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COMMENTARY

Future Directions for Theory and Research with Instructional Manipulatives: Commentary on the Special Issue Papers

Scott C. Marley · Kira J. Carbonneau

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Professional organizations, educators, and researchers have proposed that instruction with manipulatives is an effective classroom teaching technique. As examples, both the National Council for the Teaching of Mathematics (NCTM 2008) and the National Association for the Education of Young Children (NAEYC 2009) recommend manipulatives play a prominent role in classroom instruction. These recommendations are intuitively appealing as there are many circumstances in which representing the world through physical action with concrete objects occurs naturally. For example, an experienced outdoorsperson providing complicated directions to a desired location in the mountains is a circumstance that lends itself to the use of manipulatives. If a verbal description fails, the outdoorsperson may gather some rocks, sticks, and other convenient objects to construct a rough model on the ground of the local topography. In this manner, the spatial relations of the landscape can be efficiently relayed to the neophyte adventurer for cognitive processing in real time. It is anticipated that the adventurer will be able to later visualize this rough mapping of landmarks with objects while in the field.

Classroom teachers often support learning in similar manners to the outdoorsperson introducing a novice to an unfamiliar region. In formal instructional contexts, teachers of academic topics may deliver instruction by encouraging learner interactions with manipulatives that represent core concepts. The underlying theoretical expectation in these circumstances is students will derive similar benefits in terms of online and offline cognitive processing as those garnered from representing a landscape with readily available objects. However, unlike the spontaneous mapping provided by the outdoorsperson, which has a fairly discrete outcome (i.e., successful navigation of a constrained region), teachers likely assume and desire generalization to a multitude of abstract learning outcomes (i.e., transfer of learning, Barnett and Ceci 2002; Martin and Schwartz 2005).

Manipulatives used in formal learning contexts may be perceptually more or less grounded in terms of representational properties (Belenky and Schalk 2014/this issue). In addition, the instructional guidance in classrooms may differ considerably from that provided by the experienced outdoorsperson in our illustration. As an example, when teaching about the

S. C. Marley (✉)
Mary Lou Fulton Teachers College, Arizona State University, Tempe, AZ, USA
e-mail: Scott.Marley@asu.edu

K. J. Carbonneau
University of New Mexico, Albuquerque, NM 87131, USA

changing of the seasons, there are many ways in which the qualities of the manipulatives, the instructions provided and learning outcomes can vary. A teacher might verbally describe how the seasons change due to the tilt of Earth's axis to the Sun. If this description fails, the teacher may have learners demonstrate how the tilt of the Earth on its axis in relationship to the Sun affects seasons using a basketball and a baseball, or with spheres that share the perceptual appearance of the astronomic bodies. In these cases, it is expected that representing the information with objects will facilitate immediate processing as well as enhance the likelihood of subsequent usage of target information. However, the papers in this special issue suggest the effectiveness of this example and similar circumstances are not necessarily guaranteed. Among other factors, the nature of instructional guidance, perceptual qualities of the objects and model failures may detract from learning target concepts. Therefore, teachers must be attentive when selecting educational activities that incorporate manipulatives and if researchers are to make positive contributions to classroom practice they are obligated to identify the boundary conditions associated with effective manipulative-based instruction.

The decision to use instructional manipulatives as part of classroom instruction should be made with careful reflection. When considering whether manipulatives should be incorporated into classroom instruction there are three interrelated considerations. First, learning conditions need to be carefully examined to determine appropriateness of a strategy. Learning conditions such as instructional guidance, materials and domain of inquiry likely moderate the efficacy of teaching with manipulatives. Second, learner characteristics need thoughtful consideration. For instance, instruction with manipulatives is often recommended for young children. However, in several cases, the empirical literature is inconsistent with regards to effectiveness with young children (DeLoache 2000; Uttal et al. 2009). Additionally, prior knowledge has been shown to play a role in the effectiveness of manipulative-based instruction (Petersen and McNeil 2013). Finally, attention is required in terms of the intended goals of instruction. Research has demonstrated that learning outcomes (e.g., recall, application, and transfer) differentially benefit from manipulative-based instruction (Carbonneau et al. 2013). Thus, knowledge of whether learning associated with manipulatives impacts immediate performance and generalizes to other circumstances is of importance. These three considerations draw from the principal areas of current manipulative-based instructional strategy research.

The papers in this special issue present nuanced views of the theoretical, empirical and educational questions currently under study in research with instructional manipulatives. Each contribution provides hypotheses that can be examined in terms of the characteristics of instruction, learners, manipulatives, and outcomes. The following sections discuss the contributions in relationship to conditions of learning, transfer of learning, and affordances and constrains of manipulatives. Additionally, we discuss future directions suggested within the papers. We conclude with a brief discussion of research credibility in instructional manipulatives research.

Conditions of Learning

The instructional characteristics present when learners interact with manipulatives are of interest and importance to researchers and educators. Based on constructivist learning theories, it is frequently recommended that no or minimal instructional guidance be provided when learners interact with manipulatives to learn target information (Martin 2009). However, empirical evidence from studies examining instructional guidance with manipulatives (Sarama and Clements 2009), and from other areas of educational research (Kirschner et al. 2006; Mayer 2004), does not support these recommendations. Instead providing moderate to high instructional guidance typically results in improved learning outcomes (Alfieri et al. 2011; Chen and Klahr 1999; Klahr and Nigam 2004).

The mathematics and science instructional strategy being developed and empirically tested by Fyfe et al. (2014/this issue) is an example of a technique that seeks to provide an appropriate level of instructional guidance. The theoretical notions supporting systematically shifting from enactive to iconic to symbolic representations (i.e., concreteness fading) have long been of interest to educators and researchers (Bruner 1964; Goldstone and Son 2005; Lehrer and Schauble 2002). A primary reason for this interest is after a period of instruction with external representations learners often fail to generalize learning (DeLoache 2000; Resnick and Omanson 1987; Schwartz and Moore 1998). Accordingly, by gradually fading from enactive to symbolic representations with iconic representations as an intermediary the understanding of the deep structure of mathematical and scientific concepts may better generalize to related circumstances. This approach is somewhat comparable to providing multiple external representations (Ainsworth 1999, 2006). Although, unlike providing multiple external representations, concreteness fading requires that the representations systematically become more abstract. Related reading and listening comprehension research with elementary-age children investigating imagined manipulation as an iconic intermediary between enactive and symbolic representations (Biazak et al. 2010; Glenberg et al. 2004; Marley et al. 2010, 2011) provides supporting evidence for the concrete fading approach. Integrating findings from the reading comprehension literature with the reviewed math and science studies could further develop applications in science, technology, engineering, and mathematic domains that require text processing.

Instructors likely select instructional experiences that are visually engaging in efforts to capitalize on their motivational qualities. A core notion of instruction with manipulatives is the characteristics of the objects promote interest, engagement, and other affective responses. These motivational responses are expected to act as mediators that facilitate learning (Ball 1992; Reimer and Moyer 2005). The perceptual qualities of external representations are also expected to support learner construction of internal representations (Scaife and Rogers 1996). However, according to Belenky and Schalk, manipulatives that contain irrelevant details may result in learners selecting or activating irrelevant information (Mayer 1996). This finding is analogous to the seductive details effect in other areas of study (Harp and Maslich 2005; Harp and Mayer 1998; Lehman et al. 2007). Surprisingly, few studies systematically examine whether the perceptual and interactive qualities of manipulatives influence interest or other motivational characteristics.

Viewing manipulatives as a particular form of external representation Belenky and Schalk discuss the groundedness of objects. On one end of the continuum are grounded representations that share the visual appearances of objects they represent (e.g., pizzas to represent fractions) while on the other end are idealized representations with superficial characteristics stripped away (e.g., plastic circles to represent fractions). Related research with manipulatives is inconsistent with studies suggesting that grounded manipulatives may inhibit learning relative to idealized representations (Kaminski et al. 2009; McNeil and Jarvin 2007).

The perceptual or interactive richness of manipulatives may not always have negative effects, rather as Pouw et al. (2014/this issue) argue these qualities may invite the learner to interact with the manipulatives in predictable ways that encourage the construction of targeted meanings. In other words, instead of supposing richness and interactivity of manipulatives detract from learning as a given, perceptual features of manipulatives may support intended online cognitive processes. Borrowing from embodied cognitive perspectives (Barsalou 2008; Glenberg 2010; Wilson 2002) the authors propose that the internalization of multimodality routines for subsequent offline processing requires extensive amounts of time interacting with manipulatives; more so than commonly examined in empirical studies. Furthermore, providing additional manipulation time to facilitate internalization is not a matter of a concrete to abstract shift as proposed by Fyfe, McNeil, Son, and Goldstone. Rather, time is necessary to allow learners opportunities to construct robust fully grounded representations with manipulatives. Although convincing evidence supporting this and related predictions is currently scarce. The theoretical argument is compelling and further empirical examinations contrasting conditions comprised of increasing levels of abstraction with conditions containing intensive interactions are warranted. In addition, the implications for effective classroom applications are considerably different than those derived from concreteness fading. If instructional strategies are focused on internalization of multimodality representations educators will be required to provide learners with substantial opportunities to physically interact with manipulatives.

Transfer of Learning

Instructional conditions are of importance with regards to transfer of learning. Ideally, learning that occurs in one setting generalizes to other relevant circumstances. If an instructional approach facilitates transfer of learning, or provides the groundwork for further instruction that subsequently promotes transfer, the technique will be of significant value to classroom teachers. Many manipulative studies show improved performance on immediate learning outcomes associated with recognition, recall, and tasks that are contextually comparable to initial learning. On outcome tasks related to transfer of learning the results are mixed. To further muddy the waters, transfer of learning is not a well-defined construct and how it is operationalized differs considerably across areas of educational and cognitive research (Barnett and Ceci 2002). Much of the discussion in this issue approaches transfer of learning from an analogical reasoning perspective (Gick and Holyoak 1980, 1983).

Studies of analogical reasoning present problem types that share deep structure but differ in surface characteristics. For example, in classical studies by Gick and Holyoak (1980, 1983), learners were presented with a medical problem that required understanding converging forces and subsequently were to recognize that the same principles can be applied to a military problem. In both problems, the superficial details were very different, but the deep structure of the problems was identical. The research on concreteness fading described by Fyfe, McNeil, Son, and Goldstone originated from math and science studies examining learner ability to solve analogous problems (Braithwaite and Goldstone 2013; McNeil and Fyfe 2012) after a period of studying problems that differ in superficial characteristics (i.e., perceptual qualities) yet contain the same deep structure. Belenky and Schalk offer a related set of explanations in relation to the groundedness of external representations, with idealized representations expected to facilitate transfer more so than grounded representations. Once again, and much like concreteness fading, the purpose of idealizing or stripping contextual features from concrete representations is to make transparent the deep structure of the underlying concept, which in turn should facilitate transfer when subsequently encountering problem analogs.

If one holds the view that transfer of learning is the ability to solve analogous problems then fading from enactive to symbolic representations or shifting from grounded to idealized concrete representations are promising approaches. However, if transfer of learning is the application of learned content to related contexts that differ in terms of time, presence of other learners, domains, and so forth, the ability to solve problem analogs is likely not sufficient. For example, in related studies, young children's improved text and listening comprehension with manipulatives has been observed when perceptually and interactively rich manipulatives are replaced with imagined manipulation (e.g., Glenberg et al. 2004; Marley et al. 2011; Marley and Szabo 2010). The learning outcomes of these studies, rather than being problem analogs, were associated with comprehension of different stories with the same characters and context. It is possible the math and science studies are examining different facets of learning from instructional manipulatives than the reading and listening studies. It should be noted that the perceptual richness of the manipulatives in the reading and listening investigations were not varied. However, in support of this speculation Pouw, van Gog, and Paas suggest that providing rich and interactive manipulatives over extended periods of time should facilitate transfer of learning under certain circumstances.

In the Pouw, van Gog, and Paas discussion of the evidence that supports the commonly held conclusion that the perceptual and interactive richness of manipulatives impedes transfer of learning, they propose alternative interpretations. Using notions from embedded (Martin and Schwartz 2005) and embodied (Barsalou 1999, 2008; Glenberg and Robertson 2000) cognitive viewpoints, the authors propose that under certain circumstances perceptually rich and interactive manipulatives may improve transfer of learning. This perspective may help clarify the contradictory findings in the literature that perceptually rich features hinder transfer while in others these features facilitate transfer (Kaminski et al. 2009; McNeil et al. 2009; Sloutsky et al. 2005). According to Pouw, van Gog, and Paas, learning with manipulatives is a complex interplay between structuring and restructuring the environment (i.e., embedded cognition) and a cognitive system consisting of manipulations of perceptual symbols (i.e., embodied cognition). If this is so, the implications for classroom instruction are quite different than those of the view that manipulatives should be used to facilitate subsequent cognitive processes using abstract symbols. Rather, if empirical research supports this view, in certain instances educators would focus on developing opportunities for students to thoroughly interact with manipulatives when the desired learning outcome is responsive to this approach.

Examinations of the ability to transfer learning often contrast learning with manipulatives to learning in symbolic control conditions on outcomes that inherently favor the symbolic controls. Put differently, if transfer of learning is defined as successful completion of measures that are in symbolic formats, control conditions will tend to perform better than manipulative-based treatment conditions. The reverse may occur when transfer of learning consists of outcomes that require manipulating objects as extensions of the initially studied concepts. In these circumstances, manipulative-based learning conditions could perform better on measures of transfer. This phenomenon is known as transfer-appropriate processing in cognitive research (Morris et al. 1977). According to research examining this topic when there is a recapitulation of the conditions of encoding during retrieval the probability of successful retrieval is greater (Levin 1989; Roediger and Karpicke 2006). Pouw, van Gog and Paas discuss this common finding in their review of manipulatives research as a potential explanation for inconsistent results with transfer of learning. This explanation requires further examination by manipulatives researchers as it has clear implications for classroom instruction; foremost of which is that outcomes may require careful matching to learning experiences.

Affordance and Constraints of Manipulatives

Affordances and constraints of manipulatives warrant further consideration by educators and researchers. There are two primary considerations in this arena. First, the affordances of the manipulatives have to be limited in some manner if learners are to construe knowledge. As described by Pouw, van Gog, and Paas the manipulations implied by the objects should coincide with the intended learning outcomes. Second, manipulatives should be able to sufficiently represent a problem space. Vig et al. (2014/this issue) describe models commonly used in classrooms that may not sufficiently represent a problem space, highlighting the notion that models may break when learners encounter more complex extensions of the concepts.

It is unclear from the research how educators respond when encountering constraints of this nature in physical models. According to Vig, Murray, and Star teachers may devise patches to work around the limitations of the model. These patches may obscure the underlying concepts to such a degree that although correct answers are arrived upon after performing manipulations the deep structure of the concept is absent. In practice, teachers may abandon manipulatives for symbolic representations at these breaking points or well in advance of encountering the limitations of the models. In addition, the reasoning of learners is unknown when external representations are no longer tightly aligned with the concepts they purport to illuminate. If breaking points are inherent, the use of models may impede transfer of learning in concreteness fading studies when the deep structure implied by manipulatives fails to generalize to applications that are on the other side of the breaks.

Future Directions

Several directions for future research can be gathered from the papers provided in this issue. Matching instruction to learners, investigating time as a factor, exploring how teachers and learners construct meaning when models fail, and examining transfer of learning are four clear directions for future research with instructional manipulatives that have direct applications for the classroom.

Investigations of learning outcomes related to maintenance as well as near and far transfer of learning (Barnett and Ceci 2002; Pressley et al. 1990) with regards to learning conditions and subject characteristics in an aptitude-by-treatment framework (ATIs: Cronbach 1957; Cronbach and Snow 1977) are suggested by the Fyfe, McNeil, Son and Goldstone. ATI studies are challenging due to the number of participants required to have adequate statistical power to identify meaningful interactions (Cohen 1988). Empirical examinations of learning styles interacting with instructional approaches have been relatively unsuccessful (for related discussion see, Pashler et al. 2008). However, moderated effects with other individual differences such as age, prior knowledge, spatial abilities and short-term memory capacity may be substantial enough to warrant further investigations. For example, expertise reversal effects are very common in several areas of educational research (for related discussions, see Kalyuga et al. 2003; Kalyuga 2007), with findings suggesting that providing too much instructional support impedes the performance of high-knowledge learners. Comparable future studies with instructional manipulatives may find similar results.

A review of educational psychology research found that less than 5 % of studies published in peer-reviewed journals accounted for time spent learning and time between learning and outcome measurement as factors (Hsieh et al. 2005), indicating that the majority of educational psychology research is performed in abbreviated periods of time, usually less than 1 day. As discussed by Pouw, van Gog and Paas, embodied theories propose cognition consists of mental simulations that are grounded in sensorimotor experience. If so, it is likely that the successful construction of internal representations requires extensive sensorimotor experiences. In addition, proponents of discovery learning argue that robust learning requires time (Dean and Kuhn

2007; Kuhn 2007). Therefore, future studies should allow for more instructional time spent interacting with manipulatives. Time between learning and assessment should also be examined, as it is likely that the effectiveness of approaches are overestimated when instruction and assessment occur in one session.

If, as claimed by Vig, Murray and Star, all concrete models eventually break in terms of what concepts can be represented. It is essential for research to examine the reasoning processes of learners, teacher knowledge of model breaks, and approaches for overcoming model breaks. In terms of student reasoning, it is unclear what cognitive processes are invoked when model breaks are encountered by students with and without instructional support. Teacher awareness of model breaks and how they negotiate them in practice is ripe for investigation as well. Based on what is learned from student reasoning and teacher awareness studies, instructional approaches for overcoming breaks in models could be devised and scientifically tested.

Clearly, performance on various types of learning outcomes is of high interest in the instructional manipulatives literature. Primary areas for future research should focus on quantitative and qualitative factors associated with instructional activities with a focus on aspects of instruction that support performance on multiple learning outcomes (e.g., modality of outcome tasks, etc.). A potential framework for the examination of instructional support and manipulatives efficacy is Chi's Interactive-Constructive-Active-Passive (ICAP; 2009) framework. The ICAP model differentiates learning tasks by categorizing learning activities as interactive, constructive, active, or passive and links these behaviors to different cognitive processes (Chi 2009). By systematically examining learning tasks within the ICAP framework, specific recommendations for producing high performance on learning outcomes may be identified.

Prescriptive Statements and Manipulatives Research

The authors in this special issue have been very cautious with recommendations for the use of manipulatives in classrooms. Unfortunately, prescriptive statements are especially prevalent in the manipulatives literature targeted at teachers of preschool- and elementary-age children (e.g., Burns 1996; Copple and Bredekamp 2009a, b; Moch 2001). Consumers of such recommendations likely assume these endorsements are based on sound scientific evidence consisting of systematically examining the results of instructional manipulatives on learning. In other words, those who adopt these recommendations are assuming that focused research has been conducted with instructional manipulatives across varied populations of learners under authentic learning conditions and important learning outcomes have been measured. In addition, a related core expectation is the inquiry strategies used to examine the effects of instructional manipulatives control for plausible rival hypotheses (Marley and Levin 2011; Shadish et al. 2002).

At face value, stating that "manipulatives work" and recommending educators incorporate them into classroom instruction is not provocative, or particularly noteworthy. However, it is controversial when instructional approaches are recommended by professional organizations and educational researchers without, or with minimal, provision of evidence-based guidance on who, what, when and how to apply the approach. Coarse recommendations of this nature can result in unrealistic expectancies in terms of effectiveness and/or instructional misapplications. We view these recommendations as analogous to proposing the use of a particular tool for all construction purposes. Most tools work well, or well enough for their intended tasks, but when used outside of intended uses they perform miserably. Instruction with manipulatives is no different; however, the affordances and constraints of manipulatives are often not self-evident as those associated with commonly used tools. Therefore, the effectiveness of strategies that use instructional manipulatives should be carefully examined across populations, domains, outcomes and settings prior to broad professional recommendations.

A paucity of scientific research examining learner and instructional characteristics in relationship to meaningful learning outcomes exists. From what research does exist, inconsistencies between common theoretical accounts (Bruner 1964; Piaget 1962) and empirical results have been identified within this special issue and elsewhere in the literature (for examples, see Carbonneau et al. 2013; DeLoache 2000; for related discussion, see McNeil and Jarvin 2007). These inconsistencies are often overlooked in professional recommendations and suggest a conflation of theory with empirical support. Put more precisely, the professional endorsements of instructional manipulatives suggest the cross-spectrum efficacy of manipulatives is a resolved issue; a conclusion that one familiar solely with theory may harbor. This circumstance can result in practitioners misapplying manipulative-based instruction. Or worse, after experiencing difficulties associated with strategy misapplication, practitioners may indiscriminately reject promising manipulative-based approaches.

Conclusion

The authors of this special issue provide contemporary reviews of theories and evidence associated with manipulatives-based instruction. Within each contribution, several areas for future research and potential educational applications are discussed providing clear outlines of issues that should be studied to further our understandings of how to better implement manipulative-based instruction. While researchers debate the intricate details of how and why manipulatives are effective or ineffective in specific situations practitioners face the difficult challenge of effectively using manipulatives within the classroom. Therefore, in order to support and promote high-quality educational experiences for learners, it is paramount that the effectiveness of instructional strategies with manipulatives continues to be empirically examined by educational researchers.

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