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The future of mathematics education since COVID-19: humans-with-media or humans-with-non-living-things

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Abstract
The COVID-19 pandemic has changed the agenda of mathematics education. This change will be analyzed by looking at three trends in mathematics education: the use of digital technology, philosophy of mathematics education, and critical mathematics education. Digital technology became a trend in mathematics education in response to the arrival of a different kind of artifact to the mathematics classroom. It was thrust into the spotlight as the pandemic suddenly moved classrooms online around the world. Challenges specific to mathematics education in this context must be addressed. The link between the COVID-19 pandemic and digital technology in education also raises epistemological issues highlighted by philosophy of mathematics education and critical mathematics education. Using the notion that the basic unit of knowledge production throughout history is humans-with-media, I discuss how humans are connected to the virus, how it has laid bare social inequality, and how it will change the agendas of these three trends in mathematics education. I highlight the urgent need to study how mathematics education happens online for children when the home environment and inequalities in access to digital technologies assume such significant roles as classes move online. We need to understand the political role of agency of artifacts such as home in collectives of humans-with-media-things, and finally we need to learn how to implement curricula that address social inequalities. This discussion is intertwined with examples.

Keywords COVID-19 · SARS-COV-2 · Humans-with-media · Digital technology · Philosophy of mathematics education · Critical mathematics education

1 Introduction

It is not possible to predict the state of the COVID-19 crisis at the time this article reaches the reader. The effects of the pandemic, and the response to it, have been shocking—with

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lockdowns, masks, and respirators, etc.—and have left most people at a loss. Some "world leaders" say that the virus is "just a cold," while others say we may take months or years to have things "back to normal." There are even those who say that COVID-19 is just a test for a much more serious health crisis that may be still to come. What is certain is that throughout the world, things have changed dramatically and suddenly. The virus has hit all classes of society, though of course it has hit the poor harder. But what are the effects of the pandemic in mathematics education? One effect that was almost universal was a tendency to "go online": shop online, meet friends online, and learn online.

We have moved online because COVID-19 is caused by an invisible virus; it has no cure; and, without a clear pattern, it can cause the death of one person in a few days and cause almost no symptoms in another. Moreover, one may be infected and transmitting but asymptomatic for several days and then become very ill all of a sudden. Though not all "leaders" have taken their advice, most experts and the World Health Organization (WHO) recommend social isolation as the main tool to control, slow down, and hopefully stop the pandemic. All of a sudden, teachers, professors, and educational managers at all levels were put under pressure to develop (mathematics) education online, as the virus can be transmitted through physical contact—both between humans and between humans and non-living things.

Since the beginning of the official history of the International Commission on Mathematical Instruction (ICMI) in 1908, only war has interrupted the International Meetings of Mathematics Education (ICME), according to Menghini et al. (2008). This year, the ICMI decided to suspend ICME-14[1] for a different reason: due to the risk of spreading the coronavirus, traveling and gathering in groups would be unsafe. Some would say that ICME-14 was suspended due to a different kind of war: instead of generals in the background, and soldiers in the field, ready to kill or die, we have the whole of humanity trying to fight this non-living being, a virus. It is debatable whether the war metaphor is appropriate or not for this health crisis, but terminology aside, the crisis can lead us to some reflection on mathematics education. This essay will raise some questions to the mathematics education community that were caused by this non-living-thing: the virus SARS-CoV-2, which causes COVID-19.

Engelbrecht et al. (2020) reported that they had to change the conclusion of their survey paper on digital technology in March to April of this year, as it occurred to the authors that the paper could become dated even sooner than other digital technology survey papers. In normal times, such papers become old because digital technology changes so fast, and we rarely even have the time to implement a given technology in the classroom before a new one comes up. However, at this point, *everything* may become outdated, because we cannot predict the evolution of the COVID-19 crisis, nor whether a new crisis will follow it. The authors decided to include discussion about COVID-19 in the introduction and conclusion of the paper. At the end of the paper, they write:

> The question is, what has this [COVID-19] to do with mathematics education and digital technology? Besides the impact on conferences and on the transforming mathematics classroom we may have to ask broader questions: Digital technology intensified traveling and our way of living, so it is also partly responsible for the present crisis. Is it possible that the use of digital technology can generate a similar crisis in mathematics education? Conversely, if the crisis lasts for a long period, would digital technologies be able to provide alternative ways to implement mathematics education? There is not

[1] https://www.icme14.org/static/en/index.html

> much research on online mathematics education for young children, but if the crisis lasts for a long time, are we going to implement it without sufficient research? If the current crisis is over soon, are we going to develop research on mathematics education for a possible "COVID-2X" crisis? In this paper, among others, we have anthropomorphized media, talking about agency. The notion of humans-with-media as the collective that produces knowledge, may synthesize it, as we discussed in this paper. The COVID-19 virus (SARS-CoV-2) is a non-living being: can we talk about the impact (agency) of COVID-19 on mathematics education and on the world? Engelbrecht et al. (2020, p.838)

This paper will deal with the questions from this excerpt in the following sense: I will discuss how new trends of mathematics education may arise or change with the ongoing crisis, and I will draft responses to some of these questions. Trends in mathematics education can be understood as a response, an answer, to some problem, as suggested by D'Ambrosio and Borba (2010). A working group, or a conference on a given trend within mathematics education, emerges as a response to new demands. I will use the theoretical construct of humans-with-media to connect the COVID-19 crisis to three different trends: the use of digital technology, philosophy of mathematics education, and critical mathematics education. In the context of the trend of digital technology, I will discuss the possibilities and drawbacks of having more and more online education, as well as the new demand for this trend. In doing so, I will revisit the notion of humans-with-media and its perspective of collective knowledge production involving humans and non-human actors such as computers and SARS-CoV-2. This will put new issues on the agenda for philosophy of mathematics education, focusing on the agency of "things" and humans' relation to this virus thing. Finally, I will give a brief history of the trend of critical mathematics education, and I will raise an agenda provoked by COVID-19 for these three trends in mathematics education. I believe that these discussions may be important for us to understand the moment we are living in, beyond mathematics education itself. They can also help to set an agenda of research and action in the classroom for those interested in these trends and their connection to the pandemic.

2 Digital technology and mathematics education

Taking into consideration the notion of trends, presented above, the trend that studies the link of mathematics education and "new technologies"—informatics, communication and information digital technology, and alike—has been present in conferences for more than 30 years. At ERME[2] and SBEM[3] (Borba, 2018), at ICMEs (Menghini et al., 2008), and at PME[4], there are always working groups, discussion groups, and panels on the subject, because, as authors such as Jim Kaput (1991, 1992, 1998) have pointed out, we need to understand how to use computers in mathematics education. Borba et al. (2016) prepared a survey that was presented at ICME-13 and put forward four phases for the use of digital technology in mathematics education. The four phases themselves show the strength and the length of this movement, which has involved many researchers, teachers, and students.

The first two phases, symbolized, respectively, by Logo and by curriculum-topic software (e.g., Cabri-Géomèetre), are not so important for the discussion in this paper, as the Internet

[2] European Society for Research in Mathematics Education
[3] Brazilian Society of Mathematics Education
[4] Psychology of Mathematics Education Annual Meeting

became the big star during the pandemic. The third phase of the use of digital technology was characterized by the emergence of the Internet and online courses. This phenomenon became important around the turn of the century, depending on the country. Some so-called developed countries saw the Internet become popular in the mid-1990s and in some other countries, like Brazil, very early this century. Brazil was one of the first countries to start online courses at the graduate level, at a time when other countries were very protective of their face-to-face education.

The current fourth phase is characterized by the arrival of fast Internet, which reshaped the possibilities of online education. As this phase has developed, Engelbrecht et al. (2020) have pointed out that different forms of blended learning are important, in particular for teacher education. The term "hybrid" has become more important to express the combination of face-to-face mathematics education and online education:

> A wide array of media and technology is available to create new hybrid forms of teaching. The integration of technology enables educators to create learning experiences that actively and meaningfully pull students into course content. "This technology may form thinking collectives (Lévy, 1993) with teachers that can break the walls of the regular "cubic" classroom that is associated with lecturing." (Engelbrecht et al., 2020, p.838)

If we consider a trend as an effort to find answers to a given issue, COVID-19 has pushed forward the agenda of the digital technology trend in mathematics education. With the need for social isolation, it became necessary to offer education to children and undergraduates at home. In most of the world, the first semester of education in 2020 was suspended or went online. Many are now discussing different kinds of hybrid education as health conditions allow students and teachers to go back to school and universities. But although we have plenty of research on implementing education online on undergraduate education (Engelbrecht & Harding, 2002, 2004, 2005), this is not the case for education for children. In the survey articles mentioned above, and in conference working groups, hardly any research has been presented on online education for children. As this theme develops, (mathematics) education will have to deal with structural issues, such as the participation of parents or responsible others in education.

In Brazil, newspapers say that teachers are "going crazy" with demands from students coming from WhatsApp and other social networks, as students and parents in their home cannot deal with school tasks. Grading is another problem: can we grade students so young online? Is help from parents allowed? This type of question has not yet been researched. In Brazil, some research groups such as GPIMEM[5] are trying to document what is happening in some state systems as a first step for research and understanding of online education for children. In the state of São Paulo, a new app, CMSP[6], was created in less than 30 days for 200 thousand teachers and 3.5 million students to somehow have access to education. The app operates in conjunction with two preexisting TV channels, one operated by the state and another by a consortium of universities (Paz, 2020).

[5] Grupo de Pesquisa em Informática, outras Mídias e Educação Matemática [Group for research in informatics, other Media and Mathematic Education] - GPIMEM website: ttps://igce.rc.unesp.br/#!/gpimem

[6] CMSP–Centro de Mídias da Educação de São Paulo. Recovered from https://centrodemidiasp.educacao.sp.gov.br/

Teachers and administrators were able to supervise students through the app to some degree, and students were having three classes a day instead of five, as the state is trying to implement education through other platforms as well (Secretaria de Educação do Estado de São Paulo (São Paulo State Department of Education)—SEED, 2020). But this was a very complex moment: teachers had to go online without enough time to be prepared, and at the same time, they had to deal with their regular problems: São Paulo is the richest state in Brazil but pays its teachers a terribly low salary compared to other professionals, as pointed out to me in an online interview with a teacher who preferred to stay anonymous. Underpaid teachers now have to deal with students 24 h a day, 7 days a week, which includes dealing with students' "personal" problems—including problems associated with the chronic social inequality in Brazil. Teachers with low salaries are not likely to have the best mobile phones, laptops, or Internet plans. Teachers who may teach fifty 50-min classes a week may deal with hundreds of students. It is likely that such problems are occurring in other countries as well, as differences between the "haves" and "have-nots" exist throughout the world, and are amplified by COVID-19, as described by the historian Walter Scheidel (Canzian, 2020).

Crisis is also a chance for change: teachers who teach 50 classes per week will not have time to learn to use digital technology for teaching. With many states and city educational systems forced to go online because of the pandemic crisis, the argument to use technology is very strong. It is likely that we will have a lot of research associated with this new reality. For the purposes of this article, I was not able to collect data systematically, but informal reports from teachers suggest that the reality of teaching young teenagers and children online will have to be investigated. As mentioned before, there is hardly any research on online education associated with levels below high school, which can be verified in many survey papers related to the theme (Engelbrecht et al., 2020). But the focus cannot only be on teachers. How do children experience this version of home schooling? There are also many jokes on social networks about parents losing control as they become home-teachers at the same time as they had to implement the home-office, so the role of parents in online mathematics education may be another area for research. Involvement of parents in mathematics education has been a theme of some research, including involvement associated with the use of digital technology (Ford, 2015; Wilson, 2013). However, this was in informal or blended settings, such as festivals (Domingues, 2020). Now we have new challenges, including to report and discuss how online assessment was developed (or not developed). Inviting students to produce mathematical videos was a research project developed before the pandemic. Having students expressing mathematical knowledge with videos, or doing research with videos, was not a solid trend in the literature. However, video production may be an alternative for education during and after the pandemic. Instead of focusing on test results, we can have students producing videos online to express what they have learned in conditions such as the pandemic. Videos can be produced collectively, with help of parents, friends, and different media. Differences in resources, including degree of parental aid received, can be considered by teachers and school systems in a "non-ranking" type of assessment.

Production of digital mathematical videos by students and teachers is growing in Brazil (see Fig. 1 for an example), and with the onset of the pandemic, an online "library" with more than 600 videos (https://www.festivalvideomat.com/) has been used as a resource for teachers and students in their classes and as inspiration for the kind of task students and teacher may produce. Moreover, issues that have been the subject of previous research may gain new life: in a recent review paper (Engelbrecht et al., 2020), it became clear that different technologies used in a class, from the blackboard to the most modern mobile phone, are not necessarily only

Fig. 1 Mud Sea: Modelling and Mathematics Education. Source: https://www.youtube.com/watch?v=YpCteGqjxd0&list=PLiBUAR5Cdi63gZoTSrJ9qXeiZQEH2wFBL

mediators but also actors. This is an epistemological issue, and it is part of a trend that has been discussed within the psychology of mathematics education and the philosophy of mathematics education.

3 Philosophy of mathematics education and agency in the notion of humans-with-media

"Why do we have education? What are the relations between education and society? How do we know?" These are the basic questions of philosophy of education. For more than 20 years, there have been working groups on the philosophy of mathematics education (Bicudo & Garnica, 2001). "How do we learn?" is connected to "How do we know?," and thus questions regarding epistemology, the theory of knowing, have also been debated by psychology of mathematics education discussion groups. Both domains of research may be seen as trends, as they seek foundations for mathematics education, and they discuss how mathematics education is articulated in the classroom, the research that is developed about it, and its "return" to practical settings: settings, like the classroom, which for many months have been on hold by the coronavirus pandemic. Several authors have discussed classrooms and schools and the artifacts produced there. For example, Villarreal and Borba (2010) have shown how mathematics is produced by collectives of humans-with-artifacts throughout the history of mathematics.

D'Ambrosio and Borba (2010), besides conceptualizing a "trend" as a response to a given problem, have argued that trends are intertwined, using the metaphor of a tapestry. It is unsurprising, then, that the discussion about who is the agent of knowledge is discussed in more than one trend: in digital technology working groups and in philosophy of mathematics education and psychology of mathematics education discussion groups or conferences. Different mathematics education authors (e.g., Faggiano et al., 2017) have claimed that computers, for instance, have agency. Inspired by the work of Lévy (1993) and on the phenomenological approach that humans are "being-with-others," the notion of humans-with-media has been developed over the course of many years. The notion of reciprocal modeling was the first step (Borba, 1993). My early work on this showed not only that different media shape humans (an idea shared with many) but also gave some empirical evidence of how humans shape technology, specifically a piece of software about functions. Being part of the design software team and a mathematics educator developing research, I could see this "collaboration" between, on the one hand, a piece of software—full of the ideas of a multidisciplinary team, presented at meetings of developers, mathematics educators, teachers, and so on—and, on the other hand, how high school students would interact with the software (and with me, a teacher-researcher). A high school student, for instance, was influenced by what I said and by the design of the piece of software *Function Probe* (Confrey, 1991), and he also shaped the piece of software in ways that were not predicted by the multidisciplinary team that had developed the software. This student did not use the commands the design team had created but used the size of the computer screen and other measuring artifacts to coordinate algebra and graphs. Borba and Villarreal (2005) synthesized how the notion of humans-with-media could be understood based on the work of Lévy (1993), Lave (1988), and Tikhomirov (1981). This led to the notion that knowing was not social solely in the sense that it involves more than one person, but that it also involves things.

The notion of humans-with-media was proposed to emphasize that production of knowledge is a result of a collective of humans and things. From Tikhomirov and Lave came the idea that knowing was goal oriented and that values were involved. Later, in Borba (2012), discussions about the values, emotions, and media involved in knowing mathematics with GeoGebra (or whatever software was available) were extended to the idea that media and technology themselves change notions of what humans are. Media are therefore constitutive not only of what we know but also of what we are. Kaptelinin and Nardi (2006) also analyzed the idea of extending agency to non-humans. These authors compared the capacities to produce effects, act, and fulfill intentions of different agents: things (natural), things (cultural), non-human living beings (natural), non-human living beings (cultural), and human beings as social entities.

Agency, therefore, should not be seen as binary, as either present or absent, but having different levels. I see this notion of agency as a "fuzzy" one, as in fuzzy mathematics, in which we may have degrees of agency. In such a mathematics, for instance, my jeans are not just blue or not (zero or one), but they are, for instance, 0.6 blue. Kaptelinin and Nardi (2006) suggest three dimensions of agency: based on necessity (action is taken based on biological and cultural reasons), delegated (things or living beings act as the perceived intentions that are delegated by other humans and things), and conditional (actions of things or people which result in unintended effects).

The notion of humans-with-media, which is consistent with a more complex view of agency, has been challenged, in many instances, by arguments that want to preserve the power of a human as the center of any action. In these views, intentionality and action come from somewhere that is not social. Much of mathematics education, cognitivist or not, is based on such a "one-knower" view. From such a perspective, the agent of knowing is a single person, or collective of humans, even though most researchers would recognize the influence of artifacts, environment, and social cultural factors.

The notion that both humans and non-humans have agency is part of an effort to model artifacts—in particular, pieces of software, hardware, and the Internet of Things (i.e., things that are connected to the Internet)—as the historical, social, and cultural factors in the collective that produces knowledge. It stresses a view that knowledge is produced (both from a philosophical and a psychological perspective) by humans-with-artifacts. With a perspective in which things have agency, artifacts are labeled media as they are thought to communicate. This argument was more easily applied for technologies of intelligence (Lévy, 1993): humans-with-graphing-calculators were easier to accept as having agency than humans-with-libraries or humans-with-classrooms.

Regardless of whether readers value online mathematics education or not, they may at some point use their memory of a regular classroom to claim that face-to-face interaction is fundamental to any learning that occurs in mathematics education. Alternatively, one may use the notion of a "distributed classroom": an office for one student, the bedroom for another, and some kind of computer center for others. But everyone would recognize that classrooms are changing. We have described this as a classroom in movement (Borba et al., 2014).

What constitutes the unit of knowing is an endless, philosophical discussion: is it a single person? Is it social because it involves more than one person? Is it social because it has a goal and it involves humans and non-human actors? It is an endless discussion, like most philosophical discussions. However, it seems that the emergence of SARS-CoV-2 gives strength to one perspective on knowing because, according to authors such as Racaniello (2004, p.1), "Viruses are not living things. Viruses are complicated assemblies of molecules, including proteins, nucleic acids, lipids, and carbohydrates, but on their own they can do nothing until they enter a living cell. Without cells, viruses would not be able to multiply. Therefore, viruses are not living things." Yet despite being non-living, the virus has dramatically changed the way humans live. Viruses are closely connected to us: they cannot exist for long apart from living things, like humans, who have cells; the symptoms of COVID-19 arise under certain conditions when the virus is inside human cells. We can say that the virus has agency in the sense that it has changed the way we have to do things. This analogy helps us to understand how certain things are much more likely to happen if certain actors are present. To use the metaphor of the virus, software also needs humans to "survive." Software, and later on the Internet, has changed the environment of educational settings, in a similar way to how SARS-CoV-2 has suddenly turned children's bedrooms into classrooms.

Latour (2020a, b), another inspiration for the notion of humans-with-media, presents his concern with the virus crisis in a way that relates to the discussion in this paper:

> But there is another reason why the figure of the "war against the virus" is so unjustified: in the health crisis, it may be true that humans as a whole are "fighting" against viruses — even if they have no interest in us and go their way from throat to throat killing us without meaning to. The situation is tragically reversed in ecological change: this time, the pathogen whose terrible virulence has changed the living conditions of all the inhabitants of the planet is not the virus at all, it is humanity! But this does not apply to all humans, just those who make war on us without declaring war on us. For this war, the national state is as ill-prepared, as badly calibrated, as badly designed as possible because the battle fronts are multiple and cross each one of us. It is in this sense that the "general mobilization" against the virus does not prove in any way that we will be ready for the next one. It is not only the military that is always one war behind. (Latour, 2020a, b, para.8)

Latour, without saying so explicitly, foregrounds the agency of this virus: SARS-CoV-2 spreads through humans to survive and reproduce, and this action provokes reaction—agency—from humans. Of course, every comparison or metaphor has its limits. But the coronavirus has transformed our lives—we still do not know for how long—in a dramatic way. Computers—now represented by mobile phones, which are much more potent computers than the ones used at the end of the last century by the minority of students who had access to them—have changed the way we can experience mathematics, in particular the way we can "experiment" with mathematics. The Internet has become a community, an agent, and an artifact. Videos that are produced and shared by students with digital technology soon themselves become a part of new collectives of humans and media that are involved in producing knowledge. Souto and Borba (Souto & Borba, 2016, 2018) have discussed how the notion of humans-with-media, which had its origins in activity theory (Tikhomirov, 1981), is now about to change the third generation of activity theory, breaking the rigidity of the triangles espoused by Engeström (2002) and Sannino and Engeström (2018) (Fig. 2).

This version of the humans-with-media construction has been called system-of-humans-with-media (Souto & Borba, 2018) to emphasize even more the notion that the collective of humans and non-humans is goal oriented and embedded in a community that has rules (Fig. 2). Considering media as agent has made it possible to think of the rigid triangles of the third generation of activity theory as dancing triangles, or as a GIF, in which the Internet, for instance, could be jumping from the instrument corner to the subject corner and/or to the community corner. Such an animation can be found on the GPIMEM website, in order to overcome the limits of the printed text (https://igce.rc.unesp.br/#!/pesquisa/gpimem—pesq-em-informatica-outras-midias-e-educacao-matematica/animacoes/triangulo-sannino–engestrom/).

It is hard to know, as mentioned before, where the developments of the current health crisis will take us, but it seems that thinking about agency of non-living things as discussed in this section will be part of it. Questioning what the definition of "living things" is may be another consequence, which, of course, goes beyond what has been called the psychology of mathematics education or philosophy of mathematics education. But it will be relevant to some questions that perhaps were put aside or never asked before, questions such as: What are the specific roles of spaces/artifacts such as the classroom, face-to-face environments made for the intense use of Internet in education, and the "online classroom?" If the pandemic lasts even longer, what do we really mean by "face-to-face?" What does it mean to discuss affection in mathematics education without physical contact (e.g., hand shaking, hugging, kissing the cheek), so important in many parts of the world? The whole discussion about humans-with-media may gain a new dimension, as suggested in this section, related to some of the basic questions of philosophy of (mathematics) education. The pandemic foregrounds the role of

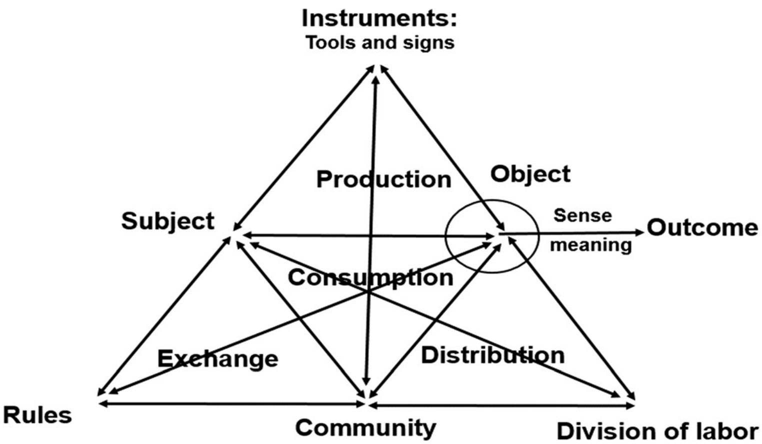

Fig. 2 The structure of an activity system. Source: Sannino & Engeström, 2018

Fig. 3 Social inequality. Source: Of "Com 1% do país concentrando 28% da renda, Brasil não tem como dar certo..." L. Sakamoto, 2020. Recovered from https://noticias.uol.com.br/colunas/leonardo-sakamoto/2020/12/15/com-1-do-pais-concentrando-28-da-renda-brasil-nao-tem-como-dar-certo.htm?fbclid=IwAR3cAed7k9bb4qhHWhi7uAtZVhgCLFz9J-yx1dPuoW5rAS1xqVfgey6YrOc

home and the role of different parents and different social conditions in collectives that construct knowledge, in activity systems that produce knowledge. The idea of seeing fuzzy agency in non-humans should be developed further to include not only good access to internet, but to housing, which is a site of brutal inequality in Brazil and elsewhere. This famous photo (Fig. 3) illustrates the extent of inequality in Brazil, which, from the educational point of view, suggests that different housing may have different agency in constructions of knowledge, in particular in situations such as the one we lived during the pandemic. Housing matters in knowledge construction. Trying to solve a mathematics problem in a crowded house in a slum is very different than doing so in a spacious, luxurious apartment with a veranda.

In this sense, SARS-CoV-2 has pushed homes into the center of a collective that produces knowledge. Once again, we ask all the basic questions of the philosophy of mathematics education and psychology of mathematics education. What is the role of mathematics education? What is the role of the different education of parents in mathematics education? What is the role of non-living things, such as viruses, pieces of software, and homes, in the way we know and learn mathematics? A question that may be more critical is: What is the role of mathematics education for resisting inequality in the world?

4 Critical mathematics education and coronavirus

The trend of critical mathematics education (CME) responds to the main problem of social inequality in (mathematics) education and struggles against the view that mathematics is a branch of science that is separate from social, cultural, and political issues. CME's role in the community of mathematics education is to remind us all about social inequality and other types of inequalities. CME may be said to have been officially born in 1990, in a meeting at the Cornell University in the USA (Powell, 2012; Torisu, 2017). There, the Critical Mathematics Educators Group was founded, with several members[7], focusing on the key phrase "social justice." Powell (2012) reports on how at ICME 6, in Budapest, Hungary, there was a meeting of researchers and how after the Cornell meeting, the group began to meet regularly, starting at ICME 7, in Quebec, Canada.

Present at the Quebec meeting was Skovsmose (1994), who also wrote about the development of critical mathematics education in Europe. Skovsmose shows the connection of this branch of CME in Europe to the Frankfurt School of Critical Education, one of the main representatives of which was Adorno, whose main issue was seeking an education that would prevent Nazism from occurring again. Today, critical mathematics education is more than important, in a moment in which countries such as the USA, Brazil, and Italy have far-right or fascist leaders, who have praised some of the fascist leaders of the twentieth century.

In the Cornell meeting, issues of social inequality, the role of mathematics in society, the ideology of certainty, and research methodologies appropriate to CME were presented (Borba, 1991; Borba & Skovsmose, 1996; Skovsmose & Borba, 2004). Since the 1990s, in Africa, authors such as Paulus Gerdes, from Mozambique, developed curricula and research about African traditions in mathematics and how to incorporate them into mathematics education (Gerdes, 2010; Torisu, 2017).

Development of curricula and pedagogical perspectives that highlight social inequality, gender and racial inequity, and the ideology of certainty was the initial focus of CME. More recently, environmental issues, and issues that were treated in other trends (e.g., mathematics education to the deaf or the blind), were brought into the agenda of CME. In sum, CME is a trend that shows that education is not neutral: it can promote equality or inequality. There are indicators already from Forbes that social inequality is growing during this pandemic: the billionaires are becoming even richer (Gavioli, 2020). The owners of Facebook and Amazon are among them! There is no need to be a mathematician to understand that this concentration of wealth upward means that the rest of the people have less. The owners of tech companies stand to gain as people move more and more online: their companies run online social networks, run online shopping services, and store digital data in online systems worldwide.

As I have already illustrated, social inequality is also growing in schools. As most schools and universities suspend face-to-face classes and go online one way or another, the issue of access has been a barrier to some and a trampoline to even more social inequality. Some universities in Brazil even opted not to resume education online because of inequitable access; but of course, as the university is not the only source of knowledge, online education also may have caused more social inequality. Here is an example from (mathematics) education in Brazil of a Catholic school located on the outskirts of a midtown city in the state of Sao Paulo: the school does not charge tuition for students, as parents do not earn enough income to feed their families; violence is also part of the daily experiences of these children. Teachers are paid above average (considering Brazilian standards), and from interviews with them, it is easy to see their engagement in fighting social inequality. Classes were first suspended in mid-March

[7] Alan Bishop, Arthur Powell, Claudia Zaslavsky, David Henderson, Dorothy Buerk, Europe Sign, George Gheverghese Joseph, Kelly Gaddis, Marcelo Borba, Marilyn Frankenstein, Marty Hoffman, Munir Fasheh, Paul Ernest, and Sam Anderson

2020 and resumed online afterwards, at different moments of April, depending on the school. Two teachers, Luiz Felipe Trovão (mathematics educator) and Karla Cristina Stropa Goulart (science educator), who were asked to answer an open question about their experience with teaching during the pandemic, reported how hard it was to communicate with students. Most students did not have access to the Internet. When they had access, they did not have the money to buy credits to operate the Internet[8]. The school tried to overcome this problem by providing chips with credits or sending printed didactical material to the children. But with less interaction with teachers, and without an environment to study in poor homes, through no fault of the teachers or the school, very little mathematics education or science education occurred. Trovão said that it is almost impossible to teach geometry online without proper interaction: homes, Internet access, etc.

The billionaires are becoming even richer; the poor are having even more difficulty accessing mathematics education: this may foreground the need that children will have, after the pandemic, to understand what happened. Mathematics educators may have to explore some tough topics: exponential functions to explain the spread of the coronavirus and how the richest grew even richer. Mathematics will not be enough, but a new agenda will be generated. Freire's (1968) work about the pedagogy of the oppressed will be even more important. Putting together the agenda for the three trends, one should consider, for example, the role that home, as a physical and emotional "thing," has in the pandemic school. We have collectives of home-parents-internet-student-teacher as the minimal unit of the collective agent who produces knowledge. Home and parents, things and humans, have added more to social inequality and to discussions about how to use digital technology in mathematics education.

Humans-with-media, seen as an activity system, provides a dynamic epistemological view that we can use to understand the different social aspects (in the micro- and macrolevels) of the research of digital technology. Simultaneously, in acknowledging agency in a wide variety of things, not only computers, it will be possible to structurally show social inequality: homes equipped differently cannot be assessed the same way. Children will suffer even more injustice than they suffer in school, if differences in Internet access, the comfort of home, etc., are not considered in assessment and teaching. Research under this frame, in digital technology, critical mathematics education, assessment, ethnomathematics, and other trends, may help to bring light to more epistemological discussion that is not value-free.

5 The three trends interacting

During the pandemic, "Lives" have become a craze in Brazil: presentations streamed over the Internet by artists, educators, and others. First, artists began holding Live presentations to incentivize people to stay at home. Soon after, other types of workers, such as mathematics educators, started holding our own Live presentations. During this pandemic, I have given many Lives produced by collectives that included Geogebra, the Internet, my home, and various broadcast software. The discussions of the mathematics of the pandemic and the sigmoid curve and its derivative were used

[8] In Brazil, most people will not have unlimited access to Internet in their cell phone. Especially if you are poor, you typically buy credits for Internet and pay as you go.

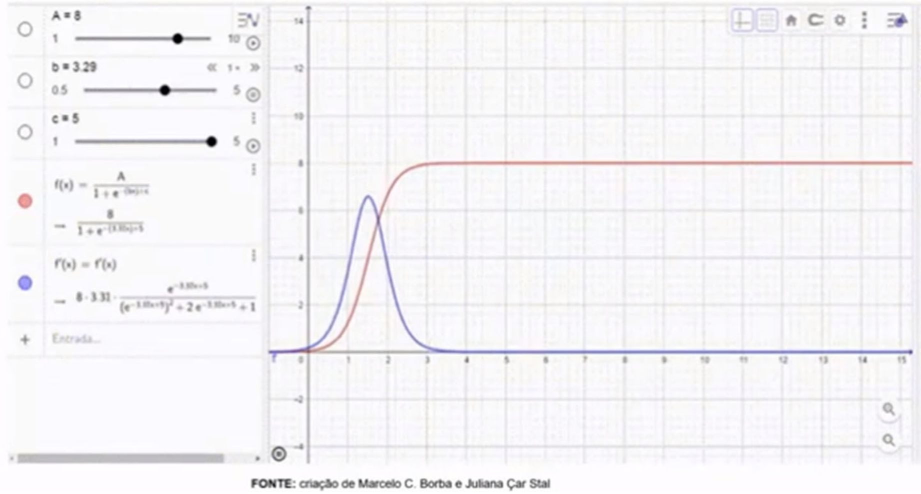

Fig. 4 COVID-19 flattening curve. Source:https://www.youtube.com/watch?v=XYNMuaPm654&list=PLiBUAR5Cdi60qXjrzAVdhOgufWVuWTdl2&index=16

in possibly thirty Lives. Figure 4 is a screenshot from a short video that shows this curve dynamically: https://igce.rc.unesp.br/#!/pesquisa/gpimem—pesq-em-informatica-outras-midias-e-educacao-matematica/animacoes/curva-epidemica-no-geogebra/.

The derivative of the sigmoid was used to explain why it was both possible and important to “flatten the curve.” Different curves, with faster or slower growth, were associated to the roles of prevention, social status, and different kinds of homes. Examples of this type of “virtual classroom,” outside of the school/university context, illustrate how much the three trends analyzed in this paper can be powerfully intertwined. This calls for research to understand what kind of mathematics education is being experienced by those who synchronously or asynchronously viewed the Lives.

6 Discussion and conclusion

Most of mathematics education is supported by empirical papers. In the 1970s, most research was quantitative, and data was used to “prove” that a given method of teaching was better than another. Empirical data had the same role it plays to this day in a good part of what is considered science: there were control groups and experimental groups, and the methodology was based on (or reduced to) statistical treatment and conclusions. Later last century, and earlier this century, qualitative research has swung the pendulum in another direction. Qualitative research sees data as a voice, as a complement that should be added to other evidence in order to make (“prove”) a point (Borba et al., 2018). Truth was assumed to be explicitly contingent and subject to change long before the COVID-19 pandemic brought so many instabilities to our beliefs. As arguments grew apart from data, a wide set of reactions, including some from powerful funding agencies, emerged. For example, there were funding agencies that require quantitative data in a project. Now the notion of mixed methods is prevalent, even though it is not clear what the role of the data or the view of "truth" is in much of the research published.

Essays such as this paper serve the purpose of discussing ideas and presenting bases for research papers, so that we can know (in the different directions briefly presented above) about mathematics education, in the different epistemological positions that characterize our community. In this sense, this paper is a result of a reflection on how three trends could have their agendas transformed by SARS-CoV-2. Of course, other trends, such as ethnomathematics or early-grades mathematics education, will also be affected. The issues raised throughout this paper should be transformed by readers and should themselves become the objects of research. In this paper, I choose to deal with digital technology, philosophy of mathematics education, and critical mathematics education because the pandemic seems to have played a significant role in the changes of the agendas of these three trends. It seems important to raise new issues in these trends.

Digital technology is now a theme of concern (or research) for everyone (Engelbrecht et al., 2020a, b). The amplification of the starkness of inequality under the pandemic cannot be ignored (except for those who believe that the Earth is flat and that hydroxychloroquine is a miracle cure for COVID-19), and the rise of the home office, associated with home schooling, confinement, and lockdown, may help many to think about philosophical issues regarding the role of "place" in knowing/learning and notions such as humans-with-media.

In the paragraphs above, I have pointed at my choices in identifying important trends. Why did I say "I" instead of "we," which would refer to a collective of humans-with-media? It is a good question, and a tentative answer, in another domain of discussion (qualitative research and its influence in the classroom), was given in Borba et al. (2018). The authorship of a paper or a book may be individual, but it is a result of a collective endeavor of "endless" humans-with-media. This paper[9] has one author, but it involved the active participation of one doctoral student (Juliana Çar Stal), three teachers who lent me their speech (Karla Cristina Stropa Gourlart, Luiz Felipe Trovão, and one who wanted to remain anonymous), the reviewers, the editors of this special issue, members of the research group I belong to, the more than 100 members of the graduate program in mathematics education at UNESP[10], Rio Claro, friends, the computer, the word processor, the home, the office, and, of course, the pandemic, COVID-19. We hope we can discuss this at the next ICME and that it does take place in 2021!

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