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Technology adoption, innovation policy and catching-up.

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Abstract

A model is proposed where economic growth is driven by innovation alongside the diffusion and adoption of technology from the frontier. Innovation investments are related to households savings, which generates multiple equilibria with low and high levels of innovation and productivity. Low-level equilibria are unstable. Starting from a position with low levels of investment and innovation, increasing investments are associated with high but decreasing dependence on international technology diffusion. A major objective of policy-making is to increase investment sufficiently in the lower end to reach the high-level steady state. An economic rationale is provided for the existence of productivity improving equilibria, where distance to the frontier is reduced based on a tax and subsidy mechanism designed to boost innovation and speed up catching-up.

Keywords Dynamic optimization · Equilibrium analysis · Fiscal policy · Technology diffusion · Innovation policy · Economic growth

JEL Classification C62 · H70 · O33 · O38 · O40

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1 Introduction

The recent literature on innovation has reignited the debate about the relationship between technology progress and economic growth over the long run. In particular, it has spurred controversy about: (a) the relative importance of technology diffusion from abroad over (local) innovation in explaining convergence across countries (Romer 1993; Barro and Sala-i-Martin 1997; Keller 2004; Acemoglu et al. 2006; Aghion and Howitt 2006; OECD 2007); and (b) the merits of fiscal policy mechanisms designed to boost innovation (Crespi 2012; Denes et al. 2020; Chu and Wang 2022).

Indeed, in addition to be a frequent topic of empirical research (Reppy 1977; Lichtenberg 1988; Lerner 1999; Pagés 2010; Nanda et al. 2015; Kerr and Nanda 2015; Howell 2017; Denes et al. 2020; Hu 2020), the theoretical reasoning that justifies the use of fiscal instruments to boost innovation and long-run growth has received considerable attention in growth literature (Romer 2000; Segerstrom 1998, 2000; Zeng and Zhang 2007; Chu and Wang 2022). In fact, the role of government in financing innovation-led growth has been popularized by Mazzucato (2013) and Mazzucato and Penna (2015), among others.

This paper seeks to contribute to these strands of the literature. We explore the long-run growth implication of innovation subsidies in a framework that distinguishes between local innovation and technology diffusion and adoption. In particular, unlike models of catching-up where the key determinant of growth in countries behind is the spread of technology from the frontier (Romer 1993; Barro and Sala-i-Martin 1997), we construct a model where production depends directly on local innovation and indirectly on the diffusion of technology from abroad. Thus, in our model, the technological catch-up is endogenous to home country investments in innovation.

Our distinction between technology diffusion and local innovation relies on widespread acceptance of the latter as a local process of *assimilation* and *entrepreneurship* (Schumpeter 1934; Baumol 1996; Nelson and Pack 1999; Jorgenson 2009) which allows countries behind to take advantage of the spread of R &D, knowledge and inventions from the technology frontier.

In this framework, innovation is boosted by a fiscal strategy that we refer to hereafter as a *"tax and subsidy mechanism"* designed to ensure a high level of diffusion and innovation. Specifically, the government in our model sets a *consumption-tax* on households income net of effective investments and uses the revenue to grant *innovation-subsidies* under a balanced budget restriction.

In our approach, we assume that households and business are separate decision units. The representative consumer maximizes the discounted sum of after tax consumption subject to the dynamics of innovation and catching-up. The increase in innovation investments after the introduction of the *"tax and subsidy mechanism"* leads the economy to jump to a new balanced growth path and hence to an equilibrium with higher levels of investment and consumption. In particular, as investment increases consumption and welfare first decrease in the short-run, but then increase over the long-run. However, the latter crucially depends on the size of the discount.

The paper proceeds as follows. In Sect. 2, we provide a brief discussion of the literature on technology and economic growth and the relationship between government's policy, business innovation and economic growth. In Sect. 3, we present the setup of the model. In Sect. 4, we give formal intuition about the *"tax and subsidy mechanism"* and its influence on innovation and economic growth over the long-run. In Sect. 5, we discuss the optimization problem and the shifting nature of steady-state trajectories induced by the tax and subsidy mechanism. Finally, Sect. 6 provides some concluding remarks.

2 Related literature

A key contribution of the model that we develop here is that it provides an analytical framework to investigate the role of fiscal incentives in influencing innovation and catching-up alongside the diffusion of technology from abroad. Thus, ours is a model of growth led by innovation which means that it emphasizes primarily innovation and only indirectly technology diffusion and adoption. This leads to policy insights on investment allocation among these activities that are very different, indeed in contrast to most models of distance to the frontier that emphasize the diffusion channel instead. Also different from the most standard approach where investment is, implicitly, modeled to offset the reduction in consumption and budgetary issues are neglected, we develop our model in a context where there is a tight balanced budget restriction and investment is subject to adjustment costs.

Our model relates to various strands of the literature on technology and economic growth. The key feature of the received theory of endogenous growth is the assumption that in less developed countries economic growth is driven primarily by the diffusion of foreign technology which boost productivity and catching-up as long as the economy behind meets some basic absorptive conditions. There are three main approaches that can be distinguished in this literature: (i) accumulation of physical and human capital, (ii) the innovation paradigm, and (iii) distance to the frontier.

The first approach, based on a mechanical association between increasing savings, accumulation (or more education) and higher growth gives little attention to innovation itself (Romer 1986; Lucas 1988; Jorgenson 2009; Damsgaard and Krusell 2010; Stokey 2015). Unlike the focus in this kind of models, our model relates to the innovation paradigm which gives preeminence to local growth enhancing creative activities (Schumpeter 1934; Baumol 1996; Nelson and Pack 1999).

Models in the second approach are set to explore either the ability to increase product variety, to improve product-quality, or simply to produce ideas (Romer 1990; Aghion and Howitt 1992; Howitt 1999; Peretto 1999; Jones 2005; Chu and Wang 2022). These models, however, are not explicitly designed to explore the driving factors of convergence across countries.

Thus, in models within the second approach, the convergence property is implicitly developed in a class of models with catching-up properties that emphasize backward advantages created by technology diffusion and imitation, rather than innovation. Backward advantages arise as long as countries behind have the right conditions– market institutions like competition, property rights, openness to trade, financial development–and sound macropolicies (Gerschenkron 1962; Barro and Sala-i-Martin 1997; Edwards 1998; Lucas 2009; Acemoglu 2015).[1] The trademark of these models is the general belief that, depending on the economy's absorptive capacity, diffusion is the driving force behind catching-up. While technology diffusion and absorptive capacities are important, in our model we consider the Schumpeterian view of (local) innovation as the true engine of growth and catching-up.

Our model relates more closely to models in the third approach, which emphasize distance to the frontier. Like other models of this kind, the model developed here builds on the interplay between technology diffusion / adoption and innovation. Acemoglu et al., 2006 study this issue assuming a sequential process whereby countries at low levels of development first follow an *"investment strategy"* to take advantage of *state-of-the-art* technologies from abroad, and switch to an *"innovation strategy"* as they approach the technology frontier. Aghion and Howitt (2006), and Lucas (2009), follow a similar approach and suggest that laggard countries would grow faster the far behind they are by *implementation* of technologies from the frontier, but shift to innovation-based strategies as they get nearer the technological frontier. Rather than *advantage of backwardness* (Gerschenkron 1962) and the implicit focus on *radical* innovations present in these models, we emphasize a more general view of innovation as the key growth enhancing activity behind the technology frontier.

Within the models of distance to the frontier, we identify a set of contributions with main focus on micro-economic settings, which however easily extend to the cross-country approach in our model. Benhabib et al. (2014, 2019), study the interaction between diffusion, adoption and innovation by endogenizing these decisions in a context where, implicitly, all agents (firms, countries) are at the frontier and optimally choose between falling back and catching-up (see also Stiglitz 2014a, b; Sampson 2015). While our model relates mostly to this approach, the key difference is our macroeconomic approach. In addition, we explicitly analyze the implications of a real-world scenario where some agents are at the frontier while others fall behind.

As mentioned, our paper relates also to the research on policy/institutional mechanism, like taxes and subsidies, that influence investment and innovation. The association between subsidies and innovation has been studied by, among many others (Romer 1990; Grossman and Helpman 1991; Jones 1995, 2005; Segerstrom 1998, 2000; Zeng and Zhang 2007; Chu and Wang 2022). Unlike the most standard approach in those models which take subsidies for granted, the *"tax and subsidy mechanism"* in our approach builds on a simple general equilibrium setting with net consumption taxes—net of effective investment—and a tight balanced budget condition.

Although they are related, in our model we do not explicitly deal with financial market imperfections that constraint innovation and growth and justify the use of fiscal instruments (Bernanke and Gertler 1990; Hall and Lerner 2010). We introduce the *"tax and subsidy mechanism"* as an exogenous innovation-policy intended to get the economy closer to the technology frontier.

[1] On the empirical approach see Coe and Helpman (1995), Hall and Jones (1999), Keller (2002), Keller (2004) and Caselli (2005).

3 Model setup

Consider a framework where productivity differences between country "*i*" and the frontier are proportional to differences in technology.

$$y_i(t)/\bar{y}(t) \approx A_i(t)/\bar{A}(t)$$

where $y_i(t) = Y_i(t)/L_i(t)$, and $\bar{y}(t) = \bar{Y}(t)/\bar{L}(t)$. We think of the frontier technology, $\bar{A}(t)$, in terms of high-tech developments that are common to all countries. Local innovation, $A_i(t)$, combines the frontier technology with indigenous ideas leading to new goods, shifts in production techniques, marketing strategies and forms of business organization. This distinction between frontier and local innovation, and the view that the latter evolves alongside the spread of developments in the most advanced countries is ubiquitous. The models of distance to the frontier discussed above, and models that emphasize general purpose technologies (Helpman 1998; Jovanovic and Rousseau 2005; Bresnahan 2010) belong to this line of research. The extent of international technology diffusion and innovation has been investigated empirically by Coe and Helpman (1995), Keller (2004) and Acemoglu et al. (2006). Our broad definition of local innovation in terms of commercial applications, production methods, market strategies and organization processes is more controversial. Romer (1993) stresses the idea of innovation diffusion from the frontier. Nelson and Pack (1999) instead put forward the importance of local assimilation, entrepreneurship and innovation to take advantage of the process of technology diffusion. Our modeling approach is more consistent with the latter view.

Thus, from the viewpoint of country "*i*", $\bar{A}(t)$ is exogenous whereas $A_i(t)$ is endogenous, determined by the ability to find new uses for the received technology.[2] Country *i*'s final output relies on the following production function

$$Y_i(t) = f\left\{A_i(t), L_i(t)\right\}$$

where *L* is labor, which equals the country's population. This feature frees us from discussing differences in productivity and welfare considerations. Furthermore, for simplicity, we assume that all the population works and all workers are allocated to the production of final output. Finally, we assume linearity in *L*. Thus, production per-worker is

$$y_i(t) = f\left\{A_i(t)\right\} \tag{1}$$

where $f\{0\} = 0$, $f' > 0$, $f'' < 0$. Final output is denoted in per-worker units, and innovation is defined in levels which implies that productivity depends on the absolute stock of technology and is subject to decreasing returns (Jones 2005; Segerstrom 1998). Notice that frontier technology, $\overline{A}$, does not show up in Eq. (1) as we assume that it influences the production of final output only indirectly through its impact on the dynamics of innovation—explained below.

[2] We assume that there are no absorptive constraints in terms of human capital, institutional infrastructure or political conditions.

3.1 The problem of the representative agent

Consider a representative agent in the private sector who wants to maximize the value of some utility function $U(C)$. In an economy without government and with balanced trade, the real value of consumption is given by the value of gross income minus total savings (hereafter, we suppress subscripts to avoid over-notation).

$$C^{\text{pt}} = Y - S \tag{2}$$

where $Y = yL$, $C^{\text{pt}} = c^{\text{pt}}L$ and $S = sL$ describe the aggregate levels of output, pre-tax consumption and savings, and "y", }}$c^{\text{pt}}\varepsilon$ and "s" are per-worker quantities.

We assume that investment is subject to *adjustment costs* (Turnovsky 1996). In particular, consider the cost function $S = b\{I\}$ with properties $b\{0\} = 0$, $b' > 0$, and $b'' > 0$, which implies that the marginal cost of innovation is positive and increases with the investment intensity. The following so-called *convex adjustment investment cost function* satisfies the above conditions:

$$S = I + \kappa I^2 L^{-1}, \qquad 0 < \kappa \tag{3}$$

we refer to the first term on the right-hand side as *"effective investments"* and the second term is the adjustment, e.g., installation costs, which we measure per-worker.

In per-worker terms savings, $s = S/L$, and investments, $\mathbf{I} = I/L$. Hence the investment adjustment constraint may be written in per-worker terms as

$$s = b\{\mathbf{I}\} = \mathbf{I} + \kappa \mathbf{I}^2, \qquad 0 < \kappa \tag{4}$$

Writing also Eq. (2) per-worker, and using Eqs. (1) and (4) yields

$$\begin{aligned} c^{\text{pt}} &= y - s \\ &= f\{A\} - \left(\mathbf{I} + \kappa \mathbf{I}^2\right) \end{aligned} \tag{5}$$

Technology diffusion is described by a logistic function that combines the dynamic interaction between the level of investment, (local) innovation and foreign technology (Barro and Sala-i-Martin 1997; Stokey 2015; Benhabib et al. 2014; Luttmer 2015; Perla et al. 2015),

$$\frac{\dot{A}^{\text{pt}}}{A^{\text{pt}}} = \mathbf{I}\left[1 - \left(\frac{A^{\text{pt}}}{\overline{A}}\right)^{\upsilon}\right] - \delta - \varphi, \qquad 0 < \upsilon, \delta, \varphi < 1 \tag{6}$$

where A^{pt} is used to denote the level of *pre-tax and subsidy* innovation, υ, δ and φ capture the rate of technology diffusion from abroad, the rate of obsolescence and the expansion of the technology frontier, respectively. By assumption all these are positive constants. The dynamics of innovation is determined by $\mathbf{I}$ and $\overline{A}$. Technology diffusion is modulated by the parameter υ, the closer it is to 1 (0), the slower (higher) the spread of technology—i.e., frontier technology does not fully nor instantly spread to other countries. Notice that Eq. (6) is negative whenever $A^{\text{pt}} = \overline{A}$. If $0 < A^{\text{pt}} < \overline{A}$, A^{pt} heads asymptotically toward $\overline{A}$.

3.2 The role of government

We assume an environment where households and business sectors are separated entities. In this context, the government sets taxes and uses the revenue to grant subsidies that boost business innovation. The government budget position (G_D) is made of taxes minus government expenditures (G_C) minus subsidy payments (TR)

$$G_D = T - G_C - TR$$

Let's assume that the government sets a flat, time invariant, *ad-valorem* tax rate ($\bar{\tau}$) on income allowing for the exemption of savings associated to *effective investments*, e.g., $S - \kappa I^2 L^{-1} = I$ (the adjustment cost is unknown to all parties and, therefore, not exempted from taxation). Let us write the tax bill as

$$T = \bar{\tau}(Y - I), \qquad 0 \leq \bar{\tau} < 1 \tag{7}$$

For simplicity, we assume that the government balances subsidy payments with tax revenues, $TR = T$ and $G_C = 0$. Thus, a net balanced budget prevails

$$0 = T - TR \tag{8}$$

From the household and the business sector view point, the *tax-and-subsidy mechanism* above influences consumption and investment decisions in two ways. First, it reduces the value of consumption as households now pay taxes

$$C = Y - S - T \tag{9}$$

Using Eqs. (3) and (7) and rearranging terms, we get

$$\begin{aligned} C &= Y - S - \bar{\tau}(Y - I) \\ &= Y - I - \kappa I^2 L^{-1} - \bar{\tau}(Y - I) \\ &= (1 - \bar{\tau})(Y - I) - \kappa I^2 L^{-1} \end{aligned} \tag{10}$$

Writing the last equation in per-worker terms and using Eqs. (1) and (4) we obtain

$$c = (1 - \bar{\tau})\Big(f\{A\} - \mathbf{I}\Big) - \kappa \mathbf{I}^2 \tag{11}$$

Taxation redefines the maximization problem as households are set to maximizing the utility of what is left for consumption after taxes and savings, i.e., investment costs, are subtracted (notice that setting $\bar{\tau} = 0$, we obtain $c^{\mathrm{pt}} = c$, e.g., Eqs. (11) and (5) are the same).

The second way the government influences private agents decision making is by increasing their resources for innovation. In the next section we analyze the likely implications of this policy approach.

4 The subsidy mechanism

A noteworthy feature of the *tax and subsidy mechanism* in our model is that it is a *discretionary policy* aimed to boost business innovation. We assume away *arbitrage* opportunities. While households pay taxes and firms receive subsidies, in practice they are distinct entities and, therefore, at least partially unable of fully assessing the cost and benefits of fiscal management policies. This may be true in a context of agents with bounded rationality or whenever one allows for surprise fiscal policies to boost innovation.

We assume that the government's policy is to fully grant the tax revenues as subsidies to support innovation in the private sector.

$$T = TR$$
$$\bar{\tau}(Y - I) = I\tau_A, \qquad 0 \leq \tau_A < 1$$

where τ_A denotes the subsidy rate:

$$\tau_A = \frac{\bar{\tau}(Y - I)}{I}$$

This subsidy rate may be written in per worker terms as

$$\tau_A = \bar{\tau}\left(\left(\frac{\mathbf{I}}{y}\right)^{-1} - 1\right) \tag{12}$$

where $(\mathbf{I}/y)^{-1}$ denotes the inverse of the investment output ratio. Notice that only *effective investments* are considered for the subsidy.

For empirically reasonable rates of investment to output such that $\mathbf{I}/y < 1/2$, the subsidy rate is proportionally larger than the tax rate and depends positively (negatively) on the income (investment) behavior. That is, $\partial\tau_A/\partial y > 0$, and $\partial\tau_A/\partial\mathbf{I} < 0$.[3]

Under the *tax and subsidy* environment, investment resources per-worker are

$$\begin{aligned}\mathbf{I}(1 + \tau_A) &= \mathbf{I}\left(1 + \bar{\tau}\left(\left(\frac{\mathbf{I}}{y}\right)^{-1} - 1\right)\right) \\ &= \mathbf{I} + \bar{\tau}(y - \mathbf{I})\end{aligned} \tag{13}$$

Where the first part of Eq. (13) is the business effective investment and the second part is the subsidy. Using this result to modify Eq. (6) we have

$$\begin{aligned}\frac{\dot{A}}{A} &= \mathbf{I}(1 + \tau_A) \quad \left[1 - \left(\frac{A}{\bar{A}}\right)^{\upsilon}\right] - \delta - \varphi \\ &= \left(\mathbf{I} + \bar{\tau}(y - \mathbf{I})\right)\left[1 - \left(\frac{A}{\bar{A}}\right)^{\upsilon}\right] - \delta - \varphi, \quad 0 < \upsilon, \delta, \varphi, \bar{\tau} < 1\end{aligned} \tag{14}$$

[3] With $\bar{\tau} = 10\%$ and $\mathbf{I}/y = 20\%$, $\tau_A = 40\%$. But with the same tax rate and $\mathbf{I}/y = 30\%$, $\tau_A = 23\%$, which is explained because increasing investments narrows the tax base.

Notice that Eqs. (6) and (14) are the same provided $\bar{\tau} = \tau_A = 0$.

There are three points worth mentioning when analyzing the macroeconomic implications of Eq. (14). Firstly, as we have noticed earlier, from the point of view of investors, the *tax and subsidy mechanism* is exogenously given. This is a key assumption. If investors are aware that they are entitled to an innovation investment subsidy on the basis of the households tax bill, they would probably adjust their consumption/savings behavior accordingly leaving investments, hence innovation, unchanged. The exogeneity of the *subsidy mechanism*, and the assumption that households and business are separated entities, precludes this kind of *arbitrage*.

Secondly, the *tax and subsidy mechanism* implicitly reflects the normative idea that the government is interested to boost a process of innovation-based growth. This is in contrast to cases where taxation precludes innovation and growth (Parente and Prescott 1999, 2002).

Finally, welfare effects matter. The effectiveness of the *tax and subsidy mechanism* hinges on its potential to increase the present net value of after tax consumption more than proportionately compared to the no tax and no subsidy scenario. Formally, one would need to show that

$$\frac{\int_0^T e^{-rt}\left[\left(1-\bar{\tau}\right)\left(f\{A\}-\mathbf{I}\right)-\kappa\mathbf{I}^2|_{\tau_A>\bar{\tau}>0}\right]dt}{\int_0^T e^{-rt}\left[f\{A\}-\mathbf{I}-\kappa\mathbf{I}^2|_{\tau_A=\bar{\tau}=0}\right]dt} \geq 1 \tag{15}$$

Providing this condition is fulfilled, resources available for consumption and investment are at least as high in the new scenario as they were in the old one. Unfortunately, we will not be able to evaluate the integrals numerically. However, we will indicate conditions under which the condition (15) holds.

5 Solving the optimization problem

The objective of the representative agent is to maximize the discounted sum of Eq. (11) subject to the dynamics of innovation established in Eq. (14). To simplify matters, we assume $\upsilon = 1$ and $\varphi = 0$. Using Eq. (1), the law of motion of innovation is

$$\frac{\dot{A}}{A} = \left(\mathbf{I} + \bar{\tau}\left(f\{A\} - \mathbf{I}\right)\right)\left[1 - \frac{A}{\bar{A}}\right] - \delta, \qquad 0 < \delta, \bar{\tau} < 1 \tag{14'}$$

The optimization problem, in per-worker terms and with future values discounted at rate r, is[4]

$$\max \int_0^T e^{-rt}\left[\left(1-\bar{\tau}\right)\left(f\{A\}-\mathbf{I}\right)-\kappa \mathbf{I}^2\right]dt$$

$$s.t.\quad \dot{A}=A\left(\mathbf{I}+\bar{\tau}\left(f\{A\}-\mathbf{I}\right)\right)\left[1-\frac{A}{\bar{A}}\right]-\delta A$$

$$A\{0\}=A_0>0,\quad \mathbf{I}\{0\}=\mathbf{I}_0>0,\qquad 0<\delta,\bar{\tau}<1$$

Assuming $\kappa = 1/2$, the current value Hamiltonian H_c is

$$H_c(\mathbf{I},A,\lambda)=\left(1-\bar{\tau}\right)\left(f\{A\}-\mathbf{I}\right)-\frac{\mathbf{I}^2}{2}+\lambda\left(A\left(\mathbf{I}+\bar{\tau}\left(f\{A\}-\mathbf{I}\right)\right)\left[1-\frac{A}{\bar{A}}\right]-\delta A\right)$$

Investment influences the objective function twice, directly, through its own value in the objective function, and, indirectly, through its impact on the evolution of the state equation. The state variable (A) evolves according to the logistic diffusion mechanism. The technology of the frontier, $\bar{A}$ influences the objective only indirectly through the state equation. Finally, the exogeneity of the *tax and subsidy mechanism*, $\mathbf{I}(1+\tau_A)=\mathbf{I}+\bar{\tau}(y-\mathbf{I})$, implies that the optimizing agent has no choices to make about optimal taxation/subsidy policy.

We aim to find an expression that reflects the dynamics of investments in innovation. The first-order conditions (FOC) for optimization are Eq. (14') and

$$\frac{\partial H_c}{\partial \mathbf{I}}=\lambda(1-\bar{\tau})A\left(1-\frac{A}{\bar{A}}\right)-(1-\bar{\tau})-\mathbf{I}\quad=\quad 0 \tag{16}$$

$$\begin{aligned}\dot{\lambda}-r\lambda=-\frac{\partial H_c}{\partial A}=\lambda\Bigg[&-\left(\mathbf{I}+\bar{\tau}\left(f\{A\}-\mathbf{I}\right)\right)\left(1-\frac{A}{\bar{A}}\right)\\&+\left(\mathbf{I}+\bar{\tau}\left(f\{A\}-\mathbf{I}\right)\right)\frac{A}{\bar{A}}+\delta\\&-\bar{\tau}f'A\left(1-\frac{A}{\bar{A}}\right)\Bigg]-(1-\bar{\tau})f'\end{aligned} \tag{17}$$

plus the usual transversality conditions, assuming $T\rightarrow\infty$

$$\lim_{t\rightarrow+\infty} e^{-rt}\lambda(t)\geq 0,\qquad \lim_{t\rightarrow+\infty} e^{-rt}\lambda(t)A(t)=0$$

[4] This formulation is based on the Stigler-Ozga model of diffusion in advertising theory (see Gould (1976) and Kamien and Schwartz (1991) Section II.9).

Equation (16) equates the marginal increase in innovation with the current increase in the investment cost. Equation (17) determines the shadow value of innovation.[5]

By log-transforming Eq. (16), we have

$$ln(1-\bar{\tau}+\mathbf{I}) = ln(\lambda) + ln(A) + ln\left(1-\frac{A}{\bar{A}}\right) \tag{16'}$$

Differencing this equation with respect to time yields

$$\frac{1}{1-\bar{\tau}+\mathbf{I}}\dot{\mathbf{I}} = \frac{\dot{\lambda}}{\lambda} + \frac{\dot{A}}{A} - \frac{\dot{A}}{\bar{A}-A} \tag{18}$$

After some algebra, we obtain[6]

$$\frac{1}{1-\bar{\tau}+\mathbf{I}}\dot{\mathbf{I}} = r + \delta\frac{A}{\bar{A}-A} - \frac{1-\bar{\tau}+\mathbf{I}\bar{\tau}}{1-\bar{\tau}+\mathbf{I}}f'A\left(1-\frac{A}{\bar{A}}\right) \tag{19}$$

Over a BGP, with $\dot{\mathbf{I}}=0$, we find that

$$\left(r+\delta\frac{A}{\bar{A}-A}\right)\left(1-\bar{\tau}+\mathbf{I}\right) = \left(1-\bar{\tau}+\mathbf{I}\bar{\tau}\right)f'A\left(1-\frac{A}{\bar{A}}\right)$$

The last expression may be written as

$$\left(r+\delta\frac{A}{\bar{A}-A}\right)\left(1-\bar{\tau}\right) + \left(r+\delta\frac{A}{\bar{A}-A}\right)\mathbf{I} = f'A\left(1-\frac{A}{\bar{A}}\right)\left(1-\bar{\tau}\right) + f'A\left(1-\frac{A}{\bar{A}}\right)\mathbf{I}\bar{\tau}$$

Solving for $\mathbf{I}$, we obtain

$$\mathbf{I} = \frac{\left[f'A\left(1-\frac{A}{\bar{A}}\right) - \left(r+\delta\frac{A}{\bar{A}-A}\right)\right]\left(1-\bar{\tau}\right)}{\left(r+\delta\frac{A}{\bar{A}-A}\right) - f'A\left(1-\frac{A}{\bar{A}}\right)\bar{\tau}} \tag{20}$$

From the state Eq. (14'), an equilibrium path satisfying $\dot{A}=0$ implies

$$A = \bar{A}\left[1 - \frac{\delta}{\mathbf{I}+\bar{\tau}\left(f\{A\}-\mathbf{I}\right)}\right] \tag{21}$$

[5] Second order conditions for optimality are satisfied also; *sufficiency* is established by checking that the conditions of the *Mangasarian's theorem* are fulfilled (Kamien and Schwartz (1991) pp. 221 ff). Notice that the production function has properties $f' > 0$, $f'' \leq 0$ and, from Eq. (16) we have

$$\frac{\partial^2 H_c}{\partial \mathbf{I}^2} = -1$$

Note, also from Eq. (16), that $\lambda > 0$. Hence, the Hamiltonian is concave in A and $\mathbf{I}$.

[6] See Appendix A for details on the derivation of Eq. (19).

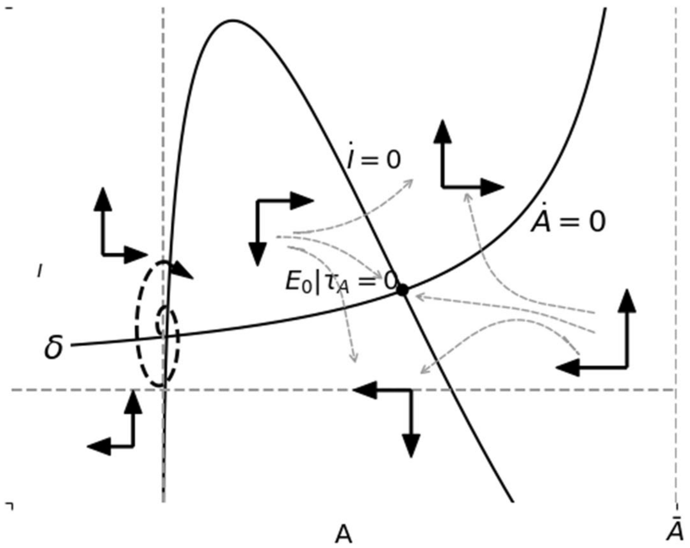

Fig. 1 A values go from 0 to 100. The arrows of motion show that the leftmost side equilibrium is unstable and that on the rightmost side is saddlepoint stable. Note that here $\bar{\tau} = \tau_A = 0$. We use $f\{A\} = A^{\alpha}$ with α=1/3, r=0.15, δ=0.2

Notice that Eq. (21) may be solved as well for **I** which yields

$$\mathbf{I} = \left(\frac{\bar{A}\delta}{\bar{A} - A} - \bar{\tau} f\{A\} \right) \left(1 - \bar{\tau}\right)^{-1} \tag{22}$$

Equations (20) and (22) describe the stationary lines of the dynamic system. These lines are drawn in Fig. 1 together with the arrows of motion. The steady state E_0, drawn for the absence of taxes and subsidies, is reached via a downward sloping saddlepoint stable trajectory. The low-level steady state is unstable, as we explain below.

Notice that, everything else constant, increasing taxation increases innovation investments by shifting up the stationary line represented by Eq. (20) in Fig. 2 compared to Fig. 1. As we explain in more detail in Section 5.2.

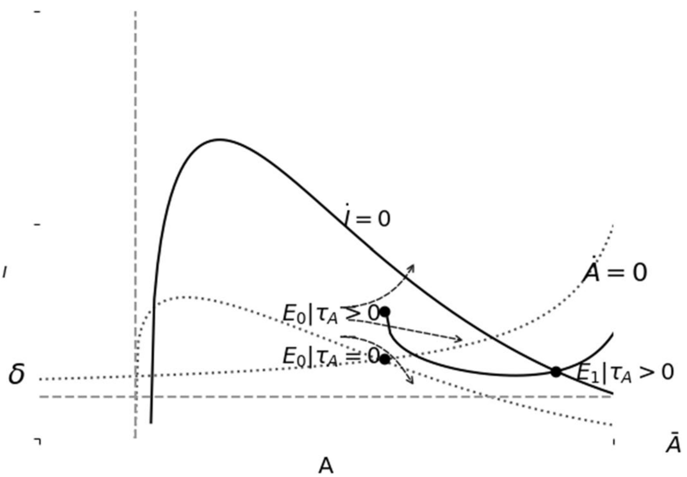

Fig. 2 A values go from 0 to $\bar{A}$=100. Equilibrium at E_0 and the corresponding stationary (dotted) lines assume $\tau_A = \bar{\tau}$=0. Equilibrium at E_1 and the corresponding stationary (solid) lines assumes $\tau_A > \bar{\tau}$=0.05. We use $f\{A\} = A^\alpha$ with α=1/3, r=0.15, δ=0.2

$$\left.\frac{\partial \mathbf{I}}{\partial \bar{\tau}}\right|_{\dot{\mathbf{I}}=0} = \frac{\left(r+\delta\frac{A}{\bar{A}-A}\right) - f'A\left(1-\frac{A}{\bar{A}}\right)}{\left(r+\delta\frac{A}{\bar{A}-A}\right) - f'A\left(1-\frac{A}{\bar{A}}\right)\bar{\tau}} + \frac{f'A\left(1-\frac{A}{\bar{A}}\right)\left[\left(r+\delta\frac{A}{\bar{A}-A}\right) - f'A\left(1-\frac{A}{\bar{A}}\right)\right]}{\left[\left(r+\delta\frac{A}{\bar{A}-A}\right) - f'A\left(1-\frac{A}{\bar{A}}\right)\bar{\tau}\right]^2} > 0 \tag{23}$$

Likewise, according to Eq. (22), increasing $\bar{\tau}$ leads to lower $\mathbf{I}$ for every given A. Hence, the $\dot{A} = 0$ isocline for positive taxes must be below the one for zero taxes.

$$\left.\frac{\partial \mathbf{I}}{\partial \bar{\tau}}\right|_{\dot{A}=0} = \frac{\left(\frac{\bar{A}\delta}{\bar{A}-A} - \bar{\tau}f\{A\}\right) - \left(1-\bar{\tau}\right)f\{A\}}{\left(1-\bar{\tau}\right)^2} < 0 \tag{24}$$

With Eq. (20) shifting up and Eq. (22) shifting down after an increase in taxation, we get a new steady state at a higher level of innovation in Fig. 2 implying catching-up to be closer to the frontier $\bar{A}$.

In general terms we find that, under the tax and subsidy mechanism, the economy could experience an increase of investments, an increase of innovation and, therefore, an increase in economic growth. Of course, the increase in taxation cannot be too large to depress consumption in current value terms. The recursive nature of this process involving taxation, investment, further innovation and economic growth is analyzed in detail in order to determining the equilibrium properties of the system.

5.1 Baseline scenario ($\bar{\tau} = 0, \tau_A = 0$)

To illustrate the main implications of our model in terms of *the tax and subsidy mechanism* on economic growth and general well being, we first analyze the baseline scenario without taxes nor subsidies.

Notice, from Eq. (12), that under a $\bar{\tau} = 0$ scenario, $\tau_A = 0$. In this case, the temporary objective function of the agent is $f\{A\} - \mathbf{I} - \kappa\mathbf{I}^2$ and the state equation $\dot{A} = \mathbf{I}A\left(1 - A/\bar{A}\right) - \delta A$. The optimal solution in Eq. (20) becomes

$$\mathbf{I} = \frac{f'A\left(1 - \frac{A}{\bar{A}}\right)}{r + \delta\frac{A}{\bar{A}-A}} - 1 \tag{25}$$

Likewise, Eq. (22) becomes

$$\mathbf{I} = \frac{\bar{A}\delta}{\bar{A} - A} \tag{26}$$

The vector field determined by Eqs. (25), (26) are plotted in the **I**-A plane in Fig. 1. From Eq. (25), the $\dot{\mathbf{I}} = 0$ locus determines a bell-shape curve. This curve is increasing for small values of A and decreasing for large values. Innovation investments rise for points above the $\dot{\mathbf{I}} = 0$ locus and fall for points below it. The vertical arrows of motion illustrate this behavior.

From Eq. (26), the $\dot{A} = 0$ curve is an increasing function that, asymptotically, approaches $\bar{A}$. This curve has an intercept on the vertical axis when $A \overset{a}{\to} 0$ at $\mathbf{I} = \delta$. A is increasing (decreasing) above (below) the $\dot{A} = 0$ locus as is shown by the horizontal arrows of motion.

There are two equilibria in Fig. 1. The leftmost equilibrium is featured by low values of innovation and investment. This is an *unstable locus* that oscillates and moves away from the equilibrium unless $A(0) = A_{ss}$ and $\mathbf{I}(0) = \mathbf{I}_{ss}$. The rightmost equilibrium, is *saddlepoint* stable. From the information provided in the Jacobian matrix, we deduce that the innovation trajectory is stable while the investment trajectory is unstable.[7] So sufficiently large disturbances on the investment dynamics will take the system away from the equilibrium.

Equation (25) suggests that investment is a declining function of the rate of return while the effect of the depreciation rate on investment is ambiguous. A higher rate of depreciation leads to lower investments through Eq. (25), but higher investments through Eq. (26). The relationship between investment and innovation, on the other side, is quite cumbersome.

Numerical calculations in Table 1 allow us to make sense of the above interrelationships between the variables in the system and the way they are affected by

[7] See Appendix B.

Table 1 Investment and productivity values for high level steady states with alternative tax and interest rates.

Tax\ Return	r=0.05			r=0.10			r=0.15			r=0.20		
	I	**I+**	A	**I**	**I+**	A	**I**	**I+**	A	**I**	**I+**	A
α=1/3, δ=5%, $\bar{A}$=100												
τ=0.00	0.240	0.240	79.180	0.217	0.217	77.000	0.197	0.197	74.700	0.179	0.179	72.050
τ=0.01	0.203	0.243	79.517	0.181	0.221	77.435	0.161	0.201	75.200	0.142	0.182	72.660
τ=0.02	0.165	0.247	79.857	0.144	0.226	77.900	0.124	0.206	75.700	0.106	0.187	73.300
τ=0.05	0.049	0.262	81.000	0.027	0.240	79.200	0.007	0.219	77.300	−0.009	0.202	75.234
α=1/3, δ=10%, $\bar{A}$=100												
τ=0.00	0.346	0.346	71.120	0.321	0.321	68.900	0.298	0.298	66.500	0.277	0.277	63.885
τ=0.01	0.311	0.349	71.400	0.287	0.325	69.300	0.265	0.302	67.000	0.244	0.281	64.500
τ=0.02	0.278	0.355	71.870	0.254	0.331	69.790	0.231	0.307	67.547	0.210	0.286	65.100
τ=0.05	0.171	0.371	73.056	0.147	0.346	71.200	0.126	0.324	69.200	0.106	0.303	67.063
α=2/3, δ=10%, $\bar{A}$=100												
τ=0.00	0.867	0.867	88.470	0.845	0.845	88.170	0.825	0.825	87.880	0.804	0.804	87.570
τ=0.01	0.715	0.907	88.975	0.690	0.881	88.700	0.674	0.865	88.450	0.656	0.847	88.200
τ=0.02	0.563	0.951	89.500	0.542	0.930	89.260	0.523	0.911	89.025	0.505	0.892	88.800
τ=0.05	0.118	1.124	91.110	0.105	1.111	91.000	0.084	1.090	90.827	0.069	1.074	90.700

Total investment (**I+**) includes the subsidy, calculated using Eq. (13) as $\mathbf{I}(1+\tau_A) = \mathbf{I} + \bar{\tau}(y - \mathbf{I})$

changes in key parameters. In particular, notice that under the $\tau = 0$ scenario, both investment and innovation decrease as the rate of return increases from low ($r = 5\%$) to high values ($r = 20\%$). On the other hand, a high rate of depreciation leads to higher values of investments and decreasing levels of innovation (as when this parameter is increased from 5 to 10% in the table). Finally, using $f\{A\} = A^{\alpha}$, and letting α to increase from 1/3 to 2/3 leads to both, higher levels of innovation and higher levels of investment.

5.2 The tax and subsidy mechanism ($0 < \bar{\tau} < \tau_A$)

The core argument of the model in this paper is that it captures an essential fact in the objectives of the innovation policy: setting a flat tax rate on consumption and using the revenues to fund additional innovation investment should lead to increasing innovation and, therefore, economic growth at the economy wide level.

In Fig. 2, we plot the original scenario $\bar{\tau} = \tau_A = 0$ jointly with a plot of the alternative scenario $0 < \bar{\tau} < \tau_A$. We focus on the right-hand region *saddlepoint* equilibria.

Assume that we start from the equilibrium without policy, $E_0|\tau_A = 0$, the investment subsidy granted under the *tax and subsidy mechanism* causes the economy to suddenly jump up to a high value of investment for given A at $E_0|\tau_A > 0$. Then a new equilibrium trajectory takes over eventually reaching a new *steady state* at $E_1|\tau_A > 0$. Below, we provide a more formal analysis of the system dynamics.

We pointed out, from Eqs. (23) and (24), that the investment loci shifts up and the innovation loci shifts down as the tax rate increases. Formally, the shift of the $\dot{\mathbf{I}} = 0$ locus is obtained from Eq. (23) where

$$\left.\frac{\partial \mathbf{I}}{\partial \tilde{\tau}}\right|_{\dot{\mathbf{I}}=0} > 0$$

in turn, the shift of the $\dot{A} = 0$ is obtained from Eq. (24)

$$\left.\frac{\partial \mathbf{I}}{\partial \bar{\tau}}\right|_{\dot{A}=0} < 0$$

Thus, the dynamical process triggered by the *tax and subsidy mechanism* involves changing linear combinations of investment and innovation until they reach equilibrium trajectories that finally are joined in the new saddlepoint at $E_1|\tau_A > 0$.

From our graphical approach, the transition to the new equilibrium seems to be consistent with an increase in innovation, hence output. However, the behavior of investment in the new equilibrium is not clear. Intuitively, the *tax and subsidy mechanism* should lead to a higher level of investment under the new *steady state* $E_1|\tau_A > 0$ relative to the origin at E_0. The numerical solutions provided in Table 1 for various parameters values show higher total investment I+ (including the subsidy value) in spite of lower privately paid investment I (excluding the subsidy).

The dynamic system outlined above is one way to illustrate how public subsidies, alongside many other policy mechanisms in this direction, may be

Table 2 Output, consumption and investment values for high level steady states with alternative tax and interest rates

Tax\ Return	r=0.05			r=0.10			r=0.20		
	c	**I**+	f{A}	c	**I**+	f{A}	c	**I**+	f{A}
α=1/3, δ=5%, $\bar{A}$=100									
τ=0.00	4.025	0.240	4.294	4.014	0.217	4.254	3.966	0.179	4.161
τ=0.01	4.036	0.243	4.300	4.024	0.221	4.262	3.980	0.182	4.173
τ=0.02	4.045	0.247	4.306	4.034	0.226	4.271	3.992	0.187	4.185
τ=0.05	4.063	0.262	4.327	4.054	0.240	4.294	4.019	0.202	4.222

We rely on the steady state values from Table 1 and use $f\{A\} = A^{\alpha}$. Total investment (**I**+) includes the subsidy. Consumption values are generated using Eq. (11): $c = (1-\bar{\tau})\left(f\{A\} - \mathbf{I}\right) - \kappa\mathbf{I}^2$

self-sustainable strategies to boost a virtual cycle of innovation and growth. In fact, as mentioned earlier, the *tax and subsidy mechanism* formalized in our model has been actual–and often controversial–practice in the innovation policy followed by both countries at the frontier and successful catching-up countries. Examples include the strategies put in place to ease catching-up by developmental Asian states (Nelson and Pack 1999; Hu 2020), initiatives like the Small Business Investment Company (SBIC) and Small Business Innovation Research (SBIR) in the US, and essentially similar program in other advanced economies (see Lichtenberg 1988; Lerner 1999; Hall and Lerner 2010; Howell 2017; Denes et al. 2020).

A final step, for the overall assessment of this mechanism, regards its potential to improve social welfare.

5.3 Welfare effects

A further implication in the shift of the equilibrium point from $E_0|\tau_A = 0$ to $E_1|\tau_A > 0$ in Fig. 2 is that, in the first instance, consumption, hence social welfare, declines. But then, along the new optimal path, consumption increases along with investment given the higher values of productivity in the new scenario.

More specifically, as investment increases at the jump between $E_0|\tau_A = 0$ and $E_0|\tau_A > 0$, consumption declines via taxes. However, as the economy moves from $E_0|\tau_A > 0$ to the new equilibrium $E_1|\tau_A > 0$, it exhibits a larger amount of productivity, and hence a larger amount of output. Following from Eq. (15) and the A-values in Table 1, we verify the condition that consumption (and investment) are higher under the *tax and subsidy mechanism* than otherwise, as may be seen from the result presented in Table 2.

From Eqs. (11) and (26), the baseline scenario $\tau_A = \bar{\tau} = 0$ implies,

$$c\Big|_{\tau_A=0} = f\{A\} - \mathbf{I} - \kappa\mathbf{I}^2$$
$$= f\{A\} - \frac{\bar{A}\delta}{\bar{A}-A} - \kappa\left(\frac{\bar{A}\delta}{\bar{A}-A}\right)^2 \tag{27}$$

where **I** is expressed in terms of A-values when going from the first to the second line. Notice, from Eq. (27), that consumption increases in $A \to \bar{A}$ as f(A) increases and private investment I falls, according to Table 1.

By relying, in the new steady state, on the *tax and subsidy mechanism*, using the steady state property established in Eq. (22), we obtain

$$c\Big|_{\tau_A>0} = (1-\bar{\tau})\Big(f\{A\} - \mathbf{I}\Big) - \kappa\mathbf{I}^2$$
$$= (1-\bar{\tau})\left(f\{A\} - \left(\frac{\frac{\bar{A}\delta}{\bar{A}-A} - \bar{\tau}f\{A\}}{1-\bar{\tau}}\right)\right) - \kappa\left(\frac{\frac{\bar{A}\delta}{\bar{A}-A} - \bar{\tau}f\{A\}}{1-\bar{\tau}}\right)^2 \tag{28}$$
$$= f\{A\} - \frac{\bar{A}\delta}{\bar{A}-A} - \kappa\left(\left(\frac{\frac{\bar{A}\delta}{\bar{A}-A} - \bar{\tau}f\{A\}}{1-\bar{\tau}}\right)\right)^2$$

Using again $\kappa = 1/2$, and deriving Eq. (28) with respect to $\bar{\tau}$, we obtain the following steady state result

$$\frac{\partial c}{\partial \bar{\tau}}\Bigg|_{\tau_A>0} = -\frac{-f\{A\}(1-\bar{\tau}) + \left(\frac{\bar{A}\delta}{\bar{A}-A} - \bar{\tau}f\{A\}\right)}{(1-\bar{\tau})^2}$$
$$= \frac{f\{A\} - \frac{\bar{A}\delta}{\bar{A}-A}}{(1-\bar{\tau})^2}$$

which implies that consumption is a positive and increasing function of $\bar{\tau}$.

Whether the welfare benefits of increasing consumption in the future are worth the sacrifice incurred by reducing consumption in the earlier phase, after the introduction of the policy, may be evaluated from the steady state values that clearly are higher in the equilibrium–and a bit earlier– under the *tax and subsidy mechanism*. Based on a subset of the steady state values calculated in Tables 1 and 2 shows higher consumption values under the tax/subsidy policy.

An important implication of this analysis is that implementing the *tax and subsidy mechanism* gives rise to an early phase with reduced consumption and utility and a later phase with increased consumption and utility. Discounting rates which give weights to these phases matter. A high discount rate means that the later positive phase gets a low weight. If the discount is high enough, the negative impact on consumption and utility of the initial phase dominates. Conversely, if the discount is sufficiently close to zero, the increased consumption of the later phase dominates because it lasts until infinity.

Summing up, in our view, the model that we have developed here captures an essential aspect in the use of fiscal instruments to increase the availability of investable resources to promote innovation, hence economic growth and catching-up. Economies that have a low discount rate will benefit from a policy that brings them closer to the technological frontier. Economies with a high discount rate stay more behind.

As we mentioned earlier, taxing current consumption and using the unconsumed resources to grant subsidies in order to boost economic growth is a policy arrangement that has been actually implemented, to some extent, in many countries. A notable and well documented exception seems to occur in a selected group of LAC (Crespi 2012). The limited application/understanding and relative lack of success of innovation-based growth policy mechanisms in this case is a testimony of the need to improve our current understanding on the rewarding benefits of a well-designed fiscal program using tax incentives to support innovation.

6 Concluding remarks

Studying the interaction between the adoption of foreign technology and the process of local innovation is crucial for the research on the ability of backward countries to catching-up, and for the design and implementation of innovation policy. We have set up a model where innovation, alongside technology trajectories that are associated with state-of-the-art inventions and working practices that are common to all countries, leads to a higher level of productivity closer to the frontier countries.

The key feature of the model that we have developed above is that it provides a formal framework for the analysis of the government when it seeks to manipulate policy instruments to obtain more favorable outcomes in knowledge leading to innovation. In particular, we have suggested that countries with high discount rates will not be willing to accept the taxation and the temporary consumption loss implied by the innovation policy under the *tax and subsidy mechanism*. But countries with low discount rates will be more likely willing to do so. But this is ultimately an empirical question for future research.

More generally, while the case for a *tax and subsidy mechanism* has been a limited practice, particularly in less developed countries, and this sort of mechanisms has been a subject of mostly empirical academic research for a longtime, we hope that the theoretical framework presented here shall become a basis for further theoretical and empirical work on the crucial relationship between technology diffusion, innovation and the process of catching-up; and a technical basis for the modern discussion on the design, implementation and evaluation of public policies spurring innovation.

Appendix A

To go from Eqs. (18) to (19), note that from Eq. (17)

$$\begin{aligned}\frac{\dot{\lambda}}{\lambda} =& r - \left(\mathbf{I} + \bar{\tau}\left(f\{A\} - \mathbf{I}\right)\right)\left(1 - \frac{A}{\bar{A}}\right) + \delta \\ &+ \left(\mathbf{I} + \bar{\tau}\left(f\{A\} - \mathbf{I}\right)\right)\frac{A}{\bar{A}} - \bar{\tau}f'A\left(1 - \frac{A}{\bar{A}}\right) - \frac{(1-\bar{\tau})f'}{\lambda} \\ =& r - \frac{\dot{A}}{A} + \left(\mathbf{I} + \bar{\tau}\left(f\{A\} - \mathbf{I}\right)\right)\frac{A}{\bar{A}} - \bar{\tau}f'A\left(1 - \frac{A}{\bar{A}}\right) - \frac{(1-\bar{\tau})f'}{\lambda}\end{aligned} \tag{17'}$$

From Eq. (14')

$$\frac{\dot{A}}{\bar{A} - A} = \left(\mathbf{I} + \bar{\tau}\left(f\{A\} - \mathbf{I}\right)\right)\frac{A}{\bar{A}} - \delta\frac{A}{\bar{A} - A}$$

Inserting these expressions into Eq. (18), and collecting terms, yields

$$\frac{1}{1 - \bar{\tau} + \mathbf{I}}\dot{\mathbf{I}} = r + \delta\frac{A}{\bar{A} - A} - \bar{\tau}f'A\left(1 - \frac{A}{\bar{A}}\right) - \frac{(1-\bar{\tau})f'}{\lambda} \tag{18'}$$

Thus, from Eq. (16)

$$\lambda = \frac{1 - \bar{\tau} + \mathbf{I}}{(1-\bar{\tau})A\left(1 - \frac{A}{\bar{A}}\right)}$$

Using this expression in Eq. (18'), we obtain

$$\begin{aligned}\frac{1}{1 - \bar{\tau} + \mathbf{I}}\dot{\mathbf{I}} =& r + \delta\frac{A}{\bar{A} - A} - \bar{\tau}f'A\left(1 - \frac{A}{\bar{A}}\right) - \frac{(1-\bar{\tau})f'}{\lambda} \\ =& r + \delta\frac{A}{\bar{A} - A} - \bar{\tau}f'A\left(1 - \frac{A}{\bar{A}}\right) - \frac{(1-\bar{\tau})^2}{1 - \bar{\tau} + \mathbf{I}}f'A\left(1 - \frac{A}{\bar{A}}\right) \\ =& r + \delta\frac{A}{\bar{A} - A} - \frac{\left(1 - \bar{\tau} + \mathbf{I}\right)\bar{\tau} + (1-\bar{\tau})^2}{1 - \bar{\tau} + \mathbf{I}}f'A\left(1 - \frac{A}{\bar{A}}\right)\end{aligned} \tag{18''}$$

From where we finally obtain Eq. (19).

Appendix B

To characterize the two equilibria in Fig. 1 we linearize the model around the steady state $(A_{ss}, \mathbf{I}_{ss})$ getting

$$\begin{bmatrix}\dot{A} \\ \dot{\mathbf{I}}\end{bmatrix} = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22}\end{bmatrix}\begin{bmatrix} A - A_{ss} \\ \mathbf{I} - \mathbf{I}_{ss}\end{bmatrix}$$

The first entry in the righthand side matrix above is the two-by-two Jacobian matrix (}}Δε) whose elements are the partial derivatives of the system around the equilibrium, which are obtained as follows[8]

$$j_{11} = \frac{\partial \dot{A}}{\partial A}\Big|_{\dot{A}=\bar{\tau}=0} = -\frac{A_{ss}\mathbf{I}_{ss}}{\bar{A}} \qquad < 0$$

$$j_{12} = \frac{\partial \dot{A}}{\partial \mathbf{I}}\Big|_{\dot{A}=\bar{\tau}=0} = A_{ss}\left[1 - \frac{A_{ss}}{\bar{A}}\right] \qquad > 0$$

$$j_{21} = \frac{\partial \dot{\mathbf{I}}}{\partial A}\Big|_{\dot{\mathbf{I}}=\bar{\tau}=0} = \left[\frac{2\delta A_{ss}}{\left(\bar{A} - A_{ss}\right)^2} + \frac{2rA_{ss} - r\bar{A}}{A_{ss}\left(\bar{A} - A_{ss}\right)} - \frac{f''}{f'}\left(r + \frac{\delta A_{ss}}{\bar{A} - A_{ss}}\right)\right]\left(1 + \mathbf{I}\right) \qquad \gtrless 0$$

$$j_{22} = \frac{\partial \dot{\mathbf{I}}}{\partial \mathbf{I}}\Big|_{\dot{\mathbf{I}}=\bar{\tau}=0} = r + \delta\frac{A_{ss}}{\bar{A} - A_{ss}} \qquad > 0$$

The characteristic roots of the matrix Δ, ε_1 and ε_2, are obtained as usual

$$\varepsilon_{1,2} = \frac{tr(\Delta) \pm \sqrt{[tr(\Delta)]^2 - 4|\Delta|}}{2}$$

where $tr(\Delta)$ and $|\Delta|$ are, respectively, the trace and the determinant of Δ. To analyze the equilibrium, we assume $r, \delta, \mathbf{I} > 0$.[9] Note that for high values of A

[8] To obtain j_{21} notice that at the steady state and under the condition $\bar{\tau} = 0$, the following expressions are equivalent

$$f'\left(1 - \frac{A}{\bar{A}}\right) = \left[\frac{r}{A} + \frac{\delta}{\bar{A} - A}\right]\left(1 + \mathbf{I}\right)$$

and,

$$\frac{f'A}{\bar{A}} = \left[\frac{r}{\bar{A} - A} + \frac{\delta A}{(\bar{A} - A)^2}\right]\left(1 + \mathbf{I}\right)$$

[9] Note that $r = \delta = \mathbf{I} = 0$, or sufficiently close to zero, implies $\varepsilon_{1,2} = 0$. On the other hand, $r = \delta = 0$ and $\mathbf{I} > 0$ implies $\varepsilon_1 = 0 > \varepsilon_2$. In these cases, Eqs. (25)-(26) are either inconsistent or redundant: in the first case there are no equilibria; in the later, with a root equal to zero, any point may be an (*knife-edge*) equilibrium. Also, note that when $r = 0 < \delta$ and $\mathbf{I} > 0$, the roots are real and distinct with $\varepsilon_1 > 0 > \varepsilon_2$. In this case, the solution hinges on the value of δ: a large value of this parameter leads the solution to be dominated by the positive root and both A and $\mathbf{I}$ grow without bound; if the value of δ is small, on the other hand, the solution converge to a *saddlepoint* equilibrium. Finally, when $\delta = 0 < r$ and $\mathbf{I} > 0$ there is also a *saddlepoint* equilibrium. But, in the latter case there are roots that are complex conjugates—as explained in the main text.

$tr(\Delta) = j_{11} + j_{22} > 0$ and $|\Delta| = j_{11} \times j_{22} - j_{21} \times j_{12} < 0$, hence if follows that the system is *saddlepoint* stable for the intersection of the loci $\dot{A}$ and $\dot{\mathbf{I}}$ in the rightmost part of Fig. 1. From the Jacobian matrix, $j_{11} < 0$ and $j_{22} > 0$, thus we deduce that the innovation process is stable while the investment dynamics is unstable.

The leftmost equilibrium, on the other hand, is featured by low values of A which implies that $j_{21} < 0$, $tr(\Delta) > 0$ and $|\Delta| > 0$. Moreover, we obtain that $tr(\Delta) < |\Delta|$ which implies that the roots are complex conjugates. The (imaginary) roots are written as[10]

$$\varepsilon_{1,2} = \frac{tr(\Delta)}{2} \pm \frac{\sqrt{4|\Delta| - [tr(\Delta)]^2}}{2}$$

Thus, on the lefthand region we have two complex roots with positive real parts, $tr(\Delta) > 0$ and $|\Delta| > 0$. The solution is an *unstable locus* that oscillates and moves away from the equilibrium unless $A(0) = A_{ss}$ and $\mathbf{I}(0) = \mathbf{I}_{ss}$.

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Declarations

Conflict of interest The authors declare that there is no conflict of interest

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[10] Complex roots are written as

$$\varepsilon_{1,2} = a \pm di$$

where $a = tr(\Delta)/2$ and $d = \frac{\sqrt{[tr(\Delta)]^2 - 4|\Delta|}}{2}$. To obtain the expression on the text, we use the property of imaginary numbers $i^2 = -1$.

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