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Enhancing Swedish Toddlers' Learning Opportunities Through Interactions with Pictures and Narrative Designed for Numerical Learning Purposes

Camilla Björklund[1] · **Hanna Palmér**[2]

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Abstract
The aim of this study is to deepen the understanding of how preschool teachers can use representations of different kinds to bring fore the mathematical content that may be afforded in pictures and narrative designed for numerical learning purposes. Seventy-three video documentations of reading sessions with 27 toddlers (1–3 years of age) over the course of three semesters were analyzed with a focus on number-oriented actions that, theoretically, would facilitate toddlers' learning of numbers. In the reading sessions the teachers extended the content of the pictures and the narrative in different ways, and how one and the same picture was handled was shown to impact on what was made possible for the children to discern. Three different objects of learning—*identifying numbers, comparing numbers*, and *operating on numbers*—were made visible. Further, the use of gestures strengthened the possibility for the children to make connections within and between representations. This contributes to our understanding of the importance of using representations and gestures with a pedagogical purpose.

Keywords Gestures · Numbers · Learning · Pictures · Toddlers · Early childhood education

Introduction

As mathematics in general and numbers in particular are abstract notions, they need to be represented in order to be perceived. Thus, one challenge in education involves supporting children in seeing phenomena like numbers that can only be mediated through representations (Duval, 2006). Such representations are often symbols, but can also be pictures or words, for example.

The present study is a continuation of studies showing that children's attention to numbers and the frequency and quality of "number talk" significantly increased when teacher and child were engaged in interactive conversation including representations (Björklund & Palmér, 2020, 2022). However, children's learning is more complex than simply engaging in activities including mathematical representations. To attain positive learning results, it has been shown to be crucial for the teacher to comprise and extend the children's responses (Hassinger-Das et al., 2015; Jennings et al., 1992). In earlier studies of teacher-child interaction with pedagogical purposes (Björklund & Palmér, 2020, 2022, 2023), how the teacher directs attention to the mathematical content and a certain *meaning* of the content has been shown to be of great importance but also a challenging task. The aim of the current study is to deepen this strand of research, by specifically focusing on how preschool teachers use representations of different kinds to bring fore the mathematical content that may be afforded in pictures and narrative designed for numerical learning purposes. The specific research question is: What is made possible to learn about numbers, and how is this learning of numbers facilitated based on teacher actions?

Background

Learning about numbers is a complex endeavor that goes beyond merely learning to recite the counting rhyme in a stable order. Knowing the meaning of the counting words

✉ Camilla Björklund
camilla.bjorklund@ped.gu.se

[1] Department of Education, Communication and Learning, Faculty of Education, University of Gothenburg, Gothenburg, Sweden

[2] Department of Mathematics, Faculty of Technology, Linnaeus University, Växjö, Sweden

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and making use of numbers in situations that include quantities requires knowledge of the meaning that numbers (and counting words) may entail and how representations can be used to communicate and work with numbers. The research in this area is broad; as a starting point, there are presumed innate cognitive processes that help a child to perceptually discriminate and quantify small sets of objects (subitizing, e.g., Clements et al., 2019) and to approximately determine larger amounts (ANS, e.g., Bonny and Lourenco, 2013). These intuitive processes are presumed to be a foundation for developing an understanding of how quantities are represented by finger patterns, counting words, or graphic symbols.

Most commonly, numbers are related to determining an exact quantity. Answering the question "how many" with a counting word is an indication of a child perceiving numbers' "cardinality" meaning. In general, children have acquired this understanding by their fourth birthday; that is, they understand that "three" means a set of exactly three objects and that if you add another object, it will make one (and only one) more than three: "four" (Sarnecka & Carey, 2008). Solving problems in which quantities change or are missing requires the knowledge that numbers are constituted of smaller numbers (or units) in a relational way. One could say that numbers need to be regarded as objects themselves, which thereby makes it possible to perceive their meaning (Radford, 2003) and allows the child to mentally manipulate, decompose, recompose, and relate them to other numbers. But as numbers are not objects in the same sense as physical ones, their meaning is only accessible through representations. Representations refer to some meaning they are taken to have, but these meanings are accessible for interpretation, manipulation, and discussion through the representations themselves (Goldin, 2014).

Even though the theoretical foundations and wordings in research on representations differ, they share an emphasis on the fact that the learning of mathematics involves and can be strengthened by the ability to make connections within and between representations (see Lesh et al., 1987; Duval, 2006; Goldin, 2014). However, children do not automatically realize such connections but must rather see them demonstrated (Björklund & Palmér, 2020, 2022). The quality of teachers' verbal communication about numbers—that is, how the connections between and meaning of numbers are demonstrated—has been empirically shown to play a significant role in children's ways of reasoning about numbers (e.g., Björklund & Palmér, 2020; Levine et al., 2010). A recent study on how modes of representation are made into resources for number learning in book reading sessions with toddlers revealed that each mode of representation had to be presented to the children in ways that highlighted the specific intended meaning, and that variation within one mode of representation was more likely to bring to the fore a certain meaning of numbers for the child than was variation between different modes of representation alone (Björklund & Palmér, 2022).

Gestures, often used together with representations, have been shown to promote mathematical learning among young children (Ping & Goldin-Meadow, 2008; Valenzeno et al., 2003; Robutti, 2014). Gestures have communicative power when used to convey information about ideas and relations, and provide additional cues when verbal language induces comprehension difficulties, for example when young learners have not yet fully developed verbal skills (Hostetter, 2011). Arzarello et al. (2009) suggest that gestures may help to bridge between different representations as well as between worldly experiences and mathematical ones. According to Radford (2003), gestures are in this way a resource for supporting the making of connections from one representation to another.

There are different kinds of gestures to be used for different purposes (see, e.g., Alibali & Nathan, 2012; McNeill, 2005): *pointing gestures*, which serve to indicate objects, for instance pointing with one's index finger to a tower of blocks to draw attention to and ground the communication in the physical environment; *representational gestures*, depicting aspects of meaning either literally or metaphorically, such as making a circular movement with one's hand as if to group objects together; and *beat gestures*, which motorically and rhythmically align with speech, such as counting out loud and pointing at the counted objects in a one-to-one correspondence. In mathematics teaching, pointing gestures have been found to be used by teachers in order to ground verbal utterances in the physical environment (Alibali et al., 2014). Even though gestures are often used spontaneously in daily interaction between teacher and children, there are reasons to more closely investigate how gestures and representations fuel thoughts and knowledge development, for instance in teaching acts (McNeill, 2005).

In research on children's mathematical learning, picture books are often used as a pedagogical tool to implement mathematical content in familiar and concrete contexts. The picture books are sometimes designed for mathematical learning purposes and are sometimes simply ordinary picture books. Regardless of the type of book, however, it cannot be taken for granted that mathematical meaning is mediated even if mathematics is highlighted in both its text and pictures. Also, picture books may contain multiple distractors, which may draw attention from the mathematics toward other issues (Ward et al., 2017). Consequently, the mathematical content that can be found in pictures and narratives do not afford meaning itself; it is rather how the mathematical content is framed that is important (Elia et al., 2010; Björklund & Palmér, 2020, 2022), why teaching actions are a significant aspect to pay attention to.

The Study

The empirical material in this study originates from a longitudinal intervention conducted in Swedish preschools. The intervention, carried out over the course of three semesters in close collaboration between three teachers working in three different preschools and two researchers, had the primary aim of designing and studying activities and teaching principles in order to facilitate young children's learning of the basic meaning and use of numbers. Planning and evaluating teaching activities were done in collaboration between teachers and researchers. The teaching was however conducted by the teachers in their respective preschool settings. All three teachers have university degree (preschool teacher exam) and 10–20 years of teacher experience. Twenty-seven toddlers (1–3 years old) from the three preschools participated in the study, with their legal guardians' informed consent. These toddlers represent typical Swedish preschools, located in similar suburban areas and constituting a heterogeneous group of both multilingual and only Swedish speaking toddlers and a mix of socioeconomic status.

Theoretical Foundation for the Specially Designed Activity

Several activities and artefacts were developed in the collaboration, one of which is analyzed in this paper: reading sessions using specially designed pictures and narrative aiming to make it possible for the children to discern numbers with a special focus on cardinality and part-whole relations. In line with Ward et al. (2017), we wanted reading material that did not contain distractions or attention to irrelevant features. To accomplish this, the authors of this article designed a narrative (text) and illustrations in the shape of a picture book called "Dutten bygger torn" (translated to English: "Dutten is building towers"). The material constitutes 14 pages with one picture and one text line on each page. The narrative centers around a single character "Dutten" who sits on a mat with building blocks that s/he makes different constructions ("towers") of. The intention was to use the material as a pedagogical tool by presenting small quantities (1–3), based on the theoretical principles of the variation theory of learning (VT) (Marton, 2015).

VT conjectures that learning is a function of differentiating necessary aspects of an object of learning. Depending on what aspects a child discerns, this constitutes how they experience the phenomenon and thereby limits or liberates what they can do with it. Further, VT describes learning and teaching in commensurable terms, which means that learning outcomes are understood as related to what the teaching has enabled the child to discern; in this study, what the teachers' actions and interaction with the child and the pedagogical material's affordances enable the child to notice and act upon. Through carefully selected patterns of variation—contrast and generalization—teachers can make it possible for a child to discern certain aspects, in our case the cardinality of numbers and how numbers are compositions of smaller numbers in a part-whole relation. Contrasting the aspect of cardinality means that sets of similar objects varying in number are presented while as many non-numerical features as possible are kept invariant. Thus, what varies (e.g., the number of objects in two otherwise identical sets) is what is noticed. Once the child has discerned differences in number between similar sets of objects, this can be generalized to comparing other kinds of objects (similar in number). In the study, we designed such patterns of variation embedded in the pictures, and developed principles for the teachers' actions, gestures, and use of representations in order to emphasize the patterns of variation. The pages were designed to attract comparison between pages being shown at the same time. For example, in Fig. 1, the same quantity of items appears on the two opposite pages, but they are arranged differently and thereby offer contrast and generalization regarding the sets of blocks that are to be discerned simultaneously.

The principles of contrast and generalization were also embedded in the activity, with modes of representation (Lesh et al., 1987) like symbols and manipulatives elaborated on together with the pictures. This is also in line with VT principles; that is, the teacher emphasizes contrasts and generalizations of quantities through the present modes of representation. For example, when manipulatives were used, there were always three blocks used that looked similar to those in the picture (red blocks equal in size and shape) but the number of blocks were varied and thereby drawing attention to similarities and differences in number.

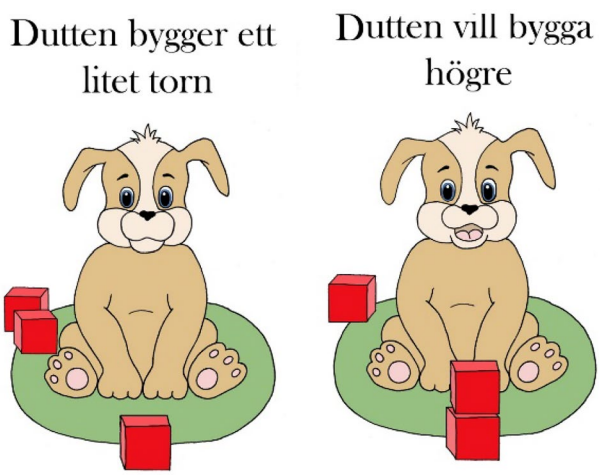

Fig. 1 Translated to English, the text reads "Dutten builds a small tower" (left) and "Dutten wants to build higher" (right)

Procedure

Based on theoretical and empirical groundings (see Björklund & Palmér, 2020, 2022), we presume that learning is made possible when the content appearing in the pictures is extended, represented, and explored in interaction between child, teacher, and mathematical content. Therefore, this study focuses on how certain content provided by the pictures is extended in the interaction; that is, extended in the sense of contrasts in order to present a conceptual meaning such as the cardinality or part-whole relation of numbers, or in the sense of using multiple representations in order to generalize the meaning.

The activity was conducted by the three preschool teachers several times to each child, both individually and in groups, during the three semesters. Some of the reading sessions were video-recorded, to be used partly for evaluating the activity's potential to facilitate learning in the iterative design-trial-evaluate-redesign process, and partly for research purposes such as the kind of micro-analyses reported here.

Analysis

Seventy-three video documentations of reading sessions (7 h 13 min) comprise the analyzed data, with the two pages shown in Fig. 1 chosen as the object of analysis. The narrow selection of only one spread was made, in order to allow for a systematic analysis of teachers' actions and interaction with the material's affordances. Through qualitative analysis we identified what was made possible for the children to discern through afforded patterns of variation in pictures, gestures, props, and verbal and physical symbols (fingers) expressed by the teacher. To capture these communicative actions, the video documentations of the two selected pages were transcribed. Interpretations were made regarding what objects of learning emerged, based on the teachers' actions that foregrounded some aspects of numbers while other possible aspects were not attended to (in line with VT; see Marton, 2015). All 73 observations were analyzed in this way, ending up in three main objects of learning. These emerged in the analysis of what was foregrounded, which means that the same picture and manipulatives could be used for different objects of learning to come to the fore. In this way, we identified the enacted objects of learning and thereby found answers to the question of what learning opportunities were provided through the interactive reading. In some observations there are not only one but two objects of learning that emerge; in the analysis, these are separated into two observations.

Results

In the analysis, three main objects of learning emerged: *identifying numbers, comparing numbers*, and *operating on numbers.* In the following it will be illustrated how these objects of learning are explored in the activity, and how the teacher, using gestures, symbols, and props, extends what the pictures afford. The chosen examples are representative for the whole set of observations.

Identifying Numbers—Foregrounding a Composite Set of Objects

The most common object of learning that emerged was identifying numbers. This means that the attention is drawn to a set of objects, with the teacher encouraging the child to enumerate or see the objects as one coherent whole. The cardinality of numbers is foregrounded, and because of the invariant features of the number of blocks in the pictures, sets should be possible to accept as a group of items that belong together (constituting a coherent set). On the one hand there are challenges in discerning cardinality, particularly when sets are spatially separated; on the other, this separation may be used deliberately as a pattern of variation to call attention to the aspect of cardinality. How the teachers act in order to make it possible to experience the object of learning in a cardinal sense is described in the following three examples of settings. Identifying numbers is attended to in all these examples, but with the difference of the numbers referred to as being *in a coherent whole*, *split sets*, or *separate sets*.

Identifying Numbers in a Coherent Whole

Based on the narrative provided in the material, the teachers sometimes encourage the children to build a similar tower with blocks. In the example below, the teacher has first directed the child's attention to the number of blocks needed by asking "how many" the character Dutten has used. She then returns to the number question by starting to count the blocks the child uses, and the child continues the counting sequence on her own.

Excerpt 1	Reading	Talking	Doing
Teacher		"Look here, Dutten is building a higher tower. How many blocks has he used? Can you build one too?"	
Esther			Takes the blocks on the table, places two as a foundation and one on top
Teacher		"One, two"	
Esther		"Three"	
Teacher		"Three yes"	Shows finger pattern for three and circles her hand over the blocks

By using physical blocks that can be moved on the table and grouped together, attention is directed to the objects as one coherent set: They belong together. But how the blocks are to be labeled emerges in the way the teacher uses various representations for the same phenomenon. The counting word "three" is connected to the group of blocks through the teacher's circling gesture. The cardinality of "three" is supported by the finger pattern for three that the teacher shows.

In the next example, the child is encouraged to reproduce the tower in the picture using physical building blocks, which the teacher labels with same counting word while pointing at the picture's blocks and the child's blocks.

Excerpt 2	Reading	Talking	Doing
Teacher	Page 2: "Dutten wants to build higher"	"He builds a tower with two"	Points at the picture's block tower
Alma		"Look"	Removes one block from her tower of three blocks
Teacher		"Now, Alma has also built a tower with two blocks"	

By using different modes of representation as ways to connect a common meaning, the teacher generalizes the numbers. However, for the children to make sense of the shared meaning of finger patterns, blocks, and pointing movements, they need to already have some knowledge of numbers. This means that what the teacher is doing, using the material and representations in this way, is intended to confirm what the children already know (or whether they already know something), and mainly generalizes to broaden the repertoire of representations to the children.

Identifying Total Number of Split Sets

The pictures analyzed here present blocks in two sets, depicted in front of and partly behind the main character (see Fig. 1). This means that the objects are spread out and the children may not necessarily regard them as parts of one coherent set. However, this is sometimes made an object of learning in the activity, with the teacher emphasizing the blocks constituting one coherent set and disregarding their spatial distance.

Excerpt 3	Reading	Talking	Doing
Teacher		"How many blocks does Dutten have here?"	
Otilia		"One, two"	Points at the blocks in the picture (Page 2)
Teacher		"There was another one behind him" "One, two, three. Dutten has three blocks."	Points at the picture on Page 2 Circles over all three blocks in the picture

The question of "how many" posed by the teacher does not specify which of the sets (or the sets together, for that matter) is/are referred to. The gesture of circular movement becomes an action to connect the spread-out blocks, thereby offering the child the opportunity to perceive them as one set. The pointing and circling gestures thereby become important ways of directing the children's attention to seeing the sets as a composed unit.

Identifying Numbers of Separate Sets

In the observations, the most common way to identify numbers is to establish the quantity of two sets in a simultaneous act. It is not a comparison of the quantities in terms of exact numbers but rather a recognition of the numerosity that is perceived to be different. The variation within the same mode of representation is intended to emphasize the numerical meaning to the child, introducing the potentially new aspect of the cardinality of numbers to be discerned. The main act conducted by the teacher involves pointing to the sets while saying the respective counting words.

The identification of sets of different quantities is done within the same mode of representation, with particularly the objects to be perceived as units being similar but in different quantities.

Excerpt 4	Reading	Talking	Doing
Teacher		"Here Dutten has built a tower with one block, and here Dutten has built a tower with two blocks"	Points at both pictures when addressing the towers in front of Dutten

In this seemingly simple action, the teacher directs attention to the similar-looking red blocks on the two visible pages and particularly those in front of the character in the pictures. As the pictures are almost identical, the different number of blocks stands out and is further pointed out through the teacher's pointing gesture. The difference

between the sets appears in the use of counting words, a different word for each set.

In general, the teachers also make connections between modes of representation, which requires that the children already have some understanding of the meaning that is generalized across representations. This can be seen, for example in Excerpt 5, when numbers are identified in the picture and among the blocks on the table.

Excerpt 5	Reading	Talking	Doing
Teacher	"Dutten wants to build higher" (Page 2)		
Nelly			Builds a tower with two blocks
Teacher		"Look: one, two" "One, two. Two blocks." "And Ossian only has one" "One block, just like Dutten has behind him"	Points at the tower with two blocks on Page 2 Points at Nelly's tower of building blocks (see part 1 in the picture series below), shows finger patterns for two (part 2) Turns to Ossian, who is sitting next to Nelly; points at Ossian's block (part 3), then shows finger pattern for one (part 4) Points at the single block in the picture (part 5)

In Excerpt 5, the teacher points at the blocks in the picture, expressing the quantity using a counting word, and then does the same with the two sets of blocks in front of the two children. She also uses finger patterns to support the similarities and differences between the sets.

Comparing Numbers—Foregrounding the Relation Between Sets

The second kind of object of learning that was observed concerns comparing numbers; thus, the numerical relation between sets is foregrounded. Reasoning about differences and number values, such as more or less and whether a block should be added or removed in order to make the sets equally large, requires the child to identify sets as composed units. One way of experiencing the comparison of numbers emerges in the enacted teaching: *identifying and comparing numbers of sets*. There are fewer observations of comparing numbers than identifying numbers. One reason for this may be that comparing numbers, as in making the relations between sets discernible, implies advancing the knowledge of numbers' meaning and possible use, which builds on the child already being able to identify numbers.

Identifying and Comparing Numbers of Sets

Making number relations discernable involves both an understanding that sets are composed of units, which makes them possible to compare, and the ability to determine *how* the sets are numerically related. Thus, the focus needs to be directed toward the numerical feature of the sets to be compared. To enable discernment of the relation the teacher makes use of representations that bridge and illustrate this relation, one of which being mapping using one's fingers.

Excerpt 6	Reading	Talking	Doing
Teacher		"But what is Dutten doing now? How many blocks has Dutten used to build his tower?"	
Assra			Builds a tower with three blocks
Teacher		"How many blocks has Dutten used to build his tower?"	Circles the tower with two blocks on Page 2 (see part 1 in the picture series below)
Assra		"This many"	
Teacher		"Has Dutten built with that many? Are there equally many?" "No, that one was left over" "How many blocks has Dutten used to build his tower?"	Puts two blocks on the picture. Points at the single block left on the table, picks it up and holds it in her hand (part 2)
Assra			Puts one block on each block in the picture on Page 2
Teacher		"How many do you have there?"	
Assra		"Three" "Yes!"	Takes all blocks in her hands, puts two on the right side of the book and one on the left side of the book (part 3) Claps her hands
Teacher		"Yes, how many are there?"	Touches the two blocks on the table
Assra		"Onetwothree"	Points above the two blocks on the right
Teacher		"Two"	Shows finger pattern for two
Assra		"One" "Mommy, daddy" "Baby"	Points at the single block on the left Points at the two blocks Points at the single block

It appears to be challenging for the child to compare numbers of different representations, in this case comparing two blocks in the picture with three blocks on the table, even though the teacher circles the sets. To make the numerical relation visible, the teacher connects the units one-to-one, block-to-picture, and points out the extra block. The child seems to experience some relation, as she also separates the unit of three, mirroring the picture's sets but labeling them in her own way.

This category differs from all the others found in the study, as it is the only one in which there are examples of teachers *not* using verbal counting words. In this case, comparing numbers is explored through perception (mapping one-to-one in order to experience similarities and differences in number) rather than through counting words that may not hold any numerical meaning for the child (for example in the above, in which the child labels the blocks "mommy, daddy, baby" while using counting words in the sense of a rhyme).

There is one observation that also refers to the *comparing numbers* object of learning, but differs in that the sets to be compared are parts within sets.

Excerpt 7	Reading	Talking	Doing
Teacher	"Dutten builds a small tower" (Page 1)	"But Dutten has equally many blocks to build with. Here he has one in front of him." "And two behind him" "There, he has two in front of him" "And one behind him"	Points at the block in front of Dutten on Page 1, then circles the single block in the picture Circles the two blocks in the picture Circles the two blocks in the picture on Page 2

This observation is one of few in which the teacher acts on both pages simultaneously, here by directing attention to how the two sets are arranged differently, which appears both spatially and numerically. She uses counting words to label the parts within sets as well as circling gestures, thereby offering a comparison of what is similar and what is different between the sets. It becomes a challenge to discern this, because of the crossing connections that are pointed

out in regard to both how the numbers vary (contrasting one versus two blocks in each set) and their different spatial arrangements that call for generalization (the set of two can be placed differently on the pictures but still remain the same in number). Nevertheless, comparing numbers in this way does bring to the fore cardinality and opportunities to explore number relations.

Operating on Numbers—Foregrounding the Part-whole Relations Within a Set

Based on the narrative embedded in the material, the teachers often introduce operations on numbers. For example, the number of blocks changes between pages, blocks are added, sets are spread out (the tower is falling) or, as in the target pages in this analysis, constituting parts can be considered one coherent set. In many cases adding, separating, or subtracting can be done with concrete props with no further numerical reasoning but simply solving a problem: adding more blocks to make a higher tower without considering how many are added or how many the tower is made of. In the context of toddlers' mathematics learning, operating on numbers is a rather advanced skill as it requires that the child already have a sense of numbers' cardinality. What comes to the foreground when the object of learning is operating on numbers is experiencing numbers as part-whole relations. Within these observations we find a difference between *adding parts* and *making numerical changes to sets.*

Adding Parts

Adding parts is the most common way to work with operating on numbers, which may be because the narrative offers opportunities to handle the objects (both depicted and physical) as additions. It then becomes necessary to experience sets as being composed of smaller units and the notion that units can be composed to form a larger set. This requires attention to units and their relation to other units in a part-part-whole relation, which the teachers do by using, in particular, gestures that demarcate units.

Excerpt 8	Reading	Talking	Doing
Teacher	"Dutten wants to build higher" (Page 2)	"Then he has one more block, two blocks"	Circles around the tower with two blocks on Page 2 and maps with finger pattern for two on the picture's blocks
Gustav		"On the same spot"	
Teacher		"Two blocks there and one block there" "How many blocks are there together?"	Maps with finger pattern for two and one on the picture's blocks on Page 2 Circles around all blocks in the picture
Gustav		"One, two. One two there."	Points at the two-block tower, then the single block, and then the two-block tower again
Teacher		"One"	Points at the single block and then the two-block tower, slowly counting out loud
Gustav		"Two, three"	
Teacher		"Three pieces, yes" "Two there and one there. Three pieces."	Circles around all three blocks in the picture Points with two fingers and one finger, circles around all with her hand

Pointing at single units, circling composed units, and mapping fingers to blocks one-to-one are all gestures that direct attention to both parts and whole. When circling around the sets, it is demarcated that the blocks form one composed unit. The teacher partly emphasizes the number of this unit by pointing at and counting them one by one, but more specifically by connecting the counting to the meaning of a composed set when mapping a finger pattern to each set. The circling movement aims to direct attention to the coherent set, to which the units are related in an additive way.

Excerpt 9	Reading	Talking	Doing
Teacher		"Dutten has put blocks behind him"	Puts two blocks beside the book
Aina			Takes one block in her hand
Teacher		"How many blocks does Dutten have here?"	Points and circles around the single block in the picture on Page 1
Aina		"One"	Points at Page 1 and puts her block on the picture's block

Excerpt 9	Reading	Talking	Doing
Teacher		"How many does Dutten have here then?"	Points at the two blocks behind Dutten in the picture
Aina		"That one and that one"	Takes one block at a time and puts them on the picture's blocks
Teacher		"Yes, Dutten has two blocks there" "And one there" "Together they make three blocks: one, two, three"	Shows finger pattern for two (see part 1 in the picture series below) Points and shows finger pattern for one (part 2) with the other hand, holding the two hands close together (part 3) Points at and counts the blocks

close together). The operation of adding all sets is further extended through the enumeration of *all* the blocks.

Making Numerical Changes to Sets

In addition to adding parts, there are instances of other kinds of operations in which the changes made to a set are specifically emphasized. To direct attention to these changes, similar to the above, gestures, symbols, and props are used. The challenge, however, is to find ways to represent the change. In the observations, the teachers do this in their attempts to bring to the fore the relation between the parts and the coherent set by pointing out contrasts between the sets.

Excerpt 10	Reading	Talking	Doing
Teacher	"Dutten wants to build higher" (Page 2)	"He takes one of those blocks to make a tower" "One, two"	Points at the tower behind Dutten on Page 1 Points at the tower with two blocks on Page 2

The teacher extends what the picture illustrates by setting out physical blocks close to those in the picture. The blocks represent the set shown in the picture. This action seems to inspire the child to connect a single block, which she has labeled "one", directly on the picture showing the single block, and then doing the same with the other two blocks on the table. The child and the teacher have now, together, created sets of two blocks and a single block. The teacher emphasizes the differing cardinality of the sets using counting words and finger patterns while pointing to the respective sets with different hands. This latter construct becomes an important key in the teaching, as it represents both sets at the same time and the teacher's finger patterns can visually be moved close together (as the blocks are often used to add sets, resulting in a coherent set of items being

In Excerpt 10, the teacher extends the narrative that includes an operation of moving one part of a set to another set, thus subtracting from one and adding to another. Through the simultaneous comparison of the two pages (see Fig. 1), the contrast between the sets (one and two blocks) is highlighted and the new compositions (two and one) can be seen at the same time. In this example the operation is presented only in the picture, but the simultaneity of the pictures makes it possible to perceive how the relations between the numbers are altered. In other cases, the teacher uses props to represent and illustrate an operation on numbers in which the changes to a set are in focus.

Excerpt 11	Reading	Talking	Doing
Aina			Builds a tower with three blocks
Teacher		"Now there are three blocks"	Lifts off one block
		"Now there are two blocks, just like Dutten's: one, two"	Points at the blocks on the table and then at those on Page 2

The teacher picks up on the child's tower building, then connects the number of blocks to the picture's depicted two blocks and makes a concrete change to the child's tower. The operation is to make the sets equal in number, which is brought into the foreground by first directing attention to the difference in number using the counting words "three" and "two", and then making the change to the larger set and comparing the result by pointing and enumerating. The picture then serves as a reference point that keeps the intended number fixed while the operations are performed on the movable set of blocks.

Overview of Ways of Extending the Designed Material

The analysis in this study was done on the video documentations of reading two opposite pages in the picture material. The pages were designed to trigger comparisons between the pages visible at the same time. However, the teachers made such connections between the pages in only six observations, five of which are found to focus on comparing whether the sets on each page are equal in number.

As Table 1 shows, most of the time the teachers focus on helping the children identify numbers (46 observations); that is, recognizing the exact quantity of a set. On only eight occasions are the teachers observed directing attention to a comparison between sets, which induces a more advanced understanding of the cardinality of numbers for reasoning about differences or equality in number. Quite many (28) observations involve operating on numbers, mainly as the addition of units into a larger whole unit.

In most of the observations the teachers use *gestures*, such as pointing with their index finger at some object they wish to direct the child's attention to, or circling around a group of objects with their index finger or hand. In only a few cases do they use gestures such as marking (demarcating) objects, for example lifting blocks one after another to indicate units, or mapping objects with their fingers in a one-to-one-correspondence. Gestures are usually combined with counting words to direct attention to units.

Counting words are the most frequently used *symbol* for extending the content in the picture and narrative, but finger patterns are also quite frequent in the observations. Finger patterns entail raised fingers shown simultaneously to represent an exact number of objects (with a cardinality meaning). Fingers can also be used to keep track of counted units, but this was not observed being done deliberately in the current intervention.

The teachers added building blocks to the reading sessions, thereby extending the pictures' blocks with physical manipulative *props*. Blocks were frequently used in the observations. On four occasions there are no counting words connected to gestures or props; instead, the teachers talk about "equally many" objects in two sets, thereby indirectly attending to the numerical feature. A reason for the absence of counting words as an extension of the reading in these observations may be that there were no counting words in the text, while blocks are visible in the pictures and gestures are common in any book reading situation.

Table 1 Overview of ways of extending the picture affordances related to enacted objects of learning. In one observation, several ways of extending content may appear

	Gestures				Symbols		Props
	Pointing	Circling	Demarcating	Mapping	Counting words	Finger patterns	Blocks
Identifying numbers (46 observations)							
Identifying numbers in a coherent whole (19)	8	6	1	0	19	4	13
Identifying total number of split sets (5)	3	3	0	0	5	0	1
Identifying numbers of separate sets (22)	18	5	0	2	22	4	17
Comparing numbers (8 observations)							
Identifying and comparing numbers of sets (8)	5	4	0	0	4	2	3
Operating on numbers (28 observations)							
Adding parts (26)	17	17	0	4	26	8	21
Making numerical changes to sets (2)	2	0	0	0	2	0	1
	53	35	1	6	78	18	56

Discussion

The affordances in the pictures and narrative, and the extensions of content in terms of symbols, gestures, and props, are at the center of attention in this inquiry. More specifically, we address what learning opportunities the material and the teachers' actions when reading it may offer the participating children. While this means that conclusions regarding actual learning will not be drawn, the study contributes to our understanding of the potential impact of interaction with the material as a pedagogical tool.

The results indicate that the way teachers act in the activity, and how one and the same picture is (or opposite pictures are) handled, impact the object of learning that the children are afforded the opportunity to explore (see Marton, 2015). Three different objects of learning emerged: *identifying numbers, comparing numbers*, and *operating on numbers.* This has significant educational implications as it shows the importance of the teachers' actions, as contrasts within and between pictures and other modes of representation rely on teacher actions in relation to characteristics and opportunities in the material used. Different actions by teachers imply different objects of learning to explore, even when the same pictures are discussed. This also contributes to our scientific theoretical understanding of the importance of teachers' actions when for example reading picture books with a pedagogical purpose.

The representations used by the teachers in the reading sessions here are manipulatives (blocks), counting words, and finger patterns. One way to bridge between these representations is to use gestures (Arzarello et al., 2009; Radford, 2003). Different kinds of gestures are described in the literature (see, for example, Alibali & Nathan, 2012; McNeill, 2005), and the teachers in this study are observed using all of them: *pointing gestures* serving to indicate objects; *representational gestures*, such as making a circular movement with one's hand to demarcate a group of objects; and *beat gestures*, such as counting out loud and pointing at the counted objects in a one-to-one-correspondence. The teachers use gestures to draw attention to some object, whereas symbols are used to represent a certain meaning and props are used to represent particular objects or illustrate an occurrence. They use gestures such as pointing with their index finger, circling around a group of objects with one hand, marking objects by lifting props one after another to indicate units, or mapping objects with their fingers in a one-to-one-correspondence. These gestures are typically combined with counting words aiming to direct attention to units. This study thereby adds to previous knowledge of how to work on representations when teaching young children, by adding knowledge of how the use of gestures may further facilitate children's learning of numbers. In line with previous studies, the gestures seem to have communicative power with these very young learners, who have not yet fully developed verbal skills (Hostetter, 2011). Future studies could preferably focus on how mathematically meaningful gestures are used when using pictures and narratives with a pedagogical purpose, both when aiming to make connections within and between representations but also in combination with other signs, for example gazes. Additionally, these studies could draw on embodiment in mathematics, of which gestures are considered a part (Robutti, 2014).

This study has several didactical implications for teachers in preschool. One is to seize the opportunity to explore a picture's content to its full potential. Another one is to consider the design of books and the possibility to make connections between opposite pictures. The small number of observations in our study showing teachers making such connections, along with the large number of observations in the *identifying numbers* category, indicates the need for reading instructions in order to enable a book to be read to its fullest pedagogical potential; not instructions in the sense of "do or say this", however, but in the sense of possibilities when reading books whereby the teacher's professionality can be further expanded. Such reading instructions could also include the possibility to use different representations (counting words, finger patterns, blocks of the same color) and explain how connections within and between modes of representation can be strengthened using various gestures.

In sum, the study reported here contributes knowledge of not only what is made possible to learn when a material designed to draw attention to numbers is used, but also of *how* this learning is facilitated by teachers through patterns of variation provided by gestures, symbols, and props.

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References

Alibali, M. W., & Nathan, M. J. (2012). Embodiment in mathematics teaching and learning: Evidence from learners' and teachers'

gestures. *Journal of the Learning Sciences*, *21*(2), 247–286. https://doi.org/10.1080/10508406.2011.611446.

Alibali, M. W., Nathan, M. J., Wolfgram, M. S., Church, B., Jacobs, R., Martinez, S. A. J., C., & Knuth, E. J. (2014). How teachers link ideas in mathematics instruction using speech and gesture: A corpus analysis. *Cognition and Instruction*, *32*(1), 65–100. https://doi.org/10.1080/07370008.2013.858161. https://doi-org.ezproxy.ub.gu.se/.

Arzarello, F., Paola, D., Robutti, O., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics*, *70*(2), 97–109. https://doi.org/10.1007/s10649-008-9163-z.

Björklund, C., & Palmér, H. (2020). Preschoolers' reasoning about numbers in picture books. *Mathematical Thinking and Learning*, *22*(3), 195–213. https://doi.org/10.1080/10986065.2020.1741334.

Björklund, C., & Palmér, H. (2022). Teaching toddlers the meaning of numbers - Connecting modes of mathematical representations in book reading. *Educational Studies in Mathematics*, *110*, 525–544. https://doi.org/10.1007/s10649-022-10147-3.

Björklund, C. & Palmér, H. (2023). *Teachers' strategies in helping children to discern cardinality when playing a designed lottery game*. Paper presented at CERME 13th July 2023, Budapest, Hungary.

Bonny, J., & Lourenco, S. F. (2013). The approximate number system and its relation to early math achievement: Evidence from the preschool years. *Journal of Experimental Child Psychology*, *114*(3), 375–388. https://doi.org/10.1016/j.jecp.2012.09.015.

Clements, D. H., Sarama, J., & MacDonald, B. L. (2019). Subitizing: The neglected quantifier. In A. Norton & M. W. Alibali (Eds.), *Constructing number* (pp. 13–45). Springer Nature. https://doi.org/10.1007/978-3-030-00491-0_2

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*(1), 103–131.

Elia, I., van den Heuvel-Panhuizen, M., & Georgiou, A. (2010). The role of pictures in picture books on children's cognitive engagement with mathematics. *European Early Childhood Education Research Journal*, *18*(3), 275–297. https://doi.org/10.1080/13502930X.2010.500054.

Goldin, G. A. (2014). Gestures in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 556–572). Springer.

Hassinger-Das, B., Jordan, N. C., & Dyson, N. (2015). Reading stories to learn math: Mathematics vocabulary instruction for children with early numeracy difficulties. *The Elementary School Journal*, *116*(2), 242–264. https://doi.org/10.1086/683986.

Hostetter, A. B. (2011). When do gestures communicate? A meta-analysis. *Psychological bulletin*, *137*(2), 297–315. https://doi.org/10.1037/a0022128.

Jennings, C. M., Jennings, J. E., Richey, J., & Dixon-Krauss, L. (1992). Increasing interest and achievement in mathematics through children's literature. *Early Childhood Research Quarterly*, *7*(2), 263–276. https://doi.org/10.1016/0885-2006(92)90008-M.

Lesh, R., Post, T. R., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics*. Lawrence Erlbaum Associates.

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology*, *46*(5), 1309–1319. https://doi.org/10.1037/a0019671.

Marton, F. (2015). *Necessary conditions of learning*. Routledge.

McNeill, D. (2005). *Gesture and thought*. University of Chicago Press.

Ping, R. M., & Goldin-Meadow, S. (2008). Hands in the air: Using ungrounded iconic gestures to teach children conservation of quantity. *Developmental Psychology*, *44*(5), 1277–1287.

Radford, L. (2003). Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. *Mathematical Thinking and Learning*, *5*(1), 37–70.

Robutti, O. (2014). Gestures in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 311–315). Springer.

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition*, *108*(3), 662–674. https://doi.org/10.1016/j.cognition.2008.05.007.

Valenzeno, L., Alibali, M. W., & Klatzky, R. L. (2003). Teachers' gestures facilitate students' learning: A lesson in symmetry. *Contemporary Educational Psychology*, *28*(2), 187–204.

Ward, J. M., Mazzocco, M. M., Bock, A. M., & Prokes, N. A. (2017). Are content and structural features of counting books aligned with research on numeracy development? *Early Childhood Research Quarterly*, *39*, 47–63. https://doi.org/10.1016/j.ecresq.2016.10.002.