



# Escaping Damocles' Sword: Endogenous Climate Shocks in a Growing Economy

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## Abstract

We consider a growing economy which is subject to recurring, random, uninsurable, and potentially large and long-lasting climate shocks leading to destruction of infrastructure, land degradation, collapse of ecosystems or similar loss of productive capacity. The associated damages and the hazard rate are endogenously driven by the stock of greenhouse gases. We highlight the important role of the relative risk aversion and provide analytical solutions for the optimal climate policy, the growth rate and the saving propensity of the economy. We stress the importance of jointly determining these variables, especially if the objective is to formulate meaningful policy prescriptions. If, for example, the growth rate or the saving rate are assumed to be exogenous, and thus independent of the characteristics of climate shocks and economic fundamentals, then future economic developments in the face of climate change and, consequently, the future mitigation efforts will deviate from the optimal paths. In a quantitative assessment we show that with log-utility and under favorable technological and climatic conditions the abatement expenditure represents only 0.5% of output, equivalent to \$37 per ton carbon. Under less favorable conditions, coupled with a relative risk aversion which exceeds unity, the abatement propensity increases to 2.9%, equivalent to \$212 per ton carbon, and it jumps to a striking 16% in the pessimistic scenario involving severe shocks and low efficiency of abatement technology.

**Keywords** Climate policy · Uncertainty · Natural disasters · Endogenous growth

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# 1 Introduction

## 1.1 Economics and the Climate

The search for an optimal policy response to climate change has preoccupied the minds of economists for over three decades. While pricing carbon at its social cost, so as to limit carbon emissions, is a generally accepted policy proposal, a global consensus on an optimal carbon price or an efficient mitigation level is still lacking. It is nonetheless widely acknowledged that a comprehensive approach should include consideration of important risks and uncertainties associated with global warming. What is especially troublesome is that planet's excessive warming may lead to severe and unforeseeable natural catastrophes (IPCC 2014) the impact of which may be long-lasting or even irreversible (Lenton and Ciscar 2013). Such catastrophes may result in potentially huge damages to economic activity, biodiversity, eco-systems and geophysical processes, e.g. loss of productive capacity, loss of plants and species varieties, melting of ice sheets, changes in El-Nino and Atlantic thermohaline circulation. The focus of the present paper is on the design of climate policy in the context of such natural disasters.

The threat of severe catastrophes puts the world economy in a situation similar to the one of Damocles, an obsequious courtier of King Dionysius of Syracuse in the 4th century BC. According to the moral anecdote, Dionysius offered Damocles to sit on his throne in order to taste the fortune of a great man of power and authority, an opportunity which Damocles eagerly accepted. The throne was surrounded by every luxury but the King arranged that a sword should hang above it, held by a single hair of a horse's tail. An impending calamity restrained by some probability can be used as an allegory for the world population facing the climate catastrophe. The analogy ends with an important difference however: Damocles had no power to deal with the imminent disaster, while the planet's warming can be controlled by our current policies.

The policy response to climate change needs to be informed by appropriately-designed economy-climate models. However, the complexity of both the economic and the climate parts of the problem pose considerable modeling challenges. Many of them are being addressed by relatively complex numerical Integrated Assessment Models (IAMs), which have provided many useful insights with respect to the social cost of carbon (SCC) but also produced diverging results.<sup>1</sup> To gain further insights into the central mechanisms at work—especially those related to the uncertainties and risks associated with climate change and its economic impact—a framework of investigation that relies on explicit analytic solutions and provides clear-cut implications for the optimal climate policy in a dynamic economy appears to be desirable.

Within such a framework, a number of important questions need to be addressed. Given the uncertain nature of environmental disasters caused by climate change, how should an economy appropriately balance its production, consumption, investment, and reduction of greenhouse gas (GHG) emissions? What is the optimal rate of output growth and the optimal emissions abatement in the uncertain environment? How do these key variables interact and respond to changes in the underlying economic and climatic fundamentals? In the present paper we examine these questions within a model of a growing economy

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<sup>1</sup> Deterministic IAMs, such as the widely used DICE (Nordhaus 2008), produce lower estimates of SCC than IAMs which include uncertainty, such as e.g. Lontzek et al. (2015), Cai and Lontzek (2019). Estimates of SCC are also sensitive to the modeling of the climate system (Dietz and Venmans 2019).

which features uncertainty about arrivals of climate shocks. We assume that an occurrence of a disaster (also referred to as an "event") follows a random process, and when a disaster strikes, some part of the economy's productive capacity is destroyed. The magnitude of the damage is assumed to be an increasing function of the stock of GHG. It follows that the process of accumulation of the productive capacity is both endogenous and stochastic. Natural catastrophes induced by climate change are large in scale and have a profound negative impact on both national and global economy. Unlike in the case of relatively small idiosyncratic shocks, the risk of such events cannot be insured.<sup>2</sup>

In our model the world neither ends nor becomes deterministic after an environmental disaster, as it is often assumed in the literature on irreversible catastrophic events (see Sect. 1.2). We consider development with recurring shocks over time, which reflects a likely pattern of climate-induced events in the future.<sup>3</sup> Optimal reduction in emissions, and the implied reduction in damages, can be achieved by appropriately balancing two types of activities: capital accumulation and abatement. In an extension of the model we endogenize the event hazard rate, in addition to damages, by letting it be an increasing function of pollution stock (IPCC 2014).

## 1.2 Related Literature

There is a growing literature on random catastrophic events causing *irreversible* damage.<sup>4</sup> Tsur and Zemel (1998) are the first to explicitly analyze reversible events focusing on the optimal steady state policy and transitional dynamics of an economy which is not engaged in any investment activity.<sup>5</sup> Van der Ploeg (2014) analyzes the optimal carbon tax in an economy subject to a random shock which reduces the nature's capacity to absorb greenhouse gases. Van der Ploeg and de Zeeuw (2018) examine policy adjustment with respect to how fast a tipping point may materialize.<sup>6</sup> Several recent contributions incorporated a

<sup>2</sup> In a globalized world, large-scale natural disasters affect not only the economic activity of the country where they strike but also other economies by virtue of either close geographic location or trade relations, FDI, etc. When it comes to relatively small idiosyncratic shocks, they can be insured against by trading insurance claims within a group of regions subject to such shocks. Our focus, however, is on a global economy where an insurance contract against a large natural catastrophe (e.g., Indian Ocean tsunami, meltdown of Greenland ice-sheet, etc.) is challenging to design and implement. Additionally, stability of contractual arrangements on such a large scale seems to be difficult to ensure, especially in light of the long time horizon and asymmetric probabilities of disasters across countries. Insurance companies seem to perceive climate change and the associated disasters as a threat rather than profitable business opportunities. As a prominent example, the strategy of one of the world's leading reinsurance companies, Swiss Re, is to raise awareness about climate-change risks through dialogue with clients, employees, and the public and to advocate a worldwide policy framework for climate change (Swiss Re 2015, p.119,ff.).

<sup>3</sup> Recurring climate-induced capital destruction may result in a new form of poverty traps for developing countries. See Le Van et al. (2010) for an analysis of poverty traps where the natural environment offers opportunities while in our case it may pose a threat to development.

<sup>4</sup> For early theoretical contributions see Clarke and Reed (1994), Tsur and Zemel (1996) and for recent numerical models Lemoine and Traeger (2014), Lontzek et al. (2015).

<sup>5</sup> de Zeeuw and Zemel (2012) provide a dynamic characterization of an optimal emission policy when the time of the regime switch from low to high damage is uncertain. One of their key findings is that, due to precautionary reasons, emissions in the low-damage regime may be lower than in the case where the system is already in the high-damage regime.

<sup>6</sup> In a model with uncertain pollution stock dynamics, Athanassoglou and Xepapadeas (2012) find that investment in damage control is increasing in the degree of uncertainty and that control might be a substitute for mitigation when it is sensitive to changes in uncertainty.

possibility of crossing a tipping point into integrated assessment modeling (e.g. Lemoine and Traeger 2014; Lontzek et al. 2015). Cai and Lontzek (2019) use DSICE, a stochastic IAM, to analyze the effects on SCC of (economic) uncertainty in productivity coupled with (climatic) uncertainty about triggering a tipping process. The results from these studies unambiguously indicate that the current SCC should be larger by at least a factor of two, as compared to deterministic IAMs, when the possibility of an irreversible tipping is taken into consideration.

Our work relates to several important papers in the field of climate and growth economics. Mueller-Fuerstenberger and Schumacher (2015) developed a decentralized version of a neoclassical growth model where agents face stochastic extreme events which are relatively small-scale and thus insurable. A very interesting and policy-relevant result of their paper is that the insurance industry, being the main provider of reactive adaptation, can induce agents to undertake more abatement by signaling the consequences of climate change through insurance premiums.<sup>7</sup> In a recent paper, Gerlagh and Liski (2017) introduce uncertain events in a climate-economy model where the impacts of climate change are not known but learned over time. As a consequence of the belief updating the optimal carbon tax does not develop in lock-step with income but depends on temperature levels. Dietz and Venmans (2019) incorporate the most recent advances in climate science with respect to the mapping from  $CO_2$  release into temperature change. They show that minimizing both the abatement costs and damages requires a much more aggressive emissions reduction policy than what has been recently suggested by Lemoine and Rudik (2017) based on a model with strong inertia in the climate system.<sup>8</sup> Our paper also relates to recent contributions on endogenous growth under environmental restrictions, in particular Peretto and Valente (2015) and Lanz et al. (2017), where economic dynamics constitute the center of the analysis, while we focus more on the environmental policy aspect.

Finally, we would like to highlight important links between our macroeconomic approach and the extensive microeconomic and case study literature deriving the parameters we use in our quantitative analysis. In a recent empirical study on alterations in risk aversion, Liebenehm et al. (2023) combine individual-level panel data with historical rainfall data for rural Thailand and Vietnam and find that rainfall shocks increase individuals risk aversion; this confirms the "risk vulnerability" hypothesis proposed by Gollier and Pratt (1996), which we will adopt when modelling the effects of climate shocks in our analysis. Brown et al. (2018) studied the effect of the 2012 cyclone on Fijian households' risk attitudes, they equally find that being struck by an extreme event substantially changes individuals' risk perceptions as well as their beliefs about the frequency and magnitude of future shocks. That risk preferences are heterogeneous among individuals, changing over time or with policy, is a main result of Koundouri et al. (2009) who analysed Finnish

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<sup>7</sup> Ikefuji and Horii (2012) examine the optimal growth rate and the carbon tax in an economy where private capital is subject to stochastic depreciation due to climate change. They assume, however, that these stochastic shocks are idiosyncratic and reflect a large number of independent small climate events. Soretz (2007) analyzes efficient pollution taxation within an endogenous growth model where environmental quality has a stochastic impact on factor productivity, which is driven by a Wiener process. Similarly to these contributions we assume that international policy coordination is feasible, as has recently been demonstrated by adoption of the Paris Agreement. If such a mechanism turns out to be inefficient, it has been suggested to replace global carbon taxes by international trading of the rights to exploit fossil-fuel deposits, see Harstad (2012).

<sup>8</sup> A recent paper by Mattauch et al. (2020) presents further arguments against the conclusions of Lemoine and Rudik (2017).

farm-level data. In a broad survey experiment on risk attitudes Dohmen et al. (2011) find that gender, age, height, and parental background have a significant impact on the willingness to take risk.<sup>9</sup> Our macroeconomic model does not distinguish between different household types as it would go beyond the scope of the present paper; but we see a promising future research avenue when applying some of our model elements in a multi-region approach, with local differences in climate vulnerability and climate events, which would also enable to extend regional policy analysis under uncertainty (see Brock and Xepapadeas 2020, and Englezos et al. 2022). Uncertainty may also affect social discounting. Like our paper, Hepburn et al. (2009) argue that over longer horizons, as in the case of climate change, the topic of uncertainty becomes more and more important. These authors estimate different models of real interest rates to determine certainty-equivalent discount rates in Australia, Canada, Germany and the UK and find that social cost-benefit analysis should be conducted with a declining discount rate to place a substantially higher weight on events in the distant future. Our framework argues along the same lines but uses a simpler approach by assuming low social discount rates from the very beginning.

### 1.3 Contribution to the Literature

The present paper contributes to formulating efficient climate policies using an approach which has been identified as an urgent priority in recent assessments of the field. Farmer et al. (2015) conclude that the first of four major issues inadequately addressed by economic models of climate change is uncertainty. An even more stringent view is expressed by Stern (2007, 2015, 2016) who states that many economic models do not account for catastrophic changes and possible tipping points which would make a crucial difference for policy assessment in his view. Addressing these issues forms the core of our contribution.

To the best of our knowledge our paper is the first to provide closed-form solutions for the abatement policy and the optimal growth rate of the economy subject to random climate shocks with damages endogenously driven by the accumulation of GHG.<sup>10</sup> Our climate-policy instrument can be conveniently expressed as a fraction of output which depends on the fundamental characteristics of the climate and the economy. This result is parallel to that of Golosov et al. (2014), who derive a simple formula for the optimal carbon tax, showing that under specific simplifying assumptions on damages and saving propensity the tax is proportional to GDP and depends on just a small number of key parameters. In the present paper we relax some of those assumptions and, in particular, endogenously derive the optimal saving rate of the economy to show how it is affected by exposure to uncertainty associated with climate change.

The second contribution of the paper is to highlight the importance of *joint* determination of the climate policy, the growth rate and the saving rate of the economy. If, for example, the saving rate or the growth rate are assumed to be constant and exogenous (as, e.g., in Golosov et al. 2014 and Barro 2015, respectively), then by construction they are independent of the economic and climatic characteristics and therefore fail to appropriately reflect their optimal counterparts. When the mitigation policy relies on these assumed

<sup>9</sup> The heterogeneity of risk attitudes is confirmed by Drichoutis and Koundouri (2012); the authors highlight the importance of adopting a proper characterization of risk attitudes when using experimental techniques.

<sup>10</sup> Bretschger and Vinogradova (2019) analyzed the optimal abatement policy in a simplified model which did not include any climate dynamics.

exogenous rates, it will deviate from the welfare-maximizing outcome. If the growth rate of consumption can endogenously respond to changes in climatic characteristics, let's say an increase in the frequency of shocks, the optimal climate policy may become less stringent than in the case with the fixed growth rate because the economy will optimally choose to grow at a lower rate. This is because the growth rate and the associated accumulation of productive capacity determine (i) the growth of the stock of GHG, (ii) security buffer in the event of a climatic hazard, i.e. how much loss the economy can withstand, and (iii) how much resources can be devoted to mitigation.

Third, and extending the contribution of Barrage (2014), we show analytically that when preferences are logarithmic, important links to climatic characteristics, including the arrival rate of disasters, efficiency of abatement technology and damage intensity, disappear from the optimal rules.

While the closed-form solution allows us to clearly disentangle the effect of each climatic and economic characteristic of the model, we are also interested in quantitative predictions with respect to the optimal policies. Numerous existing studies proposed required emissions-mitigation schemes (or a carbon tax) to ensure that the global temperature does not exceed a specific threshold. Other studies asked what the optimal carbon tax should be, given the various types of uncertainty associated with climate change (Gerlagh and Liski 2017; Golosov et al. 2014; Van der Ploeg 2014; Van der Ploeg and de Zeeuw 2018; Van Den Bremer and van der Ploeg 2018). Our investigation falls into the second category. One of the frequent messages delivered by the recent literature is that the utility discount rate appears to be the main driver of the optimal tax but not a possibility of a climate disaster.<sup>11</sup> This observation motivates us to quantitatively assess the impact of possible multiple and random climate shocks on the optimal growth rate, abatement propensity and, by extension, on the price of carbon.

We examine several scenarios based on alternative assumptions about abatement technology, risk aversion, size of damages and probability of occurrence. The gist of our findings is that it matters whether disasters are low-impact (causing a loss of, say, not more than 0.1% of GWP) occurring relatively frequently or high-impact (loss of up to 10% of GWP) even if they occur relatively rarely, say, less than once in a 100 years. On the one hand, low-impact shocks do not require a substantial increase in abatement efforts even with a relatively high RRA. On the other hand, we find that it is the simultaneous reaction of *both* abatement and growth which is decisive. The growth rate of the economy essentially works as a stabilizer of emissions and thus to some extent substitutes for the climate policy instrument. With a lower growth rate, less stringent abatement policy is needed. With high-impact shocks the abatement propensity increases almost 6-fold and the growth

<sup>11</sup> Golosov et al. (2014) calculate the optimal carbon tax to be \$25.3 and \$489 per ton carbon when the realized damages amount to 0.48% and 30% of GDP (based on Nordhaus and Boyer 2000), respectively, assuming a 1.5% discount rate and log-utility. With a much lower discount rate of 0.1% (advocated by Stern 2007), the tax jumps up 8–9 fold to \$221 and \$4,263, respectively. In another important contribution, Van der Ploeg (2014) considers a possibility of a climate catastrophe, such that a positive carbon feedback occurs at some random (and possibly endogenous) point in time. He finds that the optimal after-catastrophe carbon tax is \$29 and \$216 per ton carbon with the discount rate of 1.5% and 0.1%, respectively. The before-catastrophe tax is slightly lower at \$27.6 and \$198.6. When the hazard rate is endogenous, the tax is only slightly higher. One exception is Van den Bijgaart et al. (2016) who show in the context of a deterministic IAM that damage sensitivity plays just as important role as discounting in determining the social cost of carbon.

rate is cut by a factor of 3. The quantitative section also provides a comparison of our findings with those in the recent studies.

The remainder of the paper is organized as follows. In order to set the stage for our stochastic growth model with climate-driven disasters, we start by presenting in Sect. 2 a deterministic counterpart of this model and its full analytical dynamics. Then Sect. 3 develops our baseline stochastic framework. In Sect. 4, we present the main results with respect to the optimal growth rate, abatement, and saving propensity. Section 5 provides an extension of the baseline model by endogenizing the hazard rate. Section 6 offers quantitative implications and Sect. 7 concludes.

## 2 Deterministic Model

We start by presenting a deterministic model, similar to the one in Sect. 6 of Michel and Rotillon (1995), which we are going to use as our benchmark. We consider a global economy which produces a composite consumption good under constant returns to scale using as input broadly defined capital, denoted by  $K_t$ . The production process is polluting: every instant  $t$  a flow of greenhouse gas emissions,  $E_t$ , is released into the atmosphere. The stock of GHG, denoted by  $P_t$ , is thus augmented every instant by  $E_t$  and reduced by  $\alpha P_t$ , where  $\alpha \in [0, 1)$  represents the natural absorption rate of greenhouse gases (e.g., by biosphere or deep oceans) and is assumed to be very small or can be set to zero.<sup>12</sup> Cumulative emissions cause deterioration of the natural environment and an increase in the global temperature, resulting in a loss of welfare.<sup>13</sup> In our model, we refer to  $P$  as pollution stock, however, it can be interpreted more generally as the reciprocal of environmental quality. In the same vein, we refer to  $E$  as emissions but it can be also interpreted as any type of negative externality of production which harms the environment.

The output, denoted by  $Y_t(K_t)$ , can be either spent on consumption,  $C_t$ , or invested. There are two types of non-consumption spending: (i) investment to augment the capital stock and (ii) financing of emissions abatement. Specifically, we assume that an endogenous fraction,  $\theta_t$ , of output is spent on the latter, so that abatement expenditure is given by  $I_t = \theta_t Y_t$ . The remaining share  $(1 - \theta_t)Y_t$  is split between consumption and capital accumulation. Total abatement,  $Z(I_t)$ , is a positive function of the abatement expenditure,  $Z'(I_t) > 0$ . The total per period emissions are then given by emissions stemming from the

<sup>12</sup> Such parsimonious one-layer formulation of the carbon system has been widely used in the literature (Clarke and Reed 1994; Tsur and Zemel 1998; de Zeeuw and Zemel 2012; Van der Ploeg 2014; Van den Bijgaart et al. 2016; Van der Ploeg and de Zeeuw 2018). Integrated assessment models typically use a more elaborate representation of the carbon cycle, with atmosphere, upper ocean layers and deep oceans as the three main carbon reservoirs. Nordhaus and Boyer (2000) calibrate the transfer rate from the atmosphere to the upper ocean layer as 0.333 per decade and from the upper oceans to the deep oceans as 0.115 per decade, implying an indirect transfer rate from the atmosphere to deep oceans of approximately 0.0038 per year, which is proxied by  $\alpha$  in our model. Golosov et al. (2014) argue that a one-dimensional formulation of the carbon cycle can well approximate the cycle used in IAMs if the depreciation structure includes a term reflecting the share of a carbon unit which stays permanently in the atmosphere and another term with a geometric decay rate applied to the share of carbon which does not exit the atmosphere immediately.

<sup>13</sup> We do not explicitly model temperature increase as it is now widely confirmed by the Earth system models that warming is proportional to cumulative emissions and that temperature response to a  $CO_2$  emission is almost instantaneous and then constant as a function of time (see Dietz and Venmans 2019 and references therein).

economic activity minus abatement. We assume that one unit of output causes  $\phi$  units of GHG,<sup>14</sup> so that total emissions are given by  $E_t = \phi Y_t - Z(I_t)$ .

The planner's objective is to maximize the expected discounted utility over an infinite planning horizon with respect to consumption,  $C_t$ , and the share of output devoted to abatement,  $\theta_t$ , subject to the stochastic capital accumulation process and the dynamics of the cumulative emissions:

$$\max_{C_t, \theta_t} \left\{ \int_0^\infty U(C_t, P_t) e^{-\rho t} dt \right\} \tag{1}$$

s.t.

$$dK_t = [(1 - \theta_t)Y_t(K_t) - C_t]dt, \quad K_0 \text{ given}, \tag{2}$$

$$dP_t = (E_t - \alpha P_t)dt, \quad P_0 \text{ given}, \tag{3}$$

$$E_t = \phi Y_t(K_t) - Z(I_t), \tag{4}$$

$$I_t = \theta_t Y_t(K_t), \tag{5}$$

where  $\rho > 0$  is the constant rate of time preference. The utility function is twice continuously differentiable in both arguments. We also require that the capital stock, pollution stock, and consumption per unit of time are non-negative and  $\theta_t \in [0, \bar{\theta}]$ , where  $\phi/\sigma < \bar{\theta} < 1$ .

## 2.1 Assumptions

### 1. Production Technology

We assume constant returns to scale in aggregate production and as capital is the only input, output is produced with an  $AK$  technology, a frequently adopted specification in climate-economy models (see, e.g., Mueller-Fuerstenberger and Schumacher 2015). Parameter  $A$  denotes the constant factor productivity and  $K_t$  is interpreted as a broad measure of capital in the economy, including physical and human capital, intangibles, etc.<sup>15</sup>

$$Y_t = AK_t. \tag{6}$$

### 2. Abatement Technology

We also assume constant returns in abatement, i.e. total abatement is directly proportional to the resources allocated to emissions control, with the proportionality parameter  $\sigma > 0$  representing the efficiency of abatement technology:

$$Z(I_t) = \sigma I_t. \tag{7}$$

<sup>14</sup> With appropriate choice of units, polluting intensity  $\phi$  can be restricted to lie between zero and unity.

<sup>15</sup> Despite its formal simplicity, the  $AK$  model unites all the desirable properties of an aggregate production function in a dynamic climate model. It generates sustained growth endogenously, results in the same implications for investment and growth as if we included different capital components such as physical, human, and knowledge capital separately, and is consistent with the empirically observed strong positive relationship between investment rates and growth rates across countries and time periods (see McGrattan 1998).



### 3. Preferences

How to represent society's preferences over consumption and environmental quality has long been a cornerstone question for economists. Over the last few years of research three main trends have emerged. First, the non-expected utility and homothetic preferences of Epstein-Zin type (Epstein and Zin 1989) have gained momentum (Barro 2015; Lemoine and Traeger 2014; Van der Ploeg and de Zeeuw 2018; Van Den Bremer and van der Ploeg 2018, Douenne 2020) due to their ability to disentangle the elasticity of intertemporal substitution (EIS) from the relative risk aversion (RRA) coefficient.<sup>16</sup> Second, logarithmic preferences have been widely used due to their tractability (Golosov et al. 2014; Mueller-Fuerstenberger and Schumacher 2015). The third option, adopted here (as well as in Barrage (2014), Dietz and Venmans (2019), Van der Ploeg and de Zeeuw (2018)), is to assume a constant relative risk aversion (CRRA) utility function, also encompassing the logarithmic version as a special case. We assume that utility is additively-separable in consumption and environmental quality, with the latter being inversely related to the stock of GHG. Finally, the utility function is increasing and concave in consumption and decreasing and concave in the stock of GHG, exhibiting risk aversion with respect to both arguments:

$$U(C_t, P_t) = \frac{C_t^{1-\varepsilon}}{1-\varepsilon} - \chi \frac{P_t^{1+\beta}}{1+\beta}, \quad \varepsilon > 0, \beta > 0, \chi > 0 \quad (8)$$

where  $\chi$  is the relative weight of pollution. We will show in Sects. 3 and 5 the importance of considering a general CRRA structure, as opposed to its knife-edge case of log-utility, which ignores important effects in a climate-growth context.

### 2.2 Solution

We are going to distinguish three cases depending on the initial level of  $P_0$  and  $K_0$ . These initial conditions will determine the behavior of the optimal abatement, for which we will have a case with an interior solution and two border cases such that it is optimal to abate nothing or the maximum amount.

The current-value Hamiltonian associated with the optimization problem is given by

$$H = U(C_t, P_t) + \lambda_t[(1 - \theta_t)AK_t - C_t] + \mu_t[(\phi - \sigma\theta_t)AK_t - \alpha P_t]$$

and the associated Lagrangian by  $L = H + v_t(\bar{\theta} - \theta_t) + \nu_t\theta_t$ , where we explicitly attached a non-negativity constraint on  $\theta$  and a constraint specifying that  $\theta$  cannot exceed some maximally feasible abatement propensity  $\bar{\theta} < 1$ . The linearity of  $H$  in  $\theta$  makes the model analytically tractable and leads to the existence of the three above-mentioned abatement regimes:  $\theta = 0$ ,  $\theta = \bar{\theta}$  and an interior regime  $\theta = \theta^*$ , which we describe below. We show that maximal/zero abatement policy is applied for a finite time only and then the dynamics is switched to the interior regime, which is of our primal interest (we include border abatement regimes for completeness in the appendix). This is why we are going to focus only on the interior regime when we introduce our stochastic model in Sect. 3.

<sup>16</sup> This property, however, comes at a cost. It has been recently shown that Epstein-Zin preferences are non-monotonic with respect to the first-order stochastic dominance meaning that under some parameter constellations they lead to choices of dominated strategies (Bommier et al. 2017).

## 2.2.1 The Interior Solution

The dynamics of the economy in the interior regime are characterized by the following equations:

$$P_t = P_0 e^{-\frac{r-p}{\beta}t}. \quad (9)$$

$$K_t^* = \frac{c_0}{r-g} e^{gt} - \frac{P_0 \left( \alpha - \frac{\varepsilon g}{\beta} \right)}{\sigma \left( r + \frac{\varepsilon g}{\beta} \right)} e^{-\frac{\varepsilon g}{\beta}t} \quad (10)$$

$$\theta_t^* = \frac{\phi}{\sigma} + \frac{\left( \frac{\varepsilon g}{\beta} - \alpha \right) P_t}{\sigma A K_t^*}, \quad (11)$$

$$c_t = c_0 e^{gt}, \quad c_0 = \left( \frac{\sigma \chi P_0^\beta}{r + \alpha} \right)^{-\varepsilon} \quad (12)$$

where  $g = \frac{1}{\varepsilon}(r - \rho)$ , and it is assumed that  $g < r$ . Equation (11) describes the dynamics of the abatement propensity. We would like to make two comments on these dynamics. First, the abatement propensity converges to  $\frac{\phi}{\sigma}$  in the long run. This is because pollution declines at the rate  $g_p \equiv \frac{\varepsilon g}{\beta}$  over time and thus converges to zero in the long run. Second, if the natural decay rate is relatively large, in particular if it is larger than the rate of pollution decline, i.e.  $\alpha > \frac{\varepsilon g}{\beta}$ , then the abatement propensity converges to its limit from below. The intuition is that a large natural decay “helps” the economy to clean up emissions and while the pollution stock is large, this effect is also large. However, as the pollution stock falls over time, the natural pollution removal also becomes smaller. Thus, initially the abatement propensity is below the zero-emissions threshold  $\frac{\phi}{\sigma}$  and it increases over time as pollution stock falls. If the natural decay is relatively small, in particular it is smaller than the rate of pollution decline, i.e.  $\alpha < \frac{\varepsilon g}{\beta}$ , then the opposite occurs. The abatement propensity converges to the threshold  $\frac{\phi}{\sigma}$  from above. The intuition is that with a small  $\alpha$  (think about zero natural decay as an extreme case) all of the pollution removal takes place through abatement. Hence, abatement in a way needs to “compensate” for the small (or absence of) natural decay and thus initially exceeds  $\frac{\phi}{\sigma}$ : the second term in (11) is positive. As pollution stock falls over time, however, the need for compensation falls as well and as pollution converges to zero, so does the second term in (11). This peculiar scenario can be thought of a situation where not only all the current emissions are abated but some of the existing pollution is removed as well, which would correspond to “improving” environmental quality. Given the peculiarity of this case<sup>17</sup> we are going to focus only on parameter constellations such that  $\alpha > \frac{\varepsilon g}{\beta}$ , so that abatement is initially below (or equal to) the zero-emissions threshold.

Overall, the dynamics of this economy can be summarized as follows. Consumption grows over time at the rate  $g$ , assumed positive. Capital stock grows at the rate smaller than

<sup>17</sup> We have not seen such a case in reality yet, although it is not unthinkable, given the developments in the carbon capture and geoengineering technologies.

$g$  but its growth rate converges to  $g$  in the long run. Pollution falls over time at the rate  $\frac{\epsilon g}{\beta}$  and converges to zero in the long run. Note that the dynamics of pollution are governed by the structure of preferences, which is also visible in (9). Optimal abatement propensity is such that it converges to the zero-emissions threshold  $\frac{\phi}{\sigma}$  in the long run. The convergence path, however, depends on the parameters of the model, in particular on the relationship between the natural decay rate,  $\alpha$ , and the speed of pollution decline,  $\frac{\epsilon g}{\beta}$ . If the environment is relatively efficient (resp., inefficient) in removing pollutants, then the optimal abatement starts below (resp., above) the zero-emissions threshold. Note that in the interior regime, the initial capital stock and the initial pollution stock are linked by (10). It is, of course, only by chance that the two initial stocks align precisely to satisfy this relationship. When they do not, the economy finds itself in one of the border-control regimes (described in detail in the appendix) and eventually converges to the interior regime, as shown in subsection 2.3 and illustrated in Fig. 1.

Now that we have solved for the full dynamics of the economy in the interior regime, we can derive the explicit value function associated with the interior solution. From (10), we can express consumption as a function of the two state variables and therefore we can write the value function as

$$V(K, P) = \frac{(aK + bP)^{1-\epsilon}}{1-\epsilon} - x \frac{P^{1+\beta}}{1+\beta},$$

where the constants are given by  $a = (r - g)^{\frac{1-\epsilon}{\epsilon}}$ ,  $b = \frac{\alpha(\alpha - g_p)}{r + g_p}$ ,  $x = \frac{\beta \chi}{(1+\beta)r - \rho}$ . Knowing the shape of the value function will prove particularly useful when we introduce the stochastic case. In particular, we are going to be interested in the long-run (or asymptotic) value function such that  $t \rightarrow \infty$ . We provide a derivation of this function in the appendix and proceed by discussing the optimal trajectory under alternative values of  $K_0$  and  $P_0$ .

### 2.3 Optimal Trajectory with Different Initial Conditions

Equations (9) and (10) define the trajectory  $S_{int}$  in the state-space, illustrated in Fig. 1 by the blue solid line.

This line splits the state space in two regions. In the region above  $S_{int}$ , the maximum abatement is applied, while in the region below  $S_{int}$  zero abatement is chosen. The initial conditions determine the initial position of the economy in the state space and consequently which control from the set  $\{0, \theta_t^*, \bar{\theta}\}$  is applied from the outset. The blue line thus corresponds to the set of "interior initial conditions"  $(K_0, P_0) = (K_0, P(K_0))$  for which interior controls are applied  $\forall t > 0$ . The state vector  $(K_t, P_t)$ , with  $(K_0, P_0)$  on  $S_{int}$ , at the optimal trajectory  $(K_t, P_t, C_t, \theta_t)$ , moves from left to right along the blue solid line. For the initial conditions  $(K_0, P_0)$  not on  $S_{int}$ , such as those marked by  $(K_0, P_0)'$  and  $(K_0, P_0)''$ , optimal trajectory requires switching at some future time ( $c$ -control also switches). For instance, if the initial conditions are given by  $(K_0, P_0)''$ , the optimal trajectory is such that no abatement takes place for some finite time  $\tilde{T}$ , while pollution and capital stock both increase, as shown by the dashed line. For  $(K_0, P_0)$  above the blue line, such as for example  $(K_0, P_0)'$ , the optimal trajectory, shown by the dash-dotted curve, is such that pollution declines faster than under "interior trajectory" and it converges to the blue solid line after some time  $T$ . In

the long run, the economy converges to the interior path regardless of which path it started on initially.

### 3 Model with Climate Shocks

#### 3.1 Baseline Model

The stochastic model that we present below is a modification of the deterministic model of the previous section. We continue to assume that the production process is polluting. In addition, we are going to assume that cumulative emissions cause deterioration of the natural environment and an increase in the global temperature, leading to a random occurrence of natural disasters.<sup>18</sup> We assume that an arrival of a natural disaster (we shall also refer to it as an "event") follows the Poisson process with the mean arrival rate  $\lambda$ . In the next section we extend our model to include endogenous arrivals, while for the moment we assume that  $\lambda$  is constant. When an event occurs, an endogenously-determined amount of the existing capital stock is destroyed. We denote by  $\omega_t(P_t, K_t) \in (0, 1)$  the fraction of capital which survives the shock. In fact, recent floods, as the one in Pakistan in 2010 or in the Philippines in 2013, had a profound effect on infrastructure and the capital stock (both physical and human). According to the predictions of climate sciences, the magnitude of the damage is likely to increase in the future due to climate change and hence we model it as a positive function of the stock of GHG or, equivalently, the higher is the pollution stock the lower is the after-shock share of survived capital,  $\frac{\partial \omega_t}{\partial P_t} < 0$ . In addition, we assume that the damages are larger, the larger is the amount of capital exposed to destruction, i.e.  $\frac{\partial \omega_t}{\partial K_t} < 0$ , capturing the notion of the value at risk (Bouwer 2011). In particular, we assume that the survived share of capital,  $\omega_t$ , can be represented by a function of a single argument,  $v_t$ , which depends on  $P_t$  and  $K_t$  such that  $v_t = P_t^\xi K_t^\eta$ , where the constants  $\xi > 0$  and  $\eta > 0$  govern the relative importance of the climate change component and the exposure component, respectively. We choose a negative exponential function for  $\omega$ , so that  $\omega(v_t) = e^{-\delta v_t} \in (0, 1)$ , where the parameter  $\delta > 0$  can be interpreted as the damage intensity.<sup>19</sup> This specification has several desirable characteristics. First, it ensures that  $\omega$  is indeed a fraction, i.e. lies between zero and unity. Second,  $\omega$  is decreasing in  $P$  and  $K$ , i.e.  $\frac{\partial \omega}{\partial P} = \xi \omega \ln \omega / P < 0$ ,  $\frac{\partial \omega}{\partial K} = \eta \omega \ln \omega / K < 0$  to capture the effect of climate change and the capital exposure, respectively. In general, any function satisfying these properties would be suitable for describing  $\omega$ . Note that when  $\xi = \eta = 0$  the model reduces to a special case with constant damages.

Recent empirical literature on attitude to risk in the presence of disasters has largely confirmed the "risk vulnerability" hypothesis first proposed by Gollier and Pratt (1996). The hypothesis states that agents operating in risky environments characterized by a possibility of a loss on average, i.e. agents who are exposed to unfair risks, tend to exhibit a

<sup>18</sup> We do not explicitly model temperature increase as it is now widely confirmed by the Earth system models that warming is proportional to cumulative emissions and that temperature response to a  $CO_2$  emission is almost instantaneous and then constant as a function of time (see Dietz and Venmans 2019 and references therein).

<sup>19</sup> Exponential damages, although without the exposure component, are commonly used in the literature, e.g. Golosov et al. (2014), Dietz and Venmans (2019), Van der Ploeg and de Zeeuw (2018).

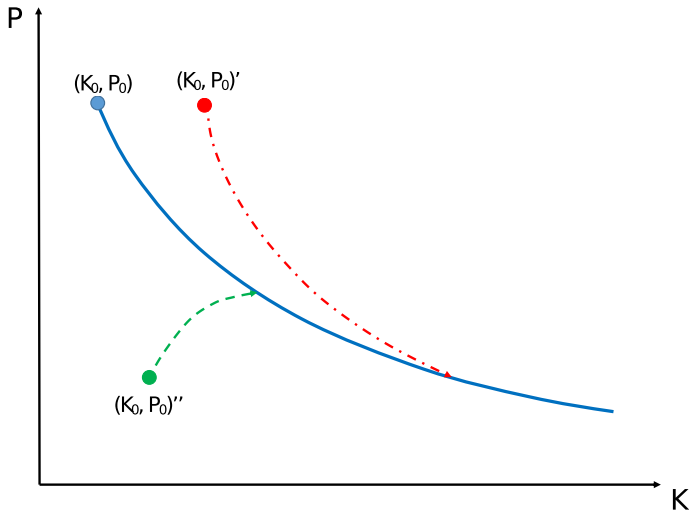


Fig. 1 Optimal paths for different initial conditions

more risk-averse behavior than otherwise (see, e.g. Harrison et al. 2007; Guiso and Paiella 2008; Cameron and Shah 2015). Moreover, risk aversion also increases in the magnitude of damages (Cameron and Shah 2015). We include this property in our model by linking the parameters of the damage function to the parameters governing attitude to risk in the utility function:  $\epsilon = \epsilon(\eta)$  and  $\beta = \beta(\xi)$  with  $\epsilon'(\eta) = \beta'(\xi) = \gamma > 0$ , i.e. the mappings are linear with equal slope such that the relative importance of the climate change component and the exposure component is preserved. This property will prove useful in obtaining explicit analytical solution. In the rest of the analysis we shall assume that  $\gamma$  is equal to unity. This is done for simplicity, as the magnitude of this parameter does not have any qualitative effects on the results. In the numerical part of the paper we do consider alternative values of  $\gamma$  as robustness checks.

The planner's objective is to maximize the expected discounted utility over an infinite planning horizon with respect to consumption,  $C_t$ , and the share of output devoted to abatement,  $\theta_t$ , subject to the stochastic capital accumulation process and the dynamics of the cumulative emissions:

$$\max_{C_t, \theta_t} \mathbb{E}_0 \left\{ \int_0^\infty U(C_t, P_t) e^{-\rho t} dt \right\} \tag{13}$$

s.t.

$$dK_t = [(1 - \theta_t)Y_t(K_t) - C_t]dt - [1 - \omega_t(P_t, K_t)]K_t dq_t, \quad K_0 \text{ given}, \tag{14}$$

$$dP_t = (E_t - \alpha P_t)dt, \quad P_0 \text{ given}, \tag{15}$$

$$E_t = \phi Y_t(K_t) - Z(I_t), \tag{16}$$

$$I_t = \theta_t Y_t(K_t), \tag{17}$$

where  $\mathbb{E}_0$  is the expectations operator,  $dq_t$  is an increment of the Poisson process, and  $\rho > 0$  is the constant rate of time preference. The utility function is twice continuously differentiable in both arguments. We also require that the capital stock, pollution stock, consumption and emissions rates are non-negative and  $\theta_t \in [0, 1)$ . In what follows, we only describe the solution on the interior path.

### 3.2 Solution

Denoting by  $V(K, P)$  the value function associated with the interior solution to the optimization problem described in (13)–(17), the Hamilton-Jacobi-Bellman (HJB) equation may be written as

$$\rho V(K, P) = \max_{C, \theta} \left\{ U(C, P) + V_k[(1 - \theta)Y - C] + V_p[(\phi - \theta\sigma)Y - \alpha P] + \lambda[V(\tilde{K}, P) - V(K, P)] \right\}, \tag{18}$$

where  $V_k \equiv \partial V(K, P)/\partial K$ ,  $V_p \equiv \partial V(K, P)/\partial P$  and the new capital stock  $\tilde{K} = \omega(P, K)K$ . In what follows, the subscripts  $c, k, p$  refer to partial derivatives with respect to  $C, K, P$ , while we shall always use the subscript  $t$  to indicate that a variable is time-dependent. We relegate the detailed derivations to the Appendix A.1, while focus only on the key results in the main text. The solution method is largely based on Sennewald and Waelde (2006), however we are going to study only the long-run behavior. The asymptotic value function can be found explicitly as (see appendix)

$$V(K_t, P_t) = \frac{\psi^{-\varepsilon} K_t^{1-\varepsilon}}{1 - \varepsilon} - \frac{x P_t^{1+\beta}}{1 + \beta}.$$

The coefficients  $\psi$  and  $x$  depend on the parameters of the model and can be found by substituting optimal controls into the HJB:

$$\varepsilon \psi^* - \rho + (1 - \varepsilon)A \left( 1 - \frac{\phi}{\sigma} \right) + \lambda [(\omega(\psi^*))^{1-\varepsilon} - 1] = 0, \tag{19}$$

where  $\omega(\psi, x) = e^{-\delta[\sigma x \psi^\varepsilon]^{-1/\gamma}}$  and

$$x = \frac{\chi}{\rho + \alpha(1 + \beta)}. \tag{20}$$

The coefficient  $\psi$  is important for our further analysis since it represents the optimal ratio of consumption to capital.

In the next step, we obtain the stochastic consumption growth by applying the Ito’s formula for jump processes:

$$\frac{dC}{C} = \frac{1}{\varepsilon} \left\{ A \left( 1 - \frac{\phi}{\sigma} \right) - \rho + \lambda [\omega^{1-\varepsilon} (1 + \eta \ln \omega) - 1] \right\} dt + (\omega - 1) dq. \tag{21}$$

Given that  $\tilde{K} = \omega K$ , while  $C = \psi^* K$  and  $\tilde{C} = \psi^* \tilde{K}$ ,  $\omega$  also represents the ratio of post- to pre-shock consumption rates. Therefore, the last term on the RHS of (21) is negative, reflecting the downward jump in consumption at the time of a disaster. The first term on the RHS, multiplying  $dt$ , represents what we label as "trend" consumption growth rate,  $g^*$ , which prevails in-between climate shocks:

$$g^* \equiv \frac{1}{\varepsilon} \left\{ \underbrace{A \left( 1 - \frac{\phi}{\sigma} \right) - \rho}_{\text{KR term}} + \underbrace{\lambda(\zeta - 1)}_{\text{precautionary effect}} \right\}, \quad (22)$$

where  $\zeta = \omega^{1-\varepsilon}(1 + \eta \ln \omega)$ . The expression has a familiar Keynes–Ramsey form albeit with some additional elements. The standard, deterministic Keynes–Ramsey formula states that the growth rate of consumption equals the difference between the real interest rate (usually the marginal product of capital) and the rate of pure time preference, adjusted by the elasticity of intertemporal consumption substitution. First, note that in Eq. (22) the economy's implicit real interest rate, given by the first term inside the parentheses, is not equal to just the marginal productivity of capital but is reduced by the emission intensity of output, adjusted by the abatement efficiency, i.e., the term  $\phi/\sigma$ . It follows that in our framework pollution has an unambiguously negative effect on the real interest rate. It may be dampened by either increasing the abatement efficiency,  $\sigma$ , or decreasing the polluting intensity,  $\phi$ .

Second, expression in (22) accounts for the effect of uncertainty, represented by the last term  $\lambda(\zeta - 1)$ , labeled as "precautionary effect", which includes the disaster hazard rate ( $\lambda$ ), the exposure effect ( $-1$ ) and the pollution-stock effect ( $\zeta$ ). We are especially interested in the magnitude of  $\zeta$  relative to unity. If  $\zeta > 1$ , then the presence of uncertainty speeds up consumption growth. In this case, the optimal stochastic consumption path is tilted counterclockwise, as compared to the consumption path in a deterministic Keynes–Ramsey model. Therefore, the economy starts with a relatively low consumption rate at the beginning of the planning horizon, which implies the presence of the precautionary-saving motive, including saving for financing of emissions control.<sup>20</sup> By contrast, when  $\zeta < 1$ , the risky environment contributes to a growth slowdown, tilts the consumption profile clockwise and thus implies a precautionary *dissaving* motive. The peculiarity of the current setting is that the gross savings are endogenously split between two purposes: capital accumulation and abatement, both of which serve to protect the economy from climate disasters. It is clear that abatement reduces emissions and therefore unambiguously contributes to a reduction in damages. Capital accumulation, however, has a double-sided effect. On the one hand, more capital implies more output and more emissions. On the other hand, having more capital creates an "emergency buffer" for the rainy days—when a disaster strikes. We discuss in more detail the economy's optimal saving rate and how it is affected by climatic parameters in Sect. 4.3. For the moment we shall focus our attention on the magnitude of the precautionary effect and its relevance for the optimal growth rate.

Note that  $\zeta = \omega^{1-\varepsilon}(1 + \eta \ln \omega) \geq 1$  is composed of two terms. First, the ratio of post-to-pre shock marginal utilities of consumption,  $U'(\tilde{C})/U'(C) = \omega^{-\varepsilon}$ , which is unambiguously larger than unity since  $\tilde{C} < C$ . Second, the term  $\omega(1 + \eta \ln \omega)$ , which represents the reaction of the post-shock capital stock to a change in the pre-shock capital stock.

We provide a detailed analysis in the appendix, while here we summarize the gist of the results. If  $\varepsilon \in (0, 1)$ ,  $\zeta$  is unambiguously less than unity, while if  $\varepsilon > 1$ , a priori an

<sup>20</sup> The outcome is analogous to what has been found in the literature on precautionary savings under uncertainty in other contexts (see, e.g., Waelde 1999, Steger 2005).

ambiguity exists. We show, however, that this ambiguity can be resolved and  $\zeta$  is less than unity for the relevant range of parameters (see Fig. 3 in Appendix A.2). We conclude that  $(\zeta - 1)$  is negative and hence the presence of uncertainty contributes to a *lower* consumption growth rate than in the deterministic model. This finding points to the presence of a specific type of consumption "smoothing" such that the economy aims at reducing the slope of its consumption profile and at controlling the size of consumption jumps, in fact the percentage drop is stabilized at a constant value in the long run. By reducing the slope and by stabilizing the jumps, the economy achieves the optimal consumption trajectory which is "smoothed out" as much as possible, leaving only the unpredictable component—the timing of jumps - affect its evolution. Even though a somewhat smoother profile can be implemented, *perfect* consumption smoothing is not achievable because of random *timing* of events.

The smoothing effect can be confirmed by comparing the optimal consumption growth rate of our economy with the one prevailing in a stochastic environment but with shocks of exogenous size. Suppose that random climate disasters arrive at the same Poisson rate but the survived share of the capital stock is exogenous and constant at  $\check{\omega} \in (0, 1)$ . Assume further that the exogenous percentage drop in the capital stock is the same as the optimal one of our baseline model, so that  $\check{\omega} = \omega$ . Then the trend growth rate of consumption is given by  $\check{g} = \frac{1}{\varepsilon} \left\{ A \left( 1 - \frac{\phi}{\sigma} \right) - \rho + \lambda \left[ \frac{U'(\check{C})}{U'(C)} - 1 \right] \right\}$ , where the ratio of marginal utilities is simply  $\omega^{-\varepsilon} > 1$ . It follows immediately that  $\check{g} > g^*$ . Even though the drops in consumption are identical under both scenarios, the optimal consumption path associated with  $\check{g}$  is steeper than that associated with  $g^*$ . The reason is that in the former case the economy, being unable to affect the size of the damage, is forced to choose a relatively high growth rate in order to build up an emergency buffer. In the latter case the economy endogenously controls the size of the jump and simultaneously chooses a lower trend rate as a precautionary measure. Such a growth path of the economy is also fundamentally different from its "expected" version which may be interpreted as a hypothetical extreme case of consumption smoothing. The expected growth rate in our baseline model, denoted by  $g^e$ , is given by

$$g^e \equiv \frac{d\mathbb{E}_t C_t / dt}{C_t} = \frac{1}{\varepsilon} \left\{ A \left( 1 - \frac{\phi}{\sigma} \right) - \rho + \lambda [\omega^{1-\varepsilon} (1 + \varepsilon \ln \omega) - 1 + \varepsilon(\omega - 1)] \right\}. \tag{23}$$

We are now in a position to state the following

**Lemma 1** *The solution to the maximization problem (13) - (8) is characterized by the optimal trend consumption growth rate which is (i) lower than that of the deterministic model without climate change induced disasters; (ii) lower than that of a stochastic model with fixed-damage disasters; but (iii) higher than the expected growth rate.*

**Proof** Parts (i) and (ii) follow from the discussion above. Part (iii) follows from the fact that since  $\omega < 1$ , the last term inside the square brackets in (23) is negative and thus  $g^e < g^*$ . □

As an illustration of the optimal path in the baseline model, we show in Fig. 2 the consumption rate as a function of time. The solid line represents the stochastic path, which exhibits a trend growth rate  $g^*$  in the absence of climate events. At times  $t_1$  and  $t_2$ , environmental shocks are assumed to occur causing an immediate downward jump, followed by a subsequent period of growth at the previous rate. The dashed line shows



the time profile of consumption under the *expected* growth scenario. There is a fundamental difference between the dashed and the solid curves in that the former smoothes out the jumps and discontinuities of the latter, creating an illusion of a perfect consumption smoothing and thereby ignoring the crucial effects of uncertainty.

We now turn to the optimal abatement policy. How much of the current resources to devote to emissions control is a key policy question. We show in the Appendix that the optimal abatement propensity on the interior path is given by

$$\theta^* = \frac{\phi}{\sigma} - \frac{\xi \lambda \omega^{1-\epsilon} \ln \omega}{\beta A}. \tag{24}$$

The first term on the RHS represents the zero-emissions threshold, which is the optimal abatement propensity of the deterministic model in the long run. We see that the presence of the stochastic climate shocks induces a larger long-run abatement propensity. The extra abatement is reflected in the second term in (24), which is positive since  $\ln \omega < 0$ . Clearly, in the absence of random disasters, i.e. if  $\lambda = 0$ , this term vanishes.

**Proposition 1** *The long-run solution of the maximization problem described by (13) - (8) is characterized by the following:*

- (i) *optimal consumption rate is a constant fraction of the capital stock;*
- (ii) *optimal abatement expenditure is a constant fraction of output;*
- (iii) *consumption, capital stock, output, and the overall abatement grow at the same constant rate in-between climate shocks.*

**Proof** provided in the Appendix. □

**Corollary 1** *In the case of logarithmic utility long-run consumption-to-capital ratio,  $\psi^*$ , is independent of climate parameters.*

*With log utility ( $\epsilon = 1$ ), the long-run value of  $\psi^*$  can be obtained explicitly and after substitution into (22) and (24) we obtain:*

$$\psi^* = \rho, \quad \theta^* = \frac{\delta \lambda}{\rho A x \sigma} + \frac{\phi}{\sigma}, \quad g^* = A(1 - \theta^*) - \rho.$$

*Since  $\psi^*$  does not respond to any climatic parameters, the post-to-pre shock consumption ratio,  $\omega$ , is independent of the hazard rate. Consequently, the optimal growth rate responds to  $\lambda$  only through its direct effect,  $\partial g / \partial \lambda$  but not through the jump effect,  $(\partial g / \partial \omega)(\partial \omega / \partial \psi)(\partial \psi / \partial \lambda)$ . The direct effect reflects the precautionary "dissaving" motive and is negative (equal to  $-\frac{\delta}{x \sigma \rho}$ ), contributing to a growth slowdown, while the indirect effect may either reinforce the direct effect or counteract it, depending on the value of  $\epsilon$ . When log utility is assumed, the latter element disappears from the general picture. This observation motivates us to depart from the log-utility case in the quantitative assessment of the model presented in Sect. 5. One of our questions of interest will be to quantify the discrepancy in the optimal policy rules under unitary and alternative values of  $\epsilon$ .*

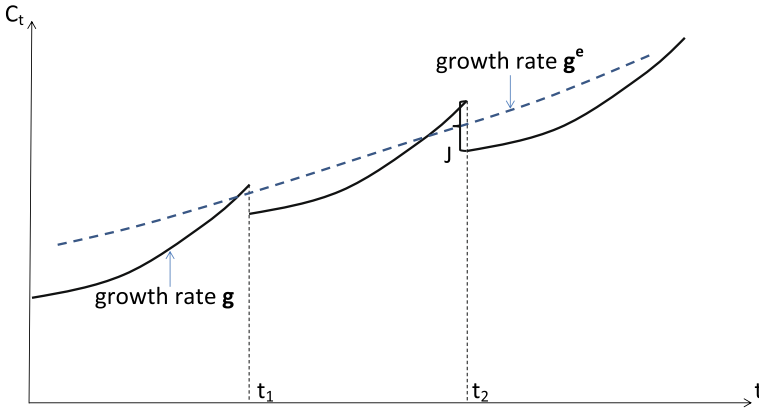


Fig. 2 Optimal path (solid) versus expected path (dashed)

## 4 Effects of Economic and Climatic Fundamentals

### 4.1 The Abatement Propensity

In order to study the responses of the economy’s optimal long-run abatement policy, characterized by  $\theta^*$ , to changes in the fundamental parameters of the model, we totally differentiate the system of Eqs. (19) and (24). Our key "climatic" parameters of interest are  $\lambda$  and  $\delta$  which reflect the expected frequency and damage intensity of climate shocks, respectively. Among the "economic" parameters we consider  $\sigma$ , representing the efficiency of abatement technology, and  $\phi$ , representing polluting intensity of production. The system becomes

$$\underbrace{\begin{pmatrix} \epsilon [1 - \lambda(1 - \epsilon)\omega^{1-\epsilon}\psi^{-1} \ln \omega] & 0 \\ \epsilon\omega^{1-\epsilon}\psi^{-\epsilon-1}\lambda\delta[1 + (1 - \epsilon) \ln \omega] & xA\sigma \end{pmatrix}}_M \times \underbrace{\begin{pmatrix} d\psi \\ d\theta \end{pmatrix}}_z = \underbrace{\begin{pmatrix} \Delta_{\psi\lambda} & \Delta_{\psi\delta} & \Delta_{\psi\sigma} & \Delta_{\psi\phi} \\ \Delta_{\theta\lambda} & \Delta_{\theta\delta} & \Delta_{\theta\sigma} & \Delta_{\theta\phi} \end{pmatrix}}_{\Delta} \times \underbrace{\begin{pmatrix} d\lambda \\ d\delta \\ d\sigma \\ d\phi \end{pmatrix}}_y,$$

where the exact expressions for  $\Delta$ 's can be found in the Appendix. We can compactly rewrite the system as  $Mz = \Delta y$ . It is useful for future analysis to find the determinant of  $M$ :

$$|M| = M_{11}M_{22} = \epsilon [1 + \lambda(1 - \epsilon)\omega^{1-\epsilon}\psi^{-\epsilon-1}\delta(x\sigma)^{-1}]xA\sigma = \epsilon xA\sigma \left[ 1 - \frac{\lambda(1 - \epsilon)\omega^{1-\epsilon} \ln \omega}{\psi} \right].$$

Since  $M_{22}$  is unambiguously positive, the sign of  $|M|$  hinges on the sign of  $M_{11}$ , which is in general ambiguous. If  $\epsilon \in (0, 1]$ ,  $M_{11}$  is positive, while if  $\epsilon > 1$ ,  $M_{11}$  may become negative under some constellations of parameter values, in particular, if  $\epsilon > \epsilon^{M_{11}} \equiv 1 - \frac{\psi\omega^{\epsilon-1}}{\lambda \ln \omega} > 1$ . If  $\epsilon$  is viewed as a parameter reflecting opinions and objectives of policymakers, its estimates may range from 1 to 3. In what follows we shall focus primarily on this empirically-relevant range of  $\epsilon$ . In order to get a rough idea whether  $\epsilon^{M_{11}}$  is anywhere near the empirically relevant range, we compute it assuming a plausible range of values for  $\lambda$ ,  $\omega$  and  $\psi$ . It turns out (see Appendix) that  $\epsilon^{M_{11}}$  is beyond the empirically-relevant range of  $\epsilon$  and thus we shall exclude the case  $M_{11} < 0$ .

For the moment we are interested in the effects of the arrival rate ( $\lambda$ ), damage intensity ( $\delta$ ), abatement efficiency ( $\sigma$ ), and polluting intensity ( $\phi$ ). The effects of the remaining parameters can be computed in a similar manner. Applying the Cramer's rule, we have:  $\frac{d\theta}{di} = \frac{|M_{\theta i}|}{|M|}$ , where  $i = \lambda, \delta, \sigma, \phi$ , and  $M_{\theta i}$  is the matrix  $M$  with the second column replaced by the column of  $\Delta$  corresponding to  $i$ . Similarly,  $\frac{d\psi}{di} = \frac{|M_{\psi i}|}{|M|}$ , where  $M_{\psi i}$  is the matrix  $M$  with the first column replaced by the column of  $\Delta$  corresponding to  $i$ .

**Proposition 2** *The long-run abatement propensity is:*

- (i) *an increasing function of the arrival rate,*
- (ii) *an increasing function of the damage intensity,*
- (iii) *a decreasing function of the abatement efficiency and*
- (iv) *an increasing function of the polluting intensity.*

**Proof** provided in the Appendix. □

**Corollary 2** *In the case of logarithmic utility function ( $\varepsilon = 1$ ), the abatement propensity is an increasing linear function of the hazard rate, of the damage intensity, and of the polluting intensity, and a decreasing convex function of the abatement efficiency.*

**Proof** provided in the Appendix. □

## 4.2 The Growth Rate

The effects on the growth rate are found as  $\frac{dg^*}{di} = \frac{\partial g^*}{\partial i} + \frac{\partial g^*}{\partial \omega} \frac{\partial \omega}{\partial \psi} \frac{d\psi}{di}$ .

**Proposition 3** *The optimal trend consumption growth rate is:*

- (i) *a decreasing function of the arrival rate,*
- (ii) *a decreasing function of the damage intensity,*
- (iii) *an increasing function of the abatement efficiency and*
- (iv) *a decreasing function of the polluting intensity.*

**Proof** provided in the Appendix. □

While the results (i), (ii), and (iv) are self-explanatory, the statement in (iii) deserves a short interpretation. In general, the effect of the abatement efficiency on  $g^*$  is ambiguous and consists of two terms (see Eq. (D.10)). The first term is clearly positive, while the sign of the second term depends on  $\varepsilon$ . If  $\varepsilon = 1$  it vanishes and if  $\varepsilon > 1$  it is unambiguously positive. The fact that a higher abatement efficiency has an ambiguous bearing on economic growth is due to two effects—the interest-rate effect and the jump-smoothing effect—which work in opposite directions. On the one hand, an improvement in efficiency of abatement increases the economy's real interest rate and thus enhances the growth rate through the first term in Eq. (22). On the other hand, it increases the post-event consumption rate, shrinking the pre- to post-event consumption gap and thus contributes to a growth

slowdown through the last term in Eq. ( 22). It turns out that with a relatively high RRA the latter effect is dominated by the former.

Polluting intensity also affects the growth rate through two channels. The first represents the direct effect stemming from a decline in the real interest rate. The second, the indirect effect, takes into account the change in the consumption jump,  $\omega$ , through  $\psi$ . A higher polluting intensity requires a higher abatement share (Proposition 2(iv)) and thus it reduces the share of consumption in total capital. Both direct and indirect effects are negative and contribute to a growth slowdown.

**Corollary 3** *In the case of logarithmic utility function the optimal trend consumption growth rate is a decreasing linear function of the damage intensity, of the arrival rate, of the polluting intensity, and an increasing concave function of the abatement efficiency.*

**Proof** provided in the Appendix. □

### 4.3 Saving Propensity

We define the propensity to save,  $s$ , as the non-consumption share of output, i.e. a share of output spent on augmenting the capital stock plus on abatement (the latter being equal to  $\theta^*$ ):  $s^* = 1 - \psi^*/A$ . It follows that the effects of all our parameters of interest ( $\lambda, \delta, \sigma, \phi$ ) on  $s^*$  are simply the opposite of those on  $\psi^*$ , divided by  $A > 0$ .

**Proposition 4** *The long-run propensity to save is:*

- (i) *an increasing function of the arrival rate,*
- (ii) *an increasing function of the damage intensity,*
- (iii) *a decreasing function of the abatement efficiency and*
- (iv) *an increasing function of the polluting intensity.*

**Proof** provided in the Appendix. □

**Corollary 4** *In the case of logarithmic utility function, the saving propensity is independent of the arrival rate, damage intensity, abatement efficiency and polluting intensity.*

**Proof** provided in the Appendix.

In the case of log utility, all the parameters of interest lose their relevance (the derivatives of  $\psi$  and thus  $s$  become zero). An increase in the abatement efficiency  $\sigma$  causes a decrease in  $\theta^*$ , implying that with unchanged  $s^*$  the share of output devoted to augmenting the capital stock increases. A better abatement technology in an economy with logarithmic preferences results in a smaller abatement share but a larger capital stock. On the other hand, an increase in the polluting intensity causes  $\theta^*$  to rise, while  $s^*$  remains unchanged, implying that capital investment must fall. The same is true for an increase in the arrival rate of disasters and their damage intensity.

Note that when  $\varepsilon \neq 1$  these effects might be mitigated because of the impacts on  $s^*$  which may go in the same direction as those on  $\theta^*$ . For instance, an increase in the polluting intensity increases both  $s^*$  and  $\theta^*$ . It can be shown that an increase in  $s^*$  is smaller

than that in  $\theta^*$ , implying that the abatement propensity rises at the expense of the investment share, which falls. The reduction in capital investment is, however, smaller than under logarithmic preferences.

The key endogenous variables,  $\theta^*$ ,  $g^*$  and  $s^*$ , respond non-linearly to changes in the economic and climatic fundamentals when we depart from log utility, although the direction of the effects is intuitively clear. A worsening in the shock's characteristics, i.e. an increase in expected frequency and/or damage intensity, increases  $\theta^*$  and  $s^*$  and reduces  $g^*$ . An improvement in abatement efficiency or a reduction in polluting intensity decreases  $\theta^*$ ,  $s^*$ , and raises  $g^*$ . The non-linearities in responses are important for a quantitative assessment of the optimal policy which we present in Sect. 5.

## 5 Endogenous Arrivals

We have mentioned in the introductory section that climate scientists have diverging opinions on whether the frequency of natural disasters will increase in the future due to global warming or not. The IPCC (2014) explicitly states that such a possibility exists. Recent contributions by Van der Ploeg (2014), van der Ploeg and de Zeeuw (2018), and Zemel (2015) model the hazard rate endogenously, as an increasing function of the stock of carbon in the atmosphere.<sup>21</sup> In this section we explore the implications of introducing an endogenous disaster arrival rate in our benchmark model. For the moment we shall write a general function  $\lambda = \lambda(P_t, K_t)$  with both partial derivatives being positive:  $\lambda_k \equiv \partial\lambda/\partial K > 0$ ,  $\lambda_p \equiv \partial\lambda/\partial P > 0$  to capture the climate change and the exposure effects.<sup>22</sup> We subsequently specify a possible functional form for this relationship.

The HJB equation of the problem is similar to (18), except that now the arrival rate depends on the stock of carbon and on the stock of capital. The optimality conditions with respect to consumption and abatement propensity remain unchanged. Only the conditions with respect to  $K$  and  $P$  are augmented by a term representing the change in the value function due to a change in the arrival rate. The exact expressions can be found in the Appendix.

In order to make further progress, we need to specify how the arrival rate of climatic hazards is affected by economic activity. We assume that the arrival rate is an increasing and possibly non-linear function of the capital and the pollution stocks, which capture the exposure and the climate-change effects, respectively. Similarly to how we modeled

<sup>21</sup> Using the Duffie-Epstein stochastic differential utility framework, van der Ploeg and de Zeeuw (2018) find that the optimal carbon price increases in the face of a pending catastrophe to make the shock less imminent and that adjustments to saving are needed to smooth consumption. They also confirm numerically that assuming a RRA which is different from the inverse of the elasticity of intertemporal consumption substitution does not affect the results significantly. Zemel (2015) studies dynamic interactions between mitigation and adaptation activities, where the former reduces the risk of a harmful event, while the latter reduces the damage inflicted in case an event occurs nonetheless.

<sup>22</sup> We assume that the frequency of disasters may also be an increasing function of  $K$ , in addition to being an increasing function of  $P$ , because man-made infrastructure can make disasters more likely to happen. For example, building houses on the land which has been recovered from the sea certainly makes these houses more exposed to flooding; in other words, these houses face a higher probability of disaster. Deforesting a hill, e.g. for agriculture, increases the chances of a landslide. Digging for oil in the sea increases the chances of an oil spill. Destruction of rain forests for road construction and industry raises the probability of land soil degradation. These examples suggest that the built environment and the build up of specific infrastructure have also an impact on the probability of adverse events, which we include in our model.

damages in our baseline model, we adopt a Cobb-Douglas combination of  $K$  and  $P$  such that  $\lambda = \lambda(v^\mu)$ ,  $v = K^\eta P^\xi$ , as before, and  $\mu > 0$  is used to differentiate the effect of  $v$  on the arrival rate from the effect on damages. We relegate the detailed derivations to the Appendix, while focus on the final results in the main text. We use a bar above the endogenous variables to indicate the equilibrium values.

The long-run consumption-to-capital ratio,  $\bar{\psi}$ , is an implicit solution to

$$\varepsilon \bar{\psi} = \rho - (1 - \varepsilon)A \left(1 - \frac{\phi}{\sigma}\right) - \bar{\lambda}(\bar{v})[\bar{\omega}^{1-\varepsilon} - 1], \tag{25}$$

where  $\bar{v} = (\bar{\psi}^\varepsilon \chi \sigma)^{-\mu/\gamma}$  in equilibrium. It is possible that (25) has multiple solutions for some functional forms of  $\lambda$ , especially non-linear.<sup>23</sup> We discuss in the Appendix the conditions under which multiple solutions occur. When the value function is increasing in  $\bar{\psi}$ , the relevant solution is given by the largest root of (25). If the utility function is logarithmic, however, the solution for the long-run consumption-to-capital ratio is unique,  $\bar{\psi} = \psi^* = \rho$ , and identical to the solution with exogenous arrivals.

Comparing Eq. (25) with (19), we may conclude that if  $\bar{\lambda} \geq \lambda$  and  $\varepsilon > 1$ , the right-hand side of (25) is unambiguously smaller than that of (19), implying that  $\bar{\psi} < \psi^*$ . Moreover, since  $d\bar{v}/d\bar{\psi} < 0$  and  $\bar{\lambda}'(\bar{v}) > 0$ , a smaller consumption-capital ratio increases the arrival rate, which reinforces our initial conclusion. Thus it appears that when disaster arrivals are endogenous to economic and polluting activities, the optimal consumption-to-capital ratio is reduced, suggesting that abatement is increased, which complies with the general intuition. We turn next to abatement.

The optimality condition with respect to the pollution stock (see Appendix) allows us to solve for the optimal abatement share:

$$\bar{\theta} = \frac{\phi}{\sigma} - \frac{\xi \bar{\lambda} \bar{\omega}^{1-\varepsilon} \ln \bar{\omega}}{\beta A} - \frac{\bar{v} \bar{\lambda}' \mu}{A} \left[ \frac{\bar{\omega}^{1-\varepsilon} - 1}{1 - \varepsilon} \right]. \tag{26}$$

Comparing Eq. (26) with Eq. (24), we note two main differences. First, in the expression for  $\bar{\theta}$  the arrival rate depends on the consumption-to-capital ratio  $\bar{\psi}$  through  $\bar{v}$ , while in the expression for  $\theta^*$  the arrival rate is fixed. Second, the last term in (26) does not appear in (24). Similarly to Van der Ploeg (2014), and adopting his terminology, the last term represents the "risk-averting effect," while the middle term refers to the "raising-the-stakes" effect. To understand the intuition behind the expression for  $\bar{\theta}$ , consider first the second term in (26) which is similar to the second term in (24), except that the function  $\omega^*$  has  $\psi^*$  as its argument, while the function  $\bar{\omega}$  has  $\bar{\psi}$ . We have established that for  $\bar{\lambda} \geq \lambda$  we have  $\bar{\psi} < \psi^*$ . Moreover,  $d[\bar{\omega}^{1-\varepsilon} \ln \bar{\omega}]/d\bar{\psi} > 0$ , so that the second term in (26) is smaller than that of (24), working to increase  $\bar{\theta}$  compared to  $\theta^*$ . Let us now turn to the last term in (26) involving  $\bar{\lambda}'$ . Clearly, its presence is warranted by the fact that the arrival rate is endogenous. Moreover, this marginal effect on the arrival rate is weighted by its contribution to the change in the value of the program, represented by the expression in the square brackets. This is given by the value of the survived unit of capital relative to the status quo, the term  $(\bar{\omega}^{1-\varepsilon} - 1)/(1 - \varepsilon)$ . Thus, the last term is negative if  $\varepsilon > 1$  and, being subtracted from

<sup>23</sup> Van der Ploeg and de Zeeuw (2018) use a linear and a quartic specification, while Van der Ploeg (2014) uses a linear function and three alternatives (quadratic, cubic and quartic) to calibrate the hazard function. The latter paper also shows that the results with quadratic, cubic and quartic specifications do not differ significantly from the results with a simple linear function (see Table 1, p. 38).

the first two, it contributes to an increase in  $\bar{\theta}$ . We would thus expect  $\bar{\theta}$  to be larger than  $\theta^*$  when  $\bar{\lambda}$  is at least as large as  $\lambda$ , which seems to be plausible in light of the predictions from climate physicists (IPCC 2014).

We turn next to the optimal growth rate. With the optimality conditions with respect to  $C$  and  $K$  and application of the Ito's Lemma for jump processes we obtain

$$\bar{g} = \frac{1}{\varepsilon} \left\{ \underbrace{A \left( 1 - \frac{\phi}{\sigma} \right) - \rho}_{KR \text{ term}} + \underbrace{\bar{\lambda} [\bar{\omega}^{1-\varepsilon} (1 + \varepsilon \ln \bar{\omega}) - 1]}_{\substack{\text{baseline} \\ \text{precautionary effect} < 0}} + \underbrace{\bar{\lambda}' \varepsilon \mu \bar{v} \left[ \frac{\bar{\omega}^{1-\varepsilon} - 1}{1 - \varepsilon} \right]}_{\substack{\text{additional} \\ \text{precautionary effect} < 0}} \right\}. \tag{27}$$

Comparing (27) with (22), we see that the term  $A \left( 1 - \frac{\phi}{\sigma} \right) - \rho$ , representing the standard Keynes–Ramsey component, is the same. The second term in (27) is similar to the respective term in (22), while the last term in (27) has no equivalent in the benchmark model. Since  $\bar{\psi} < \psi^*$  and  $\bar{\omega} < \omega$ , the second term, labeled "baseline precautionary effect", is smaller (i.e. more negative) than the respective term in (22) and hence it contributes to a growth slowdown. Turning to the last term in (27), which appears due to the endogeneity of the hazard rate and has no equivalent in the benchmark model, we see that it is unambiguously negative for  $\varepsilon > 1$ , hence reducing the growth rate even further. It represents an additional precautionary measure due to the positive dependence of the arrival rate on economic activity. We summarize the results in the following

**Proposition 6** *When the disaster arrival rate is endogenous and is at least as large as the exogenous rate of the benchmark model,*

- (i) *the optimal consumption-to-capital ratio is smaller,*
- (ii) *the saving rate is higher,*
- (iii) *the abatement share is larger, and*
- (iv) *the growth rate is lower.*

The evidence from IPCC (2014) on rising frequencies of climate-driven natural disasters suggests that our near future might be characterized by arrival rates  $\bar{\lambda}$  which are larger than, say, the known historical average  $\lambda$ . If this is the case, we conclude that a larger abatement propensity is warranted and the optimal growth rate of the world economy will have to be lower. The intuition behind is entirely driven by the precautionary considerations which dictate a lower growth rate of polluting input and a more aggressive abatement in order to reduce the probability of disasters and the associated damages.

**Table 1** Benchmark values of parameters

$\rho$	$\chi$	$\beta$	$\alpha$	$\varepsilon$	$A$	$\phi$	$\sigma$	$\lambda$	$\delta$	$\gamma$
0.015	1	1	0.0038	1	0.05	0.0004	0.08	0.02	$1e-6$	1

## 6 Quantitative Implications

In this Section we explore the quantitative implications of our model and compare them with recent findings in the literature. Our overarching objective is two-fold: to quantitatively assess the optimal abatement propensity and the growth rate of the economy; and to provide a sensitivity analysis with respect to the key economic and climatic characteristics. In particular, we look at the sensitivity of the results to variations in risk aversion, efficiency of abatement technology, and characteristics of catastrophic events. We also ask what the implications for the abatement policy are when an event is relatively common and low-impact vs. rare but high-impact. The latter question is to a large extent motivated by Stern's critique (Stern 2016) of current economy-climate models which, in his view, fail to adequately take into account the possibility of large-scale climate shocks. In our context, this task is essentially equivalent to assessing the responsiveness of the climate-policy instrument to changes in the severity and in the frequency of natural disasters.

In a recent article developing a climate DSGE model, Golosov et al. (2014) calculate the range of the optimal carbon tax of \$56.9–\$496 per ton carbon in 2010 assuming alternative discount rates.<sup>24</sup> For a similar range of discount rates Van der Ploeg (2014) finds that the optimal tax should be roughly a half, between \$29 and \$216 per ton. There is quite some divergence in the estimates of the optimal tax, with the main conclusion from the recent research being that the size of economic damages induced by climate change or a possibility of crossing a tipping point play a secondary role, while the discount rate seems to be the key driver of policy prescriptions.<sup>25</sup> This is one of the reasons why we chose to explicitly focus on recurring random catastrophic events as the main source of economic damages in order to elucidate their role in shaping the abatement policy. Within our framework, one may ask, for example, what the cost of a carbon unit would be if the world economy were to abate all of its current emissions. This can be obtained by dividing the abatement expenditure by total emissions in a given year.<sup>26</sup> The abatement expenditure is obtained by multiplying GWP with the optimal abatement propensity from Eq. (24).

### 6.1 Calibration

The reference unit of time is set to 1 year. In the benchmark calibration we assume the rate of time preference of 1.5% per year, as in Dietz and Stern (2015), Golosov et al. (2014);

<sup>24</sup> A tax range of \$28–\$55/tC has been obtained by Barrage (2014), see e.g. Fig. S.9 of her paper, after having introduced TFP growth, CRRA utility function and capital depreciation in the model of Golosov et al. (2014).

<sup>25</sup> Dietz and Stern (2015) amend the standard DICE model with damages to the capital stock and to the total factor productivity. They show that even with the discount rate of 1.5% the extended DICE can produce considerably higher carbon taxes than the original DICE model. See also Van den Bijgaart et al. 2016.

<sup>26</sup> This measure of a carbon unit is not perfect since we do not know how much of emitted carbon has been abated in a given year. Thus our measure represents an upper bound.



Nordhaus (2008) and Van der Ploeg (2014).<sup>27</sup> These authors also assume a CRRA utility function. Golosov et al. (2014) use  $\varepsilon = 1$  (log utility), which we adopt here as a starting point for the purpose of having a meaningful comparison. The relative weight on pollution in the utility function,  $\chi$ , is set to unity. The parameter governing the curvature of the disutility of pollution,  $\beta$ , is also set to unity, which implies a quadratic disutility, often used in the literature. The carbon absorption capacity of natural sinks is set at  $\alpha = 0.0038$  (see footnote 6).

To calibrate output emission intensity,  $\phi$ , we take the data from the World Bank series "CO<sub>2</sub> emissions per GDP" (World Bank 2016), which reports average values for the period 2011–15 ranging from 0.1 kg per dollar of GDP for Sweden and France, 0.2 for Germany and the UK, 0.4 for the US, Canada and Brazil and up to 2.1 for China. We use a world average value of 0.4 kg, so that  $\phi = 0.0004$  tons CO<sub>2</sub> per dollar of GDP.

As for abatement efficiency,  $\sigma$ , empirical studies (Hood 2011; McKinsey 2009) show that various abatement activities are inexpensive and thus relatively efficient; in the residential sector it applies to electronics, appliances, and insulation retrofit, in transportation, e.g., to hybrid cars, and in agriculture to tillage management. The marginal costs of these activities are reported to be negative or slightly positive amounting to less than \$5 per ton of CO<sub>2</sub>. Extending abatement activities through further policies, e.g. in the power sector and with reforestation, increases the costs substantially, although Hood (2011) concludes that "a significant level of emission abatement could be achieved with existing technologies at carbon prices of less than \$50 per ton of CO<sub>2</sub>". With the highest value of the marginal cost curve of \$50 per ton, the average value lies in the range of \$10 to \$15. However, for reaching the 2°C target further emission reductions might become necessary in the future, including carbon capturing and sequestration, whose costs are estimated to lie between \$50 and \$100 per ton of CO<sub>2</sub>. One should note, however, that there are considerable learning effects in these new technologies, especially over a long time horizon. We therefore choose an average value of the various abatement measures and aim to include dynamic effects (learning) by setting  $\sigma = 0.08$  in the benchmark calibration, which corresponds to \$12.5 per ton of abated CO<sub>2</sub>. We shall also consider alternative values of \$20 ( $\sigma = 0.05$ ) and \$40 ( $\sigma = 0.025$ ) when technology development is viewed in a more pessimistic way. For the total factor productivity,  $A$ , we adopt a moderate value of 5%.

Statistics for large-scale natural catastrophes over the last few decades suggest varying arrival rates and damages for different types of disasters. The Indian Ocean Tsunami in 2004 caused at least \$10 bn worth of damage and affected six countries: Indonesia, India, Maldives, Sri Lanka, Somalia, and Thailand. The damage amounted to 0.86% of the sum of GDPs in 2004 of the affected countries (Somalia not included due to lacking GDP data in WDI). Hurricane Katrina in 2005 caused \$108 bn damage which amounted to 0.825% of the US GDP. Typhoon Haiyan in the Philippines in 2013 caused \$2.8 bn damage, equivalent to 1.05% of GDP. Cavallo et al. (2013) count 2597 natural disasters (floods and storms, including hurricanes) worldwide during the period 1970–2008, which implies an average annual arrival rate of 0.34. Assuming that 10 percent of the shocks are climate-related events yields  $\lambda = 0.034$ . With respect to larger shocks, we refer to the IPCC to assume that there is a 20% chance that a catastrophic-climate outcome occurs in the next 50 years, which implies that  $\lambda = 0.004$ .

<sup>27</sup> The discount rate of 1.5% is a commonly-used value in the literature. We adopt this parametrization solely for comparison purposes and invite the reader to investigate the social and philosophical foundations of discounting in Stern (2015).

In our quantitative assessment we shall consider two scenarios: (i) relatively low damage intensity of disasters ("low"  $\delta$ ) and (ii) relatively high damage intensity of disasters ("high"  $\delta$ ).<sup>28</sup> Within each scenario we distinguish among three arrival rates, three abatement efficiencies, and two values of the relative risk aversion parameter. The arrival rates correspond to a 20% chance of a disaster occurring in the next 50, 20, and 10 years, corresponding to  $\lambda = 0.004$ ,  $\lambda = 0.01$  and  $\lambda = 0.02$ , respectively. The abatement efficiencies are  $\sigma = 0.08$  (\$12.5/t  $CO_2$ ),  $\sigma = 0.05$  (\$20/t  $CO_2$ ), and  $\sigma = 0.025$  (\$40/t  $CO_2$ ), as discussed earlier.

The relative risk aversion (RRA) coefficient,  $\varepsilon$ , is a subject of an ongoing debate in the theoretical and empirical literature. Recall that we have assumed, following the risk-vulnerability literature, that preferences for risk-taking are affected by exposure to loss-generating events. In particular, RRA is proportional to the parameter which governs the exposure effect in the damage function,  $\eta$ , with a constant proportionality coefficient  $\gamma$ . Instead of calibrating  $\eta$ , for which data are not readily available, we calibrate  $\varepsilon$  and then perform sensitivity analysis with respect to both  $\varepsilon$  and  $\gamma$ . This approach also allows us to meaningfully compare our results with those in the existing studies. We consider two calibrations for  $\varepsilon$  and check sensitivity of the results to variations in  $\gamma$  (in the Appendix), with the benchmark values set to unity for both (log utility). For any given  $\varepsilon$ , a value of  $\gamma$  larger (resp., smaller) than unity would indicate a reduction (resp., increase) in  $\eta$  and therefore a reduction (resp., increase) in damages. The benchmark parameter values are summarized in Table 1.

## 6.2 Low-impact Events

Let us analyze the results pertaining to the first scenario (low  $\delta$ ), summarized in Table 2. In the left-hand panel ( $\varepsilon = 1$ ), with a more optimistic abatement efficiency ( $\sigma = 0.08$ ) and a 20% chance of experiencing a climate-change driven disaster in the next 50 years ( $\lambda = 0.004$ ), we find the optimal (worldwide) growth rate of 3.475% and the optimal abatement propensity of 0.5%. To put the latter number in perspective and provide a comparison with the existing literature, we can infer what this abatement propensity implies for the price of 1 ton of emitted (and abated) carbon. In order to do so we multiply  $\theta^*$  with the world output and divide by tons of carbon emissions using the latest available data. Reuters and World Bank report that global emissions in 2014 amounted to 10.7 bn tons of carbon so that, with the global world output in 2014 at \$78.28 trillion, the implied world carbon price amounts to \$36.6 per ton. The caveat of this approach is that we use yearly emissions, which are in fact net emissions, that is, after some abatement has taken place in a given year. We therefore treat our estimated carbon price as an indicative upper bound and focus

<sup>28</sup> The intensities  $\delta = 1e - 6$  ("low") and  $\delta = 1e - 5$  ("high") correspond to damages of less than 0.1% of gross world product (GWP) and 5 – 10% of GWP, respectively. Recall that in our model damages are endogenous and hence depend not only on  $\delta$  but on all other parameters. Calibrating  $\delta$  proved to be a challenging task as there is no direct mapping to this parameter in the data. In order to circumvent this issue we chose the values which deliver average worldwide damages from natural disasters over the last 5 years. Several sources converge on those damages being in the range of several hundred billion USD for 2011–2015 (see [http://www.economist.com/blogs/dailychart/2011/03/natural\\_disasters](http://www.economist.com/blogs/dailychart/2011/03/natural_disasters), <http://www.theonebrief.com/the-impact-of-natural-disasters-on-the-global-economy>, <https://www.weforum.org/agenda/2015/12/how-much-do-natural-disasters-cost-the-world>, [http://www.kit.edu/kit/english/pi\\_2016\\_058\\_natural-disasters-since-1900-over-8-million-deaths-and-7-trillion-us-dollars-damage.php](http://www.kit.edu/kit/english/pi_2016_058_natural-disasters-since-1900-over-8-million-deaths-and-7-trillion-us-dollars-damage.php)). In particular, our lower value of  $\delta$  corresponds to an average worldwide damage of approximately \$370 bn which reflects the value in the CATDAT database.

on the abatement propensity as our main policy variable of interest. This value is robust to changes in the disaster arrival rate. Specifically, increasing the hazard rate from 0.004 to 0.02 has no significant impact on the optimal abatement propensity and growth. We shall show shortly that this outcome is strictly linked to the logarithmic utility assumption and, to some extent, to the low damage intensity of climate disasters. By contrast, changing abatement efficiency from a relatively high value ( $\sigma = 0.08$ ) to a lower value ( $\sigma = 0.05$ ) brings about an increase in the optimal abatement propensity from 0.5% to 0.8% and the corresponding carbon price rises to \$58.52 per ton which is comparable to the baseline value \$56.9/tC obtained by Golosov et al. (2014). Reduced abatement efficiency induces an only slightly lower optimal growth rate of 3.46%.

Exposure to climate risks may lead to a higher degree of risk aversion according to the risk-vulnerability hypothesis. This suggests that the unitary value of  $\varepsilon$  may be too low. We thus consider a higher value:  $\varepsilon = 3$ . We find that under the benchmark calibration the optimal abatement propensity increases slightly from 0.5 to 0.54%.<sup>29</sup> The optimal growth rate, however, drops from 3.47 to 1.16%. When abatement efficiency is less favorable,  $\theta^*$  rises from 0.8 to 0.87% and the growth rate is significantly reduced from 3.46 to 1.15%. To examine the sensitivity of the results to variations in  $\gamma$ , we replicate Table 2 for  $\gamma = 0.9$  and  $\gamma = 1.1$  in Appendix F.1. A higher (lower) value of  $\gamma$  for a given  $\varepsilon$  implies a lower (higher) exposure to damages and hence a lower (higher) optimal abatement propensity. Comparing the results from Table 2 with those in either Table 4 ( $\gamma = 0.9$ ) or Table 5 ( $\gamma = 1.1$ ), we find no significant changes in either  $\theta^*$  or  $g^*$  for  $\varepsilon = 1$  and only a few percentage points differences in  $\theta^*$  (but not in  $g^*$ ) for  $\varepsilon = 3$ . We anticipate that climate shocks with a larger damage intensity may profoundly alter the optimal policy. In addition, the impact of a higher  $\varepsilon$  has to be examined more carefully in the context of more severe disasters.

### 6.3 High-impact Events

We consider next Table 3 with the same parameter constellation except for  $\delta$  which is increased 10-fold to deliver damages of up to 10% of GWP.<sup>30</sup> First note that with log-utility the optimal abatement propensity and the optimal growth rate are not affected in a major way. However, when we set  $\varepsilon$  to 3, the picture changes significantly. First, looking at the top panel ( $\lambda = 0.004$ ) and the optimistic abatement efficiency case ( $\sigma = 0.08$ ), we already find that the optimal abatement propensity rises significantly from 0.541% to 0.956%, which corresponds to an increase in the carbon price from about \$40 to about \$70 per ton. The growth rate is reduced only marginally from 1.16 to 1.15%. Second, moving to the less optimistic abatement efficiency scenario ( $\sigma = 0.05$ ) increases the abatement propensity from 0.96 to 1.6%. Third, with more frequent disasters ( $\lambda = 0.02$ ), the abatement propensity jumps 4–5 fold from 0.706 and 1.136 (for  $\sigma = 0.08$  and  $\sigma = 0.05$  in Table 2, resp.) to 2.9 and 5.18%, respectively. The latter implies a carbon price of \$379 per ton. At the same time the growth rate is reduced from 1.16 and 1.15 to 1.11 and 1.07%, respectively.

Finally, we can assess the difference between high-impact rare events and more common low-impact events. To this end we compare the results from the top panel of Table 3 with the

<sup>29</sup> This result is consistent with Barrage (2014) who finds that an increase in RRA from 1 to 2 increases the optimal carbon tax only marginally, see Figs. S.10 and S.11 of her paper, from about \$56/tC to slightly over \$60/tC.

<sup>30</sup> Since damages are endogenous in our model, they vary between 5 and 10% of GWP, depending on the values of other parameters.

**Table 2** The optimal policy with low damage intensity:  $\delta = 1e - 6$  (up to 1% of GWP)

	$\varepsilon = 1$			$\varepsilon = 3$		
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.025$	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.025$
$\lambda = 0.004$						
$\theta^*$	0.500	0.800	1.600	0.541	0.867	1.739
$g^*$	3.475	3.459	3.419	1.158	1.152	1.137
$\lambda = 0.01$						
$\theta^*$	0.500	0.801	1.601	0.603	0.967	1.949
$g^*$	3.475	3.459	3.419	1.157	1.151	1.134
$\lambda = 0.02$						
$\theta^*$	0.501	0.801	1.602	0.706	1.136	2.305
$g^*$	3.475	3.459	3.419	1.155	1.148	1.128

results from the bottom panel of Table 2. This corresponds to moving from a scenario with a 20% chance of experiencing a 0.05% reduction in GDP in the next 10 years to a scenario with a 20% chance of experiencing a 5% drop in GDP in the next 50 years. Under log utility there is almost no change in either the abatement propensity or the growth rate, regardless of the value of  $\sigma$ . With  $\varepsilon = 3$  the growth rate reacts relatively moderately by falling by about half a basis point. The abatement propensity, by contrast, reacts more strongly with an increase of 25 basis points (from 0.706 to 0.956). An even stronger increase of 128 basis points is observed for the least optimistic abatement technology ( $\sigma = 0.025$ ). Even if we constrain the expected damage to be exactly identical in both cases (we reduce  $\lambda$  down to 0.002073), the abatement propensity is still higher ( $\theta = 1.205$ ) and the growth rate is lower ( $g = 1.145$ ) in the high-damage scenario. We conclude that a possibility of rare but high-impact events calls for a more stringent abatement policy as compared to relatively frequent but low-impact events (keeping expected damages identical). Our results provide strong support of Stern's hypothesis that optimal climate policy becomes more stringent once rare high-impact events are taken into consideration.

Our quantitative analysis leads to three conclusions. First, log-utility assumption in models of climate change is innocuous only if environmental shocks are "not too severe", i.e. they are characterized by a relatively low damage intensity. This is likely not to be the case for natural disasters induced by climate change. Damages from such catastrophes have amounted to approximately 1% of GDP of countries where they happened to strike. Intensity of disasters will worsen even further due to the planet's warming, according to predictions of climate physicists. Second, efficiency of abatement technology plays an important role as well and more so when disasters are high-impact. Decreasing abatement efficiency by about one third from  $\$12.5/tCO_2$  to  $\$20/t CO_2$  requires an increase in the abatement propensity by more than a half if shocks are low-impact and by three quarters when they are high-impact. Third, a prospect of rare but high-impact events calls for a more stringent mitigation policy than of low-impact frequent events with the same expected damage.

**Table 3** The optimal policy with high damage intensity:  $\delta = 1e - 5$  (up to 10% of GWP)

	$\varepsilon = 1$			$\varepsilon = 3$		
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.025$	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.025$
$\lambda = 0.004$						
$\theta^*$	0.502	0.802	1.605	0.956	1.589	3.587
$g^*$	3.475	3.459	3.419	1.149	1.139	1.096
$\lambda = 0.01$						
$\theta^*$	0.504	0.806	1.612	1.662	2.846	7.139
$g^*$	3.475	3.459	3.419	1.137	1.114	1.016
$\lambda = 0.02$						
$\theta^*$	0.508	0.812	1.624	2.905	5.178	16.49
$g^*$	3.475	3.454	3.418	1.114	1.068	0.792

## 7 Conclusions

An increase in the global temperature is predicted to render natural disasters, e.g. tropical storms, hurricanes, tsunamis, floods, droughts, etc., more severe and intense. Such calamities have a profound negative impact on the economy's infrastructure, physical and human capital, and they undoubtedly represent a set-back in terms of economic growth and development. An efficient and timely climate policy is necessary in order to limit damages from such devastating shocks.

In the present article we propose a model of a growing economy subject to random natural disasters, which destroy part of the economy's productive input. An important feature of our model is consideration of recurring shocks where the extent of the damage is endogenously determined through the interaction between capital accumulation process and an appropriate emissions abatement policy. Our contribution is three-fold. First, we deliver explicit analytical solutions for (i) the benchmark model, where the hazard rate is treated as exogenous and only damages depend on the accumulated emissions; and (ii) an extended model, where the hazard rate is also an increasing function of atmospheric GHG. In the case of a unitary elasticity of marginal utility (logarithmic preferences), often used in the literature on the grounds of better tractability, the consumption-to-capital ratio and the propensity to save are independent of the climatic parameters. Also, with log-preferences the endogeneity of the arrival rate does not alter the optimal mitigation expenditure. In light of these properties, policy-relevance and applicability of findings stemming from models based on the log-utility assumption seem to suffer from some limitations. Assuming a non-unitary relative risk aversion reveals important macroeconomic interdependencies and alters policy conclusions significantly.

Second, we emphasize the importance of *simultaneously* determining the optimal dynamic behavior of the economy and the abatement of GHG, since the equilibrium growth rate serves as a complement to the climate policy. When the economy is exposed to stochastic climate shocks it may choose to lower its optimal rate of growth so as to limit its GHG emissions—and thus potential damages if a shock occurs—but also to reduce its expenditure on abatement.

Third, we provide quantitative implications by calibrating our model to the recent data on global carbon emissions, output, frequency of large natural catastrophes and their damages.

With log utility, the share of output which should be devoted to emissions control is approximately 0.5% of GWP. This number is comparable to what has been found in the recent studies which relied on log-utility assumption. However, when we use a higher value for the relative risk aversion parameter, we find that the abatement propensity increases significantly, reaching 3–5% of GWP, as we consider higher degrees of severity of environmental shocks. These values are equivalent to a carbon price in the range \$212–\$370 per ton. If one takes the side of climate physicists who believe that climate change will cause an increase in disaster frequency—in addition to the damage intensity—then an even more stringent climate policy becomes warranted.

By introducing random large climate shocks and tipping points, arriving with a known stochastic process, our approach deals with the role of risk. In situations where objective probabilities or assessments of subjective probabilities of the underlying alternatives are not possible, we arrive at the theory of decision-making under deep uncertainty and ambiguity, see Bühren et al. (2021) for a recent survey. It has been shown that agents are ambiguity-averse, which is conceptually different from being risk-averse and may decrease the demand for self-protection, see Alary et al. (2013) and Mittelstaedt and Baumgärtner (2023) for a theoretical foundation. The framework that we proposed can be viewed as a situation where uncertainty becomes resolved over time due, for example, to learning, as in Brock and Xepapadeas (2020). The latter contribution stands out as a unique study that offers an analysis of social cost of carbon under deep uncertainty. They find that the latter contributes to more conservative climate policies and that, in addition, the economy is willing to spend part of its output on learning which allows to partially reduce this uncertainty. We conjecture that introducing deep uncertainty into our model will yield similar results, i.e. the abatement propensity would be larger, while the optimal growth rate would be lower.

## Appendix to Section 2

### Solution of the Deterministic Model

#### The Interior Solution

In what follows, we omit the time subscripts when there is no ambiguity and use subscripts to denote partial derivatives. By differentiating the current-value Hamiltonian, we obtain the following first-order conditions for the interior solution:

$$\frac{\partial H}{\partial C} = 0 \Leftrightarrow \lambda = U'_c, \quad (\text{A.1})$$

$$\frac{\partial H}{\partial \theta} = 0 \Leftrightarrow \lambda = -\mu\sigma, \quad (\text{A.2})$$

$$\frac{\partial H}{\partial P} = \rho\mu - \dot{\mu} \Leftrightarrow \chi P^\beta = \dot{\mu} - (\rho + \alpha)\mu, \quad (\text{A.3})$$

$$\frac{\partial H}{\partial K} = \rho\lambda - \dot{\lambda} \Leftrightarrow \dot{\lambda} + \lambda[(1 - \theta)A - \rho] + \mu(\phi - \theta\sigma)A = 0. \quad (\text{A.4})$$

Combined with the boundary conditions they can be written as

$$\lambda = -\mu\sigma, \tag{A.5}$$

$$\dot{\mu} - (\alpha + \rho)\mu = \chi P^\beta, \tag{A.6}$$

$$\dot{\lambda} + \lambda[(1 - \theta)A - \rho] + \mu(\phi - \theta\sigma)A = 0, \tag{A.7}$$

$$\dot{P} = -\alpha P + (\phi - \sigma\theta)Y, \tag{A.8}$$

$$\dot{K} = (1 - \theta)AK - C, \tag{A.9}$$

Equation (A.5) combined with Eq. (A.7) gives

$$\dot{\lambda} + \lambda \left[ (1 - \theta)A - \rho - \frac{1}{\sigma}(\phi - \theta\sigma)A \right] = 0$$

resulting in  $\dot{\lambda} + (r - \rho)\lambda = 0$ , where  $r = A \left( 1 - \frac{\phi}{\sigma} \right) > 0$ . Solving this ordinary differential equation (ODE), we obtain  $\lambda_t = e^{-(r-\rho)t} \lambda_0$ . Combining the latter with (A.5), we get  $\dot{\mu} = -\frac{1}{\sigma} \dot{\lambda} = \frac{r-\rho}{\sigma} \lambda_t$ . After substitution into (A.6), we get

$$(r + \alpha)\lambda_t = \sigma \chi P^\beta$$

which can be rewritten as

$$P_t = \left[ \frac{\lambda_0(r + \alpha)}{\sigma \chi} \right]^{\frac{1}{\beta}} e^{-\frac{r-\rho}{\beta}t}. \tag{A.10}$$

Next we can rewrite  $\dot{P} + \alpha P$  as  $\left( \alpha - \frac{r-\rho}{\beta} \right) P_0 e^{-\frac{r-\rho}{\beta}t}$  with  $P_0 = \left[ \frac{\lambda_0(r+\alpha)}{\sigma \chi} \right]^{\frac{1}{\beta}}$ . Using this in Eq. (A.8), we obtain  $\left( \alpha - \frac{r-\rho}{\beta} \right) P_0 e^{-\frac{r-\rho}{\beta}t} = (\phi - \sigma\theta)AK$ , which implies that  $\phi - \sigma\theta_t \rightarrow 0$ . Further (A.8) can be rewritten as  $\phi - \sigma\theta = \frac{\dot{P} + \alpha P}{AK}$  resulting in

$$\theta_t = \frac{\phi}{\sigma} - \frac{\left( \alpha - \frac{r-\rho}{\beta} \right) P_t}{\sigma AK_t}.$$

Since  $C_t = \lambda_t^{-1/\epsilon}$  by (A.1), Eq. (A.9) can be rewritten as the following ODE:

$$\dot{K} = \left( 1 - \frac{\phi}{\sigma} \right) AK + \frac{\left( \alpha - \frac{r-\rho}{\beta} \right) P_0 e^{-\frac{r-\rho}{\beta}t}}{\sigma} - C_0 e^{\frac{(r-\rho)t}{\epsilon}}$$

Using the standard transversality condition, defining  $g = \frac{1}{\epsilon}(r - \rho)$ , and requiring that  $g < r$ , we obtain

$$K_t^* = \frac{c_0}{r - g} e^{gt} - \frac{P_0 \left( \alpha - \frac{\epsilon g}{\beta} \right)}{\sigma \left( r + \frac{\epsilon g}{\beta} \right)} e^{-\frac{\epsilon g}{\beta}t} \tag{A.11}$$

resulting in abatement propensity

$$\theta_t^* = \frac{\phi}{\sigma} + \frac{\left(\frac{\varepsilon g}{\beta} - \alpha\right) P_t}{\sigma AK_t^*}. \tag{A.12}$$

Now that we have solved for the full dynamics of the economy in the interior regime, we can derive the explicit value function associated with the interior solution. From (A.11), we can express consumption as a function of the two state variables and therefore we can write the value function as

$$V(K, P) = \frac{(aK + bP)^{1-\varepsilon}}{1-\varepsilon} - x \frac{P^{1+\beta}}{1+\beta},$$

where the constants are given by  $a = (r - g)^{\frac{1-\varepsilon}{\varepsilon}}$ ,  $b = \frac{a(\alpha - g_p)}{r + g_p}$ ,  $x = \frac{\beta \chi}{(1+\beta)r - \rho}$ . We are going to be interested in the long-run (or asymptotic) value function such that  $t \rightarrow \infty$ .

**Maximum Abatement:  $\theta = \bar{\theta}$**

Suppose that the initial level of pollution  $P_0$  is relatively high, so that maximum abatement is warranted from the outset. Now the problem is

$$\max_{c, K, P} \int_0^\infty e^{-\rho t} U(C, P) dt$$

s.t.

$$\dot{K} = A(1 - \bar{\theta})K - C \quad , \quad \lambda \geq 0 \tag{A.13}$$

$$\dot{P} = (\phi - \bar{\theta}\sigma)AK - \alpha P \quad , \quad \mu \leq 0 \tag{A.14}$$

It is not difficult to see that there exists  $T > 0$  such that for  $t < T$  the adjoint variable  $v_t > 0$ , i.e. the control  $\theta = \bar{\theta}$  is applied for some finite time  $T$  only. The dynamics of the economy are such that pollution stock declines at the rate exceeding  $g_p$  because now abatement is at its maximal value, while consumption and capital stock grow at the rate smaller than  $g$ . The behavior of the state variables is illustrated by the red dash-dotted line in Fig. 1.

**No abatement:  $\theta_t \equiv 0$**

If the initial conditions are such that the capital stock and/or pollution stock are small enough, then it may be optimal to do zero abatement, at least initially. Now the problem is

$$\max_{c, K, P} \int_0^\infty e^{-\rho t} U(C, P) dt$$

s.t.

$$\dot{K} = AK - C \quad , \quad \lambda \geq 0 \tag{A.15}$$



$$\dot{P} = -\alpha P + \phi AK \quad , \quad \mu \leq 0 \tag{A.16}$$

The optimality conditions are

$$\frac{\partial H}{\partial C} = 0 \Leftrightarrow \lambda = U'_C \tag{A.17}$$

$$\frac{\partial H}{\partial P} = \rho\mu - \dot{\mu} \Leftrightarrow \chi P^\beta = \dot{\mu} - (\rho + \alpha)\mu \tag{A.18}$$

$$\frac{\partial H}{\partial K} = \rho\lambda - \dot{\lambda} \Leftrightarrow \dot{\lambda} + (A - \rho)\lambda + \phi A\mu = 0. \tag{A.19}$$

with  $\sigma < \frac{\lambda}{-\mu}$ . In the long run, pollution stock is proportional to capital stock, which implies a smaller welfare than with the interior solution with  $\theta > 0$ . Hence, the adjoint variable  $v_t = 0$  for  $t > \tilde{T}$ , i.e. this control is applied for some finite time only. The behavior of the state variables is illustrated by the green dashed line in Fig. 1.

### A Note on Asymptotic Value Function

The optimal pollution and capital stocks in the interior regime are given by (9) and (10). One can show that

$$V(K, P) = \frac{(aK + bP)^{1-\varepsilon}}{1 - \varepsilon} - x \frac{P^{1+\beta}}{1 + \beta},$$

where  $(K, P(K))$  belong to the "interior curve", which we will call curve (set)  $B$ . Curve  $B$  is parametrized with the time-variable  $t$ , which is convenient to treat now as a *parameter*.

We can substitute  $(K_t, P_t)$  into the value function. After that we can rewrite the resulting function  $V(t)$  as follows:

$$V(t) = V(K_t, P_t) = \psi_t \frac{K_t^{1-\varepsilon}}{1 - \varepsilon} - x \frac{P_t^{1+\beta}}{1 + \beta}$$

where  $\psi_t = \frac{(aK_t + bP_t)^{1-\varepsilon}}{K_t^{1-\varepsilon}} = a^{1-\varepsilon} (1 + \frac{b}{a} \frac{P_t}{K_t})^{1-\varepsilon}$ . Function  $V(t)$  defines lifetime-utility of an agent with initial condition  $(k, p) = (K_t, P_t) = (K_t, P(K_t))$  (note that mapping  $t \mapsto (K_t, P_t)$  is one-to-one, so choosing parameter  $t$  is equivalent to picking some point in  $B$ ). In other words, knowing function  $V(t)$  is equivalent to knowing  $V(K, P(K))$ , since one can always change variables back.

From (1) and (2) we have  $\frac{P_t}{K_t} = \underline{Q} (\exp(-(1/\beta + 1/\varepsilon)t))^{(r-\rho)} \searrow 0$ , when  $t \rightarrow \infty$  (or, equivalently, when  $K_t \rightarrow \infty$  or, equivalently, when  $P_t \rightarrow 0$ ).

Since  $(1 + x)^\alpha = 1 + \alpha x + o(x)$ , when  $x \rightarrow 0, \alpha > 0$ , we can write

$$\psi_t = \psi^* + (1 - \varepsilon)b^{1-\varepsilon} \frac{P_t}{K_t} + o\left(\frac{P_t}{K_t}\right),$$

where  $\psi^*$  stands for  $a^{1-\varepsilon}$ .

Keeping in mind that  $V(t)$  defines the value function in the interior regime, let's introduce another function  $\tilde{V} : \mathbb{R}_+ \rightarrow \mathbb{R}$  given by the identity  $\tilde{V}(t) = \psi^* \frac{K^{1-\epsilon}}{1-\epsilon} - x \frac{P^{1+\beta}}{1+\beta}$ . We call  $\tilde{V}$  the *asymptotic (or long-run) VF*.

The resulting (asymptotic) consumption  $\tilde{C}$  and (asymptotic) abatement policy  $\tilde{\theta} = \phi/\sigma$  are slightly different from the optimal policies  $C^*, \theta^*$  derived above. This is because  $\tilde{V}(t) \neq V(K_t, P_t)$ , but instead  $\frac{\tilde{V}_t}{V(K_t, P_t)} \xrightarrow{t \rightarrow \infty} 1$ .

Let  $V(K, P) = \psi^* \frac{K^{1-\epsilon}}{1-\epsilon} - x \frac{P^{1+\beta}}{1+\beta} + O(P/K)$  be asymptotic expansion of function  $V(K, P(K))$ , when  $K \rightarrow \infty$ . Let  $C^*, \theta^*$  denote optimal policies (with smoothness assumptions). Then there exist  $a_1$  and  $a_2$  such that for asymptotic policies  $\tilde{C}, \tilde{\theta}$  errors are bounded  $|\tilde{C} - C^*| \leq a_1 P/K$  and  $|\tilde{\theta} - \theta^*| \leq a_2 P/K$ .

### Appendix to Section 3

#### Interior Solution to the Baseline Stochastic Model

Hamilton-Jacobi-Bellman equation for the problem may be written as

$$\begin{aligned} \rho V(K, P) = \max_{C, \theta} & \left\{ U(C, P) + V_k [(1 - \theta)Y - C] + V_p [(\phi - \theta\sigma)Y - \alpha P] + \right. \\ & \left. + \lambda \left[ V(\tilde{K}, P) - V(K, P) \right] \right\}, \end{aligned} \tag{B.1}$$

and the first-order conditions with respect to the control and the state variables consist of

$$C : \quad U_c - V_k = 0, \tag{B.2}$$

$$\theta : \quad -V_k Y - V_p \sigma Y = 0, \tag{B.3}$$

$$\begin{aligned} K : \quad \rho V_k = & V_{kk} [(1 - \theta)Y - C] + V_k (1 - \theta)A + V_p (\phi - \sigma\theta)A + V_{pk} [(\phi - \sigma\theta)Y - \alpha P] + \\ & + \lambda \left( \tilde{V}_k \frac{d\tilde{K}}{dK} - V_k \right), \end{aligned} \tag{B.4}$$

$$\begin{aligned} P : \quad \rho V_p = & U_p + V_{pp} [(\phi - \sigma\theta)Y - \alpha P] + V_{kp} [(1 - \theta)Y - C] + \lambda \tilde{V}_k \frac{d\tilde{K}}{dP} - \alpha V_p + \\ & + \lambda (\tilde{V}_p - V_p). \end{aligned} \tag{B.5}$$

Suppose the long-run value function is of the form

$$V(K, P) = \frac{\psi^{-\epsilon} K^{1-\epsilon}}{1 - \epsilon} - \frac{x P^{1+\beta}}{1 + \beta},$$

then from (B.2) and (B.3),

$$C = \psi K \tag{B.6}$$

$$(\psi K)^{-\varepsilon} = x\sigma P^\beta. \quad (\text{B.7})$$

### Finding the Coefficients

Substituting (B.6) and (B.7) into the HJB equation we find:

$$\begin{aligned} \frac{\rho\psi^{-\varepsilon}K^{1-\varepsilon}}{1-\varepsilon} - \frac{\rho x P^{1+\beta}}{1+\beta} &= \frac{(\psi K)^{1-\varepsilon}}{1-\varepsilon} - \chi \frac{P^{1+\beta}}{1+\beta} + (\psi K)^{-\varepsilon}[(1-\theta)A - \psi]K - \\ &- \frac{(\psi K)^{-\varepsilon}}{\sigma}(\phi - \theta\sigma)AK + \alpha x P^{1+\beta} + \lambda \left[ \frac{\psi^{-\varepsilon}\tilde{K}^{1-\varepsilon}}{1-\varepsilon} - \frac{\psi^{-\varepsilon}K^{1-\varepsilon}}{1-\varepsilon} \right], \end{aligned}$$

To find  $\psi$  and  $x$ , we match the terms with  $K$  and  $P$ , respectively, on the left and on the right-hand sides. Collecting the terms in  $K$  we obtain:

$$\begin{aligned} \frac{\rho\psi^{-\varepsilon}K^{1-\varepsilon}}{1-\varepsilon} &= \frac{(\psi K)^{1-\varepsilon}}{1-\varepsilon} + \psi^{-\varepsilon}[(1-\theta)A - \psi]K^{1-\varepsilon} - \psi^{-\varepsilon}\left(\frac{\phi}{\sigma} - \theta\right)AK^{1-\varepsilon} + \lambda \frac{\psi^{-\varepsilon}K^{1-\varepsilon}}{1-\varepsilon}(\omega^{1-\varepsilon} - 1). \end{aligned}$$

Dividing both sides by  $\frac{\psi^{-\varepsilon}K^{1-\varepsilon}}{1-\varepsilon}$  yields

$$\rho = \varepsilon\psi + (1-\varepsilon)A\left(1 - \frac{\phi}{\sigma}\right) + \lambda(\omega^{1-\varepsilon} - 1), \quad (\text{B.8})$$

where  $\omega = \exp[-\delta(\sigma x \psi^\varepsilon)^{-1/\gamma}]$ .

Collecting the terms in  $P$ , we obtain:

$$-\frac{\rho x P^{1+\beta}}{1+\beta} = -\chi \frac{P^{1+\beta}}{1+\beta} + \alpha x P^{1+\beta}.$$

Dividing both sides by  $\frac{P^{1+\beta}}{1+\beta}$  yields:

$$-\rho x = -\chi + \alpha(1+\beta)x$$

which implies that

$$x = \frac{\chi}{\rho + \alpha(1+\beta)}. \quad (\text{B.9})$$

### Solving for Endogenous Variables

#### Consumption Dynamics

We solve the optimal growth rate by combining (B.2) and (B.4). First, let us divide (B.4) throughout by  $V_k$ :

$$\rho = \frac{V_{kk}}{V_k}[(1-\theta)AK - \psi K] + (1-\theta)A + \frac{V_p}{V_k}(\phi - \sigma\theta)A + \lambda \left( \frac{\tilde{V}_k}{V_k} \frac{d\tilde{K}}{dK} - 1 \right),$$

Using the fact that  $\frac{V_p}{V_k} = -\frac{1}{\sigma}$ ,  $\frac{\tilde{V}_k}{V_k} = \omega^{-\varepsilon}$ ,  $\frac{d\tilde{K}}{dK} = \frac{\partial \omega}{\partial K}K + \omega$ , we get

$$\rho = \frac{V_{kk}}{V_k} [(1 - \theta)A - \psi]K + (1 - \theta)A - \frac{\phi A}{\sigma} + \theta A + \lambda \left[ \omega^{1-\varepsilon} \left( \frac{\partial \omega}{\partial K} \frac{K}{\omega} + 1 \right) - 1 \right].$$

$$\rho = \frac{V_{kk}}{V_k} [(1 - \theta)A - \psi]K + A \left( 1 - \frac{\phi}{\sigma} \right) + \lambda \left[ \omega^{1-\varepsilon} (\eta \ln(\omega) + 1) - 1 \right].$$

From the above equation we obtain

$$\frac{V_{kk}}{V_k} [(1 - \theta)A - \psi]K = \rho - A \left( 1 - \frac{\phi}{\sigma} \right) - \lambda \left[ \omega^{1-\varepsilon} (\eta \ln(\omega) + 1) - 1 \right] \tag{B.10}$$

In the next step we use the Change of Variable Formula for jump process to compute the differential of  $U_c$ . Since we know that  $U_c = V_k$ ,

$$dU_c = \{V_{kk}[(1 - \theta)A - \psi]K\} dt + [U_c(\tilde{C}) - U_c(C)] dq$$

Dividing both sides by  $U_c$  or, equivalently, by  $V_k$ , yields

$$\begin{aligned} \frac{dU_c}{U_c} &= \left\{ \frac{V_{kk}}{V_k} [(1 - \theta)A - \psi]K \right\} dt + \left[ \frac{U_c(\tilde{C})}{U_c(C)} - 1 \right] dq, \\ &= \left\{ \frac{V_{kk}}{V_k} [(1 - \theta)A - \psi]K \right\} dt + \left[ \frac{U_c(\tilde{C})}{U_c(C)} - 1 \right] dq \end{aligned}$$

Using (B.10), yields

$$\begin{aligned} -\frac{dU_c}{U_c} &= \left\{ A \left( 1 - \frac{\phi}{\sigma} \right) + \lambda \omega^{1-\varepsilon} [\eta \ln(\omega) + 1] - (\lambda + \rho) \right\} dt - \left[ \frac{U_c(\tilde{C})}{U_c(C)} - 1 \right] dq \\ \frac{dC}{C} &= \frac{1}{\varepsilon} \left\{ A \left( 1 - \frac{\phi}{\sigma} \right) + \lambda \omega^{1-\varepsilon} [\eta \ln(\omega) + 1] - (\lambda + \rho) \right\} dt + (1 - \omega^{-\varepsilon}) dq \end{aligned}$$

The expression in the curly braces multiplying  $dt$  is the trend growth rate of consumption, i.e. the growth rate of  $C$  in between disasters, which we denote by  $g_c$ :

$$g_c = \frac{1}{\varepsilon} \left\{ A \left( 1 - \frac{\phi}{\sigma} \right) + \lambda \omega^{1-\varepsilon} [\eta \ln(\omega) + 1] - (\lambda + \rho) \right\}. \tag{B.11}$$

Since  $C = \psi K$ , the trend growth rate of consumption is the same as the trend growth rate of  $K$  ( $g_k$ ) in the long run. Hence, we can write

$$g_k = g_c.$$

**Pollution and Abatement Dynamics**

For now we will only be concerned with trend growth rates and will discuss the jumps later on. From (B.7) we know that

$$\varepsilon g_k + \beta g_p = 0.$$

From the dynamic motion of pollution stock we know that  $g_p = (\phi - \sigma \theta_t) A \frac{K_t}{P_t} - \alpha$ . Inserting this equation into the above yields

$$\varepsilon g_k + \beta \left[ (\phi - \sigma \theta_t) A \frac{K_t}{P_t} - \alpha \right] = 0. \tag{B.12}$$

This implies, in particular, that

$$g_p = -\frac{\varepsilon g_c}{\beta}. \tag{B.13}$$

In addition, from (B.5) we know that

$$\rho = \frac{\chi}{x} + \beta g_p - \lambda \sigma \xi \omega^{1-\varepsilon} \ln(\omega) \frac{K}{P} - \alpha. \tag{B.14}$$

To get this latter result, it is sufficient to divide (B.5) throughout by  $V_p$  and recall that  $x$  is constant. Solving for  $K/P$  and using the solution for  $x$ , we obtain

$$\frac{K}{P} = \frac{\beta(g_p + \alpha)}{\lambda \sigma \xi \omega^{1-\varepsilon} \ln(\omega)}.$$

Finally, inserting this result into (B.12) yields

$$\phi - \sigma \theta_t = \frac{\alpha \beta - \varepsilon g_c}{\beta A \frac{\beta(g_p + \alpha)}{\lambda \sigma \xi \omega^{1-\varepsilon} \ln(\omega)}} = \frac{(\alpha + g_p) \lambda \sigma \xi \omega^{1-\varepsilon} \ln(\omega)}{\beta(g_p + \alpha) A} = \frac{\lambda \sigma \xi \omega^{1-\varepsilon} \ln(\omega)}{\beta A}. \tag{B.15}$$

Hence

$$\theta_t = \frac{\phi}{\sigma} - \frac{\lambda \xi \omega_t^{1-\varepsilon} \ln(\omega_t)}{\beta A}. \tag{B.16}$$

### Disentangling the Precautionary Effect

We are interested in the magnitude of  $\zeta = \omega^{1-\varepsilon} (1 + \varepsilon \ln \omega)$  relative to unity. The ratio of marginal utilities, given by  $\omega^{-\varepsilon}$  is unambiguously larger than unity, while the effect of  $K$  on  $\tilde{K}$  can be larger or smaller than 1. This effect includes the direct positive effect, given by  $\omega > 0$ , and an indirect negative effect, given by a change in  $\omega$  itself,  $K(\partial \omega / \partial K) = \omega \varepsilon \ln \omega < 0$ . Whether the direct effect dominates the indirect effect, depends on the magnitude of  $\varepsilon$ .<sup>31</sup> If  $\varepsilon < -(\ln \omega)^{-1}$ , the direct effect dominates, implying that  $d\tilde{K}/dK > 0$ . The threshold  $-(\ln \omega)^{-1}$  may take values between approximately 3.47 and 99

<sup>31</sup> With the assumption  $\varepsilon = \eta$  we can also relate the magnitudes of the direct and indirect effects of  $K$  on  $\tilde{K}$  to the parameter governing the strength of the exposure component in the damage function. Recall that if  $\eta < 1$ , the survived share of capital is convex in  $K$ , i.e., as capital stock grows its marginal vulnerability to shocks declines. Alternatively, if  $\eta$  exceeds a specific threshold, the marginal vulnerability increases. The threshold is given by  $(1 + \ln \omega)^{-1}$ . For relatively small damages, say 1% ( $\omega = .99$ ), the threshold is approximately unity. For relatively large damages, say 25% ( $\omega = .75$ ), the value is around 1.4. Given that for the relevant range of the damage magnitude the threshold values are close to one, we shall distinguish primarily between two possibilities,  $\eta < 1$  and  $\eta > 1$ . This is also convenient for our further analysis since we need to consider alternative values of  $\varepsilon$ , with the usual cut-off value of one.

for  $\omega \in [0.75, 0.99]$ . Clearly, the condition is largely satisfied for the relevant range of  $\omega$  and empirically-plausible range of  $\varepsilon$  (see Fig. 3), hence  $d\tilde{K}/dK > 0$ .<sup>32</sup>

We still need to find out, however, whether  $\zeta = \frac{U'(\tilde{C})}{U'(C)} \frac{d\tilde{K}}{dK}$  is larger or smaller than unity. So far we have established that an increase in  $K$  will lead to an increase in the post-event capital stock but less than proportionally since  $\omega(1 + \varepsilon \ln \omega) < 1$  (recall that  $\omega \in (0, 1)$  and  $\ln \omega < 0$ ). The valuation of this increase is given by the ratio of marginal utilities  $\omega^{-\varepsilon} > 1$ . The total effect,  $\zeta = \omega^{1-\varepsilon}(1 + \varepsilon \ln \omega)$ , can thus be larger or smaller than unity, depending on the magnitude of  $\varepsilon$ .

**Proof of Proposition 1** Given the value function, (i) follows directly from (B.2), so that consumption-to-capital ratio is constant and equal to  $\psi^*$ . The statement in (ii) follows from (24) and the fact that  $\psi^*$  is constant. Then  $g^*$  is the growth rate of  $C$ ,  $K$ , and  $Y$ . Since abatement share is constant, total abatement expenditure grows at the same rate as well, which completes the proof.  $\square$

### Non-constant Marginal Abatement Cost

We now assume that the abatement efficiency,  $\sigma$ , is a function of the abatement propensity,  $\theta$ , such that  $\sigma = \sigma(\theta)$ ,  $\sigma' < 0$ ,  $\sigma'' \geq 0$ ,  $\sigma(0) = 0$ : a larger proportional expenditure on abatement makes it harder to abate, reflecting increasing marginal abatement cost. For practical purposes, we assume a power function:  $\sigma = \bar{\sigma}\theta^a$ ,  $a < 0$ . Our original model is included as a special case  $a = 0$ . The total abatement can be written as  $\bar{\sigma}\theta^{1+a}AK$ .

This modification of the model leads to changes in the first-order conditions with respect to  $\theta$ ,  $K$ , and  $P$ . The key difference to our original model is that the solution to the current model is a solution to the system of 6 equations (4 FOCs plus 2 constraints), while in our original model it was possible to separate the system into sub-components and solve them separately. After some rearrangements, the current system becomes

$$x = \frac{\chi}{\rho + \alpha(1 + \beta)} \tag{C.1}$$

$$\theta = (\psi^\varepsilon K^\varepsilon \bar{\sigma}(1 + a)xP^\beta)^{-\frac{1}{a}} \tag{C.2}$$

$$\rho = \varepsilon\psi + (1 - \varepsilon)A \left[ 1 - \theta - \frac{\phi}{\bar{\sigma}(1 + a)\theta^a} + \frac{\theta}{1 + a} \right] + \lambda(\omega^{1-\varepsilon} - 1) \tag{C.3}$$

$$g_c = \frac{1}{\varepsilon} \left\{ A \left[ 1 - \theta - \frac{\phi}{\bar{\sigma}(1 + a)\theta^a} + \frac{\theta}{1 + a} \right] + \lambda\omega^{1-\varepsilon} [\eta \ln \omega + 1] - (\lambda + \rho) \right\} \tag{C.4}$$

<sup>32</sup> Only for extremely large damages, exceeding 25% of GDP, the condition  $\varepsilon < -(\ln \omega)^{-1}$  might be violated, implying that  $d\tilde{K}/dK$  may become negative. Consequently, the term  $(\zeta - 1)$  in (22) may become negative as well, implying a precautionary dissaving motive. The intuition here is clear, since the overall effect of an increase in the capital stock on  $\tilde{K}$  is negative, the optimal response of the economy is to decrease its growth and saving rates.

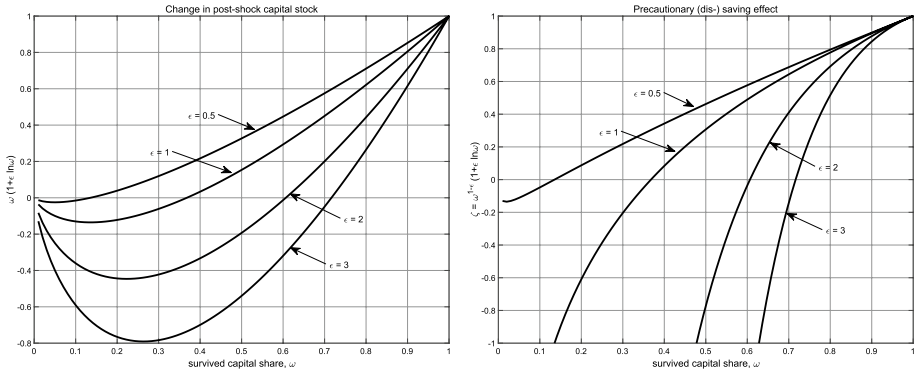


Fig. 3 Precautionary effect and risk aversion

$$0 = \varepsilon g_k + \beta \left[ (\phi - \bar{\sigma}\theta^{1+a}) \frac{AK}{P} - \alpha \right] \tag{C.5}$$

$$\theta = \frac{\phi}{\bar{\sigma}\theta^a} - \frac{(1+a)\xi\lambda\omega^{1-\varepsilon} \ln \omega}{\beta A} \tag{C.6}$$

It can be easily seen that if we set  $a = 0$  we are back to the solution of our original model. Further, by examining the last equation, one can see that if  $\bar{\sigma}$  is set equal to the abatement efficiency of our baseline model,  $\sigma$ , then the optimal abatement propensity in the model with increasing marginal abatement cost is smaller than in our baseline model, given in (24). This is because the first term in (C.6) is smaller than  $\phi/\sigma$  and, given that  $a < 0$ , the second positive term is also smaller. This result conforms with our intuition since an increasing MAC will necessarily induce less abatement.

### Appendix to Section 4

#### Comparative Statics

$$\underbrace{\begin{pmatrix} \varepsilon [1 - \lambda(1 - \varepsilon)\omega^{1-\varepsilon}\psi^{-1} \ln \omega] & 0 \\ \varepsilon\omega^{1-\varepsilon}\psi^{-\varepsilon-1} \lambda\delta [1 + (1 - \varepsilon) \ln \omega] & xA\sigma \end{pmatrix}}^M \times \underbrace{\begin{pmatrix} d\psi \\ d\theta \end{pmatrix}}^x = \underbrace{\begin{pmatrix} \Delta_{\psi\lambda} & \Delta_{\psi\delta} & \Delta_{\psi\sigma} & \Delta_{\psi\phi} \\ \Delta_{\theta\lambda} & \Delta_{\theta\delta} & \Delta_{\theta\sigma} & \Delta_{\theta\phi} \end{pmatrix}}_{\Delta} \times \underbrace{\begin{pmatrix} d\lambda \\ d\delta \\ d\sigma \\ d\phi \end{pmatrix}}_y,$$

where

$$\Delta' = \begin{pmatrix} 1 - \omega^{1-\varepsilon} & \delta\psi^{-\varepsilon}\omega^{1-\varepsilon} \\ \lambda(1 - \varepsilon)\omega^{1-\varepsilon}\psi^{-\varepsilon}(x\sigma)^{-1} & \lambda\psi^{-\varepsilon}\omega^{1-\varepsilon} [1 - \delta(1 - \varepsilon)\psi^{-\varepsilon}(x\sigma)^{-1}] \\ \frac{\varepsilon-1}{\sigma^2} (A\phi + \lambda\omega^{1-\varepsilon}\delta\psi^{-\varepsilon}x^{-1}) & \frac{1}{\sigma} [\lambda\delta\psi^{-\varepsilon}\omega^{1-\varepsilon} [(1 - \varepsilon)\delta\psi^{-\varepsilon}(x\sigma)^{-1} - 1] - \phi x A] \\ (1 - \varepsilon) \frac{A}{\sigma} & xA \end{pmatrix}$$

and

$$|M| = M_{11}M_{22} = \varepsilon [1 - \lambda(1 - \varepsilon)\omega^{1-\varepsilon}\psi^{-1} \ln \omega]xA\sigma.$$

**Determinant of  $M$  matrix**

We are interested in the minimum possible threshold and thus need to consider only a minimum value for  $\psi$ . The share of consumption in the total capital stock,  $\psi$ , is computed as consumption share in GDP, multiplied by the total factor productivity, assumed to be 5%. With the consumption share as low as 60%, this gives a value of 0.03. Historically, high-impact events (small  $\omega$ ) are associated with rare occurrence (small  $\lambda$ ) and low-impact events are more frequent. We consider a range of possibilities, from fairly common disasters to rare catastrophes. Consider first the former category. When the maximum value of  $\lambda$  is equal to 0.5 and the corresponding damage is 1% ( $\omega = 0.99$ ), the threshold value of  $\varepsilon$ , denoted by  $\varepsilon^{M_{11}}$ , is above 6.5. For a more destructive but at the same time less frequent event ( $\omega = 0.95$ ,  $\lambda = 0.1$ ),  $\varepsilon^{M_{11}} > 5.5$ . In the case of very rare and extremely damaging events the threshold remains in a similar range. For  $\lambda = 0.004$  and  $\omega = 0.7$ ,  $\varepsilon^{M_{11}} > 5.3$ . For  $\lambda = 0.001$  and  $\omega = 0.5$ ,  $\varepsilon^{M_{11}} > 4.5$ . All of these values are beyond the empirically relevant range.

**Proof of Proposition 2**

$$\frac{d\theta^*}{d\lambda} = \frac{|M_{\theta\lambda}|}{|M|} = -A^{-1}\omega^{1-\varepsilon} \ln \omega \{1 - \lambda[(1 - \varepsilon) \ln \omega + 1]\varepsilon\psi^{-1}(1 - \omega^{1-\varepsilon})M_{11}^{-1}\} > 0, \tag{D.1}$$

$$\frac{d\theta^*}{d\delta} = \frac{|M_{\theta\delta}|}{|M|} = \lambda\omega^{1-\varepsilon} [(1 - \varepsilon) \ln \omega + 1]\varepsilon\psi^{-\varepsilon} |M|^{-1} > 0, \tag{D.2}$$

$$\frac{d\theta^*}{d\sigma} = \frac{|M_{\theta\sigma}|}{|M|} = -\frac{\phi}{\sigma^2} + \frac{\lambda\omega^{1-\varepsilon} \ln \omega}{A\sigma} [(1 - \varepsilon) \ln \omega + 1] \left[1 - \frac{\varepsilon(1 - \varepsilon)\theta^*A}{\psi M_{11}}\right] < 0, \tag{D.3}$$

$$\frac{d\theta^*}{d\phi} = \frac{|M_{\theta\phi}|}{|M|} = \frac{\varepsilon}{\sigma M_{11}} [1 + \lambda\omega^{1-\varepsilon}(1 - \varepsilon)^2(\ln \omega)^2\psi^{-1}] > 0. \tag{D.4}$$

**Proof of Corollary 2** By differentiating the explicit solution for  $\theta^*$  when  $\varepsilon = 1$  we find that

$$\begin{aligned} \frac{d\theta^*}{d\lambda} &= \frac{\delta}{\rho x \sigma A} > 0, \\ \frac{d\theta^*}{d\delta} &= \frac{\lambda}{\rho x \sigma A} > 0, \\ \frac{d\theta^*}{d\sigma} &= \left(-\frac{1}{\sigma^2}\right) \left(\phi + \frac{\delta\lambda}{\rho x A}\right) < 0, \\ \frac{d\theta^*}{d\phi} &= \frac{1}{\sigma} > 0. \end{aligned}$$



**Proof of Proposition 3**

$$\frac{dg}{d\lambda} = \frac{\partial g}{\partial \lambda} + \frac{\partial g}{\partial \omega} \left[ \frac{\partial \omega}{\partial \psi} \frac{d\psi}{d\lambda} + \frac{\partial \omega}{\partial \lambda} \right] \tag{D.5}$$

$$= \frac{1}{\varepsilon} \left\{ \omega^{1-\varepsilon} (1 + \varepsilon \ln \omega) - 1 - \lambda \varepsilon \psi^{-1} \omega^{1-\varepsilon} (1 - \omega^{1-\varepsilon}) \ln \omega M_{11}^{-1} \Gamma \right\} < 0, \tag{D.6}$$

$$\frac{dg}{d\delta} = \frac{\partial g}{\partial \delta} + \frac{\partial g}{\partial \omega} \left[ \frac{\partial \omega}{\partial \psi} \frac{d\psi}{d\delta} + \frac{\partial \omega}{\partial \delta} \right] \tag{D.7}$$

$$= -\ln \omega (\lambda \omega^{1-\varepsilon})^2 (1 - \varepsilon) \psi^{-\varepsilon-1} A |M|^{-1} \Gamma < 0 \text{ for } \varepsilon > 1, \tag{D.8}$$

$$\frac{dg}{d\sigma} = \frac{\partial g}{\partial \sigma} + \frac{\partial g}{\partial \omega} \left[ \frac{\partial \omega}{\partial \psi} \frac{d\psi}{d\sigma} + \frac{\partial \omega}{\partial \sigma} \right] \tag{D.9}$$

$$= \frac{A\phi}{\sigma^2} \left[ 1 + \lambda \omega^{1-\varepsilon} (\ln \omega)^2 (1 - \varepsilon)^2 \varepsilon \psi^{-1} \right] - (\lambda \omega^{1-\varepsilon} \ln \omega)^2 \Gamma \frac{(1 - \varepsilon)}{\psi \sigma} > 0, \text{ for } \varepsilon > 1 \tag{D.10}$$

$$\frac{dg}{d\phi} = \frac{\partial g}{\partial \phi} + \frac{\partial g}{\partial \omega} \left[ \frac{\partial \omega}{\partial \psi} \frac{d\psi}{d\phi} + \frac{\partial \omega}{\partial \phi} \right] = -\frac{\varepsilon A}{\sigma M_{11}} \left[ 1 + \varepsilon \lambda \omega^{1-\varepsilon} (1 - \varepsilon)^2 (\ln \omega)^2 \psi^{-1} \right] < 0, \tag{D.11}$$

where  $\Gamma \equiv [(1 - \varepsilon)(1 + \varepsilon \ln \omega) + \varepsilon] > 0$ .

**Consumption-Capital Ratio**

$$\frac{d\psi}{d\lambda} = \frac{|M_{\psi\lambda}|}{|M|} = (1 - \omega^{1-\varepsilon}) x A \sigma |M|^{-1} \begin{cases} \succ 0, & \text{if } \varepsilon \in (0, 1) \text{ or } \varepsilon > 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta} \\ \preceq 0, & \text{if } 1 \leq \varepsilon < 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta}. \end{cases} \tag{D.12}$$

$$\frac{d\psi}{d\delta} = \frac{|M_{\psi\delta}|}{|M|} = \lambda \omega^{1-\varepsilon} A (1 - \varepsilon) \psi^{-\varepsilon} |M|^{-1} \begin{cases} \succ 0, & \text{if } \varepsilon \in (0, 1) \text{ or } \varepsilon > 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta} \\ \preceq 0, & \text{if } 1 \leq \varepsilon < 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta}. \end{cases} \tag{D.13}$$

$$\frac{d\psi}{d\sigma} = \frac{|M_{\psi\sigma}|}{|M|} = (\varepsilon - 1) [xA\phi + \lambda \omega^{1-\varepsilon} \psi^{-\varepsilon} \delta] A (\sigma |M|)^{-1} \begin{cases} \preceq 0, & \text{if } \varepsilon \in (0, 1) \text{ or } \varepsilon > 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta} \\ \succ 0, & \text{if } 1 \leq \varepsilon < 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta}. \end{cases} \tag{D.14}$$

$$\frac{d\psi}{d\phi} = \frac{|M_{\psi\phi}|}{|M|} = x A^2 (1 - \varepsilon) |M|^{-1} \begin{cases} \succ 0, & \text{if } \varepsilon \in (0, 1) \text{ or } \varepsilon > 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta} \\ \preceq 0, & \text{if } 1 \leq \varepsilon < 1 + \frac{x\sigma\psi^{1+\varepsilon}\omega^{\varepsilon-1}}{\lambda\delta}. \end{cases} \tag{D.15}$$

**Proof of Corollary 3** By differentiating the explicit solution for  $g^*$  when  $\varepsilon = 1$  we find that

$$\begin{aligned} \frac{dg^*}{d\lambda} &= -\frac{\delta}{\rho x \sigma A^2} < 0, \\ \frac{dg^*}{d\delta} &= -\frac{\lambda}{\rho x \sigma A^2} < 0, \\ \frac{dg^*}{d\sigma} &= \left(\frac{1}{\sigma^2 A}\right) \left(\phi + \frac{\delta \lambda}{\rho x A}\right) > 0, \\ \frac{dg^*}{d\phi} &= -\frac{1}{\sigma A} < 0. \end{aligned}$$

**Proof of Proposition 4**

$$\begin{aligned} \frac{ds}{d\lambda} &= -(1 - \omega^{1-\varepsilon})x\sigma|M|^{-1} > 0, \text{ for } \varepsilon > 1, \\ \frac{ds}{d\delta} &= -\lambda\omega^{1-\varepsilon}(1 - \varepsilon)\psi^{-\varepsilon}|M|^{-1} > 0, \text{ for } \varepsilon > 1, \\ \frac{ds}{d\sigma} &= (1 - \varepsilon)[xA\phi + \lambda\omega^{1-\varepsilon}\psi^{-\varepsilon}\delta](\sigma|M|)^{-1} < 0, \text{ for } \varepsilon > 1, \\ \frac{ds}{d\phi} &= -xA(1 - \varepsilon)|M|^{-1} > 0 \text{ for } \varepsilon > 1. \end{aligned}$$

**Proof of Corollary 4** In the case of logarithmic utility function, the saving propensity reduces to  $1 - \frac{\rho}{A}$  and is therefore independent of the arrival rate, damage intensity, abatement efficiency and polluting intensity.

**Endogenous Arrival**

**Solution of the Extended Model**

The HJB equation is given by

$$\begin{aligned} \rho V(K, P) = \max \{ & U(C, P) + V_k[(1 - \theta)Y - C] + V_p[(\phi - \theta\sigma)Y - \alpha P] + \\ & + \lambda(K, P)[V(\tilde{K}, P) - V(K, P)] \}, \end{aligned}$$

where  $\tilde{K} = \omega K$ .

The optimality conditions with respect to the control and the state variables consist of:

$$C : U_c - V_k = 0, \tag{E.1}$$

$$\theta : -V_k Y - V_p \sigma Y = 0, \tag{E.2}$$

$$\begin{aligned} K : \rho V_k = & V_{kk}[(1 - \theta)Y - C] + V_k(1 - \theta)A + V_p(\phi - \sigma\theta)A + V_{pk}[(\phi - \sigma\theta)Y - \alpha P] + \\ & + \lambda \left[ \frac{dV(\omega K, P)}{dK} - V_k \right] + \lambda_k[V(\omega K, P) - V(K, P)], \end{aligned} \tag{E.3}$$

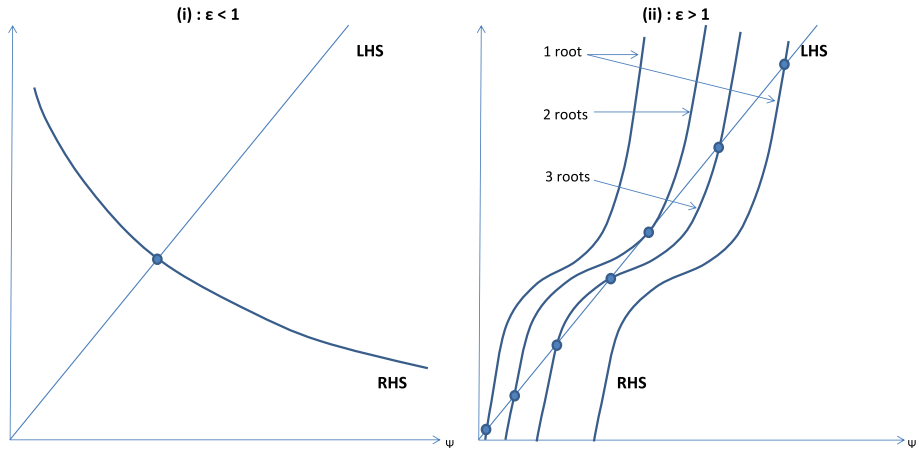


Fig. 4 Possible solutions for  $\bar{\psi}$

$$\begin{aligned}
 P : \rho V_p = U_p + V_{pp}[(\phi - \sigma\theta)Y - \alpha P] + V_{kp}[(1 - \theta)Y - C] - \alpha V_p + \\
 + \lambda \left[ \frac{dV(\omega K, P)}{dP} - V_p \right] + \lambda_p [V(\omega K, P) - V(K, P)].
 \end{aligned}
 \tag{E.4}$$

Solution proceeds along similar lines as in Sect. 2. First, we guess that the value function is of the same form as in Sect. 2. Then we derive the optimal policy from the first two optimality conditions. Recall that we also assumed  $\lambda = \lambda(v^\mu)$ , where  $v = K^\eta P^\xi$ . Substituting the latter, along with the policy, into HJB equation, we find the parameters of the value function,  $\bar{\psi}_t$  and  $x$ . Condition (E.4) is then used to find the optimal  $\bar{\theta}$ , while (E.3) allows us to obtain  $\bar{g}$ , following exactly the same steps as in A.1.2 (Fig. 4).

### Analysis of Multiple Solutions for $\bar{\psi}$

Multiple solutions to Eq. (25), which we reproduce here for convenience

$$\varepsilon\psi = \rho - (1 - \varepsilon)A \left( 1 - \frac{\phi}{\sigma} \right) - \lambda(\psi) [\omega^{1-\varepsilon}(\psi) - 1]
 \tag{E.5}$$

are feasible when the elasticity of marginal utility is above unity ( $\varepsilon > 1$ ). Define the function  $LHS(\psi) = \varepsilon\psi$  as the left-hand side of the equation and, similarly, the function  $RHS(\psi)$  as the right-hand side of the equation. The  $LHS$  is a straight line with the slope equal to  $\varepsilon$ . The  $RHS$  is, in general, a non-linear function of  $\psi$ . Differentiating it with respect to  $\psi$ , we obtain

$$\frac{dRHS}{d\psi} = -\frac{d\lambda}{d\psi} \frac{\partial v}{\partial \psi} [\omega^{1-\varepsilon}(\psi) - 1] - \lambda(1 - \varepsilon)\omega^{-\varepsilon} \frac{\partial \omega}{\partial \psi} \geq 0 \Leftrightarrow \varepsilon \geq 1,$$

so that  $RHS$  is strictly increasing in  $\psi$  for  $\varepsilon > 1$ . The second derivative is of ambiguous sign which means that the function has concave and convex segments and inflection point(s). Examining its asymptotic behavior, we see that  $\lim_{\psi \rightarrow 0} RHS = \pm\infty \Leftrightarrow \varepsilon \leq 1$  and  $\lim_{\psi \rightarrow 1} RHS$  is constant. Possible solutions to (E.5) are illustrated in the Fig. 4.

## Robustness Checks

### Sensitivity to $\gamma$

See Tables 4 and 5.

**Table 4** Optimal policy with  $\delta = 1e - 6$  (up to 1% of GWP) and  $\gamma = 0.9$

$\gamma = 0.9$	$\epsilon = 1$		$\epsilon = 3$	
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.08$	$\sigma = 0.05$
$\lambda = 0.004$				
$\theta$	0.50015	0.80024	0.62005	1.00844
$g$	3.47499	3.45998	1.15589	1.14903
$\lambda = 0.01$				
$\theta$	0.50038	0.80060	0.80158	1.32557
$g$	3.47497	3.45995	1.15218	1.14247
$\lambda = 0.02$				
$\theta$	0.50075	0.80121	1.10810	1.86662
$g$	3.47494	3.45990	1.14593	1.13128

**Table 5** Optimal policy with  $\delta = 1e - 6$  (up to 1% of GWP) and  $\gamma = 1.1$

$\gamma = 1.1$	$\epsilon = 1$		$\epsilon = 3$	
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.08$	$\sigma = 0.05$
$\lambda = 0.004$				
$\theta$	0.50015	0.80024	0.51710	0.82648
$g$	3.47499	3.45999	1.15810	1.15298
$\lambda = 0.01$				
$\theta$	0.50038	0.80060	0.54278	0.86626
$g$	3.47499	3.45998	1.15776	1.15244
$\lambda = 0.02$				
$\theta$	0.50075	0.80121	0.58565	0.93276
$g$	3.47497	3.45996	1.15718	1.15155

### Sensitivity to $\delta$ and $\beta$ .

#### Multiplicative Utility

We have also considered an alternative preference structure with multiplicative utility of the form  $\frac{(CP^{-\varphi})^{1-\epsilon}}{1-\epsilon}$ , which has also been used in Nordhaus's DICE. Because of the higher substitution with the multiplicative function, our climate policy instrument  $\theta$  is indeed lower than with the additive specification (Table 6). It is actually lower to the extent that the optimal abatement does not fully cover the current emissions—a point raised by the Reviewer in the later comments. Table 7 below shows the solution for our abatement propensity and the corresponding optimal growth rate with a multiplicative utility in various

**Table 6** Other parameters are at benchmark values and  $\varepsilon = 3$ 

$\gamma = 1$	$\delta = 1e - 7$		$\beta = 2$	
	$\sigma = 0.08$	$\sigma = 0.05$	$\sigma = 0.08$	$\sigma = 0.05$
$\lambda = 0.004$				
$\theta^*$	0.504	0.807	0.548	0.878
$g^*$	1.158	1.153	1.157	1.152
$\lambda = 0.01$				
$\theta^*$	0.510	0.816	0.621	0.996
$g^*$	1.158	1.153	1.156	1.150
$\lambda = 0.02$				
$\theta^*$	0.520	0.833	0.742	1.194
$g^*$	1.158	1.153	1.154	1.147

**Table 7** Optimal policy in low damage and high-damage scenario. Other parameters are at benchmark values and  $\varepsilon = 3$ 

		$\delta = 0.1$	$\delta = 0.3$
$\sigma = 0.08$	$\theta^*$	0.48	0.09
$(\frac{\phi}{\sigma} = 0.005)$	$g^*$	2.71	2.84
$\sigma = 0.05$	$\theta^*$	0.79	0.49
$(\frac{\phi}{\sigma} = 0.008)$	$g^*$	2.70	2.80
$\sigma = 0.025$	$\theta^*$	1.59	1.39
$(\frac{\phi}{\sigma} = 0.016)$	$g^*$	2.66	2.73

scenarios. The parameter  $\varphi$  in the utility function is set to unity; changing the value of  $\varphi$  does not bring substantial changes to the results. The calibration of the other parameters remains as in the paper, except for the damage intensity  $\delta$  which now needed to be recalibrated for the two scenarios, i.e. low damage (<1% of GDP) and high damage (up to 10% of GDP). Note that in all cases  $\theta < \phi/\sigma$ .

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## Declarations

**Conflict of interest** The authors declare that they have no conflict of interest. There are no financial or non-financial interests that are directly or indirectly related to this work.

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