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Prevalence of Number, Number Relations, and Number Operations Indicators in State Early Learning Standards

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Abstract

State Early Learning Standards (ELS) are multi-function tools that inform early childhood instruction and practices. Using an established framework of early numerical development, this study assessed the prevalence of number, number relations, and number operations indicators in ELS, specifically indicators of counting, numeral knowledge, cardinality, relations among quantity and number, and operations. The type of quantity representation and the set sizes, or upper limits, assigned to counting, subitizing, and cardinality were also summarized. State ELS were organized into two groups: Group 1 (n = 23) included states with one set of indicators for preschool (e.g., ages three to five years) and Group 2 (n = 27) included states with at least two sets of indicators for preschool (e.g., 36 to 48 months and 48 to 60 months). Key findings include: (1) how states organize their ELS is associated with level of consensus for the upper limits and set sizes associated with counting and cardinality; (2) notable variability in the prevalence of early number indicators with gaps for indicators of advanced counting, cardinal principle knowledge, symbolic number relations, and ordering; and, (3) differences in how ELS address the type of quantity representation. Results are intended to highlight strengths and shortcomings of state ELS for number, number relations, and number operations and to offer considerations for future revisions.

Keywords Early learning standards · Number · Number relations · Number operations · Counting · Cardinality

Introduction

The significance of young children's numerical competence is well-documented. Early numeracy skills are linked with post-high school outcomes including employability, earning potential, on-the-job productivity (Parsons & Bynner, 1997; Rivera-Batiz, 1992) and socioeconomic status in adulthood (i.e., occupation, housing arrangements, annual income; Ritchie and Bates, 2013). Math skills and knowledge during preschool reliably predict children's math achievement during elementary school (Duncan et al., 2007; Nguyen et al., 2016) and at 13 (Geary, Hoard, Nugent, & Bailey, 2013) and 15 years (Watts, Duncan, Siegler, & Davis-Kean, 2014), independent of family background and domain-general cognitive abilities. The crucial role of early numeracy for future math achievement and broad indicators of well-being underlies recent discoveries about core aspects of early mathematical learning and future math abilities (Geary & vanMarle, 2016) and interventions to improve early numerical thinking and skills (Raudenbush et al., 2020).

These discoveries have marked implications for early education policy, especially state Early Learning Standards (ELS) for math. Using an established framework of early numerical development (Fuson, Clements, & Beckmann, 2010; Jordan et al., 2022; National Research Council, 2009), the current study assessed the prevalence of number, number relations, and number operations indicators in state ELS. Among math education researchers, this is a familiar undertaking. Prior to (and after) the release of the Common Core State Standards for Mathematics (CCSSM, 2010), cross-state comparisons were conducted of granular-level K-8 math indicators (e.g., Dingman et al., 2013; Reys et al., 2006; Smith, Larnell, & Tarr, 2010). The current

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undertaking is situated when there are no common core standards for preschool math and is akin to research conducted on states' K-8 math indicators. It also extends other investigations of state ELS for math (Litkowski, Duncan, Logan, & Purpura, 2020a; Neuman and Roskos, 2005; Scott-Little et al., 2011;, 2012) by examining a wider array of indicators and attending to the form of quantity representation and the magnitudes that accompany select indicators. We also capture how states characterize preschool by grouping ELS on the basis of how child age is, or is not, utilized to organize the standards. Unlike K-8 standards, there are no prescribed "grades" attached to preschool and states have latitude in defining that period.

Over the past 20 years, state-funded preschool programs have more than doubled the number of three-year-olds (2.7–6.4% in n = 35 states) and four-year-olds served (14–32% in n = 45 states) (Friedman-Kraus et al., 2023). Concurrently, ELS have evolved into multi-function tools that inform teacher practices, educate parents and other early childhood professionals, and serve as a foundation for preservice training and professional development (DeBruin-Parecki & Slutzky, 2016) and standards-based instruction and curriculum (Mueller & File, 2019). ELS are valuable tools that inform the practices of many early childhood professionals (DeBruin-Parecki & Slutzky, 2016) and have great potential to influence young children's early math learning. Our goal is to provide a snapshot of current ELS for number and operations using up-to-date developmental and applied research about core numerical knowledge and competencies during preschool and to inform future revisions of state ELS.

Early Learning Standards: Background

State Early Learning Standards (ELS) can be traced to federal education policy. No Child Left Behind (NCLB) and the subsequent Good Start, Grow Smart initiative prompted a new era of accountability that increased emphasis on student testing and outcomes (Neuman & Roskos, 2005; Scott-Little et al., 2007; Mueller & File, 2019). Although NCLB applied primarily to K–12 education, as states invested in prekindergarten, a comparable emphasis on student outcomes emerged and the Race to the Top Early Learning Challenge and later Preschool Development Grant funding opportunities both kept improving outcomes for young children in the forefront. Early versions of ELS reflected new insights into children's cognitive capabilities, were framed with kindergarten "readiness", and described what children should know and be able to do during preschool (DeBruin-Parecki & Slutzky, 2016; Scott-Little et al., 2006, 2007).

Numerous studies have been conducted of state ELS. Some addressed general considerations including document title, year of publication, composition of the group that developed the ELS, and organization and number of indicators (Scott-Little et al., 2006). Others have conducted surveys, interviews, and focus groups with state education administrators to gain perspective on the value and use of ELS (DeBruin-Parecki & Slutzky, 2016; Scott-Little et al., 2007). A recent study evaluated the prevalence of content for children with special needs and developmental delays (Bruder & Ferreira, 2022). These studies highlight the variation across states' ELS, concerns with how ECE professionals use the ELS, and alignment of the ELS with other K-12 standards.

Early Learning Standards: Math Domain

Single domains of ELS have also been examined to summarize what states want children to know and be able to do in literacy (DeBruin-Parecki & Slutzky, 2016; Neuman & Roskos, 2005) or math (Litkowski et al., 2020a; Neuman & Roskos, 2005; Scott-Little et al., 2011, 2012). Scott-Little et al. (2011;, 2012) conducted a "first of its kind" in-depth analysis of math content (i.e., indicators) within math subdomains, the prevalence of the same indicators in state ELS, and alignment between indicators with the Head Start Child Development and Early Learning Framework and the Common Core Standards for kindergarten math. Notably, the authors commented, "Our work is seriously inhibited by not having established and agreed-upon learning trajectories for young children across the ages" (p. 7; Scott-Little et al., 2012). Since then, basic science on early numerical development has evolved and the current undertaking has benefitted greatly from these advancements.

What makes our study novel is that we assess the prevalence of a comprehensive set of research-based number, number relations, and number operations indicators in state ELS that are foundational to early math development (Fuson et al., 2010; Jordan et al., 2022; National Research Council, 2009), malleable to targeted instructional practices (e.g., Raudenbush et al., 2020), realistically achievable during preschool (Litkowski, Duncan, Logan, & Purpura, 2020b), and that predict future math achievement (Geary & vanMarle, 2016; Geary et al., 2018). Moreover, we attend to the type of representation associated with the indicators (i.e., non-symbolic, number words, written numerals) and the magnitudes attached to them (e.g., how high a 48-month-old should be able to verbally count), both highly relevant concepts and practical considerations for instructional practices.

To account for the increase over time in the percentage of three- and four-year-olds served by state funded preschool (Friedman-Kraus et al., 2023) and the corresponding implications for how ELS are organized with regard to child age,

two mutually-exclusive groups of ELS were created prior to analysis, one with a single set of indicators associated with "preschool" and the other with two sets of indicators, one for three-year-olds and the other for four-year-olds. We addressed only number-oriented skills and knowledge because preschool educators are encouraged to focus the majority of their instructional time on number-related topics (National Research Council, 2009) and young children's acquisition of core quantitative knowledge and skill is most predictive of future math achievement (Nguyen et al., 2016).

Early Number Sense

Early math development represents the interplay of knowledge and skills about number, number relations, and number operations (Jordan et al., 2022). Between three to five years, children are capable of learning the rules and function of counting, mapping number symbols (words and written numerals) onto quantities, understanding relations among quantities and numbers, and recognizing the outcome of simple operations using manipulatives and number words (e.g., Litkowski et al., 2020b).

Number

The principles of counting that emerge during early numerical development include: (1) stable order (i.e., reciting the count string and using number words in the same sequence when counting); (2) one-to-one correspondence (i.e., when counting sets, each item is counted one time); and, (3) cardinality (i.e., the last number applied when counting a set is the size of the set) (Gelman & Gallistel, 1978). Understanding these principles facilitates the onset of cardinal principle knowledge (CPK), or children's understanding that "four" means ■ ■ ■ ■ (Le Corre et al., 2006; Sarnecka & Carey, 2008). Accordingly, ELS were reviewed for verbal counting, enumeration, cardinality, and CPK.

Recognizing Arabic numerals and mapping them onto quantities is another task of early number development (Purpura, Baroody, & Lonigan, 2013). Cross-sectional research suggests that young children's ability to map Arabic numerals onto specific quantities follows their ability to map number words onto quantities (Knudsen, Fischer, Henning, & Aschersleben, 2015). Thus, we recorded the prevalence of numeral-oriented indicators, such as identifying and writing numbers and demonstrating cardinality and CPK using number words and written numerals.

Number Relations and Number Operations

Cardinal principle knowledge (CPK) functions as a "gatekeeper" to continued mathematical progression (Spaepen, Gunderson, Gibson, Goldin-Meadow, & Levine, 2018), including understanding relations among numbers (Geary et al., 2018; Geary & vanMarle, 2018). Because CPK typically emerges before four years (Litkowski et al., 2020b), the prevalence of number relations and number operations indicators was also coded in ELS. For number relations, indicators included distinguishing more, less, and equal using different types of quantity representation and ordering non-symbolic representations and numerals from smallest to largest. For number operations, we searched for indicators of understanding number operations, both non-symbolic (i.e., combining produces *more* and separating creates *less*) and symbolic (i.e., *solving* an operation using number words).

The Current Study

With prior examinations of K-8 state standards for math and state ELS for math as guides (Dingman et al., 2013; Litkowski et al., 2020a; Reys et al., 2006; Smith et al., 2010; Scott-Little et al., 2011; 2012), our study examines state ELS for the prevalence of conceptually and empirically grounded indicators of early number sense. Our approach is meritorious for several reasons; it was conducted against a backdrop of increasing public preschool enrollments among three and four year olds (Friedman-Kraus et al., 2023) and the lack of common core standards for "preschool" math, and was based on current research about the trajectory of early numerical development and children's future mathematics achievement (Geary & vanMarle, 2016; Geary et al., 2018). Our analysis began by accounting for how state ELS are organized by child age. Two mutually-exclusive groups of ELS were created: one included a single set of indicators and the other included at least two sets of indicators for preschool. Next, we searched for a broad range of number, number relations, and number operations indicators and the type of quantity representation (i.e., non-symbolic, number words, written numerals) and magnitudes associated with select indicators (e.g., counting, subitizing, and cardinality). Our analysis is intended to provide a snapshot of current strengths and shortcomings of state ELS for number, number relations, and number operations and to offer considerations for future revisions.

Methods

Data Sources

State Early Learning Standards (ELS) from all 50 states plus the District of Columbia (n = 51 total) were our data source. Internet searches were conducted to locate and download

the most recent version of states' ELS. Appendix A contains a list of states' ELS document titles, the publication year, the URL for the downloaded ELS, the states' ELS home page, and the date of download. All URLs were double-checked on March 1, 2023 and updated where necessary.

State ELS Groups

State ELS were grouped on the basis of how child age was utilized to organize indicators. As shown in Table 1, Group 1 was comprised of n = 23 states that had one set of indicators for preschool (e.g., 36 to 60 + months, ages 3–5). Group 2 included n = 27 states that had at least two sets of indicators for the two years before kindergarten (e.g., 36 to 48 months and 48 to 60 months).

ELS Coding System

To develop our coding system of number, number relations, and number operations indicators (see Table 2), we consulted Geary et al. (Geary & vanMarle, 2016;, 2018; Geary et al., 2018), Gelman and Gallistel (1978), Knudsen et al. (2015), Purpura et al. (2013), and Raudenbush et al. (2020). Only ELS content that was identified as indicators (or standards) and described an observable behavior or verbal response was coded. Examples of hypothetical child behavior that demonstrated the indicator were not coded. Where applicable, the coding system distinguished the type of representation and the magnitude or set size referenced. One state's ELS was not included for coding. Specifically, Michigan's Early Childhood Standards of Quality for Birth to Kindergarten, Early Learning and Development Standards was not coded because the indicators were drafted in very broad terms and accompanied by examples of child behavior, birth through 5 years, that were further sub-divided into seven discrete age groupings. This approach did not lend itself to coding with our system.

Number

The number domain included three sub-domains: counting, numerals, and cardinality. *Counting* included child can recite the count string (WORD), count in a one-to-one manner (ONE), subitize (SUBZ), count on from a number other than one (CON), and count flexibly (FLEX; i.e., counting forward and backward, or backward only, from a number other than one); magnitudes were recorded for counting aloud, one-to-one counting, and subitizing. *Numerals* included child identifies numerals as different from letters (DIFF), correctly identifies written numerals (NUMR), and writes or copies some numerals (WRITE). *Cardinality* included four indicators that distinguished between answering "how many?" and producing sets, and the type of representation (number words, numerals). Specifically, child can answer "how many?" using number words (C1) or by matching numerals to sets (C2); and, child can produce a set using a number word (C3) or with a written numeral (C4). The set sizes for cardinality were recorded.

Number Relations

This domain addressed two categories of number relations. First, child can determine more, less, and equal without symbols (MAGN1); child can determine more, less, and equal with number words (e.g., story problems; MAGN2); and, child can determine magnitudes when comparing written numerals (MAGN3). The second area addressed ordering, or child ability to arrange depictions of magnitude from smallest to largest. Specifically, child can arrange different images or set sizes from smallest to largest (ORD1) and child can arrange numerals from smallest to largest (ORD2).

Table 1 Age-related organization of state ELS indicators for number, number relations, and number operations (n = 50)

	Common age-related definitions accompanying Early Learning Standards' indicators	State Abbreviations (year of ELS publication)
Group 1 (n = 23)	36 to 60 + months; ages 3–5; preschool; 3 years – kindergarten enrollment; 3 and 4 year olds; four-year-old students; pre-kindergarten (3–5 years)	AK (2020), AZ (2018), CO (2019), DC (2019), DE (2010), IA (2017), ID (2019–2020), IL (2013), KY (2013), MA (2019), ME (2015), MO (2021), MT (2014), NJ (2014), NV (2010), NY (2019), OH (2014), OK (2019), PA (2014), TN (2018), WI (2017), WV (2019), WY (2020–2021)
Group 2 (n = 27)	Year 1 of Preschool: 48 months; by 48 months; 37–48 months; 3 to 4 years; 36 to 48 months; three-year-olds (36 to 48 months); younger preschoolers; early preschool (34–48 months); 3-year-olds; age 3 Year 2 of Preschool: 60 months; by 60 months; 49–60 months; 4 to 5 years; 4 years – kindergarten (48 months – kindergarten); older preschool; four- and five-year-olds; age 4–5/not yet in kindergarten; older preschoolers (45 to 60 + months); later preschool (44–60 months)	AL (2020) [a], AR (2016), CA (2008), CT (2014), FL (2017), GA (2013), HI (2014), IN (2015), KS (2014), LA (2013), MD (2015), MN (2017), MS (2018), NC (2013), ND (2018), NE (2018), NH (2016), NM (2014), OR (2016), RI (2013) [a], SC (2017), SD (2017), TX (2022), UT (2020), VA (2021), VT (2015), WA (2012)

[a] Alabama and Rhode Island's ELS include three discrete sets of indicators, one each for 36, 48, and 60 months of age. For the current research, only the 48- and 60-month indicators were coded

Table 2 Prevalence of number, number relations, and number operations indicators in state ELS

Indicators	Definitions	Number (%) of state ELS that include indicator		
		Group 1	Group 2	
		"Preschool" (n = 23)	Year 1 of Preschool (n = 27)	Year 2 of Preschool (n = 27)
Number				
Counting				
WORD	Number word list: child verbally recites count string, starting with one and in correct order.	22 (96%)	24 (89%)	25 (93%)
ONE	One-to-one counting: child points to and labels objects with number words, starting with "one" and in correct order, and by touching each object only once.	22 (96%)	24 (89%)	22 (81%)
SUBZ	Subitizing: child quickly answers "how many?" without appearing to count.	12 (52%)	14 (52%)	22 (81%)
CON	Count on: child recites number sequence *forward*, in correct order, from a number other than one, *outside of the context of operations*.	3 (13%)	1 (4%)	5 (19%)
FLEX	Flexible counting: child recites number sequence *forward and backward*, or *backward only*, in correct order, from a number other than one.	2 (9%)	4 (15%)	10 (37%)
Numerals				
DIFF	Identifies numerals as different than letters.	4 (17%)	1 (4%)	0
NUMR	Written number symbols: child correctly names written numerals.	19 (83%)	12 (44%)	18 (67%)
WRITE	Writes numbers: child writes/copies some numbers. Does not include marks or scribbles that might be numbers.	9 (39%)	6 (22%)	11 (41%)
Cardinality				
C1	Cardinality 1: child indicates *how many* objects are present using *number word*; child answers, "How many?" with a number word.	18 (78%)	15 (56%)	22 (81%)
C2	Cardinality 2: child indicates *how many* objects are present using *written numeral;* includes matching numerals to sets of objects.	11 (48%)	6 (22%)	13 (48%)
C3	Cardinality 3: child *produces* a set of objects using *number word*.	6 (26%)	6 (22%)	12 (44%)
C4	Cardinality 4: child *produces* a set of objects using *written numeral*.	2 (9%)	1 (4%)	2 (7%)
Number Relations				
MAGN1	Identification of more than, less than, and equal to (non-symbolic): child selects which is more than, less than, and equal to from an assortment of objects, cards with dots, fingers on a hand, etc. Includes matching to determine if sets are equal or not.	18 (78%)	12 (44%)	8 (30%)
MAGN2	Identification of more than, less than, and equal to (symbolic: number words): child selects which is more than, less than, and equal to *from hearing number words*, such as in a story problem; also includes counting to compare set sizes.	3 (13%)	5 (19%)	16 (59%)
MAGN3	Identification of more than, less than, and equal to (symbolic: numerals): child selects which is larger, smaller, and equal to from *written numerals.*	1 (4%)	0	3 (11%)
ORD1	Ordering (non-symbolic): child orders cards with different numbers of dots or images in order from smallest amount to largest amount or vice versa.	0	0	1 (4%)
ORD2	Ordering (symbolic): child orders cards with numerals written on them; similar to constructing a number line.	2 (9%)	0	1 (4%)
Number Operations				
OPER	Understands or recognizes outcome of addition (e.g., combining, joining) and subtraction (e.g., separating, taking away), does not include word "solves"	12 (52%)	15 (56%)	16 (59%)
OPERADD	Understands or recognizes outcome of addition (e.g., combining, joining); does not include word "solves."	2 (9%)	4 (15%)	2 (7%)
OPERSUB	Understands or recognizes outcome of subtraction (e.g., separating, taking away); does not include word "solves."	1 (4%)	2 (7%)	2 (7%)
SOLVE	Solves addition (e.g., combining, joining) and subtraction (e.g., separating, taking away); includes explicit mention of providing the answer using number words.	6 (26%)	5 (19%)	10 (37%)
SOLVEADD	Solves addition problems (e.g., combining, joining); includes explicit mention of providing the answer using number words.	2 (9%)	2 (7%)	3 (11%)
SOLVESUB	Solves subtraction problems (e.g., separating, taking away); includes explicit mention of providing the answer using number words.	1 (4%)	1 (4%)	2 (7%)

Number Operations

Number operations captured indicators of understanding of and ability to engage in addition and subtraction. The indicators distinguished between conceptual understanding (i.e., addition results in *more* and subtraction results in *less*) and ability to solve, using number words, when adding and subtracting. The conceptual indicators included child recognizes outcome of addition and subtraction (OPER), child recognizes outcome of addition alone (OPERADD) and subtraction alone (OPERSUB). Lastly, the solution-oriented indicators included child *solves* addition and subtraction problems (SOLVE) and child separately solves addition (SOLVEADD) and subtraction problems (SOLVESUB).

Coding Process

All ELS were coded by the two authors. ELS were first coded independently; each coder highlighted indicators and comments were inserted describing the code, magnitude or set size, and ELS group. During weekly meetings, ELS indicators of number, number relations, and number operations were compared, and disagreements were discussed and resolved. Data were entered into Excel and read into R for later summary.

Results

Prevalence of Number-Related Indicators

Counting

As shown in Table 2, several indicators of counting are highly prevalent in both Group 1 (G1; the n = 25 state ELS that used a single age category) and Group 2 (G2; the n = 26 state ELS that utilized two age categories). Specifically, WORD (reciting the count string) and ONE (one-to-one counting) appear in 96% of G1 ELS, 89% of G2Y1 ELS and 93% and 81%, respectively, of G2Y2 ELS. SUBZ (subitizing) was located in more than half of state ELS; specifically, 52% of G1 ELS, 52% of G2Y1 ELS, and 81% of G2Y2 ELS. CON (counting on) and FLEX (counting forward and backward, or backward alone) were less prevalent in ELS. For G1, CON appeared in 13% and FLEX appeared in 9% of ELS. Among G2 ELS, CON and FLEX were less frequent for Y1 (4% and 15%, respectively) compared with Y2 (19% and 37%, respectively).

Numerals

Table 2 also presents the prevalence of numeral indicators. DIFF (identifies numerals as different than letters) appears in a total of n = 5 ELS, four (17%) from G1 and one (4%) from G2Y1. Child correctly names written numerals (NUMR) is more prevalent and was located in 83% of G1 ELS, 44% of G2Y1 ELS, and 67% of G2Y2 ELS. Child writes or copies some numbers (WRITE) appeared in 39% of G1 ELS, 22% of G2Y1, and 41% of G2Y2 ELS. At the G2 ELS state level, great variability was evident for inclusion of NUMR; n = 7 states did not include NUMR, n = 2 states included NUMR only for Y1 and n = 8 states included NUMR only for Y2 and, n = 10 G2 states included NUMR for both Y1 and Y2.

Cardinality

The most frequently appearing indicator of cardinality across groups is C1 (child indicates *how many?* using a number word; Table 2). C1 is included in 78% of G1 ELS, 56% of G2Y1 ELS, and 81% of G2Y2 ELS. C2 (child indicates *how many?* using a numeral) appears in 48% of G1 ELS, 22% of G2Y1 ELS, and 48% of G2Y2 ELS. The other indicators of child understanding of cardinality were less prevalent in state ELS. C3 (child produces a set using number words) and C4 (child produces a set using a numeral) were included in 26% and 9% of G1 ELS, respectively. For G2Y1, C3 and C4 appeared in 22% and 4%, respectively; for G2Y2, C3 and C4 were located in 44% and 7% of state ELS, respectively.

Prevalence of Number Relations Indicators

The prevalence of indicators of relations among quantity and number point to differences between groups (Table 2). MAGN1 (determining more than, less than, or equal; *non-symbolic*) is present in 78% of G1 ELS, 44% of G2Y1, and 30% of G2Y2 ELS whereas MAGN2 (determining more than, less than, or equal with number words; *symbolic*) appears in 13% of G1 ELS, 19% of G2Y1, and 59% of G2Y2 ELS. The remaining indicators of number relations, which focus on identification of more than, less than, or equal to with written numerals (MAGN3) and ORD1 and ORD2 (ordering both symbolic and non-symbolic quantities) were rare and found in n = 3 or fewer ELS documents.

Prevalence of Number Operations Indicators

The prevalence of indicators of children's understanding of and ability to solve number operations appear in Table 2. OPER (child understands or recognizes the outcome of addition and subtraction; does not include solving) was found in

52% of G1 ELS, 56% of G2Y1, and 59% of G2Y2 ELS. Whereas some states separate understanding of addition and subtraction, the prevalence of these more specific indicators in state ELS is low. OPERADD and OPERSUB appear in 9% and 4%, respectively, of G1 ELS and in 15% and 7%, respectively, of G2Y1 and 7% and 7% of G2Y2 ELS.

Child ability to solve addition and subtraction problems using number words (SOLVE), such as in a story problem, is more prevalent among G2 ELS than G1 ELS. SOLVE was located in 26% of G1 ELS, 19% of G2Y1, and 37% of G2Y2 ELS. The prevalence of SOLVEADD and SOLVESUB was somewhat rare in G1 ELS (9% and 4%, respectively) and G2Y1 (7% and 4% respectively). These two indicators were found in 11% and 7%, respectively, of G2Y2 ELS.

State ELS Upper Limits for Counting and Cardinality Indicators

Counting

As shown in Table 3, state ELS included a range of upper limits for WORD (verbal recitation of the count string) and ONE (one-to-one counting). For G1 WORD, 10 appeared most frequently in n = 8 ELS, followed by 20 in n = 7 ELS. For G2Y1 WORD, 10 was the most prevalent upper limit (n = 20 states) and, for G2Y2 WORD, 20 was included in n = 17 of state ELS. At the state level, among G2 ELS that included WORD for both Y1 and Y2 (n = 23), the most common combination of magnitudes was 10 (Y1) and 20 (Y2) in n = 13 states; the data revealed no clear second place preference. The largest gap between magnitudes for reciting

Table 3 State ELS upper limits for number-related indicators

Indicators	States' Upper Limit by Child Age		
	Group 1	Group 2	
	"Preschool" (n = 23)	Year 1 of Preschool (n = 27)	Year 2 of Preschool (n = 27)
Verbal recitation of the count list (WORD)	Of 22 states (96%) No limit: 4 states (18%) 5: 2 states (9%) 10: 8 states (36%) 20: 7 states (32%) 30: 1 state (5%)	Of 24 states (89%) No limit: 0 states (0%) 5: 2 states (8%) 10: 20 states (83%) 15: 1 state (4%) 20: 1 state (4%)	Of 25 states (93%) No limit: 2 states (8%) 10: 1 state (4%) 20: 17 states (68%) 29: 1 state (4%) 30: 3 states (12%) 31: 1 state (4%)
One-to-one counting (ONE)	Of 22 states (96%) No limit: 9 states (41%) 4: 1 state (5%) 5: 3 states (14%) 10: 7 states (32%) 20: 2 states (9%)	Of 24 states (89%) No limit: 6 states (25%) 3: 1 state (4%) 4: 1 state (4%) 5: 11 states (46%) 10: 5 states (21%)	Of 22 states (81%) No limit: 4 states (18%) 5: 1 state (5%) 10: 15 states (68%) 15: 2 states (9%)
Subitizing (SUBZ)	Of 12 states (52%) No limit: 2 states (21%) 3: 1 state (7%) 4: 2 states (14%) 5: 5 states (36%) 6: 1 state (7%) 10: 1 state (7%)	Of 14 states (52%) No limit: 2 states (14%) 3: 6 states (43%) 4: 6 states (43%)	Of 22 states (81%) No limit: 3 states (14%) 3: 2 states (9%) 4: 6 states (27%) 5: 8 states (36%) 6: 1 state (5%) 10: 2 states (9%)
How many? with number words (C1)	Of 18 states (78%) No limit: 9 states (50%) 5: 4 states (22%) 10: 5 states (28%)	Of 15 states (56%) No limit: 7 states (47%) 5: 8 states (53%)	Of 22 states (81%) No limit: 10 states (45%) 10: 12 states (55%)
How many? with numerals (C2)	Of 11 states (48%) No limit: 7 states (64%) 5: 3 states (27%) 10: 1 state (9%)	Of 6 states (22%) No limit: 1 state (17%) 3: 1 state (17%) 5: 4 states (67%)	Of 13 states (48%) No limit: 2 states (15%) 5: 2 states (15%) 10: 8 states (62%) 12: 1 state (8%)
Produce set with number words (C3)	Of 6 states (26%) No limit: 0 states (0%) 5: 3 states (50%) 10: 3 states (50%)	Of 6 states (22%) No limit: 1 state (17%) 4: 2 states (33%) 5: 3 states (50%)	Of 12 states (44%) No limit: 1 state (8%) 5: 4 states (34%) 10: 6 states (50%) 20: 1 state (8%)
Produce set with numerals (C4)	Of 2 states (9%) No limit: 1 (50%) 10: 1 state (50%)	Of 1 state (4%) No limit: 1 state (100%)	Of 2 states (7%) No limit: 1 (50%) 10: 1 state (50%)

the count string was five (Y1) and 30 (Y2) and appeared in n = 1 G2 state ELS.

The upper limit attached to the indicator of one-to-one counting skill (ONE) showed a range of values. Almost half of G1 ELS did not include a magnitude for ONE and for G1 ELS that did, 10 appeared most often in n = 7 ELS. Among G2 ELS, for those that included an upper limit for ONE, five was most frequently included for Y1 (n = 11 states) and 10 was the most common for Y2 (n = 15 states). Among G2 state ELS that included ONE for both Y1 and Y2 (n = 20), the most common combination of magnitudes was five (Y1) and 10 (Y2) in n = 9 states; similar to WORD, there was no clear second preference. The largest gap in magnitudes for counting in a one-to-one manner was three (Y1) to 10 (Y2) and appeared in n = 1 G2 state ELS.

For G1 ELS, the most frequent set size associated with SUBZ (subitizing) was five and it appeared in n = 5 ELS. For G2, the range of set sizes for subitizing was narrower. Fourteen G2Y1 ELS included SUBZ and the set sizes included three (n = 6 states) and four (n = 6 states). For G2Y2, n = 22 states included SUBZ and a set size of five (8 states) was most common followed by four (6 states). A state-level comparison of G2 ELS that included SUBZ for both Y1 and Y2 (n = 14) indicated that the most common pair of magnitudes was three (Y1) and four (Y2) in n = 4 states followed closely by four (Y1) and five (Y2) in n = 3 states. The largest gap in magnitudes for determining "how many?" without appearing to count was four (Y1) and 10 (Y2) and appeared in n = 2 G2 state ELS.

Cardinality

Table 3 also presents the set sizes for cardinality. Although C1 (*how many* using number words) was the most prevalent indicator of cardinality in G1 and G2 ELS, between 45% and 50% of states did not specify a set size. For G1, 10 was the most common (n = 5 states) followed by five (n = 4 states). For G2Y1, five was most common (n = 8 states) and for G2Y2, ten appeared in n = 12 states. At the state level, among the G2 states that included C1 for Y1 and Y2 (n = 13), the most common magnitude pairs were five (Y1) and 10 (Y2) in n = 7 states.

For C2 (*how many* with a written numeral), of the n = 11 G1 ELS that included C2, 64% did not indicate a set size. The most common set size was five (three states). For G2Y1, n = 6 states included C2 and the most common was a set size of five in n = 4 states. For G2Y2, n = 13 states included C2 and the most common was 10 in n = 8 states. The set sizes for producing sets using number words (C3) and written numerals (C4) had more limited ranges and appear in Table 3.

Discussion

The current study resembles similar research conducted on states' K-8 math indicators (Dingman et al., 2013; Reys et al., 2006; Smith et al., 2010) but differs, in part, because it is limited to a single epoch – "preschool" – versus of range of grade levels. Our approach was new in that it focused on foundational indicators of early math development (Fuson et al., 2010; Jordan et al., 2022; National Research Council, 2009) and, building from a strong empirical base, expanded the number of indicators examined from prior investigations (Litkowski et al., 2020a; Scott-Little et al., 2011, 2012), acknowledged the type of quantity representation, summarized the magnitudes or set sizes associated with select indicators, and compared two groups of state ELS, one with a single set of indicators for "preschool" and the other with two sets of indicators for the two years preceding kindergarten. Similar to past research (Litkowski et al., 2020a; Scott-Little et al., 2011, 2012), findings revealed considerable variation in the prevalence of some indicators, with more complex indicators of counting, cardinality, and number relations and operations less prevalent. Among ELS with one set of indicators, there was less consensus for magnitudes and set sizes associated with counting, subitizing, and cardinality than among state ELS with two sets of indicators.

Key Themes from Study

The Discussion will consider three themes that emerged from the results. First, *how states organize indicators by age matters* for illustrating the progression of mathematical development and supporting instructional practices that meet a range of child math knowledge and ability. Second, *more advanced indicators of early number knowledge were not prevalent* and in some instances were rare throughout state ELS. And third, *type of representation is an important consideration in the progression of early number sense*. These themes will be discussed in turn.

Organization of State ELS by Child Age

To our knowledge, this is the first examination of ELS for math that has compared how states organize their ELS for the preschool period (i.e., one set of indicators versus two sets of indicators) and utilized this information to create mutually-exclusive groups for comparison. This is a relevant undertaking that reflects changes over time in how states organize ELS. Scott-Little et al. (2011) reported that four states (8%) included separate standards for three- and four-year olds whereas the current analysis indicated that 27 states (54%) had more than one set of standards for the

preschool years. These structural differences in ELS have marked implications for teachers and children, especially when considering the magnitudes, or set sizes, attached to select indicators. When compared with G2 ELS, G1 ELS were less likely to include upper limits or set sizes and, when they did, state ELS revealed a wider range and less consensus among recommendations.

Providing at least two sets of indicators with recommendations for magnitudes or set sizes that increase with child age signals to teachers that younger children, or older children who are struggling, can practice early number skills, such as one-to-one counting, subitizing, and producing sets of objects using number words, with smaller sets before training their skills on larger magnitudes or set sizes. Young children tend to develop their non-symbolic and symbolic quantitative skills with smaller sets first before moving on to larger ones (Jiménez-Lira, Carver, Douglas, & LeFevre, 2017; Jordan et al., 2022). Relatedly, progress toward cardinal principle knowledge (CPK), a more advanced form of cardinality, occurs in numerical sequence, with children first learning that one is ■, two is ■ ■, and so on (Carey, 2004). In sum, two sets of indicators more accurately captures early mathematical progression, provides richer guidance to teachers and more space for children to develop their number knowledge and skills, and better aligns with the 20-year trends in state-funded preschool and the doubling of three-year-olds who attend programs (Friedman-Kraus et al., 2023).

Limited Prevalence of More Advanced Indicators of Number, Number Relations, and Number Operations

Another theme that emerged is that more advanced indicators of early numerical development are not prevalent and, in some instances, are rare among state ELS. Similar to Scott-Little et al. (2012), our findings revealed that indicators of counting (one-to-one and verbal recitation of the count list), identification of written numerals, "cardinality," and non-symbolic magnitude comparisons were the most prevalent among states' indicators. However, in our study, numeral identification and non-symbolic magnitude comparisons were more prevalent among G1 state ELS than G2 state ELS. Additionally, although the skills are framed here as *more advanced*, it is important to note that they are skills that preschool-aged children are capable of learning (Litkowski et al., 2020b). For instance, cardinality indicators associated with answering "how many?" with number words were more prevalent in both groups' ELS (G1 = 78%, G2Y1 = 56%, and G2Y2 = 81%) than the two indicators representing cardinal principle knowledge (CPK). CPK with number words was present in 26% or fewer of G1 ELS, 22% or fewer of G2Y1 ELS, and 44% or fewer of G2Y2 state ELS. Moreover, cardinality and CPK using written numerals were both less common among state ELS than the same indicators using number words.

In the early number sense framework adopted by this study (Fuson et al., 2010; Jordan et al., 2022; National Research Council, 2009), much hinges on the emergence of CPK. The onset of CPK with number words develops from knowledge of the verbal count list, procedural skill with counting one-to-one, and the understanding that the last number counted is the set size (Le Corre et al., 2006; Sarnecka & Carey, 2008) and functions as a "gatekeeper" to continued mathematical progression (Geary et al., 2018; Geary & vanMarle, 2018; Spaepen et al., 2018). Overlooking the central role of CPK in state ELS for math may limit the inclusion of more advanced number and number relations indicators.

For example, our analysis indicated that the more advanced skill of "counting on" (i.e., the ability to count forward from a number other than one) was included in 13% of G1 ELS, 4% of G2Y1 ELS, and 19% of G2Y2 ELS, an increase from Scott-Little et al. (2012), who reported that "counting on" was present in 3 states (6%). "Counting on" is considered an outcome of cardinal principle knowledge (Paliwal & Baroody, 2018). A game-based intervention that helped 4- and 5-year-olds "count on" one or two times while proceeding along a number line (Ramani & Siegler, 2008) produced improvements in numeral magnitude comparisons and number line estimation, both advanced skills of number relations, and was included in Raudenbush and colleagues' successful numerical thinking skills adaptive assessment (2020). Interestingly, among G2 ELS, counting on was more prevalent than among G1 ELS and was more likely to appear in Y2 than Y1. This is another example of how two sets of indicators provides early childhood teachers with a more accurate depiction of the progression of children's counting skill and knowledge.

In both groups, the prevalence of magnitude comparisons and ordering, overall, was infrequent to rare. These findings reveal scant attention to the number relations strand of early number sense. Distinguishing symbolic magnitudes, such as numeral comparisons, improves after the onset of CPK (Geary & vanMarle, 2018), is responsive to intervention (Ramani & Siegler, 2008; Raudenbush et al., 2020), and makes possible learning about and performing operations. Because the prevalence of operations-related indicators in both G1and G2Y2 ELS was above 50%, a marked increase in prevalence from Scott-Little et al's, 2012 analysis [12 states (24%) included non-symbolic addition and subtraction and 9 states (18%) included "addition and subtraction"], it was surprising that advanced indicators of counting and number relations indicators were not more prevalent in state ELS.

Type of Representation is an Important Consideration

A final theme is that the type of quantity representation is an important consideration for state ELS. Young children develop quantitative competence with mapping number words to quantities first and mapping written numerals to quantities second (Jiménez-Lira et al., 2017; Knudson et al., 2015; Purpura et al., 2013). Similar to Scott-Little and colleagues (2011;, 2012), indicators of early math that captured children's ability to use number words were more prevalent throughout both groups' state ELS than indicators that relied on written numerals, This was especially the case for mapping written numerals onto sets and determining magnitude differences between pairs (i.e., magnitude comparisons) or sets of numerals (i.e., ordering). Incorporating representations of quantity that utilize Arabic numerals is more feasible when ELS have at least two sets of indicators and would also permit age-graded distinctions between comparing pairs and ordering sets.

Limitations

It must be acknowledged that this study is a snapshot of state ELS for math. ELS undergo review and revision continually and the analysis presented here will look very different in the future. That said, there are several limitations to the approach we took to coding state ELS for number and operations. The indicators were coded as written, with no effort to gain additional knowledge about possible intent or other meaning. For example, although some state ELS explicitly included the Common Core Standards for kindergarten math as an extension of the preschool indicators, we did not address this as a possibility for how states made decisions about what to include or exclude from their ELS. Additionally, our approach to indicators of ordering focused only on non-symbolic and symbolic ordering of images or numerals and did not consider ordinal language (e.g., first, second, etc.), a consideration in prior research (Scott-Little et al., 2011;, 2012). New research on the role of math language as a mediator between the home math learning environment and children's math skills (King & Purpura, 2021) suggests increased attention to the vocabulary of math as a relevant topic in state ELS for number and operations.

Conclusions and Future Research

As the number of young children attending public preschool increases (Friedman-Kraus et al., 2023), state ELS for number and operations warrant continual review and revision, especially in light of the absence of common core standards for preschool math. Even before kindergarten, young children vary widely in their numerical skills and knowledge (Garcia, 2015) and currently, about 20% of U.S. 4th graders do not meet basic competency levels in math, with this percentage increasing among 8th (30%) and 12th (38%) graders (p. 103; McFarland et al., 2019). Improving teacher knowledge and instruction of early numeracy is one solution to elevate preschoolers' math knowledge and to mitigate poor math achievement later. Math-related talk and activities in preschool are positively correlated with children's math achievement over time (Bachman et al., 2018; Klibanoff et al., 2006; Rudd et al., 2008). State Early Learning Standards are valuable tools that inform the practices of early childhood professionals (DeBruin-Parecki & Slutzky, 2016) and indirectly influence young children's math learning.

However, some teacher educators report that early childhood teachers and caregivers are plagued by low math-related confidence that interferes with their ability to learn how to effectively guide young children's math learning (Ryan, Whitebook, & Cassidy, 2014). This observation raises questions about the content and quality of teacher preparation programs, on-the-job supports, and professional development opportunities centered on facilitating early numeracy (Whitebook & Ryan, 2011). Future research should examine the role of state ELS in higher education's teacher preparation programs and ongoing inservice training, and how teacher educators, preservice teachers, and inservice teachers interpret the standards and utilize them in the classroom.

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Declarations

Competing interests The authors have no competing interests to disclose.

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