0.06)
Mean number of coded questions per act*	9.24 (2.63)	5.53 (1.28)

*Reflects a statistically significant difference (alpha < 0.05)

Table 4 Correlations of narrative characteristics with student interest for all 12 lessons

Independent variable	Intercept	Slope	R^2
Characteristics of the formulated questions			
Mean story arc length (in acts)	2.35	0.19	0.28*
Mean arc length as proportion of story	2.02	4.04	0.66**
Max arc length as proportion of story	2.35	0.68	0.26*
Proportion of extended story arcs	2.23	1.41	0.27*
Percent of story arcs with at least one uncoded act	2.48	1.45	0.34*
Characteristics of the acts			
Mean number of questions open per act	2.18	0.07	0.66**
Mean degree of inquiry per act	2.38	0.07	0.72**
Mean number of coded questions per act	2.22	0.09	0.50*

*Indicates a moderate association, according to Ferguson (2009)

**Indicates a strong association

the degree of inquiry of captivating lessons is almost three times that of non-captivating lessons. Of course, when a question is open during an act, the focus of the lesson is not necessarily related to the question. Therefore, it is also important to note that the average number of *coded* questions per act, which reflects how many interconnected questions are simultaneously addressed within an act, is also higher for captivating lessons when compared to non-captivating lessons.

To learn whether the identified characteristics help to explain the improved student experiences in the group of captivating lessons, we also studied the strength of the relationships between those plot measures and the measures of student lesson interest for all 12 lessons (see Table 4). One characteristic of the formulated questions had a strong association with student interest: *mean arc length as a proportion of story* ($R^2 = 0.66$). Moreover, two of the act-related characteristics (*mean number of questions open per act*) ($R^2 = 0.66$), the *mean degree of inquiry per act* ($R^2 = 0.72$)) were also strongly associated with the students' levels of interest in a lesson. However, the other characteristics of the mathematical plots only moderately explained student aesthetic reports.

4.2.2 Contrasts in misdirection

Overall, the captivating lessons from all six classes provided more opportunities for enhanced aesthetic student experiences in a lesson through misdirection (i.e., snare, equivocation, and jamming) when compared to the non-captivating lessons. The frequencies of each special code for the 12 lessons are reported in Table 5. Collectively, instances of misdirection were found more frequently in captivating lessons (28) than in non-captivating lessons (5). This was also true individually for five of the six teachers. Only one teacher, Mr. Palm, did not have an increase. The use of misdirection in the other five teachers' non-captivating lessons was very low (0 or 1 instance). All three types of misdirection occurred more frequently in captivating lessons than in non-captivating lessons, with some types of misdirection being particularly common in captivating lessons (e.g., both equivocation and jamming appeared in all but one of the captivating lessons). In contrast, snares only occurred in half of the captivating lessons.

Table 5 Instances of misdirection by teachers in captivating and non-captivating lessons

Teacher	Lesson type	Equivocation	Snare	Jamming	Combined	Delta
Mr. Ash	Captivating	3	1	1	5	+4
	Non-captivating	0	1	0	1	
Ms. Cherry	Captivating	6	3	2	11	+11
	Non-captivating	0	0	0	0	
Ms. Elm	Captivating	1	1	3	5	+4
	Non-captivating	1	0	0	1	
Mr. Palm	Captivating	3	0	0	3	0
	Non-captivating	1	0	2	3	
Ms. Spruce	Captivating	2	0	1	3	+3
	Non-captivating	0	0	0	0	
Ms. Willow	Captivating	0	0	1	1	+1
	Non-captivating	0	0	0	0	
Combined	Captivating	15	5	8	28	+23
	Non-captivating	2	1	2	5	

5 Discussion

This study offers clear evidence that the lessons that held broad appeal for students had mathematical plots with distinctive narrative characteristics, namely, their mathematical questions stayed open for more acts, spanned more of the story, and offered incremental progress periodically throughout the lesson. Furthermore, as a group, captivating lessons offered a far greater number of instances of misdirection (i.e., snares, equivocations, and jammings). In contrast to lessons with low aesthetic value, the captivating lessons offered a dramatic rise in how many questions were open simultaneously, reflecting a thickening of the plot. These differences indicate that the increased aesthetic value of the captivating lessons was not merely because of the conditions of the study, since teachers used similar classroom practices (i.e., warm-ups, group work, discussion) in both groups of lessons, whereas the structure of the mathematical plots of the two groups of lessons were markedly different. Instead, these findings suggest that these narrative characteristics could help address current curricular challenges by informing the design of high school mathematics lessons that motivate students to maintain engagement throughout complexity.

However, we caution educators from applying the lessons learned in this study without considering their teaching context. These lessons were designed and taught by experienced teachers who received professional development and had extensive time to collaborate. In addition, since student experiences reflect multiple factors, including their prior experiences and their individual and collective personalities, those who wish to impact the experiences of other students would need to consider those particular students and their learning context. Therefore, it is unlikely that these captivating lessons would have the same aesthetic effects in other classrooms or with other teachers. For example, although our findings show that misdirection was much more common in the captivating lessons, we do not recommend that teachers should constantly use misdirection to make their lessons more interesting. With different students and different classroom cultures, these instances of misdirection might not have improved students' experiences or may have even had detrimental effects (e.g., causing a student to feel foolish). What our findings do suggest is that misdirection *can* be used in ways that can captivate students. That is, when used appropriately, instances of misdirection can temporarily increase tension, which enables relief or surprise later when the truth is eventually revealed.

Some may suspect that Ms. Spruce's lessons were different due to the difference in the mathematical topics, assuming, for example, that all lessons on graphing linear inequalities may be inherently boring. However, the MCLEs reflected in this study often focused on content that is not typically thought of as inherently interesting for students. As shown in Table 2, the high-captivation lessons include abstract and decontextualized topics such as the rational root theorem and logarithmic identities. In contrast, the low-captivation lessons include topics that can be viewed as more potentially interesting for students, because of connections to students' lived experiences (e.g., word problems involving percent change). The data presented in this paper and in the Online Resources demonstrate that it is possible to design lessons about topics typically thought of as dull or uninspiring.

Yet we are not suggesting that the mathematical content does not matter. In fact, this current study is significant because it shows that a student's experience depends not only on *what* a story is about (i.e., its mathematical content) but also on *how* it is told (i.e., the ways tension between what is known and unknown dynamically shifts across the lesson). We suspect that when teaching decontextualized or abstract topics, many teachers may resort to enacting stories that involve direct instruction, which can reveal information before students have much opportunity to become curious. Our study demonstrates that an alternative approach is possible.

Like a rich literary story that offers a reader a new world to explore, a captivating mathematical story provides increased opportunities to ask and pursue questions that interest the student. Thus, one possible reason the captivating lessons have such distinct structures is the nature of the questions raised during these lessons (e.g., allowing for multiple approaches or solutions). Although there was no explicit focus on the quality of teacher questions in the planning of the captivating group of lessons, we acknowledge that the quality of questions found between these two groups of lessons are different. While the qualitative difference between the mathematical questions within these lessons was outside the scope of this paper, our analysis elsewhere suggests that both teachers and students in the captivating lessons asked questions that were more related to exploration and less related to known facts or procedures (Singh et al., 2021). Of course, although the captivating lessons offered an increased number of questions to consider, we do not assume that every student actively wondered about every open question. Rather, we suspect that dense mathematical plots increase the likelihood that a given student will find some question interesting to ponder. Further research is needed to understand the relationship between mathematical questions and student aesthetic experiences.

The current study, therefore, offers new insight into the changes that may have enabled Ms. Spruce's non-captivating lesson to hold similar broad appeal. For example, mathematics teacher educators would likely recommend that Ms. Spruce pose open-ended questions and press for reasoning. However, our study further demonstrates that the point at which these questions appear in relation to the unfolding mathematical content of the lesson is also important; just adding open-ended questions is not enough to offer new types of aesthetic experiences for students. If students cannot yet answer these questions, these can open story arcs that persist throughout the lesson, creating opportunities for students to become curious and wonder about the mathematical content. On the other hand, if the questions appear too obvious, or if they are raised at a point when other parts of the story allow students to quickly dismiss them (e.g., when the question seems impossible to answer), then the questions would likely not create opportunities for curiosity. As a field, we have encouraged teachers to ask open-ended questions without attending to how and when these questions are asked; our research suggests that because of this, teachers may not be leveraging the full potential of these questions. Further research could explore how redesigning specific lessons that students describe as non-captivating by attending to their narrative characteristics can improve aesthetic opportunities for students.

Furthermore, the design of this study has important considerations for the implications of our findings. For example, we do not suggest that designing lessons as mathematical stories is the only way to create lessons that are interesting to students. Since all the captivating lessons we analyzed were MCLEs, we also cannot predict whether other captivating lessons that are *not* designed as mathematical stories would have similar characteristics, such as longer story arcs or increased question density. Other studies have analyzed the mathematical plots of lessons that were not designed as MCLEs that had evident positive student aesthetic reactions (e.g., Richman et al., 2019). Further research would need to explore whether lessons such as these also contain the same narrative characteristics.

We also acknowledge the likely role of the teachers' knowledge and intentions in this study. Since the teachers knew about mathematical stories and the Barthes framework, they may have intentionally withheld information from the students or incorporated misdirection within their MCLEs. Yet we argue that this does not weaken our results. Students showed through their aesthetic reports a preference for lessons with these characteristics. That teachers can intentionally shift the way content unfolds is promising; it gives us hope that the poor aesthetic qualities of typical mathematics lessons experienced by high school students can be improved. Teachers' knowledge of the framework likely impacted their everyday lessons as well; yet the difference in aesthetic value of the selected lessons along with their differences in mathematical structure point to the impact of how content unfolds during a lesson, rather than the knowledge of the teachers. More research could explore whether these narrative characteristics also distinguish high school lessons with broad student appeal that are taught by teachers who do not know about the mathematical story framework.

For a long time, too many students have been given little reason to think of mathematics as intriguing or thought-provoking. In our view, this is inexcusable. Even within the confines of teachers' given curricula and school contexts, we were able to observe students enjoying an expanded set of aesthetic experiences, such as the suspenseful climax of Ms. Spruce's lesson. Secondary mathematics classrooms *can* be sites of delightful claims ("Yo! N and I go together!"), surprise ("Woah!"), and excitement ("That's crazy!") for students. We argue that, much like how audiences enjoy suspenseful movies that captivate until the concluding scenes, students deserve lessons that provide them with opportunities for wonderment and surprise as they engage with mathematics. If more lessons were designed with their aesthetic affordances in mind, we may prevent students from building a lasting impression of mathematics as boring.

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Data availability All coded mathematical plots are available in the Online Resources 2. Access to de-identified transcripts can be provided on reasonable request.

Declarations

Conflict of interest The authors declare no competing interests.

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