



# Social Costs of Methane and Carbon Dioxide in a Tipping Climate

Anthony Wiskich<sup>1</sup>

Accepted: 7 March 2024  
© The Author(s) 2024

## Abstract

Social costs for methane and carbon dioxide emissions, from the risk of climate tipping events and deterministic damages, are derived in an analytically tractable model. In the core model: social costs from tipping risks rise with income, just as they do for deterministic damages, and depend on only a few parameters. Consequently, methane's weight (its social cost relative to carbon dioxide) is constant and independent of temperature projections. But other damage and tipping probability formulations assumed in the literature imply methane's weight varies over time and with temperature projections. (JEL H23, O44, Q40, Q54, Q56, Q58).

**Keywords** Climate change · Tipping points · Optimal policy · Social costs · Global warming potential

Climate change includes the risks of irreversible events, referred to as tipping points. These events can have a material impact on optimal policy. One potential consequence of a risk of tipping is a change in the relative, as well as absolute, social costs of different greenhouse gases because the timing of effects of these gases differ. While carbon dioxide is long-lived, methane decays relatively quickly. Under current Intergovernmental Panel on Climate Change policy, 100-year Global Warming Potentials (GWP100) are used to aggregate methane and carbon dioxide. This paper investigates how tipping risks may affect the social cost of carbon dioxide and the methane weight (its social cost relative to carbon dioxide), compared to the weight determined under GWP100.

The social costs of greenhouse gases equal the sum of discounted economic losses from deterministic damages and stochastic tipping events, which I assume are linked to atmospheric temperature. As atmospheric carbon dioxide and methane lifetimes differ, one may expect their social costs to vary over time in absolute and relative terms and depend upon temperature projections. However, this is not the case in my benchmark core model building on Golosov et al. (2014): I assume the risk of tipping rises linearly with temperature

---

✉ Anthony Wiskich  
u6311308@anu.edu.au

<sup>1</sup> Centre for Applied Macroeconomic Analysis (CAMA), Crawford School of Public Policy, ANU College of Asia & the Pacific, Australian National University, J.G Crawford Building No. 132, Canberra, ACT 2601, Australia

and will persist long into the future, and a fixed proportional economic impact from a tipping event after some delay.<sup>1</sup> Social costs consist of two components that rise with income: one from deterministic damages and the other from the risk of tipping. This latter component depends on the discount rate, long-run damages from tipping, the delay in onset and ramp-up of impacts, and how much the tipping probability rises with each degree of warming. So the methane weight is constant, and an increase in deterministic damages can lead to the same social cost (as a ratio of income) as the inclusion of a tipping risk.

While this core model provides a useful benchmark, the underlying assumptions do not cater for our understanding of different potential tipping events; Sect. 3 discusses other assumptions made in the literature. My objective is not to present estimates based on distributions of parameters and the latest knowledge of potential tipping events (Cai & Lontzek 2019; Instead, I derive and discuss analytical equations for the sensitivities and illustrate their effect in Sect. 4 using temperature projections from Representative Concentration Pathways (RCP) scenarios 2.6 and 4.5 that, in my view, bound likely temperature outcomes. The first sensitivity assumes the tipping probability rises quadratically with temperature rather than linearly. Social costs increase for the higher temperature scenario RCP 4.5 relative to RCP2.6. The methane weight is higher than the core model for RCP2.6, as temperature peaks quickly, and lower than core for RCP4.5. Identical social cost-to-income ratios could be obtained using deterministic damages that are quadratic in temperature.

The second sensitivity restricts the number of possible tipping events to one: social costs are lower and an interesting “inevitability” effect emerges, where higher projected temperature outcomes reduce costs, similar to the dead-anyway effect for valuing a statistical life (Pratt & Zeckhauser 1996). Social costs become zero after a tipping event and cannot be replicated using deterministic damages. In the third sensitivity, exponential-linear (in temperature) damages from tipping events introduce a differential welfare impact, proportional to the difference in social costs before and after an event (Lemoine & Traeger 2014). This effect raises social costs and gives more weight to the benefits of reduced temperature in the distant future as the cumulative probability of tipping rises, lowering the methane weight. The fourth sensitivity assumes a tipping risk only exists if temperature rises to new levels (threshold formulation): social costs depend on the timing of peak temperature (the maximum projected temperature level) and the methane weight rises markedly prior to the peak. The fifth sensitivity discusses the increase in social costs when risk aversion is in line with the literature.

## 1 Previous Literature

Engström and Gars (2016) use a similar model approach to consider different tipping impacts, but do not consider methane and focus on extraction rates and the green paradox, rather than social costs. Nævdal (2006) considers the optimal regulation of methane and carbon dioxide under a threshold tipping risk and finds a temporary boost in the ratio of methane to carbon dioxide stock above the steady state, consistent with an increasing methane weight in a decentralised model. Table 1 in Appendix A lists previous literature that consider tipping points and their approaches – see also Cai (2021) for a recent review.

<sup>1</sup> A collapse of major ice sheets leading to severe sea-level rise is an example of a shock that would have long-term and direct economic impacts.

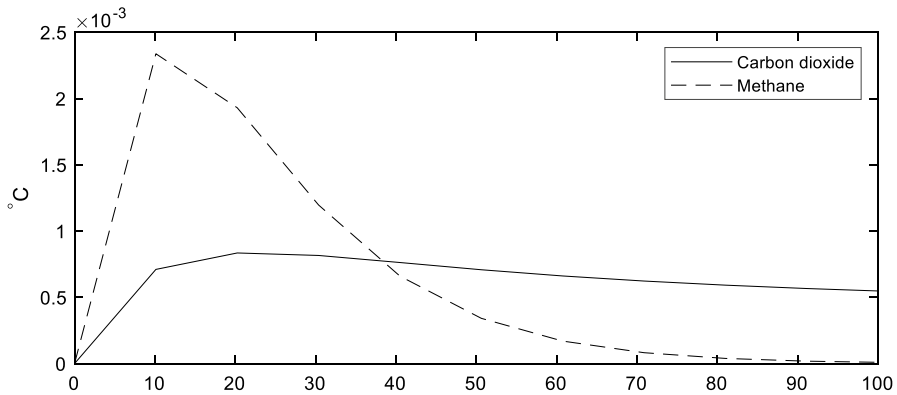
**Table 1** Climate tipping approaches adopted in the literature

Literature	Single or multiple tipping events		Tipping probability formulation		Impacts of tipping	
	One only	Multiple	Threshold	Other	Fixed	Temperature-dependent or climate response
Cropper (1976)	✓		✓		✓	
Clarke and Reed (1994)	✓			✓	✓	
Tsur and Zemel (1998)	✓	✓	✓	✓	✓	
Keller et al. (2004)	✓		✓			✓
Polasky et al. (2011)	✓			✓	✓	✓
Lemoine and Traeger (2014)	✓		✓			✓
van der Ploeg (2014)	✓			✓		✓
Cai et al. (2015)	✓			✓	✓	
Lontzek et al. (2015)	✓			✓	✓	
Cai et al. (2016)		✓		✓	✓	✓
Diaz and Keller (2016)	✓			✓	✓	✓
Engström and Gars (2016)	✓		✓		✓	✓
Lemoine and Traeger (2016)		✓	✓			✓
van der Ploeg and de Zeeuw (2016)	✓					
Gerlagh and Liski (2018)	✓			✓	✓	
van der Ploeg and de Zeeuw (2018)	✓			✓	✓	
Cai and Lontzek (2019)		✓		✓	✓	
W. Nordhaus (2019)	✓		Deterministic			✓
van der Ploeg and de Zeeuw (2019)	✓			✓	✓	✓
Dietz et al. (2020b)		✓		✓		✓
Taconet et al. (2021)	✓	✓	✓			✓

Under deterministic damages, Marten and Newbold (2012) find that the methane weight rises by up to 50% by 2050, partly due to their climate model, where the marginal forcing of methane decreases slower than carbon with the increasing atmospheric stock. Azar et al. (2023) also find an increasing weight for methane from deterministic damages that are quadratic in temperature if temperatures fall after 2100, and review estimates of the social costs of methane from deterministic damages in the literature.

Another stream of literature investigating different greenhouse gases imposes a maximum temperature and uses a cost-minimisation approach, suggesting a very low weight of methane today, which rises over time.<sup>2</sup> This paper does not support such a policy, but the threshold formulation induces a similar rise in the methane weight before peak temperature. The framework outlined in this paper can consider other actions with different temporal characteristics, including geoengineering (Bickel & Agrawal 2013; Goes et al. 2011;

<sup>2</sup> Cost-minimisation (also called cost-effectiveness) references include Manne and Richels (2001), O'Neill (2003), Aaheim et al. (2006), and Johansson, Persson, and Azar (2006). A growing methane weight as a target stock of emissions is approached was perhaps first illustrated by Michaelis (1992).



**Fig. 1** Temperature impact years after a pulse (1 GtCO<sub>2</sub>e) emission

Heutel et al. 2018) and leakage rates and risks from carbon capture and sequestration (van der Zwaan & Gerlagh 2009).

## 2 Core Model and Social Costs

The core model uses five key assumptions that lead to analytical tractability: (i) logarithmic utility; (ii) full one-period depreciation of capital; (iii) an exponential-linear deterministic impact of historical emissions on output; (iv) Cobb–Douglas production; and (v) a risk of tipping events linear in temperature with fixed proportional impacts. Golosov et al. (2014) use assumptions (i) to (iv) and find a constant optimal tax-to-income ratio for carbon dioxide, independent of economic growth and climate outcomes. This result occurs because the assumptions imply a constant savings rate, so consumption is proportional to output, and damages are exponential-linear, so emissions lead to a linear reduction in log output and thus welfare. Barrage (2014) and Rezai and van der Ploeg (2015) find the results are reasonably robust to variation in these assumptions. Golosov et al. (2014) satisfy (iii) by assuming that atmospheric carbon concentration is a linear function of historical emissions and an exponential impact of carbon concentration on output. Instead, I assume that temperature is a linear function of historical emissions, which can replicate more complex climate-economy models well, and an exponential-linear impact of temperature on output. This latter assumption leads to an approximately linear relationship between global damages and temperature for the level of damages considered, consistent with Burke et al. (2015).<sup>3</sup>

A global representative household maximises the following in discrete time, for consumption  $C_t$  and discount factor  $\beta$ :

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \text{ where } U(C_t) := \log(C_t). \quad (1)$$

<sup>3</sup> For damages up to around 10% of output, an exponential function is approximately linear. Burke, Hsiang, & Miguel 2015 find non-linear local responses to temperature but approximately linear losses at a global level.

Atmospheric temperature above pre-industrial is a linear function of historical non-interacting carbon dioxide and methane emissions  $E_{c,t}$  and  $E_{m,t}$  in units of GtCO<sub>2</sub>e:

$$T_t = \sum_{g \in \{c,m\}} \sum_{i=-\infty}^t \psi_{g,t-i} E_{g,i} \text{ where } \psi_{g,t-i} := \frac{\partial T_t}{\partial E_{g,i}}. \tag{2}$$

Figure 1 shows temperature responses to pulse emissions under GWP100,  $\psi_{g,t}$ .<sup>4</sup> These impulse functions are central to this paper and highlight the sharp temperature responses to methane relative to the carbon dioxide pulse. Further details are in Appendix C.

Deterministic damages are exponential-linear in temperature with parameter  $\gamma > 0$ , and the function  $f_t$  represents stochastic damages from tipping. For capital  $K_t$  and a function  $F$  of emissions and a vector of other inputs  $X_t$ , such as labour, output  $Y_t$  is given by:

$$Y_t = e^{-(\gamma T_t + f_t)} K_t^\kappa F(X_t, E_{c,t}, E_{m,t}) \text{ with parameter } 0 < \kappa < 1. \tag{3}$$

The emission variables  $E_{g,t}$ ,  $g \in \{c, m\}$ , correspond to emission-intensive activities, in units of carbon dioxide equivalent, which could themselves be functions of other factors and emissions without affecting the results. However, to simplify the treatment of uncertainty, I ensure that temperature outcomes (hence emissions) are independent of whether tipping events have occurred, so emissions-intensive activities cannot be functions of capital or final output. Thus, while a tipping event will have a lasting effect on output and capital stocks, it does not affect emissions.<sup>5</sup> Capital depreciates completely after one period, so the feasibility constraint in the final goods sector is

$$C_t + K_{t+1} = Y_t. \tag{4}$$

Tipping occurs in each period with probability  $p_t$ . An event variable  $I_t$  is zero if tipping does not occur in period  $t$ , and  $\delta$  if tipping occurs. Multiple tipping events are possible, but no more than one event in a period, and the tipping probability is independent of whether events have already occurred, making things easier in a discrete-time framework.<sup>6</sup> Following a tipping event, there is a delay of  $d$  periods until the onset of impacts and awareness that an event has been triggered. Impacts then ramp up linearly over  $r$  periods so that the full impact occurs after  $d + r$  periods. The function  $f_t$  is a function of temperature, previous tipping events, the safe temperature below which there is no risk of tipping  $T_{min}$ , and parameter  $\mu$  as follows:

$$f_t = \sum_{i=-\infty}^t R_{t-i+1} I_{i-d} \text{ and } p_t = \mu \tilde{T}_t \text{ where } \tilde{T}_t := \max(0, T_t - T_{min}) \text{ and } R_i := \frac{\min(r+1, i)}{r+1}. \tag{5}$$

<sup>4</sup> The GWP of a gas is the time-integrated radiative forcing from a pulse emission, relative to an equal mass of carbon dioxide, and thus resulting weights depend on the choice of time horizon. For example, methane has a 100-year GWP of 28 and a 20-year GWP of 84 (IPCC 2014). The 100-year GWP was adopted by the United Nations Framework Convention on Climate Change and its Kyoto Protocol and is now used widely as the default metric. The clearest recommendation for 100 years is that a significant fraction of carbon dioxide is removed from the atmosphere over this time scale (Fuglestedt et al. 2003), and this period also roughly corresponds to the anticipated maximum change in temperature (WMO 1992).

<sup>5</sup> An alternative assumption is that tipping impacts directly lowers utility rather than production, as discussed in Gerlagh and Liski (2018).

<sup>6</sup> Multiple potential tipping points mean that the expected number of tipping events increases without bound as temperature rises, and there is no updating the probability function if a tipping event occurs.

## 2.1 Social Costs

The social cost-to-income ratio consists of a constant component due to deterministic damages  $\widehat{D}_g$ , and a variable component due to the stochastic risk of tipping  $\widehat{S}_{g,t}$ .

Lemma 1: Given (1) to (4), the social cost-to-income ratio for emission  $g$  is

$$\widehat{\Lambda}_{g,t} := \frac{\Lambda_{g,t}}{Y_t} = \widehat{D}_g + \widehat{S}_{g,t} \text{ where } \widehat{D}_g = \gamma\Gamma_g, \Gamma_g := \sum_{i=0}^{\infty} \beta^i \psi_{g,i} \text{ and } \widehat{S}_{g,t} = \sum_{i=0}^{\infty} \beta^i \psi_{g,i} \left( \sum_{j=0}^{\infty} \beta^j \frac{\partial \mathbb{E}_t f_{t+i+j}}{\partial T_{t+i}} \right). \quad (6)$$

The proof is in Appendix A. As described by Golosov et al. (2014), a tax equal to the social costs combined with lump-sum rebates implements the social optimum in a competitive equilibrium where production factors are freely allocated across sectors.<sup>7</sup> The weight of methane is  $\widehat{\Lambda}_{m,t}/\widehat{\Lambda}_{c,t}$ . The following proposition assumes that the risk of tipping follows (5) and is always present, which seems reasonable as temperatures will likely remain elevated above pre-industrial levels for centuries.

Proposition 1: Given (1) to (5) and assuming  $T_t \geq T_{min}$  for all  $t$ , then the social cost-to-income ratio from a tipping risk is given by:

$$\widehat{S}_g = \frac{\Omega \delta \mu}{1 - \beta} \Gamma_g \text{ where } \Omega := \beta^d \frac{1 - \beta^{r+1}}{(r+1)(1 - \beta)}. \quad (7)$$

The proof is in Appendix A. The parameter  $\Omega$  accounts for the delay in the onset of impact and the time to ramp up to full impact  $\delta$ . The effect of tipping risks on the social cost-to-income ratio is equivalent to boosting the deterministic damages parameter  $\gamma$  by  $\frac{\Omega \delta \mu}{1 - \beta}$ . Both components  $\widehat{D}_g$  and  $\widehat{S}_g$  are constant by construction: while deterministic damages combine (exponential) linear damages with a fixed (100%) probability, climate tipping combines fixed proportional damages with a probability of tipping linear in temperature and independent of previous tipping events.

Naturally, the weight of methane will be constant and equals  $\Gamma_m/\Gamma_c$ . When the discount rate is high, the weight of methane is high due to the rapid temperature effect of a methane pulse relative to carbon dioxide (Fig. 1). A discount rate of around 1% implies a methane weight of 1, corresponding to current policy using GWP100.

## 3 Sensitivities

This section discusses the effect of the following model changes on social costs: a quadratic tipping probability, limiting the risk of tipping to a single event, exponential-linear damages where the post-tipping impact increases with temperature, a threshold tipping risk formulation, and greater risk aversion. As deterministic damages are unchanged, discussions of social costs relate solely to tipping risks.

<sup>7</sup> Golosov et al. (2014) also show the optimal tax formula applies when exhaustible resource stocks apply.

### 3.1 Tipping Probability Quadratic in Temperature

If the tipping probability rises quadratically with temperature,  $p(T_t) = \mu_Q \tilde{T}_t^2$ , then social costs  $\hat{S}_{Q,g,t}$  become

$$\hat{S}_{Q,g,t} = \frac{2\Omega\delta\mu_Q}{1-\beta} \sum_{i=0}^{\infty} \beta^i \psi_{g,i} \tilde{T}_i. \tag{8}$$

Social costs now depend on temperature projections and no longer grow with income, and the methane weight will vary. Note the absence of an expectation operator, as a tipping event does not impact temperature outcomes. Consider temperature stabilisation at  $T^*$ , so that  $\sum_{i=0}^{\infty} \beta^i \psi_{g,i} \tilde{T}_i = \tilde{T}^* \Gamma_g$  and social costs become:

$$\hat{S}_{Q,T^*,g} = 2\tilde{T}^* \frac{\mu_Q}{\mu} \hat{S}_g. \tag{9}$$

### 3.2 One Tipping Event Only

While a few other papers consider the possibility of multiple tipping events, most studies consider the effect of a single event. In this case, the expectation at time  $t$  of the derivative of the tipping probability at time  $t+i$ ,  $p_{one,t+i}$ , is reduced by the chance that tipping will have occurred between  $t-d+1$  and  $t+i-1$  as follows:

$$\left( \frac{\partial \mathbb{E}_t p_{one,t+i}}{\partial T_{t+i}} \Big|_{f_i = 0} \right) = \mu \prod_{k=1}^{d+i-1} (1 - (\mathbb{E}_t(p_{t-d+k}) \Big|_{f_{t+k} = 0})) \leq \mu = \frac{\partial \mathbb{E}_t p_{t+i}}{\partial T_{t+i}}. \tag{10}$$

Thus, social costs are lower in this framework. While this result is intuitive, consider the effect of temperature projections. As the risk of tipping before period  $t+i$  increases with temperature, higher temperature projections reduce social costs today, which I call an “inevitability” effect. This effect would create positive feedback from lower emission taxes to higher temperature projections in a model with endogenous temperature.

### 3.3 Exponential-Linear Damages

A tipping event could lead to a change in the climate response rather than fixed damages, such as reduced absorption of carbon into the oceans discussed by Lenton et al. (2008) and considered by Lemoine and Traeger (2014).<sup>8</sup> Increased sensitivity to temperature can act as a proxy for a change in the climate response: an exponential-linear damages (ED) case examines the implications of both the probability of tipping and impacts increasing with temperature, so (5) becomes  $f_{ED,t} = T_t \sum_{i=-\infty}^t R_{t-i+1} I_{i-d}$ . Social costs are a function of expected temperature levels and, assuming the impacts of tipping have not occurred at time  $t$ , (6) becomes

<sup>8</sup> van der Ploeg (2014) also discusses sensitivity to the functional form of damages.

$$\left(\widehat{S}_{ED,g,t} | f_t = 0\right) = \sum_{i=0}^{\infty} \beta^i \psi_{g,i} \left( \sum_{k=1}^{d+i} R_{d+i-k+1} E_t I_{ED,t-d+k} + \beta^d \frac{\partial E_t I_{ED,t+i}}{\partial T_{t+i}} \sum_{j=0}^{\infty} \beta^j R_{j+1} T_{t+i+j+d} \right). \tag{11}$$

The first bracketed term in (11) relates to the differential welfare impact (Lemoine & Traeger 2014) and is proportional to the difference in social costs before and after tipping impacts. The second bracketed term relates to the marginal hazard effect, as exists in the core model and previous sensitivities, and captures the benefits of a marginal reduction in the tipping risk. Social costs now increase by  $\frac{\delta_{ED}}{1-\beta} \Gamma_g$  after the full impact of a tipping event as the marginal damages from temperature increase. Consider temperature stabilisation at  $\widetilde{T}^* > 0$ , so that  $E_t I_{ED,t-d+k} = \mu \widetilde{T}^* \delta_{ED}$ . Then, social costs become

$$\left(\widehat{S}_{ED,T^*,g} | f_t = 0\right) = \delta_{ED} \mu \left( \widetilde{T}^* \sum_{i=0}^{\infty} \beta^i \psi_{g,i} \sum_{j=1}^{i+d} R_j + T^* \frac{\Omega \Gamma_g}{1-\beta} \right) = \underbrace{\frac{\delta_{ED}}{\delta} \widetilde{T}^* \frac{(1-\beta)}{\Omega} \left( \frac{\sum_{i=0}^{\infty} \beta^i \psi_{g,i} \sum_{j=1}^{i+d} R_j}{\Gamma_g} \right)}_{\text{differential welfare impact}} \widehat{S}_g + \underbrace{\frac{\delta_{ED}}{\delta} T^* \widehat{S}_g}_{\text{marginal hazard effect}}. \tag{12}$$

A delay in the onset of tipping impact, or a gradual ramp-up of impacts, decreases the marginal hazard effect due to discounting: core social costs  $\widehat{S}_g$  for carbon dioxide and methane reduce equally. The differential welfare impact has an opposing force: a delay in awareness increases social costs ( $\sum_{j=1}^{i+d} R_j$  rises with  $d$ ), assuming no awareness of a tipping event to date. Further, the component  $\frac{\sum_{i=0}^{\infty} \beta^i \psi_{g,i} \sum_{j=1}^{i+d} R_j}{\Gamma_g}$  lowers the methane weight in two key ways. First, there is a greater weight on future temperature impacts because the cumulative chance that tipping occurs increases with time.<sup>9</sup> Second, a gradual ramp-up of impacts amplifies this effect by reducing the extent of short-run temperature effects. In contrast, a delay in the onset of impacts partially offsets these effects.<sup>10</sup>

### 3.4 Threshold Tipping Formulation

A threshold formulation is akin to a phase transition in physics, such as a transition from liquid to gas at a particular temperature (and pressure). The literature often suggests tipping events could occur above a temperature threshold or within a range: the collapse of Atlantic thermohaline circulation “probably requires more than 4 °C warming”; the disappearance of the Greenland ice sheet “may occur at 0.8 °C – 3.2 °C (with best estimate 1.6 °C)”; and collapse of the West Antarctic ice sheet “may be triggered at > 4 °C warming” (Lenton 2013). Assuming tipping risks are proportional to the extent that the current temperature exceeds the previous maximum, the tipping probability from (5) becomes

$$p_{T,t} = \mu_T \widetilde{T}_{T,t} \text{ where } \widetilde{T}_{T,t} := \max\left(0, T_t - \max_{k < t} (T_k)\right). \tag{13}$$

<sup>9</sup> Assuming no delay or ramp-up for clarity, the component  $\frac{\sum_{i=0}^{\infty} \beta^i \psi_{g,i} \sum_{j=1}^{i+d} R_j}{\Gamma_g}$  simplifies to  $\frac{\sum_{i=0}^{\infty} \beta^i i \psi_{g,i}}{\Gamma_g}$ . As the temperature effect of methane is relatively short-lived,  $\frac{\psi_{m,i}}{\psi_{c,i}} > \frac{\psi_{m,j}}{\psi_{c,j}}$  for  $0 < i < j$ , then  $\frac{\sum_{i=0}^{\infty} \beta^i i \psi_{c,i}}{\Gamma_c} > \frac{\sum_{i=0}^{\infty} \beta^i i \psi_{m,i}}{\Gamma_m}$ .

<sup>10</sup> Omitting a ramp-up, the component  $\frac{\sum_{i=0}^{\infty} \beta^i \psi_{g,i} \sum_{j=1}^{i+d} R_j}{\Gamma_g}$  reduces to  $\frac{\sum_{i=0}^{\infty} \beta^i i \psi_{g,i}}{\Gamma_g} + d$ , so the presence of  $d$  mitigates the contribution of  $\frac{\sum_{i=0}^{\infty} \beta^i i \psi_{g,i}}{\Gamma_g}$ .



There is a discontinuity in the temperature-derivative of the tipping probability if temperature stabilises. If we assume increasing temperatures so  $\tilde{T}_{T,t} > 0$  for all  $t$ , then the tipping social cost-to-income component is constant and equal to  $\beta^d \delta \mu_T \Gamma_g$  (note the absence of the denominator compared with (7), which will become clear). But the occurrence of peak temperature should be considered in this case. Consider the weak assumption that until period  $\tau$ ,  $\tilde{T}_{T,t} > 0$  for  $t < \tau$ , and from then on  $\tilde{T}_{T,t} = 0$  for  $t \geq \tau$ , so once temperature peaks, it never rises back above that peak. Then

$$\hat{S}_{T,g,t} = \Omega \delta \mu_T \left( \sum_{i=0}^{\tau-t-1} \beta^i \psi_{g,i} + \frac{\beta^{\tau-t} \psi_{g,\tau-t}}{1-\beta} \right). \tag{14}$$

The proof is in Appendix A. Consider no impact delay or ramp-up for clarity. A marginal increase in temperature  $T_t$  increases the chance of tipping in period  $t$  by  $\mu_T dT_t$  if  $t \leq \tau$ . In the core model, there is no effect on the tipping probability in future periods, so  $\frac{\partial \mathbb{E}f_{t+j}}{\partial T_t} = \mu \delta$  for all  $j \geq 0$ , and the infinite sum leads to the denominator  $1 - \beta$ . However, for the threshold formulation, if  $t < \tau$ , the chance of tipping in period  $t + 1$  is reduced by  $\mu_T dT_t$  so  $\frac{\partial \mathbb{E}f_{t+j}}{\partial T_t} = 0$  for  $j \geq 1$  and there is no  $1 - \beta$  denominator, while if  $t = \tau$ , then  $\frac{\partial \mathbb{E}f_{t+j}}{\partial T_t} = \mu_T \delta$  for  $j \geq 0$ .

Interestingly, initial social costs are lower if peak temperature occurs further into the future. There is no tipping risk (hence no social cost) once temperature stabilises or falls in the long run.<sup>11</sup> In contrast, tipping will (eventually) occur in the core model for any temperature stabilisation with a non-zero tipping probability. Ultimately, the best representation will depend on the nature of the specific tipping event, and may be a combination of both core and threshold (or other) formulations.<sup>12</sup>

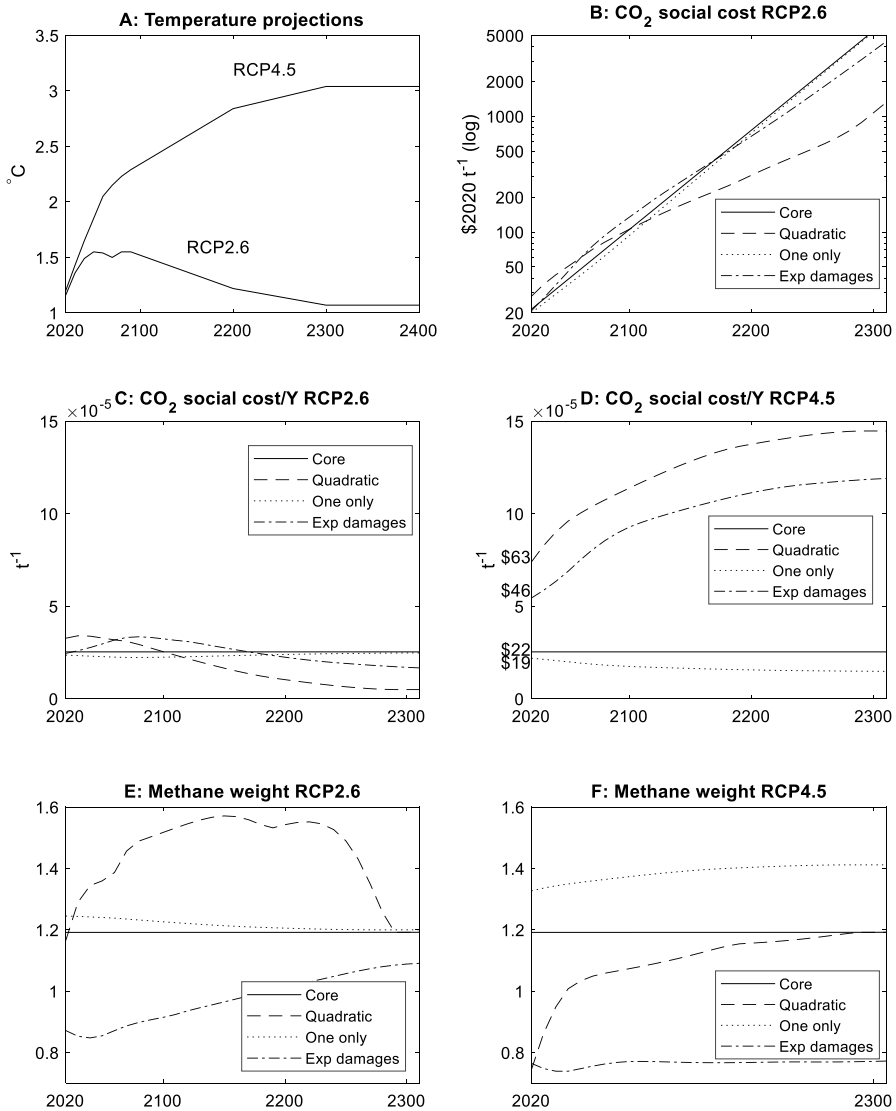
### 3.5 Risk Aversion

A logarithmic power utility is commonly used and implies an intertemporal elasticity of substitution of unity. However, some papers disentangle time preferences and risk aversion as described by Epstein and Zin (1990), including Bretschger and Vinogradova (2018), Cai and Lontzek (2019), Olijslagers and van Wijnbergen (2024) and Traeger (2018). This approach allows compliance with risk aversion estimates in the literature without leading to excessively high risk-free discount rates. An increase in risk aversion over that implied by a logarithmic utility is achieved by adding an expectation term as shown in the Bellman equation, omitting time subscripts and with parameter  $\alpha$ :

$$V(K, E, T) = \max_{K,E,T} \left( U(C) + \frac{\beta}{\alpha} \log \left( \mathbb{E} \left( e^{\alpha V'} \right) \right) \right). \tag{15}$$

<sup>11</sup> A declining optimal carbon price-to-income ratio has been found in other studies: as a consequence of uncertainty in Cai and Lontzek (2019) and Daniel, Litterman, and Wagner (2019) and of directing technical change to clean energy in Acemoglu, Aghion, Bursztyn, and Hemous (2012). Such a decline has implications for temperature and emissions outcomes and potentially on public perceptions of a carbon price. A lower tax after peak temperature has passed may help people appreciate the objective of the tax, and its temporary nature may alleviate public resistance.

<sup>12</sup> Crépin and Nævdal (2020) discuss an approach that would account for delays between temperature and the tipping probability called inertia risk not considered in this paper.



**Fig. 2** Social costs and cost-to-income ratios of carbon dioxide, and the methane weight, from a tipping risk. CO<sub>2</sub> Carbon dioxide. Methane weight of 1 is consistent with GWP100. *Quadratic*= Tipping probability quadratic in temperature. *One only*= A tipping event can only occur once. *Exp damages*= Damages from tipping are exponential-linear. Deterministic damages are infinitesimal for clarity. *One only* and *Exp damages* projections assume tipping does not occur *ex-post*. \$ values in Panel D indicate social costs in 2020

Given the simplifying assumptions detailed in Appendix A, further risk aversion increases the tipping component of the social cost-to-income ratio for carbon dioxide according to the following approximation:

$$\hat{S}_{EZ,c} \xrightarrow{\text{small } \Phi} \hat{S}_c \left( 1 + \frac{\Phi}{2} \right) \text{ where } \Phi := \alpha \varphi_f \frac{\Omega}{\beta} \delta, \varphi_f = \frac{-1}{(1 - \beta\kappa)(1 - \beta)}. \quad (16)$$

For parameters used in the next section  $\delta = 0.1$ ,  $d = 5$ ,  $\beta = 0.985^{10} = 0.86$  so  $\varphi_f = 10$ ,  $\kappa = \frac{1}{3}$  and  $d = 5$  and  $r = 5$ , so  $\Omega = 0.33$ . Traeger (2018) show that values of  $\alpha \in [-1.2, -0.7]$  are consistent with relative risk aversion values between 10 and 6 in the literature.<sup>13</sup> The uplift approximation in (16) relies on a small  $\Phi$ , between 0.27 and 0.46 given the parameters, so it is rough. The range of risk aversion uplift to match the literature is between 13% and 23%, broadly consistent with Cai and Lontzek (2019) for similar parameter values.

#### 4 An Illustration Using Temperature Projections

Consider the RCP2.6 and 4.5 temperature projections detailed in Stocker et al. (2013)<sup>14</sup>: I extrapolate to 2300 and then assume temperature stabilises (Panel A in Fig. 2). Assume that emissions lead to these temperature outcomes in each case, but allow marginal changes so that social costs are well defined by (6). For expositional clarity, deterministic damages are infinitesimal, so carbon dioxide social costs relate to a tipping risk only, and methane weights are well-defined even if the tipping risk is zero.

Global output is \$85 trillion in 2020 and grows by 2 per cent annually. Fixed damages from a tipping event are 10% of output, and the annual discount rate is 1.5%, as used in the DICE 2016R2 model. Following Lontzek et al. (2015), the tipping probability parameter is  $\mu = 0.025$  and  $T_{min}$  is set to 1 °C. A linear rise in temperature to 2 °C in 2100 leads to an expectation of 0.13 tipping events triggered by 2100.<sup>15</sup> The quadratic tipping probability and threshold sensitivities are calibrated to match this tipping expectation by 2100, so  $\mu_Q = 0.035$  and  $\mu_T = 0.16$ . In the exponential-linear damages case, 2 degrees of warming post-tipping leads to 10% damages, so  $\delta_{ED} = \delta/2$ . The delay from a tipping event to the onset of impact is 5 decades, followed by another 5 decades ramping up to full impact.

Panel B shows the social costs for carbon dioxide for the core model and the quadratic, one-only and exponential-linear damages sensitivities for the RCP2.6 projection. Panel C shows the same results as Panel B as a ratio of income  $Y$ , highlighting how social costs deviate from income growth in the sensitivities. Consistent with Proposition 1, social costs rise with income in the core formulation and are independent of temperature outcomes; thus, core results are identical in Panels C and D. The weight of methane in core is greater than one (Panels E and F): the initial methane social cost is \$26 per tonne CO<sub>2</sub>e compared with \$22 for carbon dioxide.<sup>16</sup> There are 0.11 and 0.20 expected tipping events triggered by 2100 under RCP2.6 and RCP4.5, respectively.

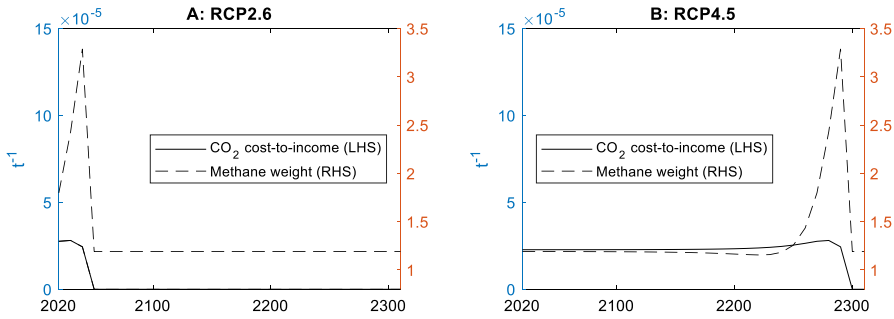
A quadratic tipping probability means higher temperatures lead to greater marginal tipping risks and higher social costs. Under RCP2.6, the social cost for carbon dioxide begins higher than core but rises slower than income due to falling temperatures, while it is higher than core and rises faster than income under the increasing temperatures in RCP4.5. The methane weight is higher under RCP2.6 as temperatures fall in the future, while the weight starts lower and rises under RCP4.5. In both cases, the methane weight matches the core

<sup>13</sup> The standard risk aversion coefficient defined in the Epstein-Zin setting is  $1 - \frac{\alpha}{1-\beta}$ .

<sup>14</sup> I consider these projections as likely bounds to the future temperature path.

<sup>15</sup> The probability of at least one tipping event by 2100 is 12.5%.

<sup>16</sup> For comparison, Nordhaus (2017) finds a social cost of carbon of \$44 (converting \$31 in 2015 using 2010 \$US) per tonne of carbon dioxide using the DICE-2016R2 model. Note I only consider social costs due to tipping risks.



**Fig. 3** Social cost-to-income ratios of CO<sub>2</sub> and the methane weight from a threshold tipping risk. CO<sub>2</sub> Carbon dioxide. Methane weight of 1 is consistent with GWP100. Deterministic damages are infinitesimal for clarity

weight as the temperature stabilises. At this point for RCP4.5, for example, social costs are  $2\tilde{T}^* \mu_Q / \mu \approx 5.6$  times the core model from (9).

If only one tipping event is possible, social costs are lower than core: initially by 7% and 13% for carbon dioxide for RCP2.6 and RCP4.5 respectively, and when temperature stabilises, by 2% and 41% respectively. The reduction in the methane social cost is smaller than for carbon dioxide, as the cumulative risk of tipping grows into the future, so the methane weight is higher than core.

If damages from tipping are exponential-linear in temperature, the profile of social costs for carbon dioxide is similar to the quadratic sensitivity. Consider the endpoint under RCP4.5. From (12), the marginal hazard effect is  $\frac{\delta_{ED}}{\delta} \tilde{T}^* \approx 1.5$  times the core social cost.

The differential welfare impact is  $\frac{\delta_{ED}}{\delta} \tilde{T}^* \frac{(1-\beta)}{\Omega} \left( \frac{\sum_{i=0}^{\infty} \beta^i \psi_{c,i} \sum_{j=1}^{i+d} R_j}{\Gamma_c} \right) \approx \frac{1}{2} 2 \frac{0.14}{0.33} (7.5) = 3.2$  times

core, making the social cost about 4.7 times core. As  $\frac{\sum_{i=0}^{\infty} \beta^i \psi_{g,i} \sum_{j=1}^{i+d} R_j}{\Gamma_g}$  is 7.5 for carbon dioxide and only 3.6 for methane, the methane weight is markedly lower and less than current policy.

In the threshold formulation, the initial social cost of carbon dioxide is 9% higher than core for RCP2.6 and 10% lower than core for RCP4.5 (Fig. 3). As a tipping risk only exists if temperature rises, this social cost ratio increases slightly and then drops to zero following peak temperature, and the methane weight is more than triple the current policy just before the peak. As social costs depend on the timing, rather than level, of peak temperature, the dynamics are identical between projections but shifted in time. The sharp changes in costs would become smooth with uncertainty of temperature outcomes, or with the effect of climate policy in an endogenous model where temperature may stabilise for an extended period.<sup>17</sup> If the threshold formulation were combined with exponential-linear damages, the social cost would drop gradually beyond peak temperature due to the delay in awareness that an event has occurred.

<sup>17</sup> As found in simulations in an earlier draft.

### 5 Conclusion

This paper examines the social costs of methane and carbon dioxide under climate tipping risks. Several formulations are considered as the nature and consequences of such risks differ between tipping events and are uncertain. The core model has restrictive assumptions that allow an easy calculation of social costs given a few parameters. A couple of temperature projections illustrate the results.

As in all work in this field, this paper has many limitations. While the risks of tipping in a stochastic framework are considered, the model and associated parameters are assumed to be known a priori. The restrictive assumptions in the economic framework do not allow precautionary capital formation considered in other papers such as van der Ploeg and de Zeeuw (2018). There is no discussion of the effect of climate policy on growth and emissions. The assumption that temperature is a linear function of previous actions can replicate the more complex climate-economy models well, but tipping impacts on climate feedback, such as a lower carbon dioxide decomposition rate, require a more complex framework.

### Appendix A: Proof and derivations

#### Proof of Lemma 1 (6)

The social cost of carbon equals the optimal carbon tax and is derived using a Lagrangian method. The social planner chooses  $C_t, K_t, E_{m,t}, E_{c,t}$  and  $X_t$  to maximise (1) subject to production and temperature constraints. Constraints on emissions technologies, which do not use capital or the final good as inputs, and other factors are omitted.

$$\begin{aligned} &\mathcal{L}(C_t, K_t, E_{m,t}, E_{c,t}, X_t, T_t) \\ &= \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t \log C_t + \lambda_{Y,t} (Y_t - e^{-(\gamma T_t + f_t)} K_t^\kappa F(E_{m,t}, E_{c,t}, X_t)) \right. \\ &\quad \left. + \lambda_{C,t} (C_t + K_{t+1} - Y_t) + \lambda_{T,t} \left( T_t - \sum_g \sum_{i=-\infty}^t \psi_{g,t-i} E_{g,i} \right) \right\}. \end{aligned} \tag{17}$$

First-order conditions for  $C_t, K_{t+1}$  and  $T_t$  are

$$\frac{\beta^t}{C_t} = -\lambda_{C,t} = -\lambda_{Y,t}, \quad \mathbb{E}_t \left( \lambda_{Y,t+1} \kappa \frac{Y_{t+1}}{K_{t+1}} \right) = -\lambda_{C,t}, \quad \text{and} \quad \lambda_{Y,t} \gamma Y_t + \mathbb{E}_t \left( \sum_{i=0}^{\infty} \lambda_{Y,t+i} \frac{\partial \mathbb{E}_t f_{t+i}}{\partial T_t} Y_{t+i} \right) = -\lambda_{T,t}. \tag{18}$$

A constant savings rate is implied by the conditions for  $C_t$  and  $K_t$ , as  $\frac{1}{C_t} = \beta \kappa \mathbb{E}_t \left( \frac{1}{C_{t+1}} \frac{Y_{t+1}}{K_{t+1}} \right)$  leads to  $\frac{Y_t - C_t}{C_t} = \beta \kappa \mathbb{E}_t \left( \frac{Y_{t+1}}{C_{t+1}} \right)$  which implies  $C_t = (1 - \beta \kappa) Y_t$ . The multiplier for temperature reflects marginal deterministic damages and the expected damages from tipping risks. The social cost of gas  $g$  in units of the final good  $\Lambda_{g,t}$  equals the sum of the future effects on temperature  $\psi_{g,i}$  multiplied by the temperature multiplier:

$$\begin{aligned}
\Lambda_{g,t} &= \frac{1}{\lambda_{Y,t}} \mathbb{E}_t \left( \sum_{i=0}^{\infty} \lambda_{T,t+i} \psi_{g,i} \right) \text{ and from (18)} & (19) \\
&= \frac{C_t}{\beta^t} \mathbb{E}_t \left( \sum_{i=0}^{\infty} \beta^{t+i} \psi_{g,i} \left( \frac{Y_{t+i}}{C_{t+i}} \gamma + \sum_{j=0}^{\infty} \beta^j \frac{Y_{t+i+j}}{C_{t+i+j}} \frac{\partial \mathbb{E}_t f_{t+i+j}}{\partial T_{t+i}} \right) \right) \\
&= Y_t \left( \sum_{i=0}^{\infty} \beta^i \psi_{g,i} \left( \gamma + \sum_{j=0}^{\infty} \beta^j \frac{\partial \mathbb{E}_t f_{t+i+j}}{\partial T_{t+i}} \right) \right).
\end{aligned}$$

### Proof of Proposition 1 (7)

From (6),

$$\begin{aligned}
\sum_{j=0}^{\infty} \beta^j \frac{\partial \mathbb{E}_t f_{t+i+j}}{\partial T_{t+i}} &= \beta^d \sum_{j=0}^{\infty} \beta^j \frac{\partial \mathbb{E}_t f_{t+i+j+d}}{\partial T_{t+i+d}} = \beta^d \delta \mu \sum_{j=0}^{\infty} \beta^j R_{j+1} & (20) \\
&= \beta^d \delta \mu \left( \frac{1}{r+1} + \frac{2}{r+1} \beta + \dots + \frac{r}{r+1} \beta^{r-1} + \frac{r+1}{r+1} \beta^r + \beta^{r+1} + \dots \right) \\
&= \beta^d \delta \mu \left( \frac{1}{r+1} (1 + 2\beta + \dots + (r+1)\beta^r) + \beta^{r+1} (1 + \beta + \dots) \right) \\
&= \beta^d \delta \mu \left( \frac{1}{r+1} \frac{(1 + \beta + \dots + \beta^r - (r+1)\beta^{r+1})}{1 - \beta} + \frac{\beta^{r+1}}{1 - \beta} \right) \\
&= \frac{\beta^d \delta \mu}{1 - \beta} \left( \frac{1}{r+1} (1 + \beta + \dots + \beta^r) - \beta^{r+1} + \beta^{r+1} \right) \\
&= \frac{\beta^d \delta \mu}{1 - \beta} \frac{1 - \beta^{r+1}}{(r+1)(1 - \beta)}.
\end{aligned}$$

### Derivation of (14)

$$\begin{aligned}
\frac{\partial \mathbb{E}_t f_{t+i+j}}{\partial T_{t+i}} &= \begin{cases} \frac{\partial \mathbb{E}_t I_{t+i}}{\partial T_{t+i}} R_1 \text{ iff } j = d \\ \frac{\partial \mathbb{E}_t I_{t+i}}{\partial T_{t+i}} R_{j-d+1} + \frac{\partial \mathbb{E}_t I_{t+i+1}}{\partial T_{t+i}} R_{j-d+2} \text{ iff } j > d \end{cases} \text{ so from (6)} \\
\widehat{S}_{g,t} &= \beta^d \left( \sum_{i=0}^{\tau-t-1} \beta^i \psi_{g,i} \left( \frac{\partial \mathbb{E}_t I_{t+i}}{\partial T_{t+i}} R_1 + \sum_{j=1}^{\infty} \beta^j \left( \frac{\partial \mathbb{E}_t I_{t+i}}{\partial T_{t+i}} R_{j+1} + \frac{\partial \mathbb{E}_t I_{t+i+1}}{\partial T_{t+i}} R_{j+2} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
 & +\beta^{\tau-t}\psi_{g,\tau-t} \frac{\partial \mathbb{E}_t I_\tau}{\partial T_\tau} \sum_{j=0}^{\infty} \beta^j R_{j+1} \Big) \text{ as } \frac{\partial \mathbb{E}_t I_{t+i+k}}{\partial T_{t+i}} = 0 \text{ if } t+i+k > \tau \text{ or } k > 1, \text{ so} \\
 \hat{S}_{g,t} & = \beta^d \delta \mu_T \sum_{i=0}^{\tau-t-1} \beta^i \psi_{g,i} \left( R_1 + \sum_{j=1}^{\infty} \beta^j (R_{j+1} - R_{j+2}) \right) + \beta^{\tau-t} \psi_{g,\tau-t} \frac{\mu_T \Omega \delta}{1-\beta}
 \end{aligned}$$

which leads to (14) as.

$$R_1 + \sum_{j=1}^{\infty} \beta^j (R_{j+1} - R_{j+2}) = \frac{1}{r+1} + \frac{\beta}{r+1} + \dots + \frac{\beta^r}{r+1} = \frac{1}{r+1} \left( \frac{1-\beta^{r+1}}{1-\beta} \right).$$

**Derivation of (16)**

For simplicity, assume a constant temperature effect for carbon  $\psi_{c,j} = \psi_{c,j} \geq 1$ , as outlined in Matthews et al. (2009) and recently adopted by Dietz and Venmans (2019). Assume the impact of a tipping event in the next period is  $\Omega\delta/\beta$ , to reflect delay in impact onset and ramp-up time used in Sect. 4. Omitting deterministic damages, time subscripts and signifying time  $t + 1$  variables using prime, the value function is

$$V(K, E, T) = \max_{K,T,E} \left\{ \log(Y - K') + \frac{\beta}{\alpha} \log(\mathbb{E}_t(e^{\alpha V'})) \right\} \text{ where } C = Y - K', \quad (21)$$

$$V' = V(K', E', T'), \quad Y = e^{-f} K^\kappa F(E), \quad T' = T + \psi_{c,1} E \text{ and } f' = f + I.$$

$$\text{So } V = \max_{K,T,E} \left( \log((1 - \beta\kappa)e^{-f} K^\kappa F(E)) + \frac{\beta}{\alpha} \log(\mathbb{E}_t(e^{\alpha V'})) \right).$$

Using a trial solution, we have:

$$\begin{aligned}
 & \varphi_K \log K + \varphi_T T + \varphi_E E + \varphi_f f \quad (22) \\
 & = \max_{K,T,E} \left( \log(Y - K') + \frac{\beta}{\alpha} \log \left( \mathbb{E}_t \left( e^{\alpha(\varphi_K \log K' + \varphi_T T' + \varphi_E E' + \varphi_f (f+I))} \right) \right) \right) \\
 & = \max_{K,T,E} \left( \log(Y - K') + \beta \varphi_K \log K' + \beta \varphi_T T' + \beta \varphi_E E' + \varphi_f f + \frac{\beta}{\alpha} \log(\mathbb{E}_t(e^{\alpha \varphi_f I})) \right) \text{ and} \\
 & \log(\mathbb{E}_t(e^{\alpha \varphi_f I})) = \log \left( p e^{\alpha \varphi_f \frac{\Omega}{\beta} \delta} + 1 - p \right) \xrightarrow{\text{small } \Phi} \varphi_f \delta \mu_T \Omega \left( 1 + \frac{\Phi}{2} \right), \quad \Phi := \alpha \varphi_f \frac{\Omega}{\beta} \delta.
 \end{aligned}$$

The first-order condition for capital leads to  $K' = \frac{\beta \varphi_K}{1 + \beta \varphi_K} Y$ , and substitution into (22) leads to

$$\begin{aligned} \varphi_K \log K + \varphi_T T + \varphi_E E + \varphi_f f &= \log((1 - \beta\varphi_K)e^{-f}K^\kappa F(E)) \\ &+ \beta\varphi_K(\kappa \log K + \log F - f) + \beta\varphi_T T' + \beta\varphi_E E' \\ &+ \beta\varphi_f(f + \varphi_f \delta\mu T\Omega(1 + X)). \end{aligned} \tag{23}$$

Equating terms for  $\log K, f$ , and the first-order conditions for  $T$  and  $E$ :

$$\log K : \varphi_K = \kappa + \kappa\beta\varphi_K \text{ so } \varphi_K = \frac{\kappa}{(1 - \beta\kappa)}. \tag{24}$$

$$f : \varphi_f = -1 - \beta\varphi_K + \beta\varphi_f \text{ so } \varphi_f = \frac{-(1 + \beta\varphi_K)}{1 - \beta} = \frac{-1}{(1 - \beta\kappa)(1 - \beta)}. \tag{25}$$

$$FOC T : \varphi_T = \beta\varphi_T + \varphi_f \mu \delta\Omega(1 + \Phi) \text{ so } \varphi_T = \frac{\varphi_f \mu \delta\Omega}{1 - \beta}(1 + \Phi). \tag{26}$$

$$FOC E : \varphi_E = \frac{F'(E)}{F(E)}(1 + \beta\varphi_K) + \beta\varphi_T \psi_{c,1}. \tag{27}$$

The shadow price of carbon energy  $\varphi_E$  consists of the benefits for production and the negative externality from temperature increase. The latter term is the social cost of carbon dioxide expressed in consumption units,  $S_{EZ,c}/C$ , so:

$$\begin{aligned} \widehat{S}_{EZ,c} &:= \frac{S_{EZ,c}}{Y} = \varphi_T \beta\varphi_T \psi_{c,1}(1 - \beta\kappa), \text{ as} \\ C &= (1 - \beta\kappa)Y, \text{ so } \widehat{S}_{EZ,c} = \widehat{S}_c(1 + \Phi) \text{ from (26), (25)} \\ &\text{and (7) and as } \Gamma_c = \frac{\beta\psi_{c,1}}{1 - \beta}. \end{aligned} \tag{28}$$

## Appendix B: Previous literature

## Appendix C: Climate model

The climate model in this paper follows Shine et al. (2005), giving a rapid temperature response to carbon dioxide emissions as recently advocated by Dietz et al. (2021). The GWP100 of methane is determined by summing radiative forcings annually up to 100 years. For carbon dioxide, temperature responses at time  $t$  after an emissions pulse (in discrete time) are

$$\psi_{c,t} := \frac{\partial T_t}{\partial E_{c,0}} = \frac{B_c}{H} \left\{ \zeta a_0 \left( 1 - e^{-\frac{t}{\zeta}} \right) + \sum_{i=1}^4 \frac{a_i \left( e^{-\frac{t}{a_i}} - e^{-\frac{t}{\zeta}} \right)}{\left( \zeta^{-1} - \alpha_i^{-1} \right)} \right\}, \tag{29}$$



**Table 2** Climate model parameters

$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$B_c$	$B_m$
0.1756	0.1375	0.1858	0.2423	0.2589	62,441	$3.81B_c$
$H$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\zeta$	$\alpha_m$
$4.2 \times 10^8$	421.09	70.597	21.422	3.4154	10.65	12

where  $H$  is the heat capacity of the system,  $\lambda$  is a climate sensitivity parameter,  $a_i$  are coefficients which sum to 1,  $\alpha_i$  reflect gas lifetimes in years,  $\zeta$  is by definition the constant  $\lambda H$  in years, and  $B_c$  is the radiative forcing due to a 1 Gt change in carbon dioxide. For methane, the equations are simpler:

$$\psi_{m,i} = \frac{B_m}{H(\zeta^{-1} - \alpha_m^{-1})} \left( e^{-\frac{t}{\alpha_m}} - e^{-\frac{t}{\zeta}} \right). \quad (30)$$

Parameter values are shown in Table 2.

**Supplementary Information** The online version contains supplementary material available at <https://doi.org/10.1007/s10640-024-00864-z>.

**Acknowledgements** Helpful comments from Warwick McKibbin, David Stern, Frank Jotzo, Jack Pezzey, Reyer Gerlagh, Cameron Eren, Nicholas Rivers, Chris Wokker, Larry Liu, Martin Quaas, anonymous referees and audiences at the Australian National University, London School of Economics, Hamburg University, Institute for International Economic Studies and AERE, EAERE and EEA conferences and MEEW workshop.

**Funding** Open Access funding enabled and organized by CAUL and its Member Institutions.

## Declarations

**Conflict of interest statement** The author declares that he has no relevant or material financial interests that relate to the research described in this paper.

**Research Involving Human Participants and/or Animals** NA.

**Informed Consent** NA.

**Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

## References

- Aaheim A, Fuglestedt JS, Godal O (2006) Costs savings of a flexible multi-gas climate policy. *Energy J* 27:485–501
- Acemoglu D, Aghion P, Bursztyrn L, Hemous D (2012) The environment and directed technical change. *Am Econ Rev* 102(1):131–166. <https://doi.org/10.1257/aer.102.1.131>
- Azar C, Martín JG, Johansson DJ, Sterner T (2023) The social cost of methane. *Clim Change* 176(6):71

- Barrage L (2014) Sensitivity analysis for Golosov, Hassler, Krusell, and Tsyvinski (2014): Optimal taxes on fossil fuel in general equilibrium. *Econometrica* supplemental material
- Bickel JE, Agrawal S (2013) Reexamining the economics of aerosol geoengineering. *Clim Change* 119(3–4):993–1006
- Bretschger L, Vinogradova A (2018) Escaping Damocles' sword: endogenous climate shocks in a growing economy. CER-ETH—Center of Economic Research at ETH Zurich, Working Paper, 18, p 291
- Burke M, Hsiang SM, Miguel E (2015) Global non-linear effect of temperature on economic production. *Nature* 527(7577):235–239. <https://doi.org/10.1038/nature15725>
- Cai Y (2021) The role of uncertainty in controlling climate change. In: *Oxford research encyclopedia of economics and finance*
- Cai Y, Lontzek TS (2019) The social cost of carbon with economic and climate risks. *J Polit Econ* 127(6):2684–2734
- Cai Y, Judd KL, Lenton TM, Lontzek TS, Narita D (2015) Environmental tipping points significantly affect the cost–benefit assessment of climate policies. *Proc Natl Acad Sci* 112(15):4606–4611
- Cai Y, Lenton TM, Lontzek TS (2016) Risk of multiple interacting tipping points should encourage rapid CO<sub>2</sub> emission reduction. *Nat Clim Chang* 6(5):520–525
- Clarke HR, Reed WJ (1994) Consumption/pollution tradeoffs in an environment vulnerable to pollution-related catastrophic collapse. *J Econ Dyn Control* 18(5):991–1010
- Crépin AS, Nævdal E (2020) Inertia risk: Improving economic models of catastrophes. *Scand J Econ* 122(4):1259–1285
- Cropper ML (1976) Regulating activities with catastrophic environmental effects. *J Environ Econ Manag* 3(1):1–15
- Daniel KD, Litterman RB, Wagner G (2019) Declining CO<sub>2</sub> price paths. *Proc Natl Acad Sci U S A* 116(42):20886–20891. <https://doi.org/10.1073/pnas.1817444116>
- Diaz D, Keller K (2016) A potential disintegration of the West Antarctic Ice Sheet: implications for economic analyses of climate policy. *Am Econ Rev* 106(5):607–611
- Dietz S, Venmans F (2019) Cumulative carbon emissions and economic policy: in search of general principles. *J Environ Econ Manag* 96:108–129
- Dietz S, van der Ploeg F, Rezai A, Venmans F (2021) Are economists getting climate dynamics right and does it matter? *J Assoc Environ Resour Econ* 8(5):895–921
- Engström G, Gars J (2016) Climatic tipping points and optimal fossil-fuel use. *Environ Resource Econ* 65(3):541–571
- Epstein LG, Zin SE (1990) 'First-order' risk aversion and the equity premium puzzle. *J Monet Econ* 26(3):387–407
- Fuglestedt JS, Bernsten TK, Godal O, Sausen R, Shine KP, Skodvin T (2003) Metrics of climate change: assessing radiative forcing and emission indices. *Clim Change* 58(3):267–331
- Gerlagh R, Liski M (2018) Carbon prices for the next hundred years. *Econ J* 128(609):728–757
- Goes M, Tuana N, Keller K (2011) The economics (or lack thereof) of aerosol geoengineering. *Clim Change* 109(3–4):719–744
- Golosov M, Hassler J, Krusell P, Tsyvinski A (2014) Optimal taxes on fossil fuel in general equilibrium. *Econometrica* 82(1):41–88
- Heutel G, Moreno-Cruz J, Shayegh S (2018) Solar geoengineering, uncertainty, and the price of carbon. *J Environ Econ Manag* 87:24–41
- IPCC (2014) *Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*
- Johansson DJ, Persson UM, Azar C (2006) The cost of using global warming potentials: analysing the trade off between CO<sub>2</sub>, CH<sub>4</sub> and N<sub>2</sub>O. *Clim Change* 77(3–4):291–309
- Keller K, Bolker BM, Bradford DF (2004) Uncertain climate thresholds and optimal economic growth. *J Environ Econ Manag* 48(1):723–741
- Lemoine D, Traeger C (2014) Watch your step: optimal policy in a tipping climate. *Am Econ J Econ Pol* 6(1):137–166
- Lemoine D, Traeger CP (2016) Economics of tipping the climate dominoes. *Nat Clim Chang* 6(5):514
- Lenton TM (2013) Environmental tipping points. *Annu Rev Environ Resour* 38:1–29
- Lenton TM, Held H, Kriegler E, Hall JW, Lucht W, Rahmstorf S, Schellnhuber HJ (2008) Tipping elements in the Earth's climate system. *Proc Natl Acad Sci* 105(6):1786–1793
- Lontzek TS, Cai Y, Judd KL, Lenton TM (2015) Stochastic integrated assessment of climate tipping points indicates the need for strict climate policy. *Nat Clim Chang* 5(5):441
- Manne AS, Richels RG (2001) An alternative approach to establishing trade-offs among greenhouse gases. *Nature* 410(6829):675

- Marten AL, Newbold SC (2012) Estimating the social cost of non-CO<sub>2</sub> GHG emissions: Methane and nitrous oxide. *Energy Policy* 51:957–972
- Matthews HD, Gillett NP, Stott PA, Zickfeld K (2009) The proportionality of global warming to cumulative carbon emissions. *Nature* 459(7248):829–832
- Michaelis P (1992) Global warming: efficient policies in the case of multiple pollutants. *Environ Resource Econ* 2(1):61–77
- Nævdal E (2006) Dynamic optimisation in the presence of threshold effects when the location of the threshold is uncertain—with an application to a possible disintegration of the Western Antarctic Ice Sheet. *J Econ Dyn Control* 30(7):1131–1158
- Nordhaus WD (2017) Revisiting the social cost of carbon. *Proc Natl Acad Sci* 114(7):1518–1523
- Nordhaus W (2019) Economics of the disintegration of the Greenland ice sheet. *Proc Natl Acad Sci* 116(25):12261–12269
- O'Neill BC (2003) Economics, natural science, and the costs of global warming potentials. *Clim Change* 58(3):251–260
- Olijslagers S, van Wijnbergen S (2024) Discounting the future: on climate change, ambiguity aversion and Epstein–Zin preferences. *Environ Resour Econ*, 1–48.
- Polasky S, De Zeeuw A, Wagener F (2011) Optimal management with potential regime shifts. *J Environ Econ Manag* 62(2):229–240
- Pratt JW, Zeckhauser RJ (1996) Willingness to pay and the distribution of risk and wealth. *J Polit Econ* 104(4):747–763
- Rezai A, van der Ploeg F (2015) Robustness of a simple rule for the social cost of carbon. *Econ Lett* 132:48–55
- Shine KP, Fuglestedt JS, Hailemariam K, Stuber N (2005) Alternatives to the global warming potential for comparing climate impacts of emissions of greenhouse gases. *Clim Change* 68(3):281–302
- Stocker TF, Qin D, Plattner G-K, Tignor M, Allen SK, Boschung J et al (2013) Climate change 2013: the physical science basis. Contribution of working group I to the fifth assessment report of the intergovernmental panel on climate change, p 1535
- Taconet N, Guivarch C, Pottier A (2021) Social cost of carbon under stochastic tipping points: when does risk play a role? *Environ Resource Econ* 78:709–737
- Traeger CP (2018) Ace–analytic climate economy (with temperature and uncertainty)
- Tsur Y, Zemel A (1998) Pollution control in an uncertain environment. *J Econ Dyn Control* 22(6):967–975
- van der Ploeg F (2014) Abrupt positive feedback and the social cost of carbon. *Eur Econ Rev* 67:28–41
- van der Ploeg F, de Zeeuw A (2016) Non-cooperative and cooperative responses to climate catastrophes in the global economy: a north–south perspective. *Environ Resource Econ* 65(3):519–540
- van der Ploeg F, de Zeeuw A (2018) Climate tipping and economic growth: precautionary capital and the price of carbon. *J Eur Econ Assoc* 16(5):1577–1617
- van der Ploeg F, de Zeeuw A (2019) Pricing carbon and adjusting capital to fend off climate catastrophes. *Environ Resource Econ* 72(1):29–50
- van der Zwaan B, Gerlagh R (2009) Economics of geological CO<sub>2</sub> storage and leakage. *Clim Change* 93(3):285–309
- WMO (1992) Scientific Assessment of Ozone Depletion: 1991, Global Ozone Research and Monitoring Project—Report No. 37. World Meteorological Organization Geneva, Switzerland

**Publisher's Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.