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Young Japanese Children's Subjectification and Objectification Through the Lens of Joint Labor in a Mathematical Activity at a Preschool: A Case Study

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Abstract
The objective of this case study was to identify the role of finger gestures in learning mathematics informally during play, especially in sociocultural settings. A mathematical activity involving addition was qualitatively analyzed at a Japanese preschool. We explored how the process of subjectification and objectification contributed to a mathematical activity at a Japanese preschool and how the role of preschoolers' finger gestures changed during the process of learning mathematics. We utilized Radford's theoretical construct of joint labor and analyzed Japanese preschool children's mathematical behaviors from a sociocultural perspective. The subjects were 15 Japanese preschool children and their teacher. We relied on both Radford's methodology and a microgenetic approach for the analysis. We found that subjectification and objectification proceeded in the scene of the conversations regarding addition; observing joint labor in a classroom activity offered valuable insights into these processes. In the activity, the children actively extended their practice of posing and answering quizzes, and learned how to resolve a conflict with the support of the teacher. Secondly, although the role of finger gestures was originally used to obtain correct answers to quizzes, it was reconstructed to solve the conflict between the children through the teacher's mediation. This showed that, even in an environment where children implicitly learn mathematics, they learn from one another, including the teacher, and that gestures in mathematical communications function well for developing mathematical thinking and skills.

Keywords Early mathematics education · Joint labor · Gestures · Sociocultural perspective · Japanese preschool children

Introduction

Recent studies on preschool children have drawn attention to the sociocultural nature of the early development of mathematical abilities from different perspectives, including the Vygotskyan approach (Dijk et al., 2004; Radford, 2020, in press). While sociocultural issues for mathematics education have not been intensively discussed in constructivist research on young children, Graham et al. (1997) argued that the context of preschools, including their settings and teachers, influenced children's mathematical development; however, previous literature has focused on individual children and neglected the broader context in which children develop their skills. This study aims to address these limitations by shedding light on gestures and other embodied actions with verbal language that are considered an integral part of children's cognitive functioning (Radford, 2012, in press).

In the Asian context, particularly in Japanese preschool education, where children only learn mathematics through informal daily life situations, the importance of hand gestures in the development of the thinking process of young preschoolers has been neither recognized nor discussed in the classroom situation. As Graham et al. (1997) insisted, educators need to know children's conceptions of mathematics and the learning context before initiating compulsory education.

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The purpose of this study is to reveal the role of finger gestures in preschoolers' learning of mathematics in informal settings, especially in sociocultural settings. To explore such roles, we refer to Radford's (2016a, b) theoretical construct of joint labor and analyze Japanese preschool children's mathematical behaviors from a sociocultural perspective. Radford (2016b) proposed the idea of joint labor as a key theoretical construct in his theory of objectification, where students and teachers work together to create common work.

The Japanese Ministry of Education, Culture, Sports, Science, and Technology's National Curriculum Standard for Kindergartens (MEXT, 2017) does not explicitly define preschool subject areas, including mathematics. It expects groups of same-aged children to acquire mathematical concepts and skills through integrated play. Individual preschools are responsible for designing such activities. Therefore, the authors believe that observing a group activity for learning mathematical concepts and skills in a Japanese preschool will provide an opportunity to analyze the role of finger gestures in the learning of mathematics in an informal or play setting.

Theoretical Framework

As stated above, Radford (2016b) proposed joint labor as a key theoretical construct in his theory of objectification. His theory is built on a Vygotskian view of activities, the aim of which is "the dialectic creation of reflexive and ethical subjects who critically position themselves in historically and culturally constituted mathematical practices, and ponder and deliberate on new possibilities of action and thinking" Radford 2016b, p. 196).

Radford (2016a, b) calls such specific activities joint labor, arguing that subjectification and objectification are two sides of the same coin; he is interested in the social cotransformative sense-making processes in mathematics classrooms (Jansen & Radford, 2015). These processes occur simultaneously during an activity. Therefore, learning can be theorized as the processes through which students gradually become acquainted with historically constituted cultural meanings and forms of reasoning and action. These processes are termed processes of *objectification* (Radford, 2015, p. 551, italics as per the original).

> Objectification is more than the connection of the two classical epistemological poles, subject and object: it is in fact a dialectical process—that is, a transformative and creative process between these two poles that mutually affect each other [...] Subjectification is the making of the subject, the creation of a particular (and unique) subjectivity that is made possible by the activity in which objectification takes place. [...] Learning is both a process of knowing and a process of becoming. (Radford, 2015, p. 553)

Thus, the concept of joint labor reconceptualizes teaching. Mathematics teachers objectify a new aspect of the mathematical concept being taught and subjectify themselves while working with their students:

> Teaching and learning not as two separate activities but as a single and same activity: one where teachers and the students, although without doing the same things, *engage together*, intellectually and emotionally, toward the production of a common work. *Common work* is the *sensuous appearance* of knowledge (e.g., the sensuous appearance of a covariational algebraic or statistical way of thinking through collective problem posing, solving, and discussion and debate in the classroom). [...] The joint labor-bounded encounters with historically constituted mathematical knowledge materialized in the classroom *common work* are termed *processes of objectification*. (Radford, 2016a, p. 5, italics in the original)
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> The concept of joint labour is a theoretical construct that allows one to move beyond the antagonism between teachers and students that has informed the transmissive teaching model and its behaviourist pedagogy as well as the various reform models and their child-centred pedagogy. (Radford, 2021, p. 92)

In this sense, the reconceptualization of teaching is crucial in the current study. The existing literature from a Vygotskian perspective interprets the classroom community as a useful resource for children's learning (e.g., Edens & Potter 2013). We interpret the whole classroom community as the target of the study. We are not interested in how each individual child learns but in how learning as a social process proceeds in the classroom. Hence, unlike other sociocultural approaches, language is not the central organizing ontological category. The ontological category of the theory of objectification is not language but activity (understood as joint labor). Radford (2021) stated that the word "activity" comes from *Tätigkeit* in German and *deyatal'nost'* in Russian (p. 29). It refers to "a dynamic system where individuals interact collectively in a strong social sense, which makes the activity's products collective as well" (p. 29). He clearly stated that activity, *Tätigkeit*, is joint labor, which can be accomplished through the teacher's and children's collaborations in class. In joint labor, the body, movements, actions, rhythm, passion, and so on are significant, as they are related to what it is to be human. In the theory of objectification, according to Radford (2021), the activity is a dynamic, complex system and the activity can be divided into "moments." The moment can be identified during different parts of a lesson, such as the teacher's presentation, children's group-work, small-group discussions between the teacher and the students, and general discussions involving the whole class (Radford, 2021, pp. 90–91).

To use Radford's notion of activity for the current study, we flexibly interpret a short-term episode of the sensory appearance of knowledge as collaborative work. Classroom practice never ends as long as classroom community exists. Today's practice continues in a whole social process until tomorrow's practice. For this reason, we need to extract a process leading to the sensory appearance of knowledge from a whole never-ending social practice to analyze empirical observational data.

Research Questions

Based on the abovementioned theoretical framework, our research questions are as follows:

A) How does the process of subjectification and objectification proceed in a mathematical activity at a Japanese preschool?
B) How does the role played by preschoolers' finger gestures in mathematical learning change during the process?

Children's Gestures in Mathematics Learning

Gestures are a type of bodily action (Radford, 2009) that express Piaget's epistemology, one of the most well-known and impactful epistemological theories of the twentieth century (Radford, 2009) emphasizes that we need to consider the cognitive role of the body and senses and that we need to identify how gestures are related to learning and thinking. Several opinions exist on how gestures, thinking, and learning are intertwined (Johansson et al., 2014; Radford, 2009). Some researchers regard gestures as facilitators of verbal expressions (Freedman, 1977). For instance, Sfard (2007)[1] suggested that, in mathematics education, only verbal language can explain abstract mathematical objects rather than gestures and other bodily actions on artifacts. Others regard both gestures and people's speech as cognitive sources (Goldin-Meadow et al., 2001; McNeill, 1992; Whitebread & Coltman, 2010), for example, insisted that gestures and speech would support young children's expressions of their thinking process. Other studies, such as Roth (2001) and Goldin-Meadow et al. (2001), have presented a similar perspective. A third view is that gestures are derived from virtual actions performed on the objects of discourse by a person in a virtual space. This view, therefore, posits that considering gestures would offer an opportunity to reveal the perspective of the mental contents of the person who speaks (Radford, 2009).

Radford's standpoint that gestures are genuine constituents of thinking is different from the above three views (Radford, 2009). He suggests that gestures come with one's thinking to restructure it. Various studies have been conducted on gestures in line with Radford's views. Johansson et al. (2014) observed that preschool children use gestures and body movements to assist their inadequate verbal expressions. Similarly, McNeill (2005) described how at about four years of age, children develop gestures almost the same way as adults do. Johansson et al. (2014) also took Radford's view and investigated how the relationship between verbal language and gestures can be considered in young children's explanations.

In the mathematics education community, gestures have been researched and investigated among older students (e.g., Arzarello et al., 2009; Meaney, 2007; Radford, 2003, 2009; Roth, 2001); however, research on gestures in young children linked with their learning and thinking have been scarce (Johansson et al., 2014). For example, Elia and Evangelou (2014) explored one kindergarten child's gestures to capture their meaning making for spatial concepts in mathematics class, applying the microgenetic approach to data analysis. Elia et al. (2014) also investigated a preschooler's gestures from a cognitive perspective in a geometric activity. Both of these studies, however, focused upon only the cognitive aspects of the children's geometrical concepts. The current study has a different perspective in that it focuses on the sociocultural standpoint. In this study, therefore, we investigated the process of objectification and subjectification through the lens of children's gestures and their learning by considering gestures as a means of objectification. In this study, we followed Radford's (2009) definition of gestures: They are a type of bodily action which shows one's cognitions, senses, and the process of learning and thinking.

Method

A Microgenetic Approach and Narrative Analysis

While the recommended approach to documenting processes of objectification and subjectification is longitudinal research on joint labor (see Radford 2015, 2011), we adopted a microgenetic approach in the current study (cf. Lavelli et al., 2008). It is a methodology for investigating developmental changes which has been proposed as a reply to the

[1] Her later works might reveal a different perspective; however, we did not deal with them because of their different perspective and focus.

criticism that a pre- and post-test approach captures only the products of change and fails to capture the processes of change. Microgenetic studies differ in terms of observation periods (short- or long-term), number of sessions (single or multiple), type of data (quantitative, qualitative, or both), and theoretical perspectives (Piaget, Neo-Piaget, Vygotsk, etc.). The spirit of microgenesis can be used flexibly for a variety of purposes (Miller & Coyle, 1999). This flexibility allowed us to use a microgenetic approach with a qualitative analysis of short-term observation based on the theory of objectification.

The microgenetic approach is suitable for answering our research questions as it has the strength to analytically describe in detail both children's learning and the changes in the functions that children's gestures express, for the short time of period, 'snapshot', compared to the longitudinal research which comsumes much time to capture children's changes (Lavelli et al., 2008). In the current study, we focused on the movements and changes of finger gestures, including, to some extent, facial expressions and postures for a short period of time. This is because the activity included addition and subtraction of numbers, and students used their fingers to calculate. They had not learned formally in the preschool setting how to calculate addition and subtraction. Given that young Japanese children do not learn formal mathematics before entering primary school but learn through play, they tend to use their fingers. Therefore, it is expected that by using the microgenetic approach to analyze a short-term video clip of a preschool activity involving finger gestures, we can understand how the role of gestures change in the social domain.

Data Collection and Contexts

As mentioned earlier, Japanese preschools design and implement an annual plan for their activities on an individual basis. The first author implemented an activity for environmental education that comprehensively integrated mathematics and nutrition education in a preschool. The preschool had its own curriculum according to which the children were to grow through playing and become capable of making their own decisions. The school's curriculum allowed children to learn mathematics informally and spontaneously, while teachers who fully understood the philosophy could intentionally implement activities (such as the one we proposed) to learn mathematics. The preschool was interested in helping children develop their capacity for self-determination and to learn how to design mathematical activities that supported this. The first author collaborated in the design of these activities to collect data on younger children's conceptions of and operations with numbers. We intentionally chose and analyzed an activity previously implemented in the school. The analysis of the activity from a sociocultural perspective inspired by Radford illustrated that the traditional cognitive perspective overlooks certain dimensions in the learning of mathematics as part of a group activity. The authors and a teacher with 16 years of teaching experience confirmed their roles before conducting the activity. The teacher's role was to organize the activity and the first author's role was to videotape the activity without intervening. All authors participated in the analysis of the video. Fifteen children (one male and 14 female) aged 5 to 6 years participated in the study. We obtained the school principal's and the parents' permission to conduct the study.

From the perspective of joint labor, we focused on a mathematical quiz activity in which students and teachers quizzed each other about the number of bananas a monkey had. The activity presented in this study is of short duration and, following Radford (2021, in press), can be a general classroom discussion between the teacher and the children.

Indeed, Radford reports a series of short-term episodes as part of the historical processes of objectification and subjectification (Radford, 2011, 2016b, 2020). Accordingly, we recorded the entire session and selected a salient segment for analysis in joint labor. We did not follow the other procedures suggested by Radford (2015) because our focus on joint labor is a relatively new application of his theory. Instead, we adopt narrative analysis, which Lavelli et al. (2008) propose as a promising qualitative method in a microgenetic approach.

Narrative Analysis

Narrative analysis, originally proposed by Polkinghorne (1995), aims at organizing "the data elements into a coherent developmental account" (p. 15) and "produces stories as the outcome of the research" (p. 15). It is distinguished from analysis of narratives, which aims at identifying "particulars as instances of general notions or concepts" (p. 13). In other words, narrative analysis describes an observed mathematical activity as an original story, rather than checking whether the activity is described as one of a certain given type of mathematical activities. Narrative analysis is suitable for answering our research questions that involve the interrogative *how*.

Following Polkinghorne's (1995) extension from an individual participant to a social community as the unit of analysis in a case study, we view the whole of a preschool mathematical activity reported in this paper as a single case. According to Radford (2015), the process of learning is theorized as a process of objectification and carried out with others. Therefore, a storyline we try to create should not be a story about an individual participant but one about a preschool community.

Lavelli et al. (2008) introduced the concept of *frame* to explain the creation of storylines in a microgenetic approach.

Originally, for capturing emotions from a social perspective, Fogel et al. (1997) defined frames as "segments of co-action that have a coherent theme that takes place in a specific location and that involve particular forms of *co-orientation* between participants" (p. 11, italics in the original). As examples of frames they mention "greetings, topics of conversation, conflicts, or children's social games" (p. 11). Lavelli et al. (2008) cite Pantoja et al. (2001) as an example of narrative analysis within a microgenetic approach. Following them, in the current study we divided a videoclip of a mathematical activity into several frames.

The analysis procedure was as follows: (1) stable and changing components of the relationship between the children and the teacher were roughly identified through repeated viewing of a clip, (2) the clip was chronologically transcribed, (3) the transcription was divided into multiple frames, and (4) a storyline that synthesized the frames to explain the stable and changing components was created. Step (4) was further divided into four steps: (4.1) joint labor based on the stable and changing components of the relationship between the children and the teacher was identified when their use of fingers changed, (4.2) the algebraic knowledge created by finger gestures was interpreted, (4.3) the objectification was interpreted, and (4.4) subjectification by the children was interpreted.

In step (3), following Fogel et al.'s (1997) definition, we separated the transcription into frames by focusing on coherent themes. Narrative analysis aims at creating a storyline that synthesizes different themes.

Lavelli et al. (2008) recommended using multiple case studies to capture the regularities between cases. However, in the present study, only a single mathematical activity was analyzed as a case study. The case study offers a new analytial way of analysing and interpreting children's learning and also shows the usefulness of the theory of objectification to understand children's learning from a different perspective. The original spirit of Polkinghorne's (1995) narrative analysis does not necessarily aim to find regularities. We focused on understanding this one activity by developing a story for it.

Results

Children's Activity

For the activity, the children watched a video clip on wild animals while seated on the floor. The teacher introduced an activity about monkeys and their lives. She told the children that they had transformed into monkeys, and some of them pretended to have become monkeys by making gestures such as raising and lowering their hands near their faces, as shown in Fig. 1.

Fig. 1 Childrenpretending to have transformed into monkeys using gestures

The teacher asked them to show their fingers and said, "Now let's try this as if you are monkeys. You have gotten hungry as we have bananas here," as shown in Fig. 2.

She asked the children, "If your fingers have become bananas, how many bananas do you have?" and the children answered, "Ten" or "Five." She confirmed by saying, "How many when we combine this and that?" and then showed her right and left hands one by one, and the children answered, "Ten." She suddenly started to sing a quiz song known to the children while clapping her hands, and the children also joined in the song with gestures and smiles. The song appeared to be popular. The teacher then showed her 10 fingers and asked, "Well, one finger is lost. How many fingers altogether now?" (Fig. 3).

The children were encouraged to use their fingers, with each finger representing one of the monkey's bananas. As she quizzed the children regarding the number of bananas, they calculated their answers by counting the fingers presented by the teacher. The children then indicated that they wished to share their quizzes by raising their hands. The teacher selected students individually, and each, in turn, came to the front of the class and took on the teacher's role. Their questions followed the sentence format for verbal expressions provided by the teacher, a common pedagogy at the school. Children provided simple addition and subtraction problems, including 10–5, 4–1, 10–8, 10–5, 10 + 2, and 11–3, to their seated peers, who listened to and answered the quizzes together by counting the presenting student's fingers. Student presentations were followed by additional presentations from the teacher, which were then followed by additional presentations from the remaining students who quizzed their peers on problems like 10–9, 10–7, 5 + 4, and 9 + 1. All of the numbers proposed by the children were less

Fig. 2 Childrenimitating the teacher's finger gestures to show bananas (fingers)

Fig. 3 Thefirst quiz that the teacher asked the children

Fig. 4 Yu becomes the teacher and quiz the other children

Fig. 5 Theteacher smiles at Yu

than 11; therefore, they used their fingers to do the calculations all the time.

The focal scene comprised the penultimate question asked by the students. Following the third step in the procedure for microanalysis, the transcription was divided into six frames. All conversations were in Japanese and were translated into English by the authors. The names of all the children are pseudonyms.

Frame 1 featured a male child's (Yu) question. He had previously asked 10–5 in his presentation, and it was his second chance to quiz the other children. He had shown his fingers when presenting his previous questions. However, for this question, he did not show any fingers, as shown in Fig. 4. In the transcriptions below, T means teacher, S means an unidentified child, and SS means all the children in the class.

182: Yu: (Singing the quiz song) Quiz, quiz.
183: SS&T: What is the quiz?
184: Yu: (Singing) Quiz for answering the number of bananas.
185: SS&T: What is the question?
186: Yu: Here are 12 bananas. A monkey put five more bananas. How many bananas are there altogether?After Yu's question, the teacher wryly smiled, likely due to the large size of the number considering the ages of the children, but decided to continue the game, as shown in Fig. 5.

187: T: It seems more difficult than before. Okay, okay. Let us try.

Fig. 6 Theteacher asks Yu the number of bananas, and he starts to count with his fingers

Fig. 7 Yuimmediately tells Hiroko “Wrong” when she says “17”

188: S: (Raising their hands) Yes, me.

Frame 2 showed the teacher’s confirmation of the content of the questions shown in Fig. 6.

189: T: How many bananas were there in the beginning?
190: Saki: 12.
191: T: 12. And how many bananas were added?

Frame 3 shows a conflict between children, as shown in Fig. 7.

192: Yu: (Pointing to a child, saying her name.) Hiroko.
193: Hiroko: 17.
194: Yu: Wrong.

Although Hiroko answered correctly and quickly, Yu did not seem to have understood what she said. After denying her answer immediately, he had a troubled look and simultaneously began moving his mouth to count, as shown in Fig. 8.

Frame 4 showed the conversation between Yu and the teacher to reconfirm the question. Figure 9 shows his gestures using finger counting.

195: T: Let us think together.
196: Yu: Wrong.
197: T: Wasn’t it right? What was the answer?
198: Yu: 16.
199: T: Well, in the beginning, the monkey had 10 and two bananas. (Showing her 10 fingers and using two of Yu’s right-hand fingers).
200: Yu: There were 12 bananas, and the monkey added five more bananas, and then ...

Frame 5 showed the start of a collaborative discussion using finger gestures.

202: T: So, there are 12 and it added five more … (Showing 10 with her hands and letting Yu show two more with his hands.)
203: Konoha: There are 16 bananas.
204: T: 16 bananas?
205: Yu: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17. (Counting his and the teacher’s fingers.)

Fig. 8 Yu anxiouslymoves his eyes

Fig. 9 Yucounts with his fingers in conversation No. 195

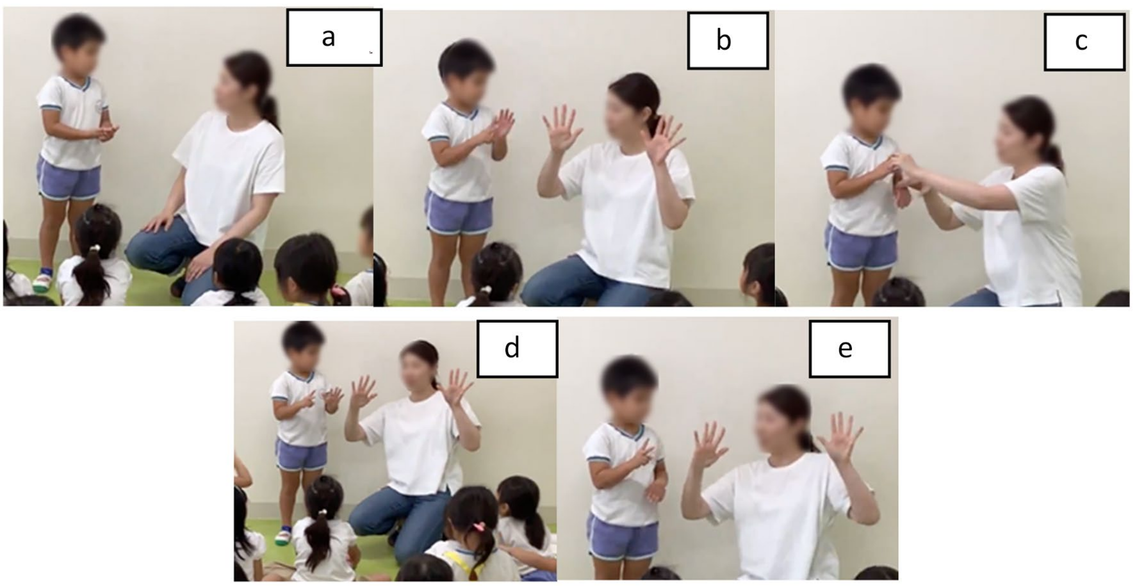

Fig. 10 The process of joint labor (1)

206: T: Yes? Right! Wonderful! (Clapping) This gradually became difficult.

Fig 10 shows the process of the joint work that the teacher and Yu did in conversationNo. 202. In Fig 10c, the teacher asks Yu to hold up two fingers. In Fig 10d,they collaboratively show 12 fingers. In Fig 10e, Yu shows five fingers withhis left hand.

In Fig. 11a, the girl who points to the teacher's fingers says, "17." In Fig. 11b, Yu starts to recount the number of fingers while the same girl shows a circle, a gesture used in the Japanese context to show that it was correct. When someone said "16," in Fig. 11c, the teacher encouraged Yu to show his fingers and in Fig. 11d, the teacher did the same. The girl on the left started counting Yu's fingers in Fig. 11d. In Fig. 11e, Yu began to count their fingers, based on the teacher's lead. In Fig. 11f, he counts the teacher's 10 fingers and then proceeds to count his fingers. In Fig. 11g, he counts "15, 16, 17." In Fig. 11h, he looks at the teacher when he finishes saying "17." Fig. 12 shows Hiroko's reasoning about the solution to 12 + 5.

Before conversation no. 207, the collaborative work from Hiroko can be observed, shown in Fig. 12. In Fig. 12a, she showed two fingers of her right hand and five fingers of her left hand. In conversation No. 207, Yu did not listen to Hiroko's explanation for the addition and tapped the other child's hand to show his happiness in getting the right answer. Immediately after recognizing that her friends obtained the same answer, 17, using finger gestures, Hiroko saw the connection between 12 + 5 and 2 + 5 and argued the point.

207: Hiroko: Well, 2 + 5 = 7. (Showing her two and five fingers together, in Fig. 12a)
208: T: Wow, well, you counted! I see, so let us do the final one? Please.

Fig. 11 The process of joint labor (2)

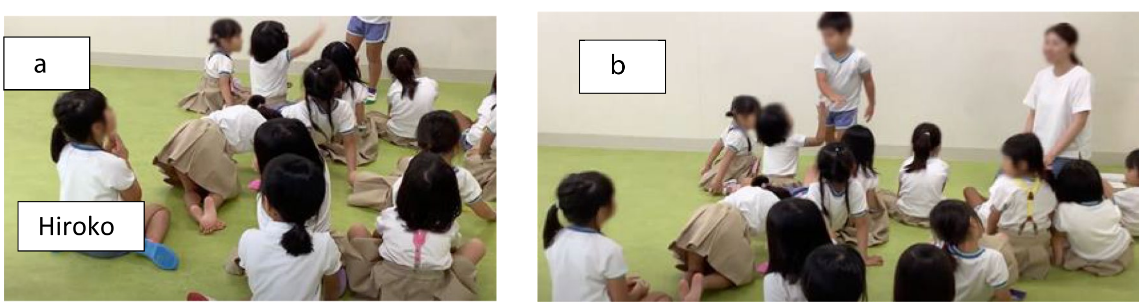

Fig. 12 The process of collaborative work

Created Storyline

The activity presented constitutes joint labor between the children and the teacher, as well as the children's number recognition. We regarded it as joint labor because its production was historically contingent. The quiz was given by Yu, a child, and not by the teacher (Frame 1). Following her song format, he invented a quiz by selecting numbers 12 and 5 by himself. Singing a song has been a good way for children to conceptualize things informally, for instance, number conceptions and calculations in this case. Thus, this environment has been historically influenced. Up to this question, the numbers the children proposed were not more than 10, thus, Yu showed his fingers as the teacher demonstrated in his first question of 10–5. However, he did not show his fingers when posing his second question. This shows that Yu implicitly recognized that 12 was bigger than 10, and it was not possible to express this number with his fingers.

His action indicates that he knew the difference in the size of numbers despite not having learned numbers officially at the preschool. Although his quiz was relatively difficult for most children besides Hiroko, they finally obtained the answer "17" with the assistance of the teacher (Frame 5). The teacher's role was to provide neither the quiz nor the answer. She only provided the format of the activity and demonstrated the use of finger gestures to assist the children in showing the correct numbers with their fingers. That is, her role in supporting the children did not change when they presented quizzes. Instead, the children were free to decide (Frame 5) whether they wished to count on their fingers as the teacher had modeled. The teacher's actions changed according to children's responses which is strongly related to the emergence of joint labor because the teacher also learned the situations and changed her way of assisting the children. Hiroko ended up not using her fingers and instead succeeded in thinking and recognizing a group of 10 in her mind. The recognition of a group of 10 was also identified in Yu's action in Fig. 11f. The children's focus actively changed following their interactions with the teacher. Their joint labor was, therefore, visible in that scene.

During joint labor, three kinds of concepts of numbers, including fundamental addition, emerged as common work. First, Yu proposed the new numbers: 12 and 5. The teacher's smile indicated that she did not expect her children to use a number as large as 12. However, contrary to her expectations, Yu realized that he could use a larger number (12) for quizzes. In the beginning, the teacher controlled the rules, but ultimately such a role was delegated to the children, which might have prompted Yu to develop the scope of the questions further. Children started with smaller numbers up to 10 in the quizzes, but they further developed to use a larger numbers and created quizzes with subtraction. In this process, the children objectified the new numbers as part of the quiz and gradually subjectified themselves as new quiz producers.

Second, the children used finger gestures to determine the number of bananas. They did so under the teacher's guidance. For example, since the number 17 was too large for the children to count quickly, Konoha counted her fingers in error (Frame 5). The finger expressions of the number, however, spatially maintained the initial assumption of the quiz that there were 12 and five bananas under the teacher's support and her mediation of the children's repeated and careful counting. Therefore, algebraic thinking involving adding and counting is re-embodied and re-mediated by the artifactual use of the fingers by the teacher. The children re-objectified finger gestures as tools for solving the conflict over the solution and re-subjectified themselves as finger gesture users in solving the quizzes.

Finally, Hiroko realized that $12+5$ was separable into 10 and $2+5$. Since she immediately answered the quiz, she might have already known how to calculate in this manner before the joint activity. Only the teacher recognized what Hiroko asserted; the other children, including Yu, did not respond to her. Her separating strategy was difficult for the others, who depended on finger gestures. She objectified the separating strategy as a tool for solving the quizzes, but her subjectification could not be determined from this observation. When she uses the strategy again in the future, her subjectification might be gradually determined, depending on the responses of her peers.

Discussion

Our observation of the children's reuse of finger gestures shows that spatial and numerical structures are linked, following Radford's (2011, 2021) claim that algebraic thinking is by nature embodied and mediated by artifacts. However, the children needed the teacher's suggestions for finger gestures to resolve conflicts. Although they repeatedly used finger gestures before the focal scene, they did not propose to use them to resolve Hiroko's and Yu's conflicting solutions. This fact does not completely fit Radford's (2008) theoretical assumption that humans have their own preserved meanings for artifacts. The children appeared to obtain help from the teacher to reconstruct the meaning of fingers as a tool for solving quizzes rather than demonstrating the preservation of its meaning in practice. Although the ability to preserve the meaning of artifacts might be built into human beings by nature, young children may need to be aided in demonstrating such ability in an appropriate context.

However, the fact that the children did not use finger gestures should not be construed negatively. Instead of focusing on the intermediate process of finger gestures, they seemed to focus on input and output. This could be the origin of flexible thinking, also called "proceptual" thinking (Gray & Tall, 1994), which is based on a focus on the relationship between input and output. It is natural and mathematically appropriate for finger gestures to lose their artifactual meaning for children as they master adding two numbers mentally. We agree with Radford's (2020) argument that social rules and mathematical content in classrooms are part of the fabric of children's subjectivities. Our interpretation and analysis corroborate this finding. We draw one possibly important implication: the role of the knowledgeable other, in this case, the teacher, in solving conflicts between learners" idiosyncratic rationalities. The observed children showed their valuable abilities: Yu's ability to generate a new quiz, Hiroko's strategy for addition without counting, and the other children's focus on the input-output relationship. However, these are still potential abilities and are not always performed in appropriate situations. The teacher's transformed assistance according to the children's actions and thinking may potentially aid them when utilizing their abilities. Therefore, we argue that the traditional constructivist focus on learners' idiosyncratic rationality (Confrey, 1991) can be more widely investigated from Radford's theoretical perspective.

In the Introduction, we raised the following two questions: (1) How does the process of subjectification and objectification proceed in a mathematical activity at a Japanese preschool? (2) How does the role that preschoolers' finger gestures play in mathematical learning activities change during the process? As an answer to the first question, two noteworthy stages could be observed in a relatively short-term activity: *extending a practice* and *resolving a conflict with the help of the teacher*. In the first stage, Yu extended his practice in Frame 1, that is, he generated a new quiz with a relatively large number. Since numbers are abstract and invisible, posing and answering quizzes increased the presence of numbers as manipulatable objects for the children. This manipulability of numbers enabled Yu to change his role from a quiz consumer to a quiz poser. It is because numbers were objectified that Yu could change his role, while it is because he subjectified himself as a quiz poser that the range of the numbers presented was further increased in the activity. Knowing objects and becoming a subject thus enhance each other and can be viewed as dual aspects of a single process. As shown in Figs. 7, 8 and 9, Yu could not solve the quiz when another child gave a different answer. Then, the children found it difficult to solve the question due to the individualization of the finger counting. The children did not notice at all that they had done so and neglected some of the processes of using their fingers.

The second stage is the children providing differing answers in Frame 3, as shown in Figs. 7, 8 and 9, in which Yu was not able to solve the quiz when another child gave a different answer from his. The children found it difficult to solve the question due to differences in their unconscious and idiosyncratic ways of omitting finger gestures. As shown in Figs. 11 and 12, the teacher assisted Yu in using his fingers again to solve the quiz correctly. Accordingly, the assistance led to the resolution of the conflict. Because the children did not notice why they obtained different answers by themselves, the teacher's help was needed to resolve this conflict. Here, we would like to emphasize that finger gestures offer a new meaning to children. It began to play multiple roles not only in individually obtaining a correct answer to a quiz, but also in socially confirming the correct answer in the children. Considering the meanings of a finger gesture from a sociocultural perspective reveals a crucial but often overlooked aspect of learning mathematics in preschool from a cognitive perspective. The cognitive stance cannot provide a clear explanation of why children who succeeded in solving calculation questions failed to solve the conflict situation. If the same finger gestures play different social roles, they might have different meanings in social practices from a sociocultural perspective. The fact that the teacher used a finger gesture to resolve the conflict enhanced both objectification of the finger gesture as a social way of confirming the correct answer and subjectification of children as persons who attempt to resolve such a conflict in this way.

As an answer to the second research question, the mathematical roles of finger gestures changed through the intervention by the teacher between the abovementioned stages, from *an individual way of obtaining a correct answer to a quiz* to *a social way of confirming what a correct answer is and how it can be shared in the process*. This observation indicates that to learn mathematical communication, a teacher is essential. While we agree with Abtahi (2017) that the concept of knowledgeable-otherness should be extended to non-human material, such as cultural artifacts from a sociocultural perspective, the concept of humans seems to be of special significance in both preschool and elementary school mathematics learning. That is, non-human material does not seem to have the ability to help children resolve conflicts between themselves. Until children grow up and come to know how they can resolve a conflict between different mathematical opinions, a teacher is needed, not as a more knowledgeable other, but as a more experienced mathematics learner. This view also fits well with Radford's theoretical view of learning and teaching as a single activity in a classroom. Thus, we argue that we succeeded in embodying Radford's theoretical foresight in a Japanese preschool and in illustrating why the theory of objectification is important in mathematics learning.

Conclusion

We analyzed Japanese preschool children's mathematical behaviors from Radford's sociocultural perspective to identify the role of finger gestures in learning mathematics in a sociocultural setting. Through a microgenetic approach, we divided their activities for posing and solving quizzes of addition into six frames. Our analysis provides possible answers to the two research questions. First, following Radford's theory, subjectification and objectification proceeded in the scene of the preschoolers' and their teacher's conversations regarding addition; observing joint labor in a classroom activity offered valuable insights into investigating these processes for the concept formation of numbers and addition. In the activity, the children actively extended their practice of posing and answering quizzes and learned how to resolve a conflict with the support of the teacher. Second, although the role of finger gestures was originally used to obtain correct answers to quizzes, it was reconstructed to solve the conflict between the children through the teacher's mediation. This was insightful to observe among children who had no formal mathematics education. This showed that from a sociocultural perspective, even in an environment where children implicitly learn mathematics, they learn from one another, including from the teacher, and gestures in mathematical communications function well for developing mathematical thinking and skills.

The study examined young children in mathematics spaces, something which is not done often enough in the field of early childhood education. Furthermore, we identified the potential of Radford's framework to shed light on an aspect of preschool children's mathematical activities that tend to be overlooked from a cognitive perspective. Since mathematics learning is essentially a social activity, future research on children's mathematical behaviors should analyze them when they are engaged as a group in a mathematical activity. Also, further work should be considered and implemented in early childhood spaces by explicitly examining children's gestures as it relates to learning in communities.

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Declarations

Conflict of interest There are non-financial interests that are directly orindirectly related to the work submitted for publication.

Ethical Approval Approval was obtained from the preschool. The study was conducted following the guidelinesof the ethics committee of Kanto Gakuin University. The procedures used in thisstudy adhered to the tenets of the Declaration of Helsinki.

Consent to Participate Written informed consent was obtained from the parentsof the children; we were given consent for publishing the photographs of the children's faces in any publicationfor research use.

Consent for Publication The parents of the study participants signed aninformed consent for the publication of their children's data and photographs.

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