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Affect in mathematical problem posing: conceptualization, advances, and future directions for research

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1 Introduction

The importance of this special issue is rooted in the idea that the cognitive and affective fields are interrelated and support each other in the learning process. For example, Pellegrino and Hilton (2012) have pointed out the centrality of both cognitive and socio-emotional dimensions in necessary skills for the twenty-first century. They list complex problem solving and creativity among the basic twenty-first-century cognitive skills that determine intellectual development, career readiness, and adaptation to exponential environmental changes. At the same time, they highlight socio-emotional skills such as positive self-evaluation and collaborative skills, responsibility and commitment, openness and flexibility to a variety of points of view, interest, and curiosity; these are usually considered to be components of affective and social development. From a dialectical perspective, Rothstein (2004) suggested that non-cognitive and cognitive abilities have the potential to mutually reinforce each other to maximize student learning, and Farrington et al. (2012) also maintained that cognitive and non-cognitive factors continually interact in essential ways to foster learning.

Although the importance of affect has recently received increased attention (e.g., Cai et al., 2017; Duckworth & Seligman, 2005; Dweck, Chiu, & Hong, 1995; Farrington et al., 2012; Hannula et al., 2019; Moyer, Robison, & Cai, 2018; Steele & Aronson, 1995), students' affect has been far less studied than their content knowledge and academic skills. Because problem posing is related to complex problem solving and creativity-directed activities (Cai et al., 2017; Leikin, 2018; Silver, 1997), it is a unique mathematical activity that provides multiple opportunities for the advancement of both cognitive and affective competencies as well as their integration. The importance of the integrative development of cognitive and non-cognitive competencies thus

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motivates this special issue's focal question: How is problem posing in teaching and learning mathematics interconnected with affective factors such as motivation, engagement, beliefs, attitudes, values, emotions, feelings, moods, and self-efficacy (Attard, 2014; McLeod, 1992)?

Ultimately, the goal of this special issue is twofold. First, it aims to reflect and analyze the state of the art in research on affective factors associated with mathematical problem posing. Second, by drawing attention to a research gap in this area, the special issue attempts to stimulate researchers to fill the gap through systematic research that integrates advances in the research on mathematical problem posing and the research on the role of affect in mathematics education. Cai et al. (2017) argued that the impact of research on students' learning should be measured through both cognitive and non-cognitive outcomes in both the near term and longitudinally. Some sociologists and economists have even argued that a well-developed set of non-cognitive traits and skills has a more durable impact on a person's school and job performance than mere academic subject knowledge (e.g., Garcia, 2014). With respect to affect and mathematical problem posing, this special issue focuses on interactions between the two as well as their integration in mathematics instruction and research on mathematics instruction. Thus, this special issue not only answers existing calls for much-needed research on mathematical problem posing (e.g., Cai et al., 2015) and affect (e.g., Goldin et al., 2016) but also presents advances in research about affect in mathematical problem posing.

2 Research on problem posing in mathematics education

Although problem-posing research is still a relatively new endeavor, there have been efforts to incorporate problem posing into school mathematics at different educational levels around the world (e.g., Brown & Walter, 1983; Cai & Hwang, 2020; Cai, Hwang, Jiang, & Silber, 2015; Felmer, Kilpatrick, & Pehkohen, 2016; Kilpatrick, 1987; Leikin, 2015; Silver, 1994; Singer, Ellerton, & Cai, 2015). These efforts indicate interest among many practitioners in making problem posing a more prominent feature of classroom instruction. Despite the interest in integrating mathematical problem posing into classroom practice, our knowledge remains relatively limited with respect to the processes involved when learners generate their own problems, the instructional strategies that can effectively promote productive problem posing, and the effectiveness of engaging students in problem-posing activities. Furthermore, we still know very little about those affective aspects of problem-posing processes that could inform and enhance the implementation of problem posing in classrooms. That said, it is clear that the mathematics education literature reflects several perspectives on problem posing linked to cognitive and affective characteristics of teaching and learning of mathematics. Note, however, that we do not directly address the social component of teaching and learning; although obviously essential, it is beyond the scope of this special issue.

In general, problem posing in mathematics education can be considered in line with activity theory, which draws connections between goals, actions, conditions, and tools in any human activity (Leont'ev, 1978). Thus, we suggest that the mathematics education research literature on problem posing, including the research presented in this special issue, can be categorized in one of four ways based on how problem posing is viewed: (1) research on problem posing as a tool for mathematics instruction, (2) research on problem posing as a goal of mathematics instruction, (3) research that uses problem posing as a tool to investigate other phenomena of interest, and (4) research that studies problem posing as a goal (i.e., that treats problem posing as a phenomenon that is itself the object of study).

Problem posing as a tool for mathematics instruction views problem posing as a means by which teachers can help their students learn mathematics. Research in this vein investigates how problem posing can serve to improve students' learning of mathematics (as measured in terms of both cognitive and non-cognitive aspects) by engaging students in problem-posing activities (Brown & Walter, 1983). Studies of this type can focus on a variety of learning goals. For example, Chen and Cai (2020) studied the development of a teacher's use of problem posing to help her students deepen their knowledge of the distributive property of multiplication over addition. Koichu (2020) analyzed problem posing in the context of teaching for advanced problem solving. Problem posing can be also employed for the development of mathematical creativity (Matsko & Thomas, 2015). Additionally, problem posing can be used to develop teachers' competencies, including their sensitivity to students' mathematical thinking (Xu, Cai, Liu, & Hwang, 2020). Problem posing also can be used as a problem-solving tool for reformulation of problems and sense-making (Cifarelli & Sevim, 2015).

Research on *problem posing as a goal of mathematics instruction* focuses on how one develops the capacity for posing good problems. This includes examining the posing of new problems as a part of other types of mathematical activities such as problem solving, proving, and investigation. Leikin (Leikin, 2015; Leikin & Elgrably 2020) examined problem posing through geometry investigations and analyzed the proof skills and creativity components that led to the creation of new geometry problems. Some research in this vein focuses on how one learns to pose certain kinds of mathematical problems. For example, Reznick (1994) described ways and rules of creation of problems for Putnam competitions, and Koichu and Andžāns (2009) described the creation of problems for the International Mathematical Olympiad as original and requiring a high level of extracurricular knowledge.

In addition to studying problem posing as a tool or goal of instruction, researchers have used *problem posing as a research tool* in studies that focus on other aspects of students' learning, thinking, reasoning, and creativity. In such work, problem posing is employed to reveal or measure the construct of interest. In addition to studying students' learning of particular mathematical concepts, researchers have used problem posing to examine the broader effects of curricula on student learning. For example, as part of a large-scale study, Cai et al. (2013) investigated the feasibility of using problem posing to assess curricular effect. In particular, they compared the effects of a standards-based middle school mathematics curriculum with those of more traditional curricula on students' algebra learning. Using parallel problem-solving and problem-posing tasks, they confirmed the association between students' abilities to solve and pose problems and found that this relationship held for students using both types of curriculum. In addition, by using qualitative rubrics to assess different characteristics of students' responses, Cai et al. (2013) found that students whose posed problems exhibited positive characteristics (such as reflecting the linearity of a given graph in their posed problem or embedding their posed problems in real-life contexts) were also strong problem solvers. Problem posing has also been also used to probe mathematical creativity by examining the variety of problems an individual can pose (Leikin & Elgrably, 2020; Singer & Voica, 2015).

Studies that include *problem posing as a research goal* focus on understanding the nature of problem posing itself, including examining and evaluating the types, quality, and quantity of posed problems (Ellerton, 2013; Kwek, 2015) as well as the competencies, strategies, skills, and other factors that allow productive problem posing (Leikin & Elgrably, 2020). In particular, because we know that there is a close relationship between affect and problem solving (McLeod & Adams, 1989), it is natural to hypothesize that affect and problem posing are also closely related. Indeed, one important direction for research on problem posing has been to investigate the links between problem posing and problem solving. Kilpatrick (1987) provided a theoretical argument that the quality of the problems that subjects pose might serve as an index of how well they can solve problems. In addition to this theoretical argument, several researchers have conducted empirical studies examining potential connections between problem posing and problem solving (Koichu, 2020).

3 Research on affect in mathematics education

Affect is a construct which is not well conceptualized in the field of mathematics education (Goldin et al., 2016; Pepin & Roesken-Winter, 2015). There are a number of perspectives on affect that suggest distinctions between different types of affect. In the past several decades, most researchers have studied affect in contrast to an individual's cognition (e.g., McLeod, 1992). This distinction can be seen in the work of Goldin (2002, 2009) who stressed that, although research in mathematics education has focused primarily on cognition and far less on affect, the affective system is central to mathematical processing. Goldin (2009) and Gómez-Chacón (2000) also distinguished between local and global affect. Global affective traits include attitudes toward mathematics in general, beliefs about the nature of mathematics and fear toward mathematics. In contrast, local affective traits include anxiety, satisfaction, enjoyment, and surprise while coping with a particular mathematical task. Hannula (2015) introduced a somewhat different perspective and defined a system that included mathematics-related emotions associated with thinking about mathematics and engaging in mathematics, emotions experienced when actually doing mathematics, and the role of mathematics in relationship to other goals. This system of affective characteristics includes a person's self-esteem in a mathematical context. In turn, self-belief, or confidence, was shown to be a predictor of academic achievement and success in mathematics (Grootenboer & Marshman 2016). In the past decade, there has been a shift toward studying affect in general and emotions in particular from a sociocultural perspective (Hernandez-Martinez & Harth, 2016; Radford, 2015; Roth, 2007). According to the cultural-historical perspective (Radford, 2015; Roth, 2007), emotions are part of "a worldview that, through our participation in cultural and social activities, we come to share" (p. 45). Emotions, mediated by cultural conceptual and ethical categories, frame attitudes toward people and events and are implicated in mathematical thinking. In short, emotions (or affect in general) and thinking are not separate entities. Taking a sociocultural perspective of affect in education, Hernandez-Martinez and Harth (2016) explored the role of emotions in students' solutions to mathematical modeling tasks (problem posing is an important component of mathematical modeling).

We fully acknowledge the complexity of conceptualizing affect as a construct. Indeed, Hannula, Pantziara, and Di Martino (2018) found much diversity in their review of research on affect that was presented in the CERME conferences. They analyzed the terminology for affect appearing in 134 published conference papers and found 51 different terms in the titles alone. Given the current state of the research on affect, in this special issue, we view affect as a system in which many constructs intertwine to make up the affective experience of a learner's participation with mathematics, including emotions, attitudes, beliefs, values, and motivations (Pepin & Roesken-Winter, 2015). In addition, self-efficacy is also considered to be an affective aspect for the purpose of examining affect in mathematical problem posing. Thus, in conceptualizing this special issue, we included a broad range of approaches to defining affect and multiple perspectives on affect in the context of problem posing. Given the categorization of problem-posing research above, we suggest applying a similar framework for considering affect in problem posing: problem posing as a tool or as a goal for instruction and for research.

With respect to affect, research on employing *problem posing as a tool for mathematics instruction* examines the use of problem posing with students in order to advance the development of positive affect (interest, curiosity, enjoyment, motivation) related to mathematical thinking and learning. Silver (1994) has suggested that there is a reciprocal expectation regarding problem posing because engagement with problem generation is thought to stimulate student interest in mathematics. For example, problem posing offers opportunities to connect mathematics to students' interests. Moreover, within a classroom community, students could be encouraged to pose problems that others in the class might find interesting or novel. Also, the integration of technological tools into mathematical instruction changes the goals and structure of the problem-posing process and thus also transforms the affective characteristics of problem posing (Leikin, 2015).

A complementary view of affect associated with *problem posing as a goal in mathematics instruction* presumes that mathematical curiosity, interest, and enjoyment can lead to the discovery of new mathematics and the posing of new problems. Indeed, one of the phenomena described in the research literature is mathematical invention by mathematicians (Goldin, 2009) and, in particular, the tendency of mathematicians to formulate problems that are perceived as elegant (Dreyfus & Eisenberg, 1986).

Researchers can also use *problem posing as a research tool* to gain better understanding of the affective characteristics linked to mathematical thinking and learning. In recent years, a number of researchers have used problem posing as a means to evaluate and understand mathematical creativity, and these researchers have started to include affect as one of the explicit dimensions in their work (Leikin & Elgrably, 2020).

Finally, treating *problem posing as a research goal* in studies of affect in mathematics education means analyzing the affective characteristics that promote or impede problem posing such as enjoyment in the problem-posing process, anxiety related to problem posing, beliefs about problem posing, and attitudes toward problem posing. As noted above, the relationship between affect and problem solving suggests that affect and problem posing are also closely related.

Table 1 draws on the foregoing analysis to present a three-dimensional analysis of cognitive and affective domains in research and instruction related to problem posing. This analysis draws on the distinction between problem posing as a tool and problem posing as a goal. Note, however, that the model does not cover all the possible mathematical activities and domains linked to problem posing. For example, problem posing by expert mathematicians for mathematical competitions and Olympiads (Reznik, 1994) does not necessarily belong to mathematical instruction. At the same time, problem posing in mathematics instruction can also be considered from a social domain rather than a cognitive or affective domain (Radford, 2015; Roth, 2007).

Table 1 3-dimensional analysis of cognitive and affective domains related to problem posing

Domain		PP role: PP as a tool	PP role: PP as a goal
Cognitive domain	*Mathematical instruction*	Employing problem posing to develop students' mathematical thinking and learning or teachers' professional competencies	Fostering better problem posers who can pose new and better problems through mathematical activities such as problem posing, problem solving, proving, and investigation
	Research	Problem posing is used to examine and better understand students' mathematical thinking and learning or teachers' professional competencies	Examining and evaluating types, quality, and quantity of posed problems and problem-posing strategies as ways to determine problem-posing skills
Affective domain	*Mathematical instruction*	Employing problem posing to develop positive affect (interest, curiosity, enjoyment, motivation) related to mathematical thinking and learning	Fostering better problem posers who can pose new and better problems through promoting positive affect (mathematical curiosity, interest, enjoyment)
	Research	Problem posing is used to examine and better understand affective characteristics linked to mathematical thinking and learning	Examining and evaluating affective characteristics that promote or impede problem posing

4 Advances in research on affect in mathematical problem posing

Recent books on problem posing (Felmer et al., 2016; Singer et al., 2015) have incorporated the work of researchers from over 20 countries. One key finding that has wide, longstanding support in the literature is that students' problem posing and problem solving are closely related (see, e.g., Cai & Hwang, 2002; Ellerton, 1986; Kilpatrick, 1987; Silver & Cai, 1996; Webber & Leikin, 2016). Although the reciprocal relationships between problem solving and problem posing have received relatively significant attention from researchers, the same cannot be said for the interplay between affect and problem posing. Indeed, there has been no report focused specifically on the relationship between affect and problem posing. Cai et al. (2015) synthesized research on problem posing using a framework of ten major questions, each of which represents a rich area for problem-posing research. However, the lack of research on affect in problem posing at that time meant that none of the ten questions focused on the affective aspect of problem posing. Fortunately, in the years since that synthesis, there has been a shift in researchers' attention to issues related to affect and problem posing.

This special issue represents some of these advances. In particular, this special issue presents empirical work investigating the interaction between affect and problem posing. Table 2 below provides an overview of affect and problem posing in each of the papers in this special issue. This table demonstrates that in the papers in this special issue, problem posing is frequently treated both as a goal and a tool in research on affect in problem posing. In contrast, problem posing in instructional practice, either as a tool or as a goal, is not as frequently represented in these studies. Indeed, this is perhaps due to the focus of this special issue on research about mathematical problem posing and affect. In what follows we provide a brief overview of each contribution with an eye to how they are situated in the 3-dimensional model.

Schindler and Bakker (2020) introduce the term *affective field* to include various affective factors (emotions, attitudes, etc.) and their interplay. Using data from an extracurricular, inquiry-oriented collaborative problem-posing and problem-solving program for secondary

Table 2 Affect and problem posing in the papers in this special issue

Author(s)	Participants	*In mathematical instruction*		*In research*	
		PP as a tool	PP as a goal	PP as a tool	PP as a goal
1. Schindler and Bakker	Secondary school students	C/A	A	A	A
2. Li, Song, Hwang and Cai	Elementary school teachers	C/A	C/A	C/A	C/A
3. Klein and Leikin	Mathematics teachers	C	C/A	C	C/A
4. Guo, Leung, and Hu	Students			C/A	C/A
5. Kontorovich	Experts in posing MO problems		C	C/A	C/A
6. Liu, Liu, Cai, and Zhang	Students			C/A	C/A
7. Aslı Çakır and Hatice Akkoc	Students and teachers	C/A		C/A	C/A
8. Fullmer, Wiezel, Tarr, Zhang, Cullicott, Middleton, and Jansen	Students			C/A	C/A
9. Bicer, Lee, Perihan, Capraro, and Capraro	Students	C	C	C/A	C/A
10. Voica and Singer	Prospective mathematics teachers	C	C	C/A	C/A

school students in Sweden, they investigated the affective and social components of problem-posing and problem-solving processes and demonstrated their dynamic nature. The analysis performed in the study leads to an explanation of the ways in which the affective field is integrated in problem solving and problem posing. The dynamic nature of the affective components of problem posing and problem solving is reflected in the shift of a participant's emotions from initial anxiety toward a feeling of safety and appreciation of the mathematical activities in which she was engaged. The anxiety of the participant during the problem solving and problem posing was rooted in her prior affective field about mathematics activities. The positive shifts, according to the authors, were linked to group collaboration. In this study, problem posing was both a goal of research and instruction, especially in the affective domain. This study emphasizes the importance of the problem-solving and problem-posing environment.

Li et al. (2020) conducted a series of workshops to foster in-service elementary-school mathematics teachers' problem-posing performance and views about teaching mathematics through problem posing. Their study focused both on affective and cognitive domains, as they advocated that in order for teachers to learn to teach mathematics using problem posing, changes are required in both beliefs and knowledge. They also treated problem posing as a goal, examining the teachers' performance on problem posing as well as their views about teaching mathematics through problem posing. They found that participating teachers were able to engage successfully in problem-posing tasks and that they held a number of different views about the advantages and challenges of teaching through problem posing. Based on their findings, Li et al. argue that any effort to integrate problem-posing instruction in school mathematics must attend to teachers' views about the advantages of teaching through problem posing and especially their views of the challenges of teaching in this way. For researchers and designers of professional development experiences, it will be important to find ways to help teachers overcome the challenges that they view as part of teaching with problem posing.

Through employing the posing of open tasks, Klein and Leikin (2020) aimed at both cognitive and affective outcomes: first, to enhance in-service mathematics teachers' understanding of the meaning and types of mathematical openness; second, to promote their positive attitudes toward the use of open tasks in instructional practices. The 44 participating teachers differed in their years of experience and in the level of mathematics they taught. The teachers were asked to pose open tasks by transforming regular textbook mathematical problems and to solve the posed tasks in order to justify their openness. This assignment was accompanied by a 5-point Likert scale questionnaire that examined teachers' affective conceptions about engaging and teaching with open tasks. Klein and Leikin introduced the term *affective conceptions* to avoid the vagueness of the distinctions between attitudes, beliefs, feelings, and emotions. Analysis in the cognitive domain drew distinctions between different types of open tasks posed by the teachers and the problem-posing strategies they used and examined the connections between them. They found that the types of tasks and strategies that teachers use are a function of teachers' experience in terms of both the level of mathematics taught and years of teaching. In the affective dimension, they found connections between teachers' conceptions regarding the difficulty of posing open tasks and their suitability for teaching and learning, their readiness to implement open tasks in their classes, and their predictions regarding teachers' and students' problem-solving behaviors. Klein and Leikin concluded that engagement with open tasks can improve teachers' ways of thinking and their habituation to posing and solving mathematical problems.

Guo et al. (2020) studied the relationships between various student affective factors (self-concept, intrinsic value, and test anxiety) and students' mathematical problem-posing performance according to different outcomes such as the complexity and accuracy of the problems posed. Self-concept (students' expectations), intrinsic value (perceived task value), and test anxiety were chosen as the relevant affective matters based on previous studies showing their significant effects on students' learning outcomes. The study's findings showed students' self-concept and intrinsic value to be positively associated with students' problem-posing performance (specifically complexity, accuracy, and quantity of problems posed), whereas their test anxiety was negatively associated with their problem-posing performance (specifically the complexity of the problems posed). This study contributes to our understanding about the relationships between affect and problem-posing performance.

Kontorovich (2020) collected reflective accounts of posing Olympiad problems as well as examples of posed problems from 26 experienced problem posers for national, regional, and international mathematical competitions such as Baltic Way Mathematical Contests, Mathematical Kangaroo, and Tournament of Towns. The semi-structured interviews that followed written communication with the participants included questions that were formulated based on the analysis of the participants' written responses. In the cognitive dimension, data analysis focused on problem-posing triggers, which are processes that lead to the problem's construction. Kontorovich identified three types of problem-posing triggers that emerged from the analysis: (i) extracting mathematical phenomena from modern elementary mathematics, (ii) abstracting mathematical phenomena from common everyday-life tasks, and (iii) situations where the participants were asked to pose a problem "here and now." Based on the analysis in the affective domain, Kontorovich found that positive affective experiences of the participants were associated with instances (i) and (ii), whereas negative emotions were linked to (iii). He argued that findings about negative affect associated with posing a problem "here and now" in expert-like problem posing explain the reluctance to pose problems upon request by novice participants. Moreover, he argues that this reluctance and its sources should be taken into account in mathematical instruction.

Liu et al. (2020) studied students' self-efficacy related to problem-posing performance in an investigation of a large sample of Chinese eighth graders, specifically focusing on students' domain- and task-specific self-efficacy. By domain-specific self-efficacy, they meant students' self-efficacy related to specific mathematical content areas, as opposed to task-specific self-efficacy, which refers to students' self-efficacy related to specific types of mathematical tasks. They found that although the vast majority of students could pose mathematical problems for all of the tasks, students' problem-posing performance was more closely correlated with their task-specific rather than domain-specific self-efficacy, and this correlation diminished as the difficulty level of the tasks students posed increased. Their findings contribute both to our understanding of self-efficacy as well as to understanding the characteristics of problem posing from a non-cognitive perspective. Methodologically, their study shows that with respect to problem-posing performance and self-efficacy, we should not assume a linear relationship.

Çakır and Akkoc (2020)[1] proposed using the lens of socio-mathematical norms to explore the affective aspects of mathematical instruction that employs problem posing as a tool. The participants of the study were 12 gifted and talented students. Analysis of 43 mathematics lessons was performed, focusing on socio-mathematical norms, which they divided into two categories: those related to the student dimension and those related to the teacher dimension. The teacher dimension is concerned with teachers' expectations and actions, whereas the student dimension refers to actions, an awareness of teachers' expectations, and the students' expectations from the teacher. Socio-mathematical norms are closely related to non-cognitive aspects as well as cognitive ones in a mathematics classroom. Çakır and Akkoc found three major socio-mathematical norms associated with problem posing: reformulations of problems, generating new problems, and evaluation and correction. In addition to these norms, the norm of raising the level of mathematical challenge through posing problems was of particular importance for gifted and talented students. The teacher's role was in establishing norms of the classroom community of practice. They suggest that performing similar research with classes of different levels could provide additional information about socio-mathematical norms that are essential for effective implementation of problem posing as a tool for mathematical instruction.

Fullmer et al. (2020) studied high school students' spontaneous problem posing and affect. They define spontaneous problem posing (SPP) as that which occurs without formal, external prompts, but rather originates from a student who is driven by curiosity to expand or improve their mathematical thinking. For affect, they focused on the emotions that students experienced in response to mathematical tasks, hypothesizing a positive relationship in which positive emotions foster SPP and, conversely, negative emotions detract from SPP. The results of the study showed that the problem posers experienced positive affect and fewer negative responses toward mathematics tasks on the days on which SPP occurred compared to days on which it did not occur. This suggests the potential for SPP as a tool to foster students' positive emotional responses to mathematics tasks as well as to explore their motivations for posing different types of problems.

The study by Bicer et al. (2020) employed problem posing and problem solving as instructional tools aimed at the development of students' creative ability from the perspective of students' cognitive outcomes (e.g., fluency, flexibility, and originality) and affective outcomes (e.g., self-efficacy). Their mathematical creativity framework conceptualizes students' mathematical creativity as consisting of both cognitive outcomes related to their

[1] This article was intended for inclusion in this Special Issue. It was, however, published in a separate issue, Volume 105, Issue 1 and is available at https://doi.org/10.1007/s10649-020-09965-0.

mathematical creative ability and the affective outcomes related to their creative self-efficacy. Creativity was examined using problem posing as a research tool that allowed the evaluation of fluency, flexibility, and originality. The findings showed that the problem-posing intervention had a positive effect on students' mathematical creative ability compared to students who did not experience problem posing, suggesting that problem-posing activities can foster students' mathematical creativity. The study also resulted in a viable tool for assessing students' mathematical creative ability and self-efficacy.

For Voica and Singer (2020), affect itself is a tool for the promotion of the problem solving and problem posing of prospective mathematics teachers, and it played a central role in the key stages that determine success in problem solving and problem posing. The research goal of the instructional activity that the prospective mathematics teachers engaged with in this study was analyzing problem posing and problem solving through analysis of the prospective teachers' affective states linked to problem solving and problem posing. At the same time, affective analysis of the activity also served as an instructional tool to promote problem posing and problem solving. Voica and Singer drew a distinction between epistemic affect, motivation, and self-efficacy. They found that posed problems created a state of comfort and enjoyment in students. Problem posing conveyed a sense of autonomy and control to a greater extent than did problem solving. Voica and Singer argue that the intrinsic motivation linked to problem posing and problem solving is different, with problem-posing activity increasing a robust sense of confidence in one's own capabilities, a phenomenon that made the prospective teachers largely perceive their work as successful.

5 Future directions for research

In this introduction to this special issue, we have outlined a three-dimensional model that makes a distinction between problem posing as a tool and problem posing as a goal in research and instructional practices in cognitive and affective dimensions. The model was theoretically driven and then used in the analysis of the papers in this special issue. In light of this model, we suggest future directions for research and practice in mathematics education related to affect and problem posing.

Problem posing as a tool in mathematics instruction is used when problem posing is employed with the goal of developing students' mathematical thinking and learning or teachers' professional competencies or of fostering positive affect (interest, curiosity, enjoyment, and motivation) related to mathematical thinking and learning. Additionally, problem posing can serve as a means for improving social skills when students are working cooperatively or establishing problem posing as a norm in the mathematics classroom. Although each of the three dimensions gets attention separately in mathematics education practice, the integration of these different dimensions into instructional practice is missing. For example:

- To what extent do teachers explicitly address the affective dimension while teaching using problem posing? To what extent is the beauty of problems posed by the students discussed in the classroom? What is the value of such discussion?
- Are teachers sensitive to students' enjoyment when engaging in problem posing?
- How can teachers learn to better develop students' mathematical thinking and learning and foster positive affect through problem posing?

Problem posing as a research tool is used to examine and reach a better understanding of students' mathematical thinking and learning or teachers' professional competencies and of affective characteristics linked to mathematical thinking and learning. Although many previous studies have addressed cognitive aspects of mathematical problem posing, this special issue puts the emphasis on affective characteristics of problem posing. Still, in both the cognitive and affective dimensions, many questions that deserve the attention of the mathematics education research community remain open. Among them are the following:

- What kinds of problem-posing tasks better reveal and unravel students' understanding of specific mathematical concepts and ideas? How can problem-posing activity be structured so that affective characteristics can be analyzed in the process of problem posing?
- Are there generic characteristics of problem-posing tasks that are effective for revealing and unraveling students' understanding of specific mathematical concepts or are such characteristics specific to each concept? What are these characteristics?
- Considering the potential role of affect in problem posing as a measure of what students learn, how could problem posing and affect complement each other as measures of learning outcomes (both cognitive and non-cognitive)?

Problem posing as a goal of mathematics instruction is linked to the promotion of mathematical activities or of positive affect (mathematical curiosity, interest, enjoyment) that encourages students and teachers to pose new and better problems. Yet, activities such as mathematical investigations, problem posing, problem solving, proving, and investigation directed at mastering problem-posing skills are not often analyzed in the mathematics education literature. At the same time, positive affect as a springboard for the development of problem-posing competencies is rare. Even in this special issue, this direction of research is missing. Mathematicians pose problems all the time based on their natural curiosity, enjoyment of mathematical discovery, and intrinsic motivation. They pose problems with deep understanding of the area of research, with long-term thinking. Hadamard (1945) stated that this activity is largely unconscious. In contrast, learners (students or teachers) pose problems upon the instructor's request; this situation is not as natural as problem posing is to research mathematicians. Thus, we propose that future research can seek answers to the following questions:

- How can positive affect be integrated in mathematics instruction as a trigger for problem posing?
- How can problem posing become natural for learners, so that they will ask mathematical questions based on their true mathematical interests and curiosity?
- How does affect play a role in the process of problem posing? How do affect and problem posing reciprocally play their roles when students engage in problem posing? How are students' problem-posing performance and affective traits related to each other?

Problem posing as a research goal is associated with examining types, quality and quantity of posed problems, and problem-posing strategies in order to determine problem-posing skills or to evaluate affective characteristics that promote or impede problem posing. Although such an examination is usually performed by focusing on the problem-posing outcomes, the analysis of cognitive and affective characteristics in problem posing is quite rare. We imagine that such research may involve conducting clinical individual interviews that address questions such as:

- What cognitive and affective processes are involved in the problem-posing process? Are there specific mechanisms that lead to the creation of elegant and original problems?
- Do different kinds of problem-posing tasks evoke different cognitive and affective mechanisms? Can such mechanisms be taught, and can teachers be educated to promote effective problem-posing processes?
- How does problem posing as a cognitive activity interact with affective processes? What affective factors support or impede problem-posing performance?

6 Conclusion

Di Martino and Zan (2015) observed that the publication of the book *Affect and Mathematical Problem Solving* (McLeod & Adams, 1989) represented a turning point for research on affective constructs and mathematical problem solving. Given the increased recognition of the importance of both affect and problem posing in recent years, we hope that this issue on affect in mathematical problem posing will represent a similar turning point for research on affective constructs and mathematical problem posing.

It is recommended that the research community actively consider affective aspects in mathematical problem-posing research. The inclusion of affect in mathematical problem-posing research is not only consistent with the fact that educational goals include both cognitive and non-cognitive goals in different nations but also consistent with the fact that affect is crucial in cognitive processes like problem posing. Although this special issue represents current thinking about affect in mathematical problem posing, there are many remaining research questions. The diverse, international perspectives in this special issue can serve as a starting point for tackling these research questions.

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