



Teachers' learning from addressing the challenges of online teaching in a time of pandemic: a case in Shanghai

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Abstract

Due to the COVID-19 pandemic in Shanghai, China, all school classes were delivered through an online environment from February 24 to May 22, 2020. To support this transition, the Shanghai Education Commission led expert teachers and specialists to develop a series of online video lessons based on the Shanghai unified curriculum, and suggested students watch the online video lessons individually from home, followed by an online synchronous lesson supported by class teachers. This study investigated what primary mathematics teachers learned from addressing these challenges through a case study. By following two purposefully selected teachers over 2 weeks during the transition, multiple data sets including online video lessons, online synchronous lessons, daily reflections, and post-online teacher interviews were collected. A fine-grained analysis of the data from the lens of the documentational approach to didactics found that teachers adaptively used online video lessons as important resources for their online synchronous lessons and virtual Teaching Research Groups as a teachers' collaboration mechanism supported them to develop online video lessons and address various technological constraints. Finally, implications of this case study for mathematics education globally are discussed.

Keywords Teacher collaboration · Online teaching · Teaching research group · Teacher professional development · Teachers' resource system

1 Introduction

Since early 2020, the COVID-19 pandemic has forced physical closure of schools and migrated the physical learning environment to online platforms. Responses to this emergency varied around the world. For example, the Chinese government launched the 'School Out but Class On' campaign (i.e., suspending classes without stopping learning) to provide platforms and models for online learning at home, while local educational practitioners

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adapted their own models and support systems based on local need for and availability of online teaching (Yao et al., 2020). Yet, the pandemic is an unprecedented context in which millions of students have needed to adapt to a new medium for learning mathematics, while teachers have had to provide teaching using online resources without any preparation (Hodges et al., 2020). This unique situation provided valuable insight into teachers' learning in response to the changing condition. Researchers have called for curating what 'people have learned or are going to learn about the current situation, for the benefit of future crises and times of stability' (Bakker & Wagner, 2020, p. 4).

To support the shift to online teaching in Shanghai, China, the Shanghai Education Commission led a team of expert teachers, who have rich school-teaching experience and who have been honored by local District Education Bureaus, to develop a series of online video lessons based on the Shanghai unified curriculum. The process of developing online video lessons was supported by the teaching research system (Yang, 2009) featuring Teaching Research Groups (教研组) at the school level for teachers' collaborative work, such as implementing curricula, mentoring teachers' research activities, organizing public lessons, and designing standardized examinations (Huang, Lai, et al., 2021), and by Teaching Research Offices at district and municipal levels (Mourshed et al., 2010, p. 89–90). The online classes consisted of students watching the online video lessons individually and then participating in an online synchronous lesson. In Shanghai, unified textbooks with associated student books and teacher guidebooks are the primary materials used for supporting teachers' teaching (Li & Huang, 2018). In the pandemic period, the online video lessons were suddenly added into the teacher's resource system. In such a context, it was unknown how the teachers would move from face-to-face to online teaching rapidly without any preparation.

Several studies have identified the difficulties and challenges in online teaching and learning in response to the emergency of the pandemic (Aldon et al., 2021; Cao et al., 2021; Chirinda et al., 2021; Clark-Wilson et al., 2020; Engelbrecht et al., 2020; Kalogeropoulos et al., 2021; Mailizar et al., 2020; Yilmaz et al., 2021). Teachers encounter challenges in four areas: technology, students, mathematics, and resources. These are (1) unfamiliarity with online teaching platform functions which are needed for online synchronous teaching, and lack of access to Internet and computers in developing countries (Aldon et al., 2021; Cao et al., 2021; Chirinda et al., 2021; Clark-Wilson et al., 2020; Engelbrecht et al., 2020); (2) difficulties in communication with students online (Kalogeropoulos et al., 2021; Mailizar et al., 2020); (3) constraints in carrying out manipulative activity and grouping activity online (Aldon et al., 2021); and (4) lack of time and support for identifying or designing appropriate electronic resources for students' learning from home (Chirinda et al., 2021). Although in some countries, governments provided unified and systematic online support resources such as video lessons, television programs, and other resources for teachers and students to use, it is hard for teachers to adapt them in online synchronous instruction to meet various students' learning needs (e.g., Yilmaz et al., 2021). These studies presented benefits, strategies, and challenges of online mathematics teaching in the pandemic. Yet, the teaching of mathematics online in a hybrid mode, both synchronously and asynchronously, in large scale is still underexplored (Di Pietro et al., 2020; Ferdig et al., 2020). Therefore, it is urgent for policymakers, school administrators, and teachers to work together systematically to ensure that all students have access to high-quality mathematics during the pandemic.

Building on previous relevant research in mathematics education which explored the potentials and challenges of online mathematics teaching, this research aimed to develop a deep understanding of what teachers struggled with and how teachers addressed various

challenges during online teaching. Thus, it may provide insight into post-COVID mathematics teaching in an emerging hybrid learning environment in general, and how teachers learn to teach online through interacting with multiple resources and collaborating with colleagues within a virtual teaching and professional learning environment in particular.

2 Theoretical considerations

In this section, we first present the theoretical framework grounding our research, then state our research questions.

2.1 The documentational approach to didactics

This paper is centered on studying teachers' learning through their interactions with resources, to address critical issues—in this case the pandemic situation. The interest in research related to teachers' resources appears clearly at the beginning of this century, resources being understood as what is provided by the curriculum (Remillard, 2005) and, more generally, what has the potential of 're-sourcing' teacher activity (Adler, 2000) at the core of instruction (Cohen et al., 2003). This interest in resources has been strengthened with an evolution of the field of technology in mathematics education, 'rethinking the terrain' (Hoyles & Lagrange, 2010), towards a view of resources beyond the technology itself. In particular, Ruthven (2009) developed a structuring features of classroom practice framework, which includes working environment, resource system, activity format, curriculum script, and time economy. The concept of resource system focuses 'on the combined operation of the mathematical tools and curriculum materials in classroom use, particularly on their compatibility and coherence of use, and on factors influencing this' (Ruthven, 2009, p. 136). Ruthven suggested that the resource system consists of aspects of the artifacts in uses that are directly and deliberately related to the teaching and learning of the specific topic or subject.

To capture the features of mathematics teacher professional development through the lens of their interactions with resources, a specific conceptual framework, documentational approach to didactics (DAD), has been developed over the past decade (Gueudet & Trouche, 2009; Trouche, Gueudet, et al., 2020). This frame extends the notion of resource given by Ruthven (2009), encompassing wider aspects of the artifacts that a teacher could use such as a textbook, a piece of software, a student's worksheet, an Internet resource, or a discussion with a colleague.

DAD calls the teacher's work with resources for achieving a given teaching goal his/her *documentation work* and identifies the result of this work as *document*: a document developed by a teacher is a hybrid entity made of resources resulting from the productive dimension of their activity and of the knowledge resulting from its constructive dimension. Knowledge is understood here in a broad way, following Gueudet and Trouche's (2009) proposition: "In his conceptualization of teachers' knowledge, Shulman (1986) distinguishes between subject matter content knowledge, pedagogical content knowledge, and curricular knowledge ... In our study, we consider teachers' knowledge as a whole, without sorting out its different kinds" (p. 201).

Three main dialectical relationships underpin the DAD: Concerning the teacher and the resources, specifically the relationship between two processes: the teacher permanently adapts the resources they are working with (*instrumentalization* process),

while these resources and their constraints and affordances contribute to configuring the teacher's activity (*instrumentation* process). These two processes are indeed inter-related: the adaptation of resources (instrumentalization) conditions their integration into the practice of teachers (instrumentation);

Concerning the teacher's activity, specifically the relationship between two dimensions: There is a *productive* dimension (the teacher produces new resources for achieving a given teaching goal) and a *constructive* dimension (the teacher develops new knowledge in searching for, adapting, and implementing resources). These two dimensions are indeed interrelated: the production of new resources leads to the development of new knowledge, and this new knowledge influences the production of resources.

Concerning the *individual* and *collective* dimensions of teachers' interactions with resources, which reciprocally impact each other: Gueudet and Trouche (2012) evidence how the development of teachers' communities and their resources goes hand in hand. From the lens of DAD, studies have shown that teachers' collaboration has an important role in their use and design of resources (e.g., Gueudet et al., 2013; Pepin et al., 2016). The study by Trouche, Rocha, et al. (2020) provided a window into the use, design, and transformation of resources by mathematics teachers in their teaching, which can help us better understand teachers' professional learning and the way collaborative contexts shape their professional work and learning. Through examining three Shanghai teachers' resources connected to their practice, their perceptions of mathematics teaching expertise, and their ways of developing their expertise, Pepin et al. (2016) found that the context of collective working enhanced mathematics teachers' expertise and capacity building that forms a strong workforce for promoting pupil learning. This finding supports that localized collaborative design efforts have an important role in generating resources appropriate for specific school settings and enhancing teachers' capability in effective use of these resources (Ruthven, 2013). However, Gueudet et al. (2013) found although teachers' routine work includes many collaborative aspects and teachers' collaboration surrounding resources is crucial for teacher professional development, productive communities of practice, which include a shared interesting domain, pursuit of their common interest, and use of in their repertoire of resources (Wenger, 1998), are scarce. Pepin et al. (2013) argued that how teachers work for adopting or adapting resources in collective contexts, and how to support teachers' collective work with resources for/in practice, is needed for further investigation.

2.2 Research questions

In this study, due to the importance of both traditional resources (e.g., textbook, student exercise book, and teacher guidebook) and the newly added ones (e.g., online lesson videos and online materials, and platforms) and due to the usefulness of the DAD utilized to examine teachers' work with resources, we adopted DAD as the theoretical framework for addressing our research questions. Specifically, this study aims to investigate the following research questions (RQ):

RQ1: What are the strengths and challenges of teachers' documentation work on online video lessons and platforms?

RQ2: To what extent do teachers' collaborations support their documentation work?

3 Methodology

In this section, we first present our methodological principle, then describe the criteria of selecting participating teachers and rationale of focused content, and further explain data collection and analysis strategies.

3.1 A reflective investigation methodology

The study is based on the reflective investigation methodology proposed by DAD (Trouche, Gueudet, et al., 2020). This methodology is based on stimulating teachers' reflectivity on their interactions with resources: teachers are asked to show and describe their resources, their structure (related to their activity), their implementation, and their evolution.

To address RQ1, we focus on the appropriation of online video lessons and platforms, analyzing how they affect teachers' documentation work, in terms of instrumentation and instrumentalization;

To address RQ2, we focus on the teachers' interactions in the Teaching Research Group and how these interactions support their documentation work.

3.2 Participants

Considering multiple factors impacting teachers' online teaching, the following three criteria were taken into account: (1) teachers' teaching experience at school (experienced or novice); (2) teachers' responsibilities and roles in the process of designing online video lessons (Yes or No); (3) teachers' experiences in the Teaching Research Group during the online teaching period. Prior to the pandemic, Teaching Research Groups typically held activities once weekly in schools. However, during the pandemic different practices in terms of frequency of activities held emerged which include three levels: remain (once a week as usual but online), strengthen (once a week online and frequent interactions on social media), and weaken (once online or none during the 2 weeks). Therefore, twelve mathematics teachers representing different situations were purposefully selected from different schools. The five teachers from a suburban district were chosen by the director of the district Teaching Research Office, and the other seven teachers from urban districts were chosen by the first author, who is a mathematics educator at a local university and has extensive collaboration with mathematics teachers in schools due to leading a longitudinal UK-China mathematics teacher exchange program since 2014 (Huang, Huang, et al., 2021).

To address our research questions, we decided to focus on two contrasting cases to represent the wide spectrum of the data (Table 1). Zi, a designer and user of the online video

Table 1 The two teachers' profiles

Name	Teaching experience	Professional position	Position in Teaching Research Group	Designer for online video lessons	Collaboration in Teaching Research Groups
Zi	23	Expert teacher	Lead	Yes	Weaken
Wei	4	Novice	No	No	Strengthen

lessons in this study, is the most experienced teacher in the teacher group. Wei, a novice teacher, generally needed more instructional support from colleagues or resources. Therefore, Zi and Wei were considered as two extreme cases among the twelve cases.

3.3 Teaching content

According to the Shanghai mathematics textbooks (Huang, 2019), the online video lessons were organized, regarding decimal content, in four parts: (1) the meanings of decimals (13 online video lessons), (2) the properties of decimals (5 online video lessons), (3) movement of the decimal point (6 online video lessons), and (4) the addition and subtraction of decimals (9 online video lessons). The online classes included two parts (20 min for each): students watched the online video lessons individually and then participated in an online synchronous lesson. The eleven lessons (numbered from L1 to L11) were chosen from the properties of decimals, and movement of the decimal point, involving some difficult topics, e.g., proportional reasoning and connections with fractions (Huang, 2006; Lamon, 2007), which are big challenges for student learning (Tempier, 2016; Takker & Subramaniam, 2019; Deliyianni et al., 2016). The Shanghai Mathematics Curriculum Standards specify the learning goals of this content, based on visual representations and practical measuring: intuitively understanding of one to three decimal numbers (0.1...0.9; 0.001 ...0.999), knowing the relationships between fractions and decimals, adding or removing 0 at the end of a decimal, and mastering the decimal size (Shanghai Education Commission, 2004).

3.4 Data collection

To address the research questions, we first collected three online video lessons (data named D0), corresponding to L5, L6, and L11, which were recommended by Zi as the designer. As Zi explained, L5 is a review lesson on the properties of decimals, providing exercises for consolidation, and L6 and L11 introduce new knowledge which is difficult for student learning (in L6 exploration of the change-pattern when moving the decimal point and in L11 using the *Wan* (万) unit (ten-thousand) or *Yi* (亿) unit (hundred-million) to express large numbers). During the 2 weeks of online teaching, we also collected the following data sets: (1) videos of online synchronous lessons during L5, L6, and L11 (D1); (2) teacher daily reflections upon their teaching in response to question prompts (D2); and (3) audio or written records of interactions in the Teaching Research Groups (D3). In addition, immediately after completion of the 2 weeks of online teaching, semi-structured interviews with the two teachers were conducted and audio recorded (D4).

3.5 Data analysis

Based on the research questions and preliminary examination of the five types of data collected for drawing our results which were triangulated by other data, we employed grounded theory (Corbin & Strauss, 2008) to generate codes from Zi's interview, including four categories: resources used in the episodes of online teaching, connection with the online lesson videos to improve student learning, tools of the platforms used to support the interactions in the online synchronous lessons, and teachers' interaction and collaboration in Teaching Research Groups. We used these codes to analyze the teacher's other types of data. When any data could not be encoded with the existing codes,

we modified or adjusted the codes for adapting to the data. By iterating back and forth in this way, an initial coding system was formed. Next, we used this coding system to analyze Wei's data. We provide the prompts for interview and reflection, and the coding system as electronic Supplementary Material A and B.

Data processing and analyzing included three steps:

- Transcription of data (both audios and videos). The three selected online video lessons were transcribed in Chinese verbatim. Then, English subtitles were added to the online video lessons for the third author's review. The teachers' videos of online synchronous lessons corresponding to the three selected online video lessons were transcribed in Chinese and then translated into English. The teachers' audio records of interviews were transcribed in Chinese verbatim;
- Data coding and analysis. Regarding teaching materials, and student texts, the first two authors used the codes of textbook, student book, and teacher guidebook in the first category of coding system to identify connections among the three selected online video lessons (D0) independently. Inter-rater reliability was checked (around 90%), and the disagreements were resolved through discussions. Both the first and second authors read the transcription of Zi's four types of data (D1, D2, D3, and D4) carefully, and coded the data independently with the coding system (see Supplementary Material B). Inter-rater reliability was checked (around 80%), and the disagreements were discussed and resolved. After that, two research assistants were trained to code, and they collaboratively coded Wei's data (D1, D2, D3, and D4). The first author then checked the codes by the research assistants and resolved some disagreements through discussion;
- Categorization and illustration. Comparing the codes across teachers, the first two authors collaboratively identified patterns, and illustrated them using original data. After that, the findings were translated into English by the first author, checked by the second author, and then sent to the third author for review. Some questions raised by the third author were discussed and clarified through weekly online meetings.
- To answer RQ1, we analyzed the three selected online synchronous lessons (D1), teachers' diaries (D2), audio or written records of interactions in the Teaching Research Groups (D3), and teacher' interviews (D4), looking for the way they considered the online video lessons as new resources in the three online synchronous lessons in terms of instrumentation and instrumentalization.

To answer RQ2, we analyzed the records kept by the two teachers about their interactions in the Teaching Research Groups (D3) and the interviews with teachers (D4) to identify how teacher collaboration in the Teacher Research Group supported their design or use of the online video lessons and online synchronous lesson implementation.

4 Results

We described each case in terms of what documentation works (including strengths and weaknesses) and how the Teaching Research Group may support the teacher's documentation works. At the same time, the three dualities of dialectic relationship are the

core complements to highlight. We present the case of Zi first and then the case of Wei, followed by a summary of the two cases.

4.1 The case of Zi as an expert teacher

The case is described in two subsections with respect to our two research questions.

4.1.1 Documentation work for interacting with platforms and the online video lessons as a designer

The dialectical relationship between instrumentalization and instrumentation is featured by the ways of making use of students' mistakes in homework and content from online video lessons as teaching resources in her online synchronous lessons. In her online teaching [D1], Zi connected students' mistakes that occurred in homework with the online video lessons. In L5, she taught the contents summarized in online video lesson 5 and elicited the students to share what they had learned. She then had an interactive discussion of the most difficult task in the homework assigned in online video lesson 5. In L6, she employed the place value system to discuss the question, which had been explained with fractions in online video lesson 6, but it was hard for students to understand why multiply (or divide) by 10, then the decimal point moves one place to the right (or left). In L11, she quickly reviewed the key points of online video lesson 11 and asked students to summarize the procedure for rewriting numbers in the unit of *wan* (i.e., a ten-thousands) or *yi* (a hundred-millions) with an accurate language: to express a number in the unit of *wan* (*yi*), one should first find the digit in the *wan* (*yi*) place and then place a decimal point in the lower right corner of it.

She recorded related events explicitly in almost all her daily reflections. For example, in the L5 daily reflection, she wrote:

Zi: Today, I reviewed the students' mistakes, which were found through watching the video and marking students' homework. [D2]

In the L11 daily reflection, she wrote:

Zi: I asked students to explain what they have learned in the video, and then assigned related tasks to elicit the students' thoughts about their errors in their homework. [D2]

The fact that Zi is so comfortable linking online video lessons to her online synchronous lessons is a result of her involvement in designing and producing these videos. Zi's experience in producing the resource (e.g., online video lessons) contributes to her constructing knowledge about using the resource such as using students' mistakes in homework and appropriately addressing the difficult contents in synchronous online teaching. In the interview, she talked about how she knew the objectives of the video lessons and was familiar with the content, so she focused on student difficulties and online interactions in her synchronous online lessons.

Zi: For the content which are not introduced well in the videos, I will make it clear in the online lessons. For the contents that are clear in the video, I will focus on students' feedback on the exercises and provide specific instructions for students. [D4]

The dialectical relationship between instrumentalization and instrumentation can be also revealed by the ways of using the platform in her online teaching. From the three videos of online synchronous lessons [D1], we also find Zi had many online interactions with the students, using the various functionalities of the platform, efficiently instrumenting her teaching. On the platform, she constantly used the name list to find students who made mistakes in their homework and asked them to answer questions. For example, she required students to use the hands-up icon to show when they completed their classroom exercises.

In using the functions of the platform, Zi found that there were some problems with the platform that prevented her from interacting with the students. This led her to try different strategies to instrumentalize the functions of the platform. For Zi, the first problem was that the teacher could not see her students, which prevented her from getting immediate feedback from students' body language about their learning.

Zi: In the previous face-to-face classroom, I could observe students' eyes, expressions, and behaviors to understand how they were learning. But now it's impossible. [D4]

She tried to get more feedback from what students did in their homework and provided the students who made mistakes with more opportunities to communicate. Due to the design of the platform, when students used the hands-up function, their names were rearranged in the list in the order in which the students pressed the icons. This made it difficult for the teacher to immediately find the target students displayed on the platform and remove their mute restrictions to engage with them in conversation. Zi, later in the interview [D4], told us that she had taken two approaches to solving this problem. The first was to improve the student-teacher relationship: before officially starting online synchronous lessons, she logged onto the platform and chatted with the students to warm them up, thus bringing them closer to the teacher. The second was to give the initiative to the students: she let the students turn on the microphone by themselves before starting online synchronous lessons; then when the teacher called on a student, she could talk directly with them.

4.1.2 Collaboration to support documentation work

Zi, as the leader of her Teaching Research Group (9 teachers), held only one online formal activity of the Teaching Research Group in the 2 weeks [D3], but she had a very intensive interaction with her Teaching Research Group during the process of designing the online video lessons. For each video lesson, she first designed a specific lesson plan independently, which she submitted to the specialists from the Teaching Research Office for review and subsequently revised in response to the changes suggested. This went back and forth until a final script was created. Zi, with her young colleagues at the school, designed the lesson slides based on the lesson plans and submitted them to the specialists for review and revision. Indeed, this constituted a collaborative design team, involving the Teaching Research Group at the school coordinated by Zi and the specialists from the Teaching Research Office. Zi mentioned that specialists provided her with advice for improving lesson plans or slides.

Zi: They have higher professional capacities and think more comprehensively than me. I designed the lesson plans based on my previous experience and personal understanding. But the specialists have rich experience and deep understanding of

the curriculum, so they provided some suggestions on the improvement of the lesson plans or slides. [D4]

Because she had no experience in creating slides for the online video lessons, she got support from her colleagues:

Zi: I invited a couple of young teachers from my Teaching Research Group to help with the lesson slides. Actually, I'm sure I couldn't do this alone. Some of the slides may involve more than a dozen animations. During this process, I need to design the teaching script, and then communicate with them what kind of visual effect I want to achieve. [D4]

Due to the complexity of the process of designing online video lessons in the lockdown phase, Zi was very stressed. However, colleagues' collaboration encouraged her:

Zi: A lot of tasks we had to get done in a short period of time, so we cheered each other on. Sometimes I think it's almost done, and then they [the specialists] tell me no, I have to start over. I was desperate. But when I think of my colleagues who are so motivated to do it with me, I have no reason to quit. I feel that through the completion of this task, we deepened our understanding of each other and felt like comrades. [D4]

The evidence above clearly shows the dialectic relationship between individual effort and collective collaboration with the specialists and her colleagues when producing the online video lessons. The teacher also claimed that she learned various knowledge from the specialists and her colleagues, including the understanding of the curriculum and the use of technology. At the same time, we argue that what the teacher learned would have a positive effect on her subsequent use and production of resources.

Based on the evidence Zi provided about the Teaching Research Group, they rarely used social media among colleagues to communicate about their work when they implemented synchronous online lessons. They did, however, hold one formal online Teaching Research Group event (about 56 min) in the 2 weeks through a platform. They shared how to use the platforms and online teaching practices in other schools. However, they did not share any experience of using online video lessons in their practice. Zi indicated that the biggest problem they encountered was technology, not related to content or instructional approach. So naturally, the issue of their Teaching Research Group focused on how to use the platform:

Zi: Online teaching is not something that teachers are familiar with at first. Even though we were trained how to use the platform before we started, when using it, a lot of problems occurred. For example, how to upload materials, how to show slides on the platform, and how to interact with students on the platform became the most pressing issues for everyone. [D4]

4.2 The case of Wei as a novice teacher

The case is described in two subsections, with respect to the two research questions.

4.2.1 Documentation work for interacting with platform and with the online video lessons as a user

Due to technology issues, Wei was not able to video record her L6. From L5 and L11, we found that the major resources Wei used in her lessons were provided by her Teacher Research Group or designed by herself. She did not directly use the online video lessons. Through analyzing the diary reflection on the 2 days, we found that Wei designed tasks for her online synchronous sessions after watching the online video lessons. For example, Wei found number lines were used in both the textbook and the video to help students understand “multiplying a decimal by 10, decimal point moving one place to the right”. But she felt that the number lines were too complicated, so she supplemented the use of the cube model, which she used to help with her students’ understanding of the base-10 system in whole numbers.

Wei: In the video, the teacher used a number line to represent the relationship between 0.1, 0.01, and 0.001. I used a cube model to represent 1. When divided equally into 10 parts to make planes, each plane would represent 0.1. Then dividing a plane equally into 10 parts to make bars, each bar would represent 0.01, and dividing a bar equally into 10 parts to make small cubes, each small cube would be 0.001. [D2]

Nonetheless, in the interview, Wei also described how the online video lessons had been one of the new resources she referred to for practice.

Wei: I will read the textbook first, then combine it with the exercise books to form initial slides based on the materials from my Teaching Research Group, and finally make adjustments and additions in conjunction with the online video lessons, possibly cutting or adding some content. [D4]

In the interview, Wei appreciated the quality of the online video lessons and believed they were quality resources for teachers to promote reflection on their own teaching.

Wei: The teachers in the video are able to highlight the key points of teaching, elicit and motivate students to think and focus on mathematical ideas and methods, which I believe are very worthwhile. Through the videos, I feel that my practice needs to be polished. Particularly, I actually don’t do well in the classroom discourse. I often record the teacher’s language in the online video lessons to learn by myself. [D4]

By viewing the online video lessons as a new teaching resource Wei was adapting and modifying her teaching practice to incorporate these materials, and the teachers’ approaches in the online video lessons also facilitated her reflection and stimulated her to improve practice. This process exemplifies the teacher’s instrumentation and instrumentalization by the online video lessons as a component of her resource system.

But Wei’s documentation work for the online video lessons was not only in a one-way direction. She reflected upon and changed her ways of using the online video lessons, which constitutes the alternative process of instrumentalizing the online video lessons. In her diaries, she offered different ideas and advice for specific lessons. For example, for online video lesson 3, she commented ‘when comparing decimals, we can arrange them vertically by aligning the decimal points to make the comparisons more intuitive’ [D2]. In online video lesson 6, a task ($0.1 \times 10 = 1$, $0.01 \times 10 = 0.1$, $0.001 \times 10 = 0.01$) was designed to generalize the pattern of the movement of the decimal point (multiplying by 10 results

in moving the decimal point one place toward the right). Wei advised to design a variation task where the same decimals were divided by 10 [D2]. In the interview, she also addressed a more general question about the design of online video lessons.

Wei: In the online video lessons, the dialogues are well designed by the teachers and are expressed in a standard and concise manner, but those may not be easy for students to understand. I sometimes have to listen to those several times to understand what they were saying. I would recommend that subtitles be added to this section and the dialogues be designed to be more verbally adapted to students. [D4]

When interacting with the online video lesson, the dialectical relationship between the instrumentalization and instrumentation further results in the dualities of productive and constructive activities. The teacher claimed that she learned a lot from watching and adopting the online video lessons (constructive activity). She used different methods from those presented in the online video lessons (productive activity) based on internet resources and self-analysis of the content in the online video lessons and self-reflection on synchronous online teaching.

In the two videos of the online synchronous lessons [D1], Wei used the supplementary exercises to carry out teaching. She presented tasks on slides. These tasks were multiple-choice questions, true–false questions, and fill-in-the-blank-questions. Then she required students to use the answer-card function to respond. Wei used the data-collection function which shows the distribution of all students' answers. Then she required students to use the answer-card function to respond. Thus, Wei knew the overall situation of students learning, and then provided corresponding feedback and explanation. In the interview she talked about why she used such a teaching method, mainly because the network was slow and there was a time delay if students and teachers talked directly. Additionally, she felt the same way as Zi about the hand-up function of the platform.

Wei: After the students raise their hands, the platform only shows the icon of 6 children raising their hands. Because of the speed issue, it often shows the same students, but slower children are not shown. [D2]

Wei felt that the platform limited her practice. When she used the slide-presentation function for online teaching, it was difficult to adjust some contents to student learning. When she used the shared interface function, it occupied part of the original interface and affected online interaction. Comparing these two methods, Wei gradually preferred to use the sharing screen to interact with her students.

Wei: If I use the slide-presentation function, I have to follow the content designed. It is difficult for me to present different ways that students come up with in the class. If I use the sharing screen, I can write on the pad, which is a little better. [D4]

In Wei's process of using the platform's functions, we can see the instrumentalization and instrumentation processes regarding the platform. That is, she would first try to use a function of the platform, and when she found that it was not conducive to her online interaction with students, she would choose another function to improve that.

4.2.2 Collaboration to support documentation work

Wei's Teaching Research Group, which included four novice and one experienced teacher, had an online meeting almost every day to discuss technical and didactical issues [D3], as

well as to appease teachers' emotions. Wei indicated that the online Teaching Research Group meeting was not restricted by time/venue and made her to be 'brave behind the screen'; 'I have strengthened the confidence in teaching through communication and sharing'. However, 'we did not have enough time for a deep discussion' [D4].

Wei's Teaching Research Group rarely discussed how to integrate the online video lessons with their online teaching [D3]. Both in her interview and diary, Wei detailed a problem she encountered with online video lesson 8.

Wei: I was impressed with the lesson on the use of moving decimal points to perform conversions between different units. Because in this lesson, the teacher in the video wrote out the solution process in detail for the previous tasks but gave the direct answer to the final exercises. I think there is a gap. Some students may not know how to make the transition. [D4]

She said that in the Teaching Research Group, they also discussed this issue, but there was no consensus, and each teacher used a different approach.

Wei: My colleagues in the Teaching Research Group employed different approaches. Some colleagues felt it was completely fine, and the solution in the complete process was not emphasized in their lesson. Some colleagues required students to write the complete process, such as multiplying or dividing decimals by 10 or 100 to gain the outcome, while I focused on moving the decimal point on the original decimal to get an outcome. [D4]

Wei found that her approach caused many errors or mistakes in the students' homework. In converting units, some students neither understood whether to multiply or divide, nor knew what to multiply or divide by [D2]. Wei needed more support from her colleagues. In Wei's case, although more individual effort was made, more strong collective collaboration is needed.

4.3 Summary addressing the two research questions

By comparing the two extreme cases with a focus on three dialectical relationships in the DAD, we were able to make the following observations in line with research questions: (1) The videos and platform as resources played a developmental role for the documentation work of both expert and novice teachers. The expert teacher developed technological skills through her online teaching experience, while she had a deep understanding of the videos because of her team's great effort in creating the videos. The novice teacher seemed to have fewer issues with the use of technology, but she developed her understanding of the content through watching and adapting the online videos which became an important source for her learning. (2) Teacher collaboration provided important support when the expert teacher designed and created the online video lessons. The novice teacher used social media to maintain rich interaction with her colleagues, for technological, didactical, and affective reasons in the lockdown period. However, she was not able to receive more support from her colleagues for adapting the online videos and supporting her synchronous online teaching.

These conclusions concerning *one* expert teacher and *one* novice teacher based on a contrasting case study may not be generalized to other situations. Yet, our observations over the whole sample of twelve teachers seem to confirm such patterns concerning the role of instructional expertise, involvement in development of online video lessons, and capacity

in using technology in such complex periods where teachers have to face the uncertainty of a new mode of teaching requiring an immediate integration of new resources.

5 Discussion

This study presented various ways to address the challenges in online teaching as literature indicated (Aldon et al., 2021; Cao et al., 2021; Chirinda et al., 2021; Kalogeropoulos et al., 2021; Mailizar et al., 2020; Yilmaz et al., 2021). Specifically in this study, we found the following: (1) Limited 'face-to-face' communication online is still a pressing technological constraint. However, the teachers were able to use their own strategies to deal with receiving students' immediate feedback. In Zi's case, she addressed students' mistakes or errors from targeted students' homework, which was marked carefully before online synchronous lessons, while Wei used technology functions to obtain students' responses immediately. (2) Learning to use unfamiliar technology was not a big challenge in this case. For example, Zi was originally afraid of technology, but through collaboration with colleagues and her attempts at, and reflection on, online teaching, she developed technology capacity. (3) The online video lessons, as resources for students to learn from home, were integrated into the online synchronous lessons by teachers in two dialectical ways (instrumentation and instrumentalization) from the DAD perspective, to promote students' learning. (4) Teachers were encouraged to participate in the development of the resources so that they could adapt them for their students. Specially, regarding the online video lessons as resources for teachers, our study revealed that: for the novice teacher, the instrumentation process was salient, which helped her to develop deep understanding of the content and pedagogy; and for the experienced teacher, the instrumentalization processes involved using online video lessons to connect with students' difficulties. However, exploration of how to develop video lessons that can promote student engagement and interaction is needed in the future. Our study also evidences the critical role of the Teaching Research Group for addressing the challenge of online teaching. The Teaching Research Group system as a well-established infrastructure of teacher collaboration had already been analyzed as a powerful means for enhancing teacher documentation work (Pepin et al., 2016). During the pandemic, the usual form of the Teaching Research Group was adapted by teachers through technology (e.g., in using social media, or a meeting platform) for addressing the challenges of online teaching. Virtual Teaching Research Groups played different roles in teachers' documentation work. In the development of online video lessons, through the events in virtual Teaching Research Groups, specialists, experienced teachers, and novice teachers were closely connected to play different roles and develop the resources through division of labor and collaboration. The Teaching Research Group looks like a project team (Bond-Barnard et al., 2018), where the interdependent individuals work together to meet their goals and share responsibility for specific outcomes in an organized context by the Teaching Research Office in Shanghai. When teachers used online video lessons and implemented their own online synchronous lessons, the virtual Teaching Research Groups opened new opportunities for the novice teacher (e.g., making it easier for her to 'be brave' in asking sometimes naïve questions) while providing new challenges for experienced teachers (e.g., addressing diverse and complex questions from novice teachers and supporting them). Due to the complexity and novelty of adapting to teaching online during this pandemic period, the collaborative work of Teaching Research Groups was focused on figuring out how to effectively use the platform, rather than discussing the content of

online video lessons and the pedagogical issues in teaching, which novice teachers need as supports from their Teaching Research Groups. The Teaching Research Group in Wei's case did not form a true community of practice (Gueudet et al., 2013), because without opening a dialogue between the inside and outside perspectives (Wenger et al., 2002), the novice teachers' interests were not shared and pursued by the community. Unfortunately, the activities in the Teaching Research Group in Zi's case also weakened during the phase of implementing synchronous online lessons. Thus, during the pandemic, when all the contexts suddenly changed, teacher collaboration in virtual Teaching Research Groups, without strong organization and coordination, faced a huge challenge.

Actually, communities of practice are emphasized by DAD, when teachers interact with resources through their collaborative work (Trouche, Gueudet, et al., 2020). According to the principles for cultivating a community of practice (Wenger et al., 2002), to strengthen teachers' collaboration to help teachers address unexpected challenges, the following two aspects need to be strategically addressed: first, participants with different roles are encouraged to work together and have open dialogues between insider and outside perspectives. In this case as teachers implement online synchronous lessons, experienced and novice teachers in a school-based virtual Teaching Research Group can mutually support each other if they share the difficulties and successful experience using the platform technology and online video lessons. If expert teachers and specialists who design online video lessons can share their experiences of designing online video lessons in cross-school virtual Teaching Research Groups and engage in dialogue between designers and users, then the difficulties experienced by teachers like Wei can be resolved. Second, all teachers should be aware of the value of teachers' collaboration for helping them address new challenges. During the pandemic, although the traditional face-to-face events disappeared in the virtual Teaching Research Group community, the established traditional Teaching Research Group community values and norms are still important. When teachers developed resources, the virtual Teaching Research Group played a great role in linking the different level participants to work together. While the novice teacher used the teaching resources from the Teaching Research Group for her synchronous online practice, she indeed wanted more support from the virtual Teaching Research Group. In order to establish a community of practice, teachers' collaboration needs to create dialogue, activities, and relationships that share and address the diverse issues and problems emerging in the community.

Additionally, this study of mathematics teachers in Shanghai features two aspects that are specific to mathematics teachers. First, the Shanghai primary mathematics teachers are specialists who mainly teach mathematics. So, they may have a profound understanding of mathematics knowledge (Ma, 1999). Second, the content of decimals requires the use of multiple, visual and dynamic representations to support students' understanding and the use of multiple approaches to solve a problem. These features may result in mathematics teachers needing more resources, as well as being adept at using them. Thus, the findings of this study could provide specific implications for mathematics teacher professional development in the post-COVID era.

Due to the difficulty following up on the quick changes in teaching modes during the period, we probably missed important data (e.g., following the online video lessons' design process itself). In addition, our analysis does not consider the students' written work due to the space limitation of the paper. The whole set of data could serve indeed for further analysis.

6 Conclusion

As a conclusion, we would highlight the potential of the documentational approach to didactics for theoretically grounding our study. The three dialectical relationships at the heart of this approach appeared particularly relevant for developing our analysis:

- The duality instrumentation–instrumentalization helped us to understand the way that the online video lessons and platform structured teacher work in a time of uncertainty, and, the way that the teachers adapted these resources as well;
- The duality of productive–constructive activity helped us to understand the link between resource design and professional development. The case of Zi as an online video lesson designer was particularly illustrative of this duality, leading to the notion of design capacity (Pepin et al., 2017);
- Finally, the duality collective–individual documentation work clearly appeared in the pandemic, where the teachers facing the complexity of online teaching developed a new form of collective work (virtual Teaching Research Groups combined with various usages of social networks).

Our study and relevant literature (‘Research questions’) evidenced how the pandemic brings the issues of resources to the fore. This focus creates new theoretical and methodological needs, aiming to deepen the processes involved in the design, use and sharing of resources for teaching mathematics. These theoretical and methodological needs call for a networking of the different scientific fields (‘The documentational approach to didactics’) and cultures studying teachers interacting with resources (Choppin, 2019; Remillard, 2019; Ruthven, 2019; Fan et al., 2018), towards a domain extending the initial scope of DAD: a ‘resource’ approach to mathematics education (Trouche et al., 2019).

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Declarations

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