People use math as a weapon: critical mathematics consciousness in the time of COVID-19

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Abstract

The emergence of the COVID-19 pandemic has brought increased attention to the critical mathematical literacy of citizens in the United States and around the world. A statistically and mathematically literate society is crucial for ensuring that citizens are able to sift through political rhetoric to maintain life-saving procedures such as social distancing and other infection dampening efforts. Additionally, recent civil unrest due to the disproportionate killings of Black men by police provokes investigation into the public's mathematical literacy. In this paper, we investigate adolescent students' critical mathematics consciousness and mathematics literacy as they reason through two interview tasks on the coronavirus and police shooting data. Drawing on Frankenstein's program of Critical Mathematics Education, we introduce an analytic framework for documenting the critical mathematics consciousness of adolescent students. We interviewed fifteen 14- to 16-year-old students as they solved five tasks designed to elicit their critical and ethical mathematical awareness. Our findings indicate that students exhibit very little critical mathematics consciousness in the context of the police problem but show awareness that data can be presented in ways that manipulate the public's emotions in the coronavirus problem. We conclude the paper with a discussion of implications for designing future instruction to support students' growth in critical mathematics consciousness.

Keywords Critical mathematics consciousness · Critical Mathematics Education · Critical agency · Teaching mathematics for social justice

People use math as a weapon in terms of creating biases for people...and I think the person who made that [graph] knew that and I think they were going to use that as a weapon for people like me who don't experience critical thinking.

~8th grade student, Tyrone

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1 Critical mathematics in the COVID-19 era

Critical mathematics literacy has been on the agenda of mathematics educators and researchers in the United States (U.S.) since the 1980s. Despite 40 years' attention, novel technologies and social media environments continue to be used to further disempower oppressed groups and spread misinformation, enabling those in power to maintain political and financial advantages. Whether intended or not, technology perpetuates and may even create new forms of discrimination since inequities can be hidden within algorithms and oppressor(s) can remain anonymous behind a computer screen (D'Ignazio & Klein, 2020; O'Neil, 2016). Additionally, mathematics, coding, and statistics have been viewed as "doable" by only a select few abstract thinkers and are perceived as inaccessible by a majority of our population (Ernest, 2018). One attempt to mitigate these circumstances in the U.S. has been the addition of "digital citizenship" courses in middle and high school. Students learn to question the reliability and accuracy of sources, statistical reports, and graphs that appear in various media formats. However, the misuse of or refusal to use data from statistical and mathematical models in favor of one's own opinion creates a danger, having devastating consequences on civic life, particularly for those who are oppressed.

The emergence of the COVID-19 pandemic has brought attention to media and critical mathematical literacy. The words "mathematical model," "flatten the curve," "peak infection," and other mathematical terms can be heard in both media outlets and public discourse. Graphs have frequently been used to communicate statistics about the pandemic, and graphs containing irregular and inaccurate scales have been used to support dubious claims (Kwon et al., 2021). A statistically and mathematically literate society is crucial for ensuring that citizens are able to sift through political rhetoric and mathematical representations to maintain life-saving procedures such as social distancing and other infection dampening efforts (Yoon et al., 2021). The disparate rates of infection and death from COVID-19 in communities marginalized by race and poverty also reveal the persistence of racism and other forms of oppression in the U.S. (data from www.cdc.gov). The point of this article is to explore the answer to the following questions: *Are students aware of the role that mathematics plays in disenfranchising certain groups in society? How does mathematical reasoning affect the ability to be critically aware?* To explore these questions, we posed mathematical tasks to 15 teenage students in one-on-one interviews. These tasks were designed to elicit students' critical mathematical consciousness as they interpreted social media messages involving the coronavirus and police shootings in the U.S.

Numerous critical scholars have exposed the past, current, and potential harms caused by technologies and mathematics. Noble (2018) warns that decisions made with algorithms often reinforce oppression and create new methods for racial profiling and other forms of discrimination. O'Neil (2016) argues convincingly that racist and discriminatory practices, whether intentional or not, are the consequence of the biases coded within the algorithms and mathematical models used to determine employment, imprisonment, college acceptance, and other programs that support civic life. The *New Jim Code*, according to Benjamin (2019), refers to the use of technologies and mathematics to reproduce existing inequities. For example, website algorithms that track a person's searches as a way to detect preferences and refer them to other similar sites can cause, what Eli Pariser called, the filter bubble or a state of intellectual isolation that fails to introduce opposing viewpoints (Pariser, 2011). In 1973, Paulo Freire (1973) warned that the influx of new technologies had the potential to develop *massified consciousness*, meaning the belief that humans act through free choice *alone*

rather than by choice *and* manipulation. Individuals with massified consciousness participate in their own domination by uncritically accepting media reports rather than through their critical reflection on the world. Freire (1970/2005) introduced the term “critical consciousness” as “learning to perceive social, political and economic contradictions and to take action against the oppressive elements of reality” (p. 36).

Drawing on Freire’s notion of critical consciousness, Frankenstein (1983), Skovsmose (1994), and Gutstein (2006) argue that *Critical Mathematics Education* and *Teaching Mathematics for Social Justice* can encourage students to challenge the contradictions and outright lies perpetuated by hegemonies via mathematics. Furthermore, Critical Mathematics Education can and should inspire students to develop *critical mathematics agency*: the recognition and commitment to disrupt discriminatory practices using mathematics. Frankenstein (1994) defined *critical mathematical literacy* as “the ability to ask basic statistical questions in order to deepen one’s appreciation of particular issues, and the ability to present data to change people’s perceptions of those issues” (p. 1). This definition involves both *critical reflection* and *critical agency* in that not only do students need to be aware of how mathematics should be questioned but also how they can communicate mathematically to make positive change.

2 Designing for critical mathematics consciousness

The development of critical consciousness primarily occurs as the result of education, by family, lived experiences, or formal schooling (Freire 1970/2005). Research indicates that students mature as they pass through different stages of moral development (Kohlberg, 1983) and primarily through formal education (Hartwell, 1995). Numerous studies provide insight into the levels of critical consciousness exhibited by students from marginalized groups (Gutstein, 2016; Rubel et al., 2016; Skovsmose, 1994). However, there is very little empirical research that documents *how to design for critical consciousness* in mathematics, especially that which attends to the complicated relationship between the mathematical knowledge needed for making life-altering decisions and one’s political ideology. Regarding this relationship, the Science Comprehension Thesis from psychology (see Kahan et al., 2013) suggests that society’s poor science comprehension exacerbates the political divide around certain decision-related science issues such as climate change, and more recently, COVID-19 vaccines and spread prevention. Surprisingly, the public’s ability to reason scientifically in order to make good life decisions is also *disabled by* cultural and political divisiveness (Kahan et al., 2013). In their study, Kahan et al. found that *highly* numerate individuals will make decisions based upon their ideological beliefs even when the mathematical data suggests a different conclusion. Thus, individuals who exhibit strong mathematical ability actually reason incorrectly in order to make the data fit their ideology. More specific to mathematics, Yoon et al. (2021) showed that during the COVID-19 pandemic, South Korean and U.S. citizens relied on political beliefs developed through the news or conversations with friends and family in lieu of believing statistical evidence. These studies suggest that research in Critical Mathematics Education must explore more closely the complex relationship between mathematical reasoning/literacy and political ideologies.

The preliminary work of the design research project described in this article was sparked by the lack of empirical research on designing for high school students’ critical mathematics

consciousness in the ways outlined above. Classroom-design researchers maintain that when studies documenting students' conceptual understanding of the desired topic are lacking, they must draw on a variety of sources to inspire their design. For instance, interviews may be used to elicit diverse student reasoning about the topic under study (Clements, 2008). Accordingly, the interview findings in this article, while small in number, represent our team's first attempt towards understanding the relationship between current students' mathematical reasoning and critical mathematics consciousness. Our intent is not to construct a robust empirical base for the work but rather to provide inspiration for designing an instructional innovation that builds on current student reasoning. Our research question was what is the relationship between high school students' mathematical reasoning with proportions/graphs and their critical mathematics consciousness? In the next sections, we outline the theoretical perspective and accompanying framework that guided both the design and analysis of interview tasks given to 15 students.

3 Methodology

3.1 Theoretical perspective

Our theoretical perspective is situated within the Critical Mathematics Education program elaborated by Frankenstein (1983) and Shor (1993). The complementary goals of Critical Mathematics Education are to promote students' *critical mathematics reflection* so that they can question mathematical arguments, models, and representations and inspire *critical mathematics agency* to act in ways that use mathematical communication to liberate, rather than disenfranchise, groups. Kokka (2020) defines critical mathematics consciousness (CMC) as the development of sociopolitical understanding, critical civic empathy, and action taking through mathematics. Our definition of CMC differs in that it emphasizes mathematics through a critical lens. Critical mathematics consciousness, for us, refers to *the awareness of the role that mathematics plays in disenfranchising or liberating oppressed groups in society and the willingness and commitment to act* (i.e., *critical mathematics agency*). CMC involves three different types of awareness:

- *Sociopolitical Mathematics Awareness*—Mathematics is used to model and interpret the real world and can be used to make decisions both at the individual and systemic levels that may be oppressive or liberatory
- *Ethical Mathematics Awareness*—Human beings do mathematics; thus, there are potential ethical dilemmas and implications of mathematical work; mathematics may be neutral but humans *doing* mathematics are not
- *Communicative Mathematics Awareness*—Mathematical communication has the power to educate and mis-educate society and encourage the masses to act in certain ways

Like Freire, we view critical mathematics consciousness as something that can be developed with instruction. Shor (1993) described Freire's levels of critical consciousness at which a person may be working at any time in their lives. We have adapted these to create a framework for CMC growth described in the next section.

3.2 Analytic framework

Freire (1970/2005) maintained that individuals are situated at different levels within a hierarchy of consciousness and that growth in consciousness develops through critical reflection and action. We consider intellectual and moral development as a dynamic and unstable process and view the constructs in Table 1 as characteristics, not levels, of consciousness that a person may exhibit differently across a variety of contexts. The first characteristic, called *intransitive*, involves a lack of critical thought and the person's belief that her own or others' condition may be changed only by God or luck (Frankenstein, 1983; Freire, 1970; Shor, 1993). Those who exhibit intransitive consciousness believe they do not possess the power or responsibility to change their own or others' circumstances and thus, would not question the mathematics. We have added the term *dysconscious* (King, 1991) to refer to a person that has a distorted vision of oppression and justifies inequity as the natural order of the world. Individuals who are dysconscious think that the oppressed are at fault and it is the oppressed's responsibility to overcome their situation. As King explains, dysconsciousness is usually taught through familial or educational influences, or by mass media (i.e., massified consciousness). Individuals who have *disempowered* consciousness think critically because they see themselves or others as oppressed and may even see the role that mathematics plays in that oppression. Yet, they believe that they have no power to change their status due to living in a society dominated by certain individuals or a group. We have split the term *semi-transitive consciousness* into two categories: isolated and systemic. Those who have isolated, semi-transitive awareness think critically, but see problems as isolated incidents or as caused by a dominant group, without a connection to the underlying *systems* that cause them. They feel empowered to act only on that single incident. Those who have systemic semi-transitive awareness recognize the systems or a dominant group as the cause of oppression but put their faith in others to incite change. People who demonstrate *critical transitive consciousness* are empowered both to consider and act on the conditions around them that are related to the broader systems that perpetuate inequity.

The constructs in Table 1 can be used to understand students' sociopolitical awareness (of systemic and individual causes of oppression/liberation), one component of our definition of critical mathematics consciousness. Sociopolitical awareness involves understanding the impact that mathematical decisions have on groups of people in society in both positive and negative ways, such as using data to identify individuals living in certain zip codes (usually communities of color) as at risk for paying back loans (O'Neil, 2016). The second component, ethical mathematical awareness, refers to more general ethical principles that may or may not be considered in the process of making mathematical decisions, such as violations of privacy, questions of who owns the mathematical data, and accuracy of the results. We acknowledge sociopolitical awareness can be considered a special case of ethical awareness; however, we believe sociopolitical awareness deserves attention in its own right. We outline a specific analytic framework for documenting students' ethical mathematics awareness in Register et al. (2021) and do not focus on this aspect in this paper. Communicative mathematics awareness refers to being cognizant that individuals who report or make decisions with mathematics have the power to inform or misinform citizens. This last aspect of CMC is the least developed in terms of analytic constructs, but we use data from this project to begin a discussion about what aspects of mathematical communication are critical for students.

Table 1 Critical Mathematics Consciousness Growth Framework

<i>Mathematical consciousness</i>	Critical thought	Perceived causes	Perceived <i>Mathematical</i> Power to act <i>Critical Mathematics Agency</i>
Critical transitive <i>Actualized CMC</i>	Yes	Systems/dominant group	Empowered to act by focusing on <i>system</i>
Systemic semi-transitive	Yes	Systems/dominant group	Put faith in others to act
Isolated semi-transitive	Yes	Isolated incidents/dominant group	Empowered to act on isolated injustices
Disempowered	Yes	Individual/dominant group	Disempowered to act
Dysconscious	No	Fault of oppressed	Onus on oppressed to act
Massified consciousness		<i>Technology/media influence</i>	
Intransitive	No	God/luck	Disempowered to act

3.3 Data collection method and participants

The work presented in this article was conducted as the first step in a design research project (Cobb et al., 2003) that focuses on designing for privileged students' critical mathematics consciousness. Interviews were conducted primarily with students who identify as White to (a) understand the CMC of 14- to 16-year-old students of privilege and (b) determine viable contextual problems that may provoke mathematics consciousness. Like Kokka (2020), we aim to work with students of privilege, meaning students from social groups who historically and currently possess disproportionate economic and political power, since most of the research in this area is done with under-represented student populations or shows very little progress with students of privilege (cf. Nurenberg, 2011).

The Design Research Team consists of two mathematics educators, two STEM educators, one doctoral student, and one assessment professional. All team members but two identify as White, with one identifying as Black/African-American and one as Asian-Indian. All team members are socioeconomically privileged and have engaged in work to develop their own socio-political consciousness. When engaging in critical work, we believe researchers are obligated to make transparent their positionality so that, as in our case, readers can draw their own conclusions about any potential researcher bias within the designed interview materials and data analysis. Our shared ideological beliefs are that the U.S. has a history of systematically oppressing Indigenous, Black and People of Color, people living in poverty, individuals who identify as Queer and Trans, and numerous other social groups, and that this oppression continues today as a result of policies and practices, many of which are not intentionally or explicitly oppressive (Kendi, 2019).

We interviewed fifteen students, nine of whom self-identified as male and six as female. Most of the students matriculated at Hill High Charter high school with the remaining from one charter and one private middle school. Charter schools in the U.S. are publicly funded, yet independently operated schools that tailor their curriculum to the specific needs of their students. A charter school operates under a "charter" and is accountable to a non-profit or government agency. The demographics of the students are presented in Table 2. Students in 9th grade typically take Math 1 unless they are above grade level (Math 2). If a student is performing below grade level in high school, they attend a second mathematics course, called Foundations, to help students attain grade level. Students on grade level in 8th grade take regular mathematics unless they are above grade level (Math 1).

3.3.1 Interview tasks

The interview tasks were designed to elicit students' critical mathematics consciousness. Interviews lasted from 25 to 45 min and took place at the student's school in a private setting during a non-academic class period such as physical education. One member of the research team conducted the interview while a second team member took field notes and video recorded the session. The interviewer began each session with general talk, attempting to build rapport with the student. Then, all students but two were asked to reveal their thinking on each of five tasks that can be found at <https://bit.ly/mathethics>. Although there were five total tasks, *It's Black and White* and *Corona Crisis* were the most relevant for this analysis because they centered on sociopolitical and communicative awareness more strongly than the other problems.

Table 2 Demographics of participants and schools

Student	Grade	Self-identified race	School	Sex	Math class
Rene*	9th	White	Hill High Charter	Female	Math 1; Foundations
Jasmine	9th	African-American**/Jamaican	Hill High Charter	Female	Math 1; Foundations
Charles, Matthew	9th	White	Hill High Charter	Male	Math 1; Foundations
Greta, Jade	9th	White	Hill High Charter	Female	Math 1
Nicholas, Collin	9th	White	Hill High Charter	Male	Math 1
Ghalen	9th	African-American	Hill High Charter	Male	Math 1
Tony	9th	White	Hill High Charter	Male	Math 2
Miya	9th	Asian	Hill High Charter	Female	Math 2
Jamal	8th	African-American	Lakeview Charter	Male	Math 1
Tyrone	8th	African-American/White	Lakeview Charter	Male	Math 1
Eduardo	8th	White/Hispanic	Lakeview Charter	Female	8th grade math
Ella	8th	White	Paradise Bluffs Private	% Black	% Hispanic
School		% proficient Math 1	% White		
Hill High Charter		30	65	22	5
Lakeview Charter		63	70	13	5
Paradise Bluffs Private		80	75	13	1

* All names are pseudonyms. ** We use the word "Black" throughout the manuscript to conform to the current preference in critical scholarly publications. In this table and elsewhere, we use the term "African-American" when the student has self-identified using that specific term or when it is used in a direct quote by a student or interviewer

It's Black and White PART 1

The following graph appeared in a YouTube news video with the following statement from the YouTuber:

It's a myth that police kill more blacks than whites.

It's Black and White Part 2

Interviewer: I was interested in the post from the YouTuber too. I went to the United States Census website and downloaded the population data into the graph below.

Interviewer: What does this graph tell you?

Interviewer: Does it change anything you thought about the YouTuber's graph and statement? Elaborate.

Interviewer: If you were going to write a comment or post a response on the YouTube video, what would you say?

Fig. 1 It's Black and White interview question

In the first part of *It's Black and White* (Fig. 1), the student is shown a graph of the number of people shot and killed by police in the United States and told that someone had posted it on YouTube with the statement: *It's a myth that police kill more Blacks than Whites*. Data on police shootings in the U.S. indicate that Black men are the victims of police violence at a disproportionate rate than Whites and Latinx (example data at <https://policescorecard.org/>). The student is asked if she thinks the graph supports the YouTuber's statement and would she post any questions to the YouTuber. After the student responds, she is shown a graph of data taken from the 2010 Census illustrating the population of the U.S. broken down by percentage of race and ethnicity. The student is then asked to compare the Census graph to the YouTuber's graph and determine if she still agrees with the statement. She is again asked if she would post a comment or question to the YouTuber's post. The intent of this interview task is to explore how students use proportional reasoning to draw conclusions about an arguably prejudicial claim made in a social media format (sociopolitical mathematical awareness) and to determine if students are aware of the way that mathematics can be manipulated to convince

Corona Crisis Part 1

The President of Commerce Bank is concerned about how the Coronavirus is impacting the economy. He tweets the following graph on Twitter to his followers:

Interviewer: Would you take his advice? How did the graph help with your decision? Explain.

Corona Crisis Part 2

The Center for Health and Disease Control posted the following information via Instagram.

130 likes
CHDC Our data shows that Coronavirus still a high risk. Stay indoors until we issue a Level 1 warning. Follow @CHDC
View all 39 comments

Add comment...

Fig. 2 The Corona Crisis interview task

the public of different possible interpretations of the media (communicative mathematical awareness).

For the *Corona Crisis* task (see Fig. 2), we capitalized on the ongoing COVID-19 pandemic that had just become major news in the United States when we began interviewing students. *Corona Crisis* is also presented in two parts; first as a tweet from the president of a fictitious bank who was using the data from Darden Data, Inc. to suggest that infections were declining. The student is asked if she would take his advice and if she used the graph to make that decision. Then, the second graph is presented as an Instagram post on the Center for Health and Disease Control's (CHDC) Instagram account with the statement that the disease is still a high risk and there is a Level 1 warning. Students are shown the second graph, told that the CHDC had used the same data set from the Darden Data, Inc. and asked to compare posts to determine whose advice they would follow and why. This task was designed to explore students' awareness that mathematics can be used to manipulate society in both positive and negative ways (communicative mathematical awareness). In the next section, we describe our data analysis method for the two problems.

3.4 Data analysis

To analyze the data, we first transcribed all 15 interviews. Since the two interviewers were White and unfamiliar to most students, we were concerned that students may be uncomfortable sharing their genuine feelings about race. Thus, the data from the two African-American students from Hill High Charter were not used in the analysis. The three remaining students of color, Jamal, Tyrone, and Eduardo, were friends of the one of the interviewers, and we were confident that their interview responses were genuine. We used the *Sociopolitical Mathematics Awareness Growth Framework* to document the CMC that arose as students analyzed the *It's Black and White* task. The researchers independently coded each transcript and met as a group to resolve conflicts in coding and determine agreement when possible. We also looked for patterns in reasons for why students may or may not post responses to the YouTuber (Critical Mathematics Agency). Finally, the research team independently looked for students' communicative mathematical awareness across the *Corona Crisis* task to identify potential themes and met as a group to compare, contrast, and finalize themes.

4 Findings

4.1 It's Black and White analysis

It's Black and White was designed to explore how students use proportional reasoning to critique a potentially prejudicial statement made on a social media news platform and to gauge students' awareness of the way that mathematics is used to manipulate public perception, emotions, and behaviors. Two major themes emerged from analyzing students' responses to the task: (1) varied sociopolitical mathematics awareness and (2) the interplay between sociopolitical mathematics awareness and mathematical reasoning.

Table 3 Summary of the students' awareness and agency as evidenced by the It's Black and White task

Student	Sociopolitical mathematics awareness	Proportional reasoning	Agency to act
Rene	Systemic semi-transitive	Not demonstrated	Not demonstrated
Jasmine	Not analyzed		
Charles	Dysconscious	Not demonstrated	Not demonstrated
Matthew	Dysconscious	Not demonstrated	Not demonstrated
Greta	Dysconscious	Not demonstrated	Not demonstrated
Jade	Systemic semi-transitive	Demonstrated	Not demonstrated
Nicholas	Dysconscious	Demonstrated	Not demonstrated
Collin	Intransitive	not fully demonstrated	Not demonstrated
Ghalen	Not analyzed		
Tony	Inconclusive	Demonstrated	Not asked
Miya	Inconclusive	Demonstrated	Demonstrated
Jamal	Intransitive	Demonstrated	Not demonstrated
Tyrone	Critical transitive	Demonstrated	Demonstrated
Eduardo	Disempowered	Not demonstrated	Not demonstrated
Ella	Intransitive	Demonstrated	Inconclusive

4.1.1 Varied sociopolitical mathematics awareness

Students exhibited varied levels of *sociopolitical mathematics awareness* while reasoning about *It's Black and White*, as identified in Table 3. Three students exhibited intransitive, four dysconscious, one disempowered, two systemic semi-transitive, and one critical transitive consciousness and two were indiscernible. Two of the students (Jamal and Collin) who exhibited intransitive consciousness did not demonstrate an awareness of oppressed groups in the interview task. For example, Collin noticed the proportionality between White populations in both graphs relative to the total population but failed to notice the disproportionality between the Black populations relative to the total, resulting in his inability to consider issues of oppression altogether. Jamal noticed the disproportional Black deaths, noting that the Black population would “take more damage since their percentage is only 12.6%”; however, he did not identify causes for the disproportionality. In contrast, the four students who demonstrated dysconsciousness (Charles, Matthew, Greta, and Nicholas) were aware of differential outcomes between Black and White citizens but attributed such outcomes to personal decisions and behaviors. For instance, two students commented on how it was “sad” that race had to be studied at all.

Greta: I feel like in some points it shouldn't matter what their ethnic is, like their ethnic background is, it just matters if they deserved it or if it like needed to be done in some ways. So, like I feel if that, if the police were to do that, then maybe it shouldn't be on their ethnic background as much as it should be if they needed it or not.

Faith in law enforcement was another characteristic demonstrated by students who exhibited dysconsciousness. Such students placed the onus of responsibility on the victims and simultaneously expressed that police officers make decisions based on their job description and responsibilities.

Nicholas: No. No. I believe cops are just trying to do their job. And you know, whatever they see as a problem that they'll take care of.

Charles: I feel like there's more news when the police kills a Black person. There's less news when police kill a White person. There's more people, there's more police that kill White people.

One student, Eduardo, discussed the oppression of Black citizens in the past, but did not discuss modern oppression outside of individually targeted racism. He did not have any questions or comments for the YouTuber; thus, his response was coded as disempowered. In contrast, Rene and Jade demonstrated systemic, semi-transitive awareness through their recognition of the Black population as a currently oppressed group in the U.S.

Jade: Um, well there's a lot of like oppression of people of color like a long time ago and there still is. So I think that people still do things based on the color of your skin... Like more like Black people getting shot because they're Black. Less White people because people trust them more for some reason.

Rene and Jade's inability to make a critical conclusion or demonstrate personal responsibility to dismantle oppressive systems is what prohibited us from concluding that they possessed critical transitive consciousness. Although all three students referenced Black and White relations in history, Rene was unable to decide if she agreed with the YouTuber or not. Jade ultimately disagreed with the statement, but her findings did not lead her to question the post, implying a lack of agency.

One student, Tyrone, demonstrated critical transitive consciousness through his attention to systemic racial oppression and his commitment to transparent communication to the public. Prior to seeing the census graph, Tyrone questioned the racial makeup of the U.S. population in order to determine if the percentages of those being shot are proportional to the racial composition of the national population.

Tyrone: Cause our population is probably bigger in percentages of Caucasian people than African American people...Without the data it would be completely unfair. Because technically, proportionally, more Black people are getting shot.

Tyrone further demonstrated critical mathematical agency by making recommendations to the author of the post. He highlighted the importance of including both graphs to show the overrepresentation of Black people among the victims of police killings and recommended that the news source include an explanation of how the census graph influences the meaning of the original data. Tyrone not only recognized that mathematical representations can be used to miscommunicate (communicative mathematical awareness) but also attempted to accommodate readers who may not possess the quantitative literacy to interpret the data.

Tyrone: I think you should talk about how that [second graph] changes the first part of the data...Um, cause some people may not get what the second part is there cause it shows all this data [and that it's a] myth that police killed more Blacks and Whites, and then here, nothing. So I think they should be like, "but here's the data on the population and that's how it changes the other part."

4.1.2 Interplay between sociopolitical mathematics awareness and mathematical reasoning

Sociopolitical mathematics awareness and mathematical reasoning were found to both support and inhibit one another. While Tyrone's proportional reasoning skills supported his existing sociopolitical mathematical awareness, reasoning about the two graphs presented an opportunity for Jade and Tony to develop sociopolitical mathematical awareness. Though Tony demonstrated dysconsciousness throughout a majority of the interview, his analysis of the *It's Black and White* task indicated that his sophisticated mathematical reasoning might support the development of sociopolitical mathematical awareness.

Tony: I mean I feel like this one [second graph] shows this [first graph] to be a little less factual. Because if there's more White people than Black people, but still like half the amount of Black people are being killed by police [referring to first graph] than White people? Over here there should be more Black people [referring to the second graph] if it was equal. I mean like the ratios for like how many White people there are to how many are shot, how many are dying are different.

Unfortunately, Tony was not asked how he might respond to the YouTuber resulting in our inability to determine whether his proportional reasoning fostered sociopolitical mathematical awareness. In contrast, Jade was able to develop systemic, semi-transitive consciousness through mathematical investigation. Though she agreed with the YouTuber's statement initially, her proportional reasoning skills supported her discovery of the racial disproportionality between Black and White gunshot victims.

In contrast to Jade and Tony, Rene's limited proportional reasoning seemed to impede her ability to make a critically conscious conclusion. Rene consistently referenced issues of Black oppression and noticed the difference in size between the White and Black populations but was unable to see that the data supported her initial claim.

Rene: I'm like, why they would shoot more, like the White, it's not trying to be, I can kind of see why. Cause there's like more Whites [pointing to the population graph]...there's less of a Black... I'm still like stuck on like the history part of like Blacks. I don't know why... I'm still a little 50-50.

For other students, lack of either sociopolitical mathematical awareness or proportional reasoning seemed to impede demonstration of the other. Three students, Miya, Collin, and Nicholas, possessed proportional reasoning skills but demonstrated a lack of sociopolitical mathematical awareness, which influenced their conclusions differently. For Collin and Miya, the absence of sociopolitical mathematical awareness left them without motivation to investigate the overrepresentation of Black victims. For instance, Miya was able to compare corresponding percentages between the graphs, but agreed with the YouTuber based on the raw data. In contrast, Nicholas was *able* to disprove the YouTuber's statement by reasoning proportionally, but did not have any desire to act. Unlike the others, he noted that the Black population would deplete faster if they continue to be killed at the rates demonstrated in the first graph, but ultimately declined the opportunity to critique the author of the post, likely due to his dysconsciousness.

Charles, Matthew, and Greta demonstrated a lack of proportional reasoning and sociopolitical mathematical awareness, resulting in their inability to recognize the racial disproportionality among Black and White gunshot victims. Charles, for instance, compared

pictures and wedge sizes but was unable to draw a mathematical conclusion about either population. He saw that there is a difference between the size of the Black population of gunshot victims and the Black population in the U.S. but did not conclude that a disproportionality exists.

In summary, students demonstrated varied levels of mathematical reasoning and sociopolitical mathematical awareness depending on their personal/mathematical experiences, which both supported and inhibited one another. Those with some sociopolitical mathematical awareness were more likely to investigate the claim of their own volition, while those who held sophisticated proportional reasoning skills were more likely to come to a critically conscious conclusion, with little agency demonstrated. Students limited in both demonstrated neither the desire nor ability to investigate and critique the claim or the mathematical representations used to support it. In contrast to *It's Black and White*, we found that the *Corona Crisis* task was more accessible for students with regard to impact and mathematical rigor. In the next section, we present an analysis of *Corona Crisis* and then a discussion of students' ethical concerns with data dissemination in both tasks.

4.2 Corona Crisis analysis

Corona Crisis was designed to elicit students' mathematical interpretation of graphs as well as their awareness of the communication strategies used by the people who create graphs. Since we had not designed the *Corona Crisis* task prior to starting the interviews, there is data on only 11 students. Two main themes emerged from analyzing students' responses to the task: (1) Making judgements about dynamic graphs and (2) exhibiting strong communicative mathematical awareness.

4.2.1 Making judgements about dynamic graphs

When critiquing the first coronavirus graph, all but one student used language that indicated the graphs were still changing; they were living, dynamic pictures. Consider several reactions by students.

Collin: I feel like not yet [going out shopping], once it gets down to like almost to zero and you can make sure everything's okay. Just to be 100% sure that it's not around or any cases are there.

Matthew: But I mean it does look kind of under control, but you don't know what could come. You have like March 30. And then you have April 10 and then that could be like [swipes a finger upward from the end of the graph as if it would sky rocket again] right back up.

Tony: I think he's [banker] speaking a little bit too soon. Cause just because the Coronavirus stats dropped one time doesn't mean it's not gonna jump back up.

The coronavirus context has the unique possibility to elicit students' thoughts about the predictive power of mathematics. The predictive power of models, in fact, is the very reason that attention to mathematical models has appeared so prominently in the public discourse around COVID-19. A model is used to describe the infection pattern that currently exists so that discussions can center on ways to "flatten the curve." Such language in both the public and students' discourse indicates that this type of model is alive and can change. That is, if the

conditions under which the number of infections is decreasing continue, the data should remain in a downward trend unless something affects the infection rate. It is not clear if the students thought that the trend of the curve was simply the natural progression of the virus or if it could be affected by social action such as a “stay at home order.” However, the students did recognize that mathematics can be used to model the real world and they can interpret it to make an informed decision (cf. Skovsmose, 2021).

4.2.2 Exhibiting strong communicative mathematics awareness

The second theme that emerged from students’ explanations concerns both their mathematical reasoning and their awareness of how individuals manipulate graphs to persuade society. Significantly, all but one student noticed that the banker and CHDC graphs contained the same data but used altered visualizations. For a few students, the observation was entirely visual-spatial where they described the difference as one of zooming in or out, much like a graphing calculator or computer. Most of the students, however, reasoned by attending to the increment size on each scale. Furthermore, the student that did not notice the data was the same that took the advice of the CHDC. In fact, the source of the graph was the primary basis upon which a majority of students made their decision about staying at home. However, when asked if they thought the person who created the graph was aware that he or she was changing the way the data was presented, all of the students but one said yes.

Jade: I feel like he’s [banker] definitely saying that because he wants to make profit and because we... we watched like the CNN 10 thing [television news station for students] and like every day in my social studies class which is really interesting...but they had a whole one [video segment] about how it’s affecting the Chinese economy specifically and how it could start to affect our economy which I thought was really interesting. So, I don’t blame him for saying that but that isn’t a very honest thing...and he doesn’t have the authority to say that.

Tony: He might have made his graph on intervals of 2000 just make it look like it’s going down now. He might have tried to trick some people knowing that, the internet is not a very smart place...I mean, he’s showing real stats. I think that he’s self-interested. He wants you to go shopping. He doesn’t really care about your health...[Interviewer: What about CHDC?] I guess they’re just trying to protect you. I would trust them.

Both Tony and Jade recognized that the scale increments had been adjusted and that this technique was being used blatantly by the banker in order to trick the public to go shopping or to profit off people. Jade went so far as to question the banker’s honesty but understood the pressure he was under to manage the economy.

While most students relied on the CHDC, two students argued that the CHDC was attempting to *scare* the public into staying at home by using inflammatory words like “Level 1 warning” and “high risk” whereas the banker was trying to make you feel safe.

Nicholas: Well, I wouldn’t go out immediately. I’d still wait like for news sources to like die down and stop overreacting. This one [CHDC] is made to look more scary. Well [it has] just a bigger spike, but the same numbers if you compare the two charts. But yeah, I would be more scared of this one [CHDC]. No, this one [banker] makes it look less dramatic. But I do think this one [CHDC] is intended to strike more fear into people. I’d

probably go with this one [banker] if I had to like go out and shop just because this [CHDC] looks like a more dramatic fall.

Charles: I scroll through here [mimicking scrolling on the phone image] and I see that: There's *warning*. The word *warning* kind of sets it off. It's scary....And so that makes you kind of want to stay indoors. [The CHDC graph was modified] to let this stress off everyone.

These excerpts show how students recognized that the graphs and, as importantly, the words used in their posts can evoke emotions one way or the other. Therefore, a majority of the students recognized that mathematics, mathematical representations, and the language that accompanies them have the power to influence society's beliefs and/or actions.

In summary, all students were able to access the mathematics in this context easily and used it to "read their world." On the other hand, most students indicated that they did not trust the data to predict the behavior of the virus, which calls into question the students' belief that mathematics can be used to model the real world accurately. Additionally, all but one student recognized that people could present the same mathematical data differently to manipulate society's actions. These learning points have consequences for designing instruction to increase students' critical mathematics consciousness, as we will explain in the "Discussion and design implications" section

4.3 Accuracy and accountability in data dissemination

Across both tasks, students demonstrated concern for the accuracy of data and accountability of those who report it. All but one student for *It's Black and White*, and close to half interviewed in *Corona Crisis*, examined the source, reliability, and/or relevance of the data represented in each graph. Several inquiries of this nature were concerned with the accessibility that the authors of the graphs had to the data, and their authority to make claims. For instance, in *It's Black and White*, Tony suggested that the graph could be incorrect or fabricated based on its source, while in *Corona Crisis*, Jade criticized the authority of the banker and the accuracy of the data:

Tony: Who did this study? Is this like a fact or did you just make this stuff up? Cause I don't think YouTube is a very factual place.

Jade:...because the graph is going down but it's not completely under control and he doesn't have the authority to say, as the president of a bank, compared to the president of the United States or China. He can't really tell everyone that like that's okay because he doesn't, he doesn't really have all the extra information that people working in the government have and that the rest of us don't know.

Although Tony was concerned with the trustworthiness of the source, Jade pondered whether the banker possessed enough authority to make public claims. In Jade's view, the banker does not have access to the most accurate data and thus the accuracy of the graph is in question. Charles and Rene took this a step further to suggest that the *Corona Crisis* graph is not representative of the entire data set. While Charles doubted the accuracy of the data, Rene recognized that the crisis is ongoing and that the graph could change.

Charles: Because, how does he know, because the graph can't tell you the truth, I guess. It doesn't, there could be cases somewhere in the world that we don't know.

Rene: So, like the data is still changing. You can't just take information from one piece of, like evidence. So, like, I mean you could, but like it might not be 100% accurate.

Their concern for the sufficiency of data was shared by students in the *It's Black and White* task, who questioned whether the data represented in the post was enough information to illustrate the phenomena accurately.

Miya: I think this is just like not enough information besides the graph. Like it could, they could tell us more about like, reason [crime], age, gender...

Miya was concerned with the context of the shooting surrounding each death and demographic variables outside of race, and Tyrone worried that the racial disproportionality of gunshot victims would be misrepresented by the purposeful omission of population data. Prior to being shown the census graph, he expressed that the public was being fed an inaccurate picture of the situation.

Tyrone: They didn't put context in it about how big the population was and said that there were more White people who are getting shot in terms of Black people. But it was really unproportional because the actual population showed that like, population-wise, Black people were getting shot more in terms of percentage of the population... I think that person who made that knew that.

Like Tyrone, Miya was concerned that the graphs may be used to mislead the public, proclaiming that sufficient information is necessary for truthful reporting.

Miya: What are you doing?! Information sir! Information. Don't forget other things. It's very important. This, I would feel like it's not telling me enough so I wouldn't necessarily say it was true. So really, I like as much information as possible.

5 Discussion and design implications

Students' reasoning on the two interview tasks revealed some important considerations for designing instruction that might develop privileged students' critical and critical mathematics consciousness. Data from *It's Black and White* provided greater opportunity to uncover students' sociopolitical mathematics awareness than did *Corona Crisis* since the former context involved a popular, public controversy about the disenfranchisement of an oppressed group (Black individuals). However, only four of the thirteen students (Tyrone, Jade, Renee, and Eduardo), two of whom are students of color, exhibited critical thought, while only three (Tyrone, Jade, and Renee) exhibited agency. Tyrone alone demonstrated actualized critical mathematics consciousness through his critique of, and proposed action on, the system. Although this finding is not surprising and could be disconcerting, our analysis of *Corona Crisis* is encouraging. Unlike the context of *It's Black and White*, we posit that students could "see themselves" in the COVID-19 graphs, perhaps enabling them to become mathematically and critically engaged. Since students were experiencing the spread of the virus at the time of the interview, their mathematical interpretation of the task had consequences for their personal safety and the safety of those for whom they care. This, combined with their ability to reason more easily mathematically about the fact that the data was the same in both graphs, allowed students to readily recognize the manipulation by the authors of the graphs. In contrast, the

context of the *It's Black and White* task was “outside” their lived experience as White students, which, along with the complexity of the mathematics involved, impeded their ability to investigate fully and interpret the overrepresentation of Black gunshot victims through a critical lens. Instead, we hypothesize that tasks such as *Corona Crisis*, in which privileged students both see themselves in the data and may be influenced by its interpretation, could serve as a viable starting point for instruction.

In addition to its tangible impact on privileged students, we conjecture further that the *Corona Crisis* task may serve as a plausible starting point since students could readily (a) engage in the mathematics and (b) assess the use of mathematical representations (graphs) as a tool of manipulation. Proportional reasoning is notoriously a difficult mathematical concept (Lamon, 2007) whereas interpreting a line graph is more accessible for students. Once students are primed to consider that mathematics has the power to (mis)educate, a subsequent task could extend the COVID-19 data to include the number of infections broken down by ethnicity to engage students in a conversation about systemic racism. After students see the disparity of COVID-19 infections and deaths within communities of color, they might begin to ask questions about other oppressed communities (e.g., people in poverty) or about other kinds of inequities perpetuated on communities of color (e.g., police brutality, credit denial).

The analysis also leads us to consider how to design instructional tasks to leverage the relationship between students’ critical mathematics consciousness and their mathematical reasoning. The data indicated that if a student did not have strong proportional reasoning, she did not exhibit any agency, even if she possessed some critical awareness. For example, Rene questioned the YouTuber’s claim because she “believes more Black people are killed by police” and that “some White people still probably believe [they are better]...and are still like killing the Blacks.” Yet, she did not have the proportional reasoning skills to be able to draw a definitive conclusion or provide a strong rationale for why the YouTuber was wrong. Had our intent been to develop her proportional reasoning skills, her critical mathematics consciousness would have likely fostered both mathematical growth and critical (mathematical) agency. The task was well positioned to serve as a learning opportunity for Tony, who possessed strong proportional reasoning that provided a potential resource that he could have used to develop sociopolitical mathematics awareness and agency. We hypothesize that the task would have been appropriate to use within classroom instruction even for students like Miya, Collin, and Nicholas, who exhibited moderate proportional reasoning but limited critical mathematical consciousness. They possessed the proportional reasoning necessary to understand comments of other students in a class who noticed the disproportionality in the Black population. We see tasks that effectively leverage students’ quantitative reasoning to develop their CMC or vice versa as uniquely situated to foster the dialectical relationship (or tension) between developing students’ mathematical understanding and critical mathematics consciousness (Gutstein, 2006).

Finally, all students showed critical mathematics consciousness in a variety of ways depending on the context of the problem. This leads us to conclude that critical consciousness, in general, and CMC, in particular, are not levels of awareness that you reach. Rather, a person’s critical reflection and action depend upon the context of the problem and her mathematical reasoning, as well as the lived experiences of the person. For example, students who showed no sociopolitical mathematics awareness or critical mathematics agency in the *Black & White* task did so in the *Corona Crisis* due to either their ability to reason mathematically in this problem or to their personal involvement in the ongoing crisis. Because

their lives were at risk in the second situation, perhaps they were more likely to reflect on the intentions of the person who graphed the data, whereas this personal involvement did not exist for most students in *Black and White*. Based upon these students' reasoning, we believe we now have a data-based starting point for designing instructional materials to grow students' critical mathematics consciousness.

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Data Availability De-identified transcripts of data can be made available upon request. Interview tasks are available at the bitly address provided in the text of the paper.

Code availability Not applicable.

Declarations

Competing interests The authors declare no competing interests.

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