



Exploring adults' awareness of and suggestions for early childhood numerical activities

Esther S. Levenson¹  · Ruthi Barkai^{1,2} · Dina Tirosh¹ · Pessia Tsamir¹

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Abstract

This study focuses on adults who are neither preschool teachers nor professional caregivers and investigates their beliefs regarding the importance of engaging young children with numerical activities. It also examines the types of numerical activities adults report having observed children engaging with, as well as the types of activities they propose as a way for promoting counting, enumerating, recognizing number symbols, and number composition and decomposition. Findings showed that participants believed to a great extent that engaging young children with numerical activities is important. Most reported that they had observed children engaging with at least some numerical activity. In general, participants relayed more activities and more detailed activities when suggesting activities for each competency, than they did when reporting observed activities. Findings also suggested a need to enhance adults' knowledge regarding the necessity to promote verbal counting, separate from object counting, as well as to increase adults' awareness of number composition and decomposition. For mathematics educators wishing to plan workshops for adults, this study offers a method for investigating adults' knowledge of early numerical activities, as well as a starting point with which to plan appropriate workshops.

Keywords Adults' knowledge and beliefs · Numerical activities · Number symbols · Counting · Enumeration · Number composition

1 Introduction

The importance of fostering mathematical development during the early years is supported by studies that found early mathematics competencies to be a predictor of later school success (e.g., Duncan et al., 2007). Studies also suggest that for children to take advantage of the

✉ Esther S. Levenson
levenso@tauex.tau.ac.il; levensone@gmail.com

¹ Tel Aviv University, 30 Haim Levanon St, 6997801 Tel Aviv, Israel

² Kibbutzim College of Education, Technology, and the Arts, 140 Mordechai Namir Rd, Tel Aviv, Israel

academic opportunities provided at preschool, some level of support from the home environment is advisable (Anders et al., 2012; Pan et al., 2006). Adults, however, do not always know how to enrich young children's mathematics competence. Some parents have openly admitted that they do not know what mathematics their child could or should learn or how to go about helping them learn mathematics (Cannon & Ginsburg, 2008). Because the nature of mathematical learning matters, it is important to increase adults' knowledge of what it means to promote early mathematics learning and how to go about providing worthwhile mathematics enrichment practices. A first step in providing intervention for adults is to investigate adults' beliefs and knowledge for promoting children's mathematics knowledge.

Taking into consideration that young children often spend much of their day in the care of parents, grandparents, aunts, neighbors, and other responsible adults (e.g., Pilarz, 2018), we have begun to investigate the knowledge and beliefs of adults who are not professional early childhood educators regarding activities that can support numerical, geometrical, and pattern competencies among young children. In this study, we report on adults' knowledge and beliefs regarding number skills. First, we investigate adults' beliefs regarding the importance of engaging young children with numerical activities. In line with Beswick (2007), we use the term belief to refer to any proposition an individual holds to be true. We then investigate their recollection of situations in which they noticed children engaging with numerical activities. These recollections can shed light on adults' abilities to notice opportunities for engaging children with numerical ideas. Finally, we investigate adults' suggestions of activities that they believe can promote four numerical competencies: verbal counting, object counting, number composition and decomposition, and recognizing number symbols. Furthermore, taking into consideration that teachers, as opposed to non-teachers, might have a heightened sensitivity to instructional activities in general, and mathematics teachers might have a heightened sensitivity to mathematical activities in particular, this study examines similarities and differences between three groups: adults who are mathematics teachers, those who are teachers but not mathematics teachers, and adults who are not teachers.

1.1 Early childhood numerical competencies

Verbal counting and object counting are related, and yet they are separate competencies, each needing attention. Verbal counting is the skill of reciting numbers in the conventional order and knowing the principles and patterns in the number system as coded in one's natural language (Baroody et al., 2006). The relationship to language may be seen in the difficulties of English-speaking (and Hebrew-speaking) children when learning the number words from 11 till 20 and going from 29 to 30 (Han & Ginsburg, 2001). According to Fuson (1988), children first learn to recite the conventional number words, produce them in order, and consistently. This is called the acquisition phase. A common error during this phase might be to skip one of the numbers consistently, for example, reciting 1, 2, 3, 4, 5, 7, 8, 9, 10 and repeating again 1, 2, 3, 4, 5, 7, 8, 9, 10. The next, elaboration phase, is when children become aware that the chain of numbers can be broken up and that parts of the chain may be produced starting from a number other than one. Thus, another counting competency is being able to count forward from some number other than one (Israel National Mathematics Preschool Curriculum [INMPC], 2010).

Beyond reciting the numbers forward, children should also be able to count backward and count by twos, fives, and tens (also called skip counting) before first grade (INMPC, 2010; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Counting backward is a prelude to learning subtraction, while skip counting

lays the groundwork for multiplication (Sarama & Clements, 2009). Counting backward can be considerably more difficult than counting forward. In one study (Howell & Kemp, 2010), although 93% of 5-year-old children could count forward from 1 to 10, only 54% could count backward from five. Related to counting forward and backward is knowing which number comes before or after some number, another competency to be developed before first grade. Knowing the number that comes before some number is more difficult than knowing which number comes after some number (Howell & Kemp, 2010).

Being able to recite the counting numbers is essential for object counting, also called enumeration. Enumeration refers to counting objects for the purpose of saying how many. Gelman and Gallistel (1978) outlined five principles of counting objects. The first principle, called the stable-order principle, is based on being able to count verbally as described above. The one-to-one principle involves assigning one count word to each object. Common, related mistakes occur when an object is skipped over, and not counted, or when one object is assigned more than one count number (Fuson, 1988). The third principle is cardinality, which involves knowing that when counting objects in a set, the last number mentioned represents the number of objects in that set. A child who has not yet understood this principle may simply state any number when asked how many objects are in a set or recount the objects which have just been counted (Fluck & Henderson, 1996). The fourth principle is the abstraction principle, indicating that any set of discrete objects can be counted. Finally, the order-irrelevance principle means knowing that one may enumerate the objects in any order (e.g., from right to left, from left to right, etc.) and that enumerating objects in different ways results in the same cardinality. Some kindergarten children readily accept that objects may be enumerated in several ways but have difficulty accepting that the cardinality stays the same (Baroody, 1984). Notably, children may show knowledge of one principle while violating another principle, for example, erring with regard to the one-to-one correspondence principle but showing understanding of cardinality (Fuson, 1988).

Related to counting objects is the notion that wholes consist of parts and that a set may be decomposed into subsets and then brought together again to compose the original set. Composing and decomposing numbers can enhance children's number sense as well as their ability to solve addition and subtraction problems (e.g., Baroody et al., 2006). Note that number composition and decomposition are not the same as addition and subtraction. For example, when adding $7 + 5$, a child may use several strategies, such as counting all, counting on from first, or decomposition. The strategy of decomposition may be seen thus: $7 + 5 = 7 + 3 + 2 = 10 + 2 = 12$. In other words, decomposing 5 into 3 and 2 allows for the strategy of completing to ten and then counting up. Furthermore, understanding the notion of number decomposition is a prelude to understanding two-digit numbers as tens and ones (Sarama & Clements, 2009).

Knowing mathematics also includes knowing the symbols with which we express mathematical concepts. According to several curricula, before first grade, children should be able to identify and write the numbers from 0 to 20, with 0 representing a count of no objects (e.g., National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Identifying number symbols includes understanding that the symbol represents a quantity and that these symbols are different than letter symbols (e.g., a, b, c, ...) or other mathematical symbols (such as the equal sign) (Gjicali et al., 2019).

1.2 Activities that promote early number competencies

Promoting numerical competencies involves engaging children in appropriate activities. Activities may be implemented with the specific aim of practicing skills, or they may occur in natural

settings. For example, verbal counting can be practiced by reciting the number words in unison with a teacher or caregiver or when playing games, such as “hide and seek,” where counting is part of the game (INMPC, 2010). The *Big Math for Little Kids* program (Greenes et al., 2004) encourages the use of body movements and sounds to represent different decades from 1 to 100. For example, children make funny faces when reciting numbers from 11 till 19 and twist their bodies when reciting twenties. Greenes et al. (2004) theorized that body movements and sounds may add motivation, which in turn facilitates the learning of oral counting.

Enumerating, as noted above, involves several sub-competencies, and thus activities that aim to promote enumeration should take this into consideration. For example, while parents often cite reading counting books as a way to promote children’s enumeration, explicit or even implicit emphasis on cardinality was found to be quite infrequent in many counting books (Ward et al., 2017) leaving it up to the adult reading the book to focus on different sub-competencies. Yet, even when parents read a number book, they rarely provide cardinal labels after counting (Mix et al., 2012). In one study (Gaylord et al., 2020), when specifically choosing a counting book to read to their children, more parents chose books based on their assessment of how fun and enjoyable it would be for the children and if the story was narrative and included main characters, than on the mathematical content and challenge it would provide for children.

The types of objects counted may impact on children’s acceptance of the abstraction principle (Gelman & Gallistel, 1978). Counting objects that are different in color, size, or function can help children recognize that these other attributes do not affect counting (Greenes et al., 2004). Counting objects in different formations may promote the order-irrelevance principle (Gelman & Gallistel, 1978). In Tsamir et al.’s (2018) study, when 4–5-year-old children were requested to count seven bottle caps placed in a circle, two children, who had previously correctly counted the bottle caps when placed in a row, claimed that they did not know what to do and gave up.

Regarding number symbols, one activity that supports this competency is matching number quantities to their number representations (Purpura et al., 2013). The *Big Math for Little Kids* program (Greenes et al., 2004) emphasizes that number may be represented in different ways (e.g., numeral, word name, sets of tallies or dots) and suggests showing children several representations at the same time. Clements et al. (2017) suggested using dot patterns to represent quantities, specifically suggesting the use of 5-group dot patterns (as we have five fingers) for numbers 6 through 10. Children then practice counting from 1 to 10 as they point to cards which have both numerals and dots. Dot patterns can also promote number decomposition, as children see that 7, for example, is made up of 5 dots in a row and then another row with 2 dots.

Additional activities to promote number composition and decomposition were suggested by Tsamir et al. (2015). In their study, children were presented with picture cards having different amounts of apples on each card. Children were requested to choose two cards in such a way that together there would be seven apples. Results indicated that children found it easiest to compose seven from seven and zero, whereas the most difficult was three and four. In a second activity, children were shown a number of identical items in the palm of the interviewer’s hand and asked to say how many were hiding in the closed second hand, given that together there were seven items. These tasks illustrate to children that there are many ways to decompose and compose seven.

1.3 Previous studies of adults

Several previous studies found that both preschool teachers and parents agree that mathematics should be and can be promoted in the years before first grade (e.g.,

Missall et al., 2015). However, when comparing the importance of learning mathematics to other subjects, Vlassis and Poncelet (2016) found that first-year prospective preschool teachers rated engaging with mathematics in preschool as less important than engaging with language, arts, and psychomotricity. Similarly, some home day care providers believe it is less important for young children to acquire mathematics skills before entering kindergarten, than other social and academic skills (Blevins-Knabe et al., 2000).

Regarding numerical activities conducted at home, previous studies investigated parents' and home care givers' numeracy practices by offering participants a list of activities to check off regarding the relative importance of such activities (Skwarchuk, 2009) or the frequencies of engaging children with such activities (e.g., Zhu & Chiu, 2019; Zippert & Rittle-Johnson, 2020). These lists included counting objects, learning simple sums, telling time, recognizing printed numbers, counting, reciting numbers in order, singing counting songs, counting by twos, ordinal numbers, and playing board games.

Another method for investigating parents' numerical activities was to focus on specific contexts within which numerical activities may take place. In one study, parents were given a set of Duplos and a kitchen set to play with their 4-year-old children, investigating which context would elicit more attention to mathematical activities (Chan et al., 2020). In another study, families were given mathematical card games to play with their children at home (Ramani & Scalise, 2020).

What is missing is requesting adults to suggest activities that have the potential to promote specific numerical competencies. Studies have reported the benefits of supplying parents with mathematical activities to be done at home (Vandermaas-Peeler et al., 2012). Yet, not all parents have access to such activities, and in fact, one study reported that parents are interested in receiving information regarding activities to do with their children at home (Sonnenschein et al., 2020). Knowledge of activities is an important element for teaching mathematics at any age (Ball et al., 2008). Thus, investigating the types of activities adults believe could enhance children's numerical knowledge can offer us a glimpse into their knowledge of how numerical concepts can be fostered at a young age. Furthermore, while previous studies have investigated differences between participants of different ethnic or socioeconomic backgrounds (e.g., Ramani & Scalise, 2020), none have taken into consideration whether participants are educators and specifically if they are mathematics educators. While being an educator may sensitize one to pedagogical activities, in many countries teacher preparation programs focus on the mathematics taught in school for children ages six and up (Tatto, 2013). Thus, teachers who are not preschool teachers may not be aware of the mathematics that children are able to learn prior to beginning first grade or in out-of-school contexts.

In this study, we address the following questions concerning adults who are not preschool teachers: (1) To what extent do adults believe in the importance of engaging young children with numerical activities? (2) What types of numerical activities do adults report having observed children engaging with? (3) What types of activities do adults propose as a way for promoting young children's competencies in four areas: counting, enumerating, recognizing number symbols, and number composition and decomposition? (4) Is there a relationship between numerical activities observed and those suggested? (5) Are there differences between adults who are not teachers, adults who are teachers, but not mathematics teachers, and adults who are mathematics teachers?

2 Method

2.1 Participants

The study was conducted with a convenience sample of 104 adults (labelled A1-A104), recruited by three researchers from acquaintances in their areas of residence (middle to high socioeconomic neighborhoods). All were between the ages of 20 and 60, 94% had an academic degree, and none were preschool teachers or professional childcare givers. There were 24 mathematics teachers (MT) (elementary and high school), 24 teachers who were not mathematics teachers (NMT), and 56 non-teachers (NT).

2.2 Tool and procedure

A three-part questionnaire was designed for this study and handed out individually to participants in a comfortable environment in the presence of the researcher. Each part was handed out separately, and when it was returned, the next part was handed out. The first part, called “Belief Questions,” contained eight Likert-scale questions ($\alpha=0.790$) related to the adults’ beliefs concerning the importance of engaging young children with numerical activities (see Appendix for the questions). The range of the scale was from 1 (I do not agree) to 6 (I fully agree). The second part, called “Observed Activities,” consisted of the following two questions: (1) Describe a situation, in which you observed young children (aged 3–6) raising numerical ideas without adult involvement. (2) Describe a situation, whereby during an interaction between a young child (aged 3–6) and an adult, number-related ideas were raised. The reason for specifying situations with and without adult interaction was to offer participants two different contexts for recollecting situations where children engaged in numerical activities.

The third part of the questionnaire, called “Suggested Activities,” focused on four central competencies of early childhood mathematics (INMPC, 2010) (we feared that including additional competencies would make the questionnaire too long) and had four questions: (1) The preschool mathematics curriculum states that by the end of kindergarten, children should be able to count to 30. What counting activities would you implement with children to promote this skill? (2) What enumerating activities would you implement with children to promote their ability to enumerate eight objects? (3) Describe activities that could promote children’s ability to identify the number symbols (numerals) for zero through ten. (4) Describe activities you would implement with children to promote their knowledge of composing and decomposing seven. There were two questions on each page, so that participants had plenty of empty space to write their responses. Participants took as much time as they needed to fill in the questionnaires.

2.3 Data coding

A mean belief score was configured from the first part of the questionnaire for each participant. While the second part of the questionnaire – Observing Activities – was open and participants could and did report on activities within a variety of numerical contexts, the third part – Suggested Activities – was specific and focused on four numerical competencies: verbal counting, enumerating (object counting), number composition and decomposition, and numeral identification. Thus, we began by coding suggested activities, separately for each

competency, forming a coding scheme that was then also used to analyze responses for the observed activities. A mixed-methods qualitative analysis was carried out beginning with a deductive approach and continuing with inductive analysis (e.g., Mayring, 2015). The deductive analysis was based on the sub-competencies for each major competency, as described in the background. For example, the verbal counting category included not only counting forward from one but also counting backward and skip counting.

Inductive analysis was carried out for responses to the suggested activities and took into consideration the different types of activities described by participants and was conducted through an iterative process, using the method of constant comparison (Glaser & Strauss, 1967). For example, when requested to suggest activities for promoting numeral recognition, some participants wrote general statements such as “I would show children numbers and go over them with the children” (A104, NMT), while others specifically noted the contexts where children may be encouraged to identify number symbols. For example, A53 (NT) wrote, “We could read the numbers on road signs or on the television remote.” Some participants suggested more than one activity from the same category type. In such cases, if the activities were of different types, for example, identifying number symbols on license plates and practicing writing number symbols, then they were counted as two different activities. However, if the activities were essentially the same, for example, identifying number symbols on license plates and identifying number symbols on cereal boxes, then it was counted as one activity.

One researcher coded responses for counting and number symbol recognition activities, and a second researcher coded responses for enumerating and number composition/decomposition activities. Each researcher then validated the codes of the other researcher, discussing the coding until full agreement was reached. At that point, a third mathematics educator, specializing in early years mathematics, validated 20% of all coding. This process is in accordance with O'Connor and Joffe (2020), who stated that a minimum of two independent coders is necessary to establish inter-coder reliability. They added that when a full data set is not coded by another researcher, then “Depending on the size of the data set, 10–25% of data units would be typical. It is important this subsample is selected randomly or using some other justifiable criteria (e.g., selecting a member of each group in a stratified sample) to ensure representativeness of the entire data set” (p. 5). Thus, samples for coding for the third researcher were chosen to proportionally represent participants of different backgrounds. Care was also taken to sample data representing various codes of categorization. Agreement between codes was 88%. At that point, the first two researchers discussed between themselves the discrepancies, deciding together on the final code.

3 Findings

3.1 Beliefs related to early numerical activities

Recall that participants responded to a 6-point Likert scale beliefs questionnaire. In general, participants strongly believed in the importance of engaging young children with numerical activities ($M=5.42$, $SD=.53$). When comparing responses between the three groups, non-teachers (NT), teachers who were not mathematics teachers (NMT), and mathematics teachers (MT), no significant differences were found between any of the groups. The only statement with a belief score less than 5 ($M=4.90$, $SD=1.05$) was the following: It is important for

children to be able to identify if a suggested method for solving a number activity/task is correct. Thus, it might be said that participants did not all agree that it is important for children to assess the correctness of solutions to numerical tasks.

3.2 Numerical activities observed by participants

Participants' observations were categorized into nine major categories. Some participants described more than one activity, but from different categories, and these were counted separately. As can be seen from Table 1, enumerating was the most often mentioned type of activity observed by participants. Taking a closer look at the enumerating activities, most participants wrote general statements such as A17 (MT), who stated, "a young child was playing make-believe with her dolls, pretending that she was the teacher and the dolls were children in preschool, and she counted how many children were at the school." A67 (NT) wrote, "My three-year old daughter tries counting things all the time, like cookies, cats, and toys." Only 12 (out of 104) adults wrote more detailed descriptions relating to at least one of Gelman and Gallistel's (1978) principles for object counting. For example, A82 (NT) wrote, "when we set the table for a meal, (I told my child to) count the number of people and then set a plate for each person." According to Tirosh et al. (2020), such an activity encourages one-to-one correspondence.

Although, in general, enumerating activities were the most described type of activity, this was not the case among the mathematics teachers. Among mathematics teachers, the most mentioned types of activities were set comparison (e.g., children compare amounts) and arithmetic problems. For example, A18 (MT) recalled children having to share equally a bag of candies. These activities, especially solving arithmetic problems, may be considered closer to school-like activities than counting or enumerating and thus perhaps more noticeable by mathematics teachers than other activities.

Arithmetic activities were categorized separately from number composition and decomposition activities. For arithmetic activities, participants wrote, for example, "My son (aged 4), recites all kinds of arithmetic exercises, like two and two equals four, or ten and one equals eleven" (A70, NT). On the other hand, activities that included physical items and were not solely abstract operations were categorized as number composition/decomposition. For example, A68 (NT) wrote, "An adult holds a different number of peanut snacks in each hand and asks: How many snacks in total do I have?" or A58 (NT) wrote: "I gave my granddaughter two containers of soap bubbles and she told me another container was missing in order to have three."

Table 1 Frequencies (in %) of numerical activities observed by participants

	NTs N=56 (100%)	NMTs N=24 (100%)	MTs N=24 (100%)	Total N=104 (100%)
Enumerating	26 (46%)	12 (50%)	4 (17%)	42 (40%)
Set comparison	11 (20%)	6 (25%)	7 (29%)	24 (23%)
Arithmetic	10 (18%)	2 (8%)	6 (25%)	18 (17%)
Counting	14 (25%)	2 (8%)	1 (4%)	17 (16%)
Money	8 (14%)	5 (21%)	2 (8%)	15 (14%)
Number symbols	3 (5%)	2 (8%)	2 (8%)	7 (7%)
Number composition	3 (5%)	1 (4%)	2 (8%)	6 (6%)
Other	3 (5%)	1 (4%)	-	1 (1%)
No response	9 (16%)	3 (13%)	8 (33%)	20 (19%)

Regarding verbal counting, all those who described a counting activity referred to counting forward from one. For example, A74 (NT) wrote, "My son likes to count to ten." None of the participants reported observing a counting activity that included additional competencies, such as counting backward or skip counting. Surprisingly, one of the least mentioned activities was identifying number symbols. Considering that many children's games require the rolling of dice, or number cards, and that essentially numerals may be found in children's natural surroundings, we had hypothesized that many adults would report observations of children identifying numerals.

3.3 Suggested activities versus observed activities

This section describes the specific activities participants suggested for promoting counting, enumerating, identifying number symbols, and number composition/decomposition. It then returns to those participants, who in response to the previous question had reported observing each of these types of activities and compares their suggested activities to their observed activities.

3.3.1 Verbal counting activities

Although participants were requested to suggest activities that could promote counting, as can be seen from Table 2, over half of the participants from all three groups suggested enumerating activities. Although verbal counting may be considered a sub-competency of enumerating, it is nevertheless a skill which needs to be promoted on its own. Among activities that consisted of counting forward from one, as can be seen from Table 2, some participants merely suggested to have children count out loud, such as A20 (MT) who wrote, "I would ask them to count out loud till whatever number they know, however high they can. If they reached 30 or 50 you can ask them to stop." Some participants suggested counting along with some rhythmical movement, such as walking and counting steps (e.g., A42, NMT). Interestingly, teachers who were not mathematics teachers suggested activities that promote additional competencies (e.g., counting backward), more than the other two groups.

Table 2 Frequencies (in %) of suggested counting activities

	NTs N=56 (100%)	NMTs N=24 (100%)	MTs N=24 (100%)	Total N=104 (100%)
Counting objects	34 (61%)	13 (54%)	14 (58%)	61(59%)
Counting forward from one	26 (46%)	6 (25%)	10 (42%)	42 (40%)
Plain counting	13 (23%)	4 (17%)	7 (29%)	24 (23%)
Counting with movement	13 (23%)	2 (8%)	3 (13%)	18 (17%)
Additional counting competencies	8 (14%)	10 (42%)	2 (8%)	20 (19%)
Counting on from some number	2 (4%)	2 (8%)	1 (4%)	5 (5%)
Counting backward	3 (5%)	3 (13%)	1 (4%)	7 (6%)
Skip counting	1 (2%)	3 (13%)	-	4 (4%)
The number that comes before/after	2 (4%)	2 (8%)	-	4 (4%)
No suggestion/irrelevant	8 (14%)	4 (17%)	3 (13%)	15 (14%)

Looking back at Table 1, when reporting observations of children engaging with numerical activities, 17 responses were coded as verbal counting activities. Of those 17 participants, eight suggested an enumerating activity. For example, A55 (NT) reported observing her son counting in the car, “When we drive from one place to another, my son counts until we reach our destination.” However, when asked to suggest an activity that could promote counting, she wrote, “to count the children who came to preschool, dolls, people in the family, and books.” Counting dolls or books is essentially an enumerating activity and less a counting activity. Only one of the 17 participants (A39, NT) suggested counting on from some number other than one. The rest (9) suggested counting from one, similar to activities they had observed. To conclude, there seemed to be little connection between counting activities that were observed and counting activities that were suggested. In fact, while none of the participants reported observing an additional counting competency activity, 19% of the participants suggested such an activity, such as A12 (MT) who suggested to have children count backward from 10 or 15 and A104 (NMT) who suggested to request that children skip count.

3.3.2 Enumerating activities

When suggesting enumerating activities (see Table 3), 80% of adults suggested at least one enumerating activity. Nearly half of the NTs suggested a general enumeration activity such as A50, who stated, “count objects such as candies, cornflakes, cards, etc.” Such activities were lacking in detail and did not refer to any of the enumerating principles mentioned by Gelman and Gallistel (1978). In contrast, over half of the teachers who were not mathematics teachers did suggest activities focusing at least on one enumerating sub-competency. For example, A36 (NMT) wrote, “I would have children count eight children standing in a row, then in a column, then eight children standing in a circle.” By having children count different formations of items, this adult is attempting to promote the order-irrelevance principle, knowing that one may enumerate the objects in any order. A43 (NMT) wrote, “to count different objects of different sizes.” By stressing that the objects should not be the same, this adult is offering children an opportunity to develop the abstraction principle. Finally, a fifth of all participants suggested a counting out activity, such as taking out eight blocks from a carton of blocks (A51, NT).

Referring back to reports of observed activities, recall that enumerating activities were the most observed activities mentioned by participants (see Table 1). Looking at the details of

Table 3 Frequencies (in %) of suggested enumerate activities

	NTs <i>N</i> =56 (100%)	NMTs <i>N</i> =24 (100%)	MTs <i>N</i> =24 (100%)	Total <i>N</i> =104 (100%)
General enumeration	28 (50%)	10 (42%)	8 (33%)	46 (44%)
Attention to enumeration principles	20 (36%)	14 (58%)	10 (41%)	44 (42%)
Order irrelevance	10 (18%)	9 (38%)	6 (25%)	25 (24%)
Cardinality	4 (7%)	2 (8%)	2 (8%)	8 (8%)
Abstraction principle	5 (9%)	2 (8%)	1 (4%)	8 (8%)
One-to-one correspondence	1(2%)	1(4%)	1 (4%)	3 (2%)
Counting out	12 (21%)	4 (17%)	6 (25%)	22 (21%)
No suggestion/irrelevant	10 (18%)	3 (13%)	6 (25%)	19 (18%)

those reports, 15 (out of 42) participants described activities that promoted specific enumeration sub-skills (Gelman & Gallistel, 1978). However, when asked to suggest activities that could promote enumeration, only six of the 15 suggested likewise detailed activities. On the other hand, 14 participants reported observing general enumerating activities, but when asked to suggest enumerating activities, they suggested more detailed activities. That being said, of the 19 participants who did not suggest any activity for promoting enumeration (or suggested an irrelevant activity), five had reported observing an enumeration activity.

3.3.3 Identifying number symbols

As seen in Table 4, most participants suggested an activity where children must visually identify number symbols, some suggested to point out numerals found in the children's immediate environment, others related to numerals in games, and others did not specify a context. However, when looking at the different groups of participants, for mathematics teachers, visually identifying number symbols was mentioned less than writing numerals and matching a numeral to an appropriate amount. This makes sense, as these activities may be considered more instructional than merely pointing to numerals in one's surroundings. Eight participants (out of 104) suggested activities that might help children remember the shape of a numeral by associating it with a recognized concept. For example, A27 (NMT) suggested, "I would use the association method. For example, the numeral two looks like a swan, an eight looks like a pair of glasses."

When comparing observed to suggested number symbol activities, the first thing to note is that only 7% of participants reported observing such an activity, whereas when asked to suggest an activity to promote numeral recognition, approximately 85% suggested an appropriate activity. Of those who reported observing a number symbol activity, four described identifying numbers in the environment, one mentioned number symbols in a game, and one referred to flash cards. None of the participants reported observing young children writing numerals, and only one reported observing a child matching a numeral to an appropriate amount.

Table 4 Tale 4 Frequencies (in %) of suggested Identifying number symbol activities

	NTs N=56 (100%)	NMTs N=24 (100%)	MTs N=24 (100%)	Total N=104 (100%)
Visual recognition of number symbols	31 (55%)	12 (50%)	6 (25%)	49 (47%)
In the environment	11 (20%)	2 (8%)	2 (8%)	15 (14%)
In a game	16 (28%)	5 (21%)	2 (8%)	23 (22%)
Non-specific	4 (7%)	5 (21%)	2 (8%)	11 (11%)
Writing/feeling the numeral shape	20 (36%)	9 (37%)	10 (42%)	39 (38%)
Ordering the numerals	4 (7%)	2 (8%)	---	8 (8%)
Matching a numeral to an amount	12 (21%)	4 (17%)	7 (29%)	23 (22%)
Associations	5 (9%)	3 (12%)	---	8 (8%)
No suggestion/irrelevant	6 (11%)	2 (8%)	6 (25%)	14 (13%)

3.3.4 Number composition and decompositions

Although participants were requested to suggest activities that could support composition or decomposition of the number seven, most activities focused on decomposition, specifically decomposing seven into two groups (see Table 5). For example, A11 (MT) wrote, “You give the child seven candies and ask her/him to divide them into two groups and then tell you how many candies are in each group. Afterwards, you ask him to divide them again in a new way.” Several participants specified different ways to decompose seven, such as A35 (NT) who wrote, “Divide seven children among two tables, two at one table and five at the other. Each time do it differently: (4+3), (6+1), (7+0).” Only four adults specifically suggested to ask the child to sum the total after the decompositions, such as A2 who wrote, “I would show the child one cookie and then six cookies, and ask how many cookies there are all together.” An example of a composition activity was, “I would give the child a bunch of blocks and tell him to make two piles such that together there would be seven blocks” (A44, MT). Note that only four (of 104) participants suggested abstract addition exercises, such as, “I would give them worksheets with addition questions” (A63, NT).

Recall from Table 1 that 6% of all adults reported observing a decomposition/composition activity, while 17% reported observing children solving arithmetic exercises. When asked to suggest composition/decomposition activities, approximately 65% of all participants made relevant suggestions, of which only 4% suggested arithmetic exercises. That such a small percent suggested to add numbers as a way for promoting number composition show that most realized that there is a difference between arithmetic exercises and number composition. This might explain why among the 34 participants who did not suggest any number composition/decomposition activity, six had reported observing children solving arithmetic questions, but did not suggest those activities when responding to the third question.

Table 5 Frequencies (in %) of suggested composition activities

	NTs N=56 (100%)	NMTs N=24 (100%)	MTs N=24 (100%)	Total N=104 (100%)
Decompose 7 into two groups	23 (41%)	9 (37%)	7 (29%)	39 (37%)
Ask how much in all	2 (4%)	1 (4%)	1 (4%)	4 (4%)
Various decompositions	21 (37%)	8 (33%)	6 (25%)	35 (33%)
Compose 7	9 (16%)	4 (17%)	4 (17%)	17 (16%)
Second addend missing	3 (5%)	3 (13%)	5 (21%)	11 (11%)
Add two numbers	3 (5%)	-	1 (4%)	4 (4%)
No suggestion/irrelevant	18 (32%)	8 (33%)	8 (33%)	34 (33%)

4 Discussion

Considering that individuals' beliefs can influence what they see and how they process these observations (e.g., Lee & Francis, 2018), as well as how they act, this study began by examining adults' beliefs regarding the importance of engaging young children with numerical activities. Furthermore, because individual experiences may also impact on beliefs (Ambrose, 2004), we hypothesized that mathematics teachers, and teachers in general, would believe more strongly in the importance of early numerical activities than non-teachers. Findings showed that participants' backgrounds were not significant and that, like previous studies (e.g., Missall et al., 2015), participants believed to a great extent that engaging young children with numerical activities is important.

In accordance with these findings, we expected and found that most participants showed an awareness of children's engagement with numerical activities. Yet, a fifth of the participants claimed not to recall observing such situations. It could be that even if adults had observed children engaging with numerical activities, they might not have considered those activities to be numerical in nature. This might explain why so few participants reported observing children identifying number symbols. Some adults might consider recognizing number symbols to be part of general symbol knowledge, such as recognizing the symbol for a "do not walk" sign. Thus, even if they believe promoting early numerical skills is important, they might not specifically promote number symbol recognition.

In general, there seemed to be little connection between activities observed and activities suggested. A few participants reported observing a specific activity (e.g., enumeration), but then did not give any suggestion for that activity. Interestingly, other participants described observing a specific enumeration activity, for example, a child counting objects, but then suggested a different kind of activity for promoting that skill, such as counting out an amount from a larger amount. Such findings hint at participants' discrimination between what they observe and what they themselves might do, suggesting that adults realize the importance of choosing appropriate activities, rather than merely mimicking observed activities, if they want to proactively enhance children's number skills.

Despite the seeming disconnections, when taking a closer look at each of the four competencies, some trends may be found. First, participants suggested more activities for promoting each competency, than they did when reporting observed activities. This might be expected. The question related to observed numerical activities was open, and not directed, while the question requesting suggestions focused participants on specific skills. This difference was most noticeable when it came to identifying number symbols and number composition/decomposition. Participants reported few observations related to those skills, yet upon request, they were able to suggest a variety of activities to promote both. Because the observation question was open and not directed, we might consider participants' responses to be an indication of the frequency different types of activities occur in the participants' surroundings, where the more the frequent an activity occurs, the more likely it will be noticed and observed. Recall that the most observed activity was enumeration. This is in line with previous studies that found counting objects to be one of the most frequently occurring numerical activities parents reported engaging in with children (Missall et al., 2015; Skwarchuk, 2009). However, in those same studies, parents also reported frequently counting aloud and reading numbers to their children, whereas in the current study, participants barely reported observing these activities.

A second trend found was that more details were given for suggested activities than for observed activities. It might be that when recalling observed activities, details were remembered less or deemed less important to report, but when suggesting activities to promote a specific skill,

participants felt that it is important to describe the details of the activity. Another reason for this disparity may be related to the specific skill. Consider, for example, counting aloud. Out of school, there are few opportunities to count backward or to count forward from some number other than one. Similarly, in previous studies, parents rated the frequency of engaging children with skip counting and counting backward among the least frequent activities (Missall et al., 2015; Skwarchuk, 2009). When requested to suggest activities, only 20% of all participants mentioned sub-competencies of counting, showing little awareness of these skills. Several participants did suggest reciting the number words along with movement. This is reminiscent of Greenes et al. (2004) who theorized that body movements and sounds may facilitate the learning of oral counting.

Focusing on the details of adults' suggested activities, we notice additional details that are in line with educators' suggestions. For example, when it came to numeral identification, some adults suggested specific contexts that might elicit such identifications. Although they did not explicitly state that the aim was to show numerals represented in different ways (in line with Greenes et al., 2004), by employing various contexts for identifying numerals, such as on playing cards, children can be exposed to different representations. Other activities mentioned were ordering numerals and matching numerals to a corresponding number of objects, in line with Purpura et al.'s (2013) suggestion. Regarding number composition/decomposition, several participants specifically mentioned that they would ask children to distribute seven items between two groups in several different ways. Knowing that there are many ways to compose and decompose each number is an important concept for children to grasp (Sarama & Clements, 2009). Some adults suggested second addend missing activities, like those found in Tsamir et al. (2015).

The last research question dealt with possible differences between adults who are not teachers, adults who are teachers but not mathematics teachers, and adults who are mathematics teachers. Although no differences were found regarding their beliefs, some differences were noticed when suggesting activities. For example, more teachers who were not mathematics teachers suggested activities that could promote additional counting competencies, as well as activities that focused on one or more of Gelman and Gallistel's (1978) principles, than the other groups. One possible reason for this could be that some of those teachers who were not mathematics teachers were elementary generalist teachers, with some mathematical pedagogical knowledge related to young children. On the other hand, mathematics teachers tended to suggest what might be considered difficult activities for young children, such as counting out (enumeration), matching a numeral to an amount (number symbol recognition), and second addend missing (number decomposition). Interestingly, these activities may also be related to more advanced mathematical notions. Counting out is related to the notion of sets and subsets. For example, counting out four items from a bag with seven items could be represented as $\{\{1, 2, 3, 4\}, 5, 6, 7\}$. The activity of second addend missing may be represented by the algebraic expression: $4+x=7$. On the other hand, non-mathematics teachers suggested more activities that related naturally to a child, such as identifying numerals in the environment and counting with movement. While differences between participants might not be generalizable, they indicate the possibility of variations in adults' knowledge regarding implementing numerical activities.

4.1 Implications

One of the reasons for conducting this study was to help mathematics educators prepare workshops for adults interested in promoting young children's numerical competencies. As teachers have knowledge of their students and content (Ball et al., 2008), workshop providers should have some knowledge of their participants. One contribution of this paper is the

method, which can provide adult educators with ways for investigating workshop participants' knowledge, to help plan appropriate workshops.

This study also points out where adults might warrant guidance. Regarding the lack of details in observed activities, while it might be said that these adults were not preschool teachers, and therefore, professional noticing (Sherin & van Es, 2009) need not be expected, a first step in providing young children opportunities to practice skills in a non-school environment is recognizing when opportunities arise to practice skills. In fact, with teachers, the emphasis is usually on being proactive and planning lessons (e.g., Paparistodemou et al., 2014), whereas other adults wishing to promote early numerical skills may have more of a need for reacting to circumstances. Thus, a workshop for adults might aim to raise adults' awareness of opportunities for promoting number skills, especially number symbol recognitions and number composition and decomposition, which were the least observed activities.

Taking a look at specific competencies, when asked to suggest counting activities, over half of the participants suggested enumerating activities. However, learning to recite the numbers consistently and in order is a competency in its own right (Baroody et al., 2006). A workshop might illustrate children's common errors in counting, reinforcing the importance of this skill, as well as introduce adults to sub-competencies of counting, such as skip counting.

Of the four competencies discussed in this study, the fewest reported observations as well as the fewest suggestions of activities were for number composition and decomposition. Regarding the lack of observations of this skill, it may be that the other domains occur more frequently in everyday circumstances. However, the lack of suggestions hints at a deeper gap. A workshop could introduce adults to this competency and explain what it is, its importance, and its relation to addition strategies and the decimal system (Sarama & Clements, 2009). For this competency, as well as for the others, a workshop could point out why the details of an activity are important, such as asking the child to say how many marbles are there all together, after decomposing them into several groups. In general, a workshop could build on adults' activity suggestions, increasing their awareness of opportunities and repertoire of activities for furthering children's number skills. As we write this paper during the COVID-19 pandemic, when many children are isolated at home with various responsible adults, we realize even more the importance of considering the potential of all adults to promote early mathematics.

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Appendix

Beliefs questionnaire

One a scale of 1 (not at all) – 6 (very much), how strongly do you agree with the following statements?

1. Children enjoy activities/games that deal with number aspects.
2. Children's number knowledge can be promoted.
3. It is worthwhile to engage children with activities/games that deal with number aspects
4. Activities/games that deal with number aspects can increase children's knowledge of number concepts.
5. It is important for children to be able to solve number tasks in various ways.
6. It is important for children to be able to identify if a suggested method for solving a number activity/task is correct.

7. It is important for children to be able to choose appropriate ways for solving number activities/tasks.
8. Interaction between a child and an adult while engaging in an activity/game can increase the child's knowledge of number.

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