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The Teaching of Numbers in Common Preschool Activities: A Delicate Balancing Act

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Abstract
This paper reports on a combined research-development project conducted in collaboration between researchers and preschool teachers in three Swedish preschools. The aim is to investigate how ongoing preschool activities may become the starting point for mathematics teaching in which toddlers are given the opportunity to distinguish necessary aspects of numbers. One challenge in preschool education is the balance between children's previous experiences and interests versus offering them new experiences and challenges towards a learning goal. In the article, empirical examples are used to illustrate how small changes in an activity may open up opportunities for toddlers to discern different critical aspects of numbers without losing the activity's initial intention. Principles for how early numeracy education can be designed to achieve this balancing act are presented and elaborated on.

Keywords Early childhood education · Numbers · Numerical development · Toddler mathematics · Play

Introduction

Today, many studies aim to simultaneously investigate and develop mathematics teaching and learning using approaches involving close collaboration between researchers and teachers (Cai, 2019). Such collaborative studies differ in both direction and size, but have the common aim of developing both theories and new forms of instruction, thus bringing value to both research and practice (Anderson & Shattuck, 2012; Bakker, 2018). Collaborative studies often address specific educational issues, such as the design of new educational materials. These materials can be specific tasks, worksheets, digital tools, or extensive professional development programmes (Bakker, 2018; McKenney & Reeves, 2014). However, in some countries such specifically designed tasks and tools are rare and even seen as controversial in early childhood education. This is for example true in Sweden, which may be due to the Swedish preschool being situated within a social pedagogy tradition where 'care, socialisation and learning together form a coherent whole' (National Agency for Education, 2018, p. 7). The Swedish preschool is offered to children between the ages one and five. As part of the formal education system, it is clearly stated that teaching is to be conducted in the preschool (Education Act 2010: 800). This teaching ought to be founded in exploration, curiosity and children's desire to play. Quantities, order and numbers are examples of mathematical content the preschool should provide each child with conditions to develop understanding of. But, an evaluation by the Swedish Schools Inspectorate (2017) shows that mathematics teaching practices need to be further developed in a majority of Swedish preschools. Many preschool teachers are observed to be uncertain of *how* to teach, which is why situations that could support children's learning are missed. Educational studies in collaboration between researchers and teachers are one way to counteract this by developing activities and artifacts that comply with the specific context and traditions of the preschool while advancing the mathematical content to meet the expected goals for learning opportunities, regardless of the educational stage.

In a longitudinal combined research-development project, we investigated how the youngest preschoolers, the toddlers (1–3-year-olds), develop numerical skills and how this development can be facilitated in preschool practice (the

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project is funded by the Swedish Institute for Educational Research, grant no. 2018-00014). While there is a large body of research within the domain of early numerical development, few studies focus on *how* this development can be facilitated. There are some indications of play-based education being beneficial for preschoolers' mathematics learning (Vogt et al., 2018). Nevertheless, few studies involve toddlers and particularly *how* the teaching of numbers can be made meaningful for them. Thus, an essential part of the project was to develop and study activities that would enhance the toddlers' opportunities to discern numbers in ways that make the numerical content meaningful and useful, by taking the toddlers' perspective.

In this paper, we address the issue of balancing children's experiences, needs, and interests as the starting point for education, while simultaneously making it possible for children to make new discoveries and acquire new knowledge. In the project, a selection of common preschool activities became the starting point for jointly designed numeracy education in which toddlers were given the opportunity to distinguish critical aspects of numbers. In this article, empirical examples from one activity are used to illustrate how small variations in the activity make it possible for the children to discern different aspects of numbers without the initial intention of the activity being lost. Using this activity as an example, principles for how preschool numeracy education can be designed to achieve the balancing act between children's previous experiences, needs, and interests, versus offering them new experiences and challenges towards a learning goal, are presented and elaborated on. More explicitly, we address the following research question:

– What features are of significant importance in preschool activities, in which toddlers are given the opportunity to distinguish critical aspects of numbers?

First in the paper, we conduct a short review of the theoretical foundations of the study, followed by a presentation of the methods and design principles to be elaborated on. Then, empirical examples are used to illustrate features found to be of significant importance for teaching that bring forth opportunities to learn about numbers. This means the focus is not on the learning of individual children but on the learning opportunities the teaching entails. Finally, we conclude our findings in a discussion and present the implications of the study.

Four Essential Aspects of Numbers

Ordinality, cardinality, part-whole relations, and representations have, in a large body of research (for example Baroody & Purpura, 2017; Carpenter et al., 1982; Fuson, 1992), shown to be essential aspects of the complex construct of numbers that children need to discern in order to develop their numerical competence. Based on previous successful interventions with 4–5-year-olds in which these four particular aspects of numbers were focused on (Björklund et al., 2021a, 2021b), it was reasonable to also focus on these four aspects in this study involving toddlers.

Ordinality implies the relation between objects in a sequence, and is directed towards single entities within a set. Every item, situation, or number word has its exclusive position in a sequence and is thus related to the others in the same sequence. Even though most preschool children learn to recite a counting rhyme up to 20 or higher, relating ordered entities to each other involves more advanced knowledge of ordinality (Fuson, 1988). *Cardinality* is instead directed towards units constituting a composite set, whereby the cardinal word principle means that the last uttered number word in a counting act includes all the counted items (Gelman & Gallistel, 1978). Learning to understand arithmetical principles is based on the child's ability to handle the *part-whole relations* of numbers (Piaget, 1952; Starkey & Gelman, 1982). An understanding of the part-whole relations of numbers allows children to develop ideas about numbers as units to operate on. These ideas are essential for the later understanding of mathematical relationships such as decomposition, commutativity, and the compensation principle (Venkat et al., 2019). Finally, *representations* are important, as numbers are abstract and thus have to be represented in some way in all communicative situations (Goldin & Shteingold, 2001). According to Lesh (1981), five modes of representation are important in early mathematics education: real-world situations, pictures (pictographic and iconic), verbal symbols, written symbols (formal and informal), and manipulatives (general and mathematically specific). According to Lesh, the learning and understanding of mathematics, including that of numbers, is reflected in the ability to make connections within and between these modes of representation. Thus, by using different modes of representation and by connecting between and within them, children can discern new aspects of numbers. As mentioned, there are few studies on how the teaching of numbers can be made meaningful for toddlers. There is a knowledge gap on *how* to teach toddlers so that they may distinguish critical aspects of numbers. The aim of our study is to address this absence and to deepen the knowledge of how the teaching of necessary aspects of numbers can be done in a way that is meaningful for toddlers.

Theoretical Foundation for Teaching About Numbers

The theoretical foundation of the study is the variation theory of learning (VT) (Marton, 2015). This theory directs attention to how a person experiences the meaning of a

phenomenon. In every situation several aspects of a phenomenon can be discerned, and those that are discerned are decisive for how the phenomenon at hand is experienced. Thus, in a situation in which numbers are included in some way, the aspects of numbers that are discerned by a child are decisive for how the child understands and handles a certain task. For instance, if ordinality is the primary aspect foregrounded in the child's awareness, the child may recite the counting rhyme in correct order, but if cardinality is not discerned, numbers (or the counting words in the counting rhyme) are not experienced as means to find out how many objects there are in a set. According to VT, learning occurs when learners discern new and necessary aspects of a phenomenon, which in this study means children discerning ordinality, cardinality, or part-whole relations through different modes of representation. Further, children need to experience several necessary aspects of a phenomenon simultaneously to be able to understand and handle it in a profitable way. For example, a child needs to experience the meaning of numbers as cardinal values as well as their ordinality in order to add units in a counting task, thus coordinating the cardinal and ordinal properties of numbers (Björklund et al., 2021b) VT offers strategies for how to design tasks to direct children's attention to the aspects intended for them to discern. According to VT, an aspect of a phenomenon is most likely to be discerned if this aspect is contrasted against other aspects that are kept invariant. This means, that the cardinality of numbers is more likely discerned when sets that are of similar items, but different in number, are made available for comparisons. If too many features vary within and between the sets, it is assumed to be more difficult to discern irrelevant from relevant features, that is, colours and shapes may attract attention rather than numerical aspects. Once the numerical feature is recognized, the child can further benefit from generalization, such as extending the awareness of cardinality of a set to differently composed sets. Through using patterns of variation and invariance (contrasts and generalization as described above) when designing teaching acts, one can increase children's opportunities to discern critical aspects of numbers. Consequently, activities may be designed and conducted based on VT principles, allowing children to discern necessary aspects of numbers. The educational challenge, though, is to implement such principles in activities that are meaningful to the toddlers and in which the mathematical content is meaningful for the progression of the activity in question.

The principles for how learning is made possible in a teaching situation as outlined above, can also be used as an analytical lens; this has been done in studies of early mathematics teaching (e.g., Alkhede & Holmqvist, 2020; Björklund et al., 2021a, 2021b; Ekdahl, 2021). When an analysis is guided by the theory, it results in thorough observations of what aspects are made discernible through offered patterns of variation. Children's responses are then considered to reflect their way of experiencing the objects of learning and, in line with VT, the aspects that are discerned (or not yet discerned), reveal what the teaching affords the child to learn.

Method

During three semesters, two researchers and three preschool teachers from three different preschools in two Swedish municipalities collaborated on this study. The selection of the three preschools was based on the teachers' interest in participating. The three teachers have a university (bachelor) level preschool teacher exam and have worked in preschool for several years, and have been recognised by their headmasters as excellent teachers. All toddlers at these three preschools were invited to participate in the study. The guardians agreed to the participation of their child. At the start of the study, the 27 participating children were between 12 and 27 months old. Like Swedish preschools in general, the children had different socio-economic backgrounds and there were both monolingual and multilingual children participating in the study. The project, including the development of tools and methods, was approved by the Swedish Ethical Review Authority (Dnr: 2019-01037).

One way to distinguish between the roles of teachers and researchers in collaborative research is to look at their specific tasks and their joint work (Cai et al., 2018). The roles of the researchers in the project presented here primarily entailed providing an outsider's perspective, synthesizing relevant research on which to base the intervention, and analysing the empirical material. The preschool teachers, on the other hand, stood for 'an insider's nuanced and reality-based perspective' (Cai et al., 2018, p. 518). Their familiarity with the children and the educational context enabled them to anticipate the children's experiences, needs, and interests. Based on these different perspectives, the researchers and teachers jointly designed and evaluated the activities within the intervention, while the teachers were the ones who implemented the activities in the preschools. Other studies (Breive et al., 2018) have shown limitations when implementing education developed solely by researchers, emphasizing the risk of the researchers becoming producers and the teachers implementers, which may result in the connection between these two becoming weak and fragmented. In this study, the researchers and teachers met for two hours every other week during three semesters to evaluate and discuss the activities and the children's responses (in total approximately 30 meetings). These frequent meetings were an important part of the collaboration and the validity of the study.

Designing Activities Within the Study

To be able to take the children's experiences, needs, and interests as the starting point for the activities, observations were conducted before the intervention was initiated. The observations were conducted by the researchers and included free play indoors and outdoors as well as joint activities planned by the preschool teachers. Thus, the observations covered the kind of activities that the children were involved in during a preschool day. These observations provided information on the activities the children engaged in spontaneously as well as on how they engaged in activities planned by the teachers. The observations were video-documented and jointly reviewed by researchers and preschool teachers. By categorising the activities and comparing their frequency, the activities in which the children at all three preschools showed high spontaneous interest were selected as a starting point for further development within the project. The chosen activities were from categories of different natures to correspond to the variety of activities in the observations, for example book reading and memory games, as well as dance and motor play. The selected activities were jointly developed and implemented with the intention to make it possible for the children to discern critical aspects of numbers (ordinality, cardinality, and part-whole relations, as well as connections between and within different modes of representation) in meaningful ways.

According to Cobb et al. (2003), design studies ideally result in a greater understanding of what they call 'learning ecology'. A learning ecology implies a 'complex, interacting system involving multiple elements of different types and levels' (p. 9). In design studies, the intention is to design and anticipate how different activities will support learning in such learning ecologies. In this study, the learning ecology is Swedish preschool mathematics education, with a special focus on the balance between children's previous experiences and interests versus offering them new experiences and challenges towards a learning goal. Based on research on numbers and the variation theory presented above, as well as on the learning ecology of Swedish preschools, the activities within this study were designed in accordance with the following three principles:

- *The principle of context* The context of the activity ought to be based on children's experiences, needs, and interests; being familiar so that they can participate, relate to, and reason about the content based on their previous social and cultural experiences. At the same time, based on VT, the interference of irrelevant elements ought to be reduced.
- *The principle of numbers* The activity ought to make it possible for the children to discern essential aspects of numbers (representations, cardinality, ordinality, and part-whole relations). These four aspects can be more or less integrated into the activities, but based on VT, different aspects can be foregrounded through afforded patterns of variation and invariance.
- *The principle of differentiation* The activities ought to allow the children to express different ways of understanding, and allow a variety of experiences and expressions both between the children and for the same child over a prolonged period of time.

The observations revealed that one activity the children enjoyed was rhymes and songs, preferably with materials being used in different ways. One of these songs chosen for further elaboration within the study, the balancing elephant song, will be used as example in this paper:

> One elephant balanced//on a tiny little spiderweb//He thought that was so interesting//so he went and fetched another elephant.
> Two elephants balanced//on a tiny little spiderweb// They thought it was so interesting//so they went and fetched another elephant.
> *Continue as long as you can, for example to nine, and then…*
> Ten elephants balanced//on a tiny little spiderweb// They thought it was so interesting//but now there were no more elephants.

The intention was not for the toddlers to learn the song by heart, instead the song served as a familiar context within which different aspects of numbers could be explored in different ways with several modes of representations. For three semesters, researchers and teachers met two hours every second week to discuss and evaluate the extent to which their expectations for the designed activity had been met. Based on these evaluations, the activity was redesigned in terms of the numerical content as well as their responsiveness to children's experiences, needs, and interests.

Analysis

The analysis for this paper is based on 121 video recordings (7 h and 50 min, average length 5 min and 7 s) of singing and playing the balancing elephant song presented above. These recordings were conducted during a period of one year. Based on these recordings, the presence of ordinality, cardinality, part-whole relations, and representations was documented in an Excel file. These documentations were not exclusive, but overlapping; thus, the same episode in a recording could result in the presence of both cardinality (for example, a child discerning the number of elephants) and representation (for example, a child showing three fingers to represent the number of elephants). Next, the documented instances were analysed based on each design principle: Did

Fig. 1 A balance beam used as the spiderweb to walk on

the context of the activity connect to the children's experiences, needs, and interests? If so, how? Were irrelevant elements reduced? If so, how? Did the activity make it possible for the children to discern necessary aspects of numbers by offering contrasts and generalisation (see VT principles described above)? If so, how? Did the activity allow the children to express different ways of understanding? If so, how? Patterns within each design principle were explored; for example, what made an activity connect to children's experiences, needs, and interests in one design and not in another? Based on this, features of significant importance within each design principle gradually emerged. These are the focus of the results to follow.

Results: The Example of the Balancing Elephants

Five examples from the activity of the balancing elephant song will be used to illustrate features of significant importance when preschool activities become the starting point for mathematics teaching, in which toddlers are given the opportunity to distinguish critical aspects of numbers. Then follows a meta-analysis focused on the significant features found, significant features that are to be understood as refined details of the three design principles presented above.

Example 1: Elephants Walking on a Balance Beam

In this example, one teacher and four children are involved. The children are acting as elephants and a low balance beam made of wood is used as the spiderweb (Fig. 1).

The teacher sings the first verse of the song while holding one child by the hand as the child walks on the balance beam. When the first verse ends, the teacher asks the child to fetch another child:

Teacher Selma, now you get Nelly.

Selma Yes!

Teacher How many of you are going to balance then? How many of you are now balancing?

The teacher takes each child's hand and helps them to stand beside each other on the balance beam.

Selma Two. *At the same time as Selma says 'two' she holds forward two fingers. …*

Teacher Two. One, two. *When the teacher says 'one' she points at Selma, and when she says 'two' she points at Nelly. Then she shows two fingers. After this, she takes one child's hand in each of her hands and starts to sing about two elephants.*

In this example, the children act as the elephants. They need to know how many (principle of numbers) elephants there are, as they need to know what to sing (principle of context). The teacher and Selma use different modes of representation: finger patterns and verbal counting words (principle of differentiation). Furthermore, the teacher contrasts two aspects of numbers: ordinality when counting up on the counting sequence 'one, two' while raising one finger for each said word, and cardinality when showing the pattern of two in a simultaneous gesture. Thus, the principles of numbers and differentiation are embedded in the activity; but still, the initial aim, to sing and play the song (principle of context), is foregrounded.

Example 2: Determine the Number of Birds to Sing About

In this example, one teacher and two children are involved. The children sit at a table, facing each other. They have decided to sing about birds instead of elephants. The teacher takes out a box containing pictures of birds:

Teacher Nelly, how many birds are we going to sing about?

Nelly Two. No, three. *At the same time as Nelly says 'two' and 'three' she shows her fingers but it is not visible on the video documentation how many fingers she shows.*

Teacher Three birds. *Shows three fingers.* Can you take three birds? *Nelly takes one bird at a time from the box and puts them on the table. When she*

has taken three birds from the box, she stops and leans back in her chair.

Teacher Is it three now?

Nelly Yes.

Teacher How nice. *Picks up the three birds from the table and gives one to Nelly.* First, we have one. Are you going to start balancing with this?

In this example, the children are to decide both what animals to sing about and how many. Nelly creates a set of three (principle of numbers). The teacher and Nelly use different modes of representation (principle of differentiation): number words and finger patterns. The teacher, furthermore, makes explicit that it is the birds that are to be three together, not only by repeating the counting word, but by connecting to the set they are to sing about. This is a kind of generalization where numbers are put into context. Thus, the principles of numbers and differentiation are included in the activity; but still, the initial aim to sing and play the song (principle of context), is foregrounded.

Example 3: Showing One Elephant with Your Fingers

In this example, one teacher and three children are involved. The children sit at a table, facing each other. The teacher has a box containing five plastic elephants:

Teacher We're going to sing about the elephants. *Puts one elephant on the table.* How many elephants is this?

Holger One. *Takes the elephant.*

Teacher One elephant. *Holds up one finger.* Can you show one elephant with your fingers? How many fingers is one?

One child plays with the elephant. One child starts to sing another song about spiders, with movements. The third child starts to do the movements to the spider song. The teacher turns to the child playing with the elephant.

Teacher Can you show one with your fingers?

Holger One. *Holds up the elephant. Takes it down and points at the elephant's trunk.*

Teacher One trunk. *The children start to talk about the elephant's legs. The teacher asks them to count the legs. One child counts three legs, another five legs.*

Teacher Five feet. *Starts to sing the elephant song.*

In this example, the teacher attempts to generalize how different representations can be used for expressing the same numerical meaning. She does this by showing the number of elephants (principle of numbers) with her fingers (principle of differentiation). However, this is apparently not experienced by the children as a necessary aspect of the activity (principle of context). Even though the question of showing 'one with your fingers' is connected to modes of representation (principle of differentiation), from the children's perspective it is not connected to the question of determining the number of elephants (principle of numbers) or to the performance of the song (principle of context). Thus, in the example the principles of numbers and differentiation are separated from the principle of the context; thus, for the children, showing one with one's fingers serves no purpose. This lack of connection to context is visible because none of the children respond to the teacher's question.

Example 4: Three Children Acting as Elephants

In this example, one teacher and five children are to sing and play the elephant song. The children are to act as elephants:

Teacher Now we're going to sing the song about the elephants. Does anyone want to be the first elephant? *All children want to be the first elephant. The teacher tells Max that he can be the first elephant.*

Teacher How many elephants do we have? *Max shows one raised finger.*

Teacher One. *Shows one raised finger.*

Together they sing the song while Max, as the elephant, walks between the other children, who are sitting on the floor. Then the teacher points at Assra.

Teacher *Picks up Assra so she is standing next to Max.* How many elephants are there? *Max shows two raised fingers.*

Teacher One, two. *Unfolds one finger for each number word.* One, two. *Points at one child for each*

number word. There are two elephants. *Shows two raised fingers.*

Together they sing the song while Max and Assra act as the elephants, walking between the other children, who are sitting on the floor.

Teacher Melanie. *Melanie stands up.* Assra, you can stand over there with the others so we can count. *Assra stands next to Max and Melanie.* Let's see if we can count. How many elephants are there now? *Max shows three raised fingers.*

Teacher How many is this then? *The teacher shows three raised fingers, not the same three fingers as Max. Max points at himself, then at Assra and then at Melanie.*

Teacher One, two, three. *Points at one child for each number word.* There are three elephants. *Shows three raised fingers. Starts to sing about three elephants.*

In this example, knowing the number of elephants (principle of numbers) is a necessary aspect of the activity. Both the cardinal and ordinal properties of numbers are coordinated when the children check the number of elephants (principle of numbers). Max uses fingers, and the teacher connects this mode of representation to both verbal and gestural representation, with different finger patterns (principle of differentiation). Thus, the principles of numbers and differentiation are included in the example; but still, the initial aim of the activity is foregrounded (principle of context).

Example 5: Bears Walking on the Spiderweb

In this example, one teacher and one child sing and play the elephant song with small toy bears (as an alternative to elephants). First, the teacher shows the child the line on the table—the spiderweb—where the bears are to balance, and then she takes out a box containing several bears:

Teacher Can you take three bears? *Holding up three fingers.* Three. (Fig. 2a)

The child puts one bear on the line on the table.

Teacher How many have you taken now?

Sander One.

Teacher One. And you were to take three.

The child nods and puts one more bear on the line on the table.

Teacher How many do you have now?

Sander *The child takes another bear from the box while the teacher asks the question, and says* 'three' while *pointing at the third bear. (Fig. 2b)*

Teacher Are there three now?

Sander Yes.

Teacher *Puts the box away.* Then we'll check.

Sander One—two—three. *Moves one bear slightly to the left on the line at the same time as he says each number word.*

In this episode, the child composes a set of three bears. The cardinal and ordinal properties of numbers are possible to coordinate when he, with help from the teacher, is to check the number of bears (principle of numbers). The child uses verbal representations when he counts the bears, and composes the set of three with manipulatives. The teacher adds one mode of representation to the activity as she shows three fingers as a finger pattern set (principle of differentiation). This brings to the fore a possible contrast between the

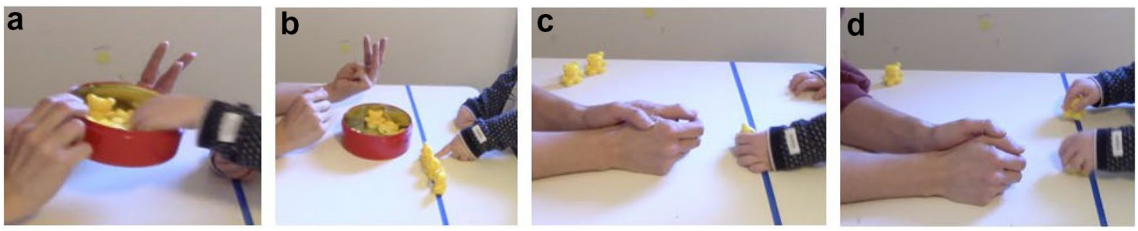

Fig. 2 **a**–**d** Composing a set of three bears and exploring the part-whole relations of three

teacher labelling a whole set of three with a single counting word (thus with cardinal meaning) and the counting sequence said by Sander 'one, two, three' (foregrounding the ordinal meaning of the counting words).

Then, the singing starts:

Teacher Now we take these two away. Now they are to stand here and watch their friend (Fig. 2c). Now one friend is to walk. *Points at the bear the boy is holding in his hand.*

The teacher (the whole song) and the child (some words) sing the song as the child walks one bear on the line.

Teacher *Moves one of the bystanding bears towards the child.* Are you going to walk this one? *The child takes the bear.* Yes. How many do you have now?

Sander Two! *Holds one bear in each hand.*

Teacher Two. Now one bear will stand by and watch. (Fig. 2d)

Sander Yes.

The teacher (the whole song) and the child (some words) sing the song as the child walks two bears, one in each hand, on the line.

In this episode, the teacher implements the idea of a part-whole relation of the set of three bears in the activity, with the number of bears kept invariant while the parts are varied between one and two (principle of numbers). This implementation is part of the activity: how many bears are to walk on the spiderweb and how many are to wait (principle of context). The teacher also introduces a generalization of the meaning of numbers in that she talks about "one" referring not to one specific bear but to any of the bears that are appearing alone. Different modes of representation are involved, as the teacher expresses the parts with number words at the same time as the parts are composed by the manipulatives (principle of differentiation). Through this part of the activity, the child may discern ideas about numbers as units to operate on.

What Features are of Significant Importance?

Together, these five examples illustrate features of significant importance within each design principle (principle of context, numbers, and differentiation). One feature of significant importance found within *the principle of context* is the delicate issue of including numbers as a possible object for learning in the activity, without losing the activity's initial context and purpose. Even when the teachers intend to make it possible for the children to discern essential aspects of numbers (principle of numbers), the aim for the toddlers in the activity is to sing and play the balancing elephant song. Together, the examples illustrate the difference between when numbers and the representations of numbers are a necessary feature of the activity and when they are not. In Examples 1, 2, 4, and 5, using numbers are necessary in the activity, as the children need to know or decide 'how many' to sing about. This differs from Example 3, in which the children do use numbers and representations, but not as a necessary or even integrated part of their singing the elephant song.

The second principle induces that the activity makes it possible for children to discern necessary aspects of numbers (representations, cardinality, ordinality, and part-whole relations). The examples illustrate how small variations in the activity facilitate the children's discernment of different aspects of numbers without the initial intention of the activity being lost. Different ways of acting out the song allow the children to discern ordinality, cardinality, and part-whole relations. Critical for discerning cardinality is the contrast between numbers, for example a single elephant in contrast to two elephants, and the set of three elephants walking on the spiderweb. The counting of the elephants is directed towards single entities within the set, at the same time as the units constitute a composite set, labelled with a counting word. Thus, the cardinal and ordinal properties of numbers are coordinated in the song, the narrative, and the teacher's acts, which all may allow the children to discern the meaning of numbers as cardinal values, as well as their ordinality in the order and relation between counting words (first comes 'one', then 'two', followed by 'three' in an additive structure).

Thirdly, the examples illustrate how the activity allows the children to express different ways of understanding. A variety of experiences and expressions is visible both between children and for the same child at different times (for instance, Nelly in Examples 1 and 2). One differentiation is how aspects of number come to the fore in the activity: ordinality, cardinality, and/or part-whole relations. Further, there is differentiation within each of these aspects. For example, regarding cardinality, the teacher can be the one who creates the set (Examples 1, 3, 4) or it can be the child/ren who do it (Examples 2, 5). When the children are to create the set, they (Example 2) or the teacher (Example 5) can decide 'how many' the set should include. Yet another feature of differentiation is the representation to be used and the difference between when the children themselves decide

this (Examples 1, 3, 4, 5) and when they are asked to use a specific representation (Example 2). Also, the elephants are represented in different ways in the play, by the children themselves (Examples 1, 4), by plastic elephants (Example 3), or by another manipulative (Example 2, 5).

Discussion and Implications

In this study, the activities were designed based on three principles: context, numbers, and differentiation. The results indicate that the main educational challenge is to implement these three principles in activities that are meaningful to toddlers and in which the numerical content *becomes* meaningful for the progression of the activity in question. This is evident in the observations where children *do not* respond to the numerical content as intended by the teacher. The analysis shows that *the three principles all have to be addressed in the same activity as they presuppose and support each other*, at the same time. Combining these three makes it possible to balance between children's previous experiences and interests and offering them new experiences and challenges towards a learning goal.

As mentioned, the aim of this study was to deepen the knowledge of how the teaching of necessary aspects of numbers can be done in a way that is meaningful for toddlers. Based on the results, it is suggested that the three principles can be used as a framework for teaching numeracy with toddlers. In the paper, features of significant importance within each of these design principles have been explored. Teaching mathematics in preschool does not solely concern children learning *how to use* numbers, but also *how numbers can be useful* for the child's interaction with others and for their making sense of the surrounding world. For example, a mathematical idea such as using numbers to determine the size of a set (principle of numbers), are foundational for further numerical development, and the generality of the number system (which is possible to apply in any context). Still, such ideas are not learnt in isolation; they are learnt when needing to make use of them, with the support of others (principle of context). Here, it is essential that teachers point out necessary aspects of numbers through modes of representation and carefully selected patterns of variation in a context in which, from the perspective of the child, it is meaningful to attend to numbers (principle of context, numbers and differentiation). This conclusion is supported by research showing that numerical meaning is not self-evident even when the mathematics is present to explore (Björklund & Palmér, 2020).

Finally, some limitations of this study. As the study is conducted in a cultural context with a play-oriented teaching philosophy it is not self-evident that these results are transferable to other educational contexts. Thus, the suggestion that the three principles can be used as a framework for teaching numeracy with toddlers is only valid for preschools within a play-oriented teaching philosophy. This, as the delicate issue of including numbers as a possible object for learning without losing the activity's initial context and purpose may not be an issue at all within another educational context. This may in turn influence both the principle of numbers and differentiation. This is why it could be interesting to replicate this study in other cultures where learning-oriented approaches are more favored in early childhood education. As the three preschool teachers participated in the study based on their own interest, someone might argue that the results are not generalizable even within the play-orientated context. However, these teachers can be understood as a 'best case scenario' which does not influence the appearance of the features of significant importance as such. However, if these features of significant importance can be realized by other teachers is yet another study to be conducted.

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