Environmental and Resource Economics (2020) 76:655–663
https://doi.org/10.1007/s10640-020-00485-2

Is the Price System or Rationing More Effective in Getting a Mask to Those Who Need It Most?

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Accepted: 13 July 2020 / Published online: 4 August 2020

Abstract
Weitzman's classic insight on the virtues of allocating a scarce good via the price system or through rationing is applied to the problem of distributing masks, when the use of a mask provides a positive external benefit. I show that if a market leaves some individuals without a mask (when potentially there is supply for all), then rationing may be the superior option. When the variation in need is small, then even if the external effect of mask wearing is approximately equal to the personal benefit, even 10–20% maskless in the population may justify rationing.

Keywords Weitzman · Covid-19 · Masks · Prices versus quantities

1 Introduction

In many countries masks have been recommended as a means of reducing the risks of being infected with the Covid-19 virus (WHO 2020). But masks, especially those of reasonable surgical quality, are in short supply. How then should they be allocated? South Korea is one of a number of countries that have used rationing to control access to face masks during the Covid-19 pandemic (Kim 2020) [see also Taiwan, Everington (2020)], while other countries have relied on the market to allocate masks outside the hospital sector. In this paper, I apply methods from the late Martin Weitzman's famous articles on rationing, Weitzman (1974) and Weitzman (1977) to examine the issue.

2 Theory

To fix ideas, I suppose the good to be a mask of reasonable quality, in the sense of providing some protection against infection in everyday use without necessarily meeting standards expected for medical workers. A higher amount of x means that a person consumes each mask for a shorter period before disposal.

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Following the notation of Weitzman (1977), let the utility function of an individual be,

$$u(x) = \frac{(A+\epsilon)x}{B} - \frac{x^2}{2B} - \lambda px \tag{1}$$

where x is the amount consumed, p is the price per unit, A, and B are parameters[1] (with B and A positive), while λ represents the inverse of the marginal utility of income and ϵ is a taste or need parameter for x. The idea behind Eq. 1 is that it represents a second order approximation to a more general utility function. Need may vary because, for example, risk aversion varies or intolerance to wearing a mask varies, but also because the likely consequences of infection are heterogeneous (Jordan et al. 2020). As in the original Weitzman (1977) model, the normalizations of $E(\epsilon) = 0$ and $E(\lambda) = 1$ are employed, where E(.) represents the mean over the n members of the society and where it is assumed for simplicity that ϵ, λ are distributed independently. For a consumer in this situation faced with an unrestricted market, the demand for x is given by, $x(\epsilon, \lambda) = A - B\lambda p + \epsilon$

Weitzman compares the price system versus rationing when the ideal is a distribution based on need and not income (i.e. not on λ) and there is a fixed supply X. The ideal is $x = \overline{x} + \epsilon$ where $\overline{x} = X/n$. Note that, as he readily admits, this is a not a standard welfare economics approach in two ways. First, it would be more normal in economic analysis to define loss using a social welfare function, which itself would be an increasing function of individual utility. Here, though it is assumed that the ideal distribution matches consumption to need, ignoring other dimensions of taste. Weitzman even uses healthcare as an example where policy often seems focused on equality of need.[2]. Secondly, there is an assumption of paucity of instruments. In the simple model, it is assumed that the government does not have the power to identify need, while at the same time it is restricted to choosing between rationing and laissez-faire.[3] In practice, with masks there may be some individuals such as nursing home workers, with a clearly identifiable need. Nevertheless, there still remains the problem of how to allocate the scarce good amongst groups of people whose needs are not distinguishable.

In this context, the loss function for the rationing system is,

$$L^r = \sum\sum \frac{1}{n}\left(\overline{x} - (\overline{x}+\epsilon)\right)^2 = V(\epsilon),$$

Where $V(\epsilon)$ is the variance of ϵ . When the pricing system is used to allocate x, then the loss function becomes:

$$L^p = \sum\sum \frac{1}{n}\left(A + B\lambda\hat{p} + \epsilon - (\overline{x}+\epsilon)\right)^2,$$

Letting $\sigma^2 = \sum\sum \frac{1}{n}\left(A + B\lambda\hat{p} + \epsilon - \overline{x}\right)^2$, then the difference in losses can be written as,

$$L^p - L^r = \sigma^2 - 2V(\epsilon), \tag{2}$$

[1] In Weitzman (1977), there is also a constant term, C, but that plays no part in the analysis, so I omit it.

[2] "Nevertheless, society sometimes seems to act as if one-dimensional equity is a valid principle, at least for some commodities. As an example, it seems perfectly meaningful to be concerned about the effectiveness of the mental health profession in reaching people with counseling needs" (Weitzman 1977) p. 519

[3] Many of the developments of Weitzman (1974) such as Roberts and Spence (1976) are concerned with widening the feasible set of instruments.

When this expression is positive, then rationing is better than the price system and vice versa.

2.1 Externalities

To apply this to the issue at hand, I suppose that use of a mask creates a positive externality, which is increasing in the extent of use. In the spirit of simplicity of Weitzman's original paper, I use a quadratic form as a second order approximation to an underlying function. Specifically, I suppose that the modified utility function is,

$$u(x) = \frac{(A+\epsilon)x}{B} - \frac{x^2}{2B} - \lambda px + \frac{DX}{Bn} - \frac{E}{2Bn}\sum\sum x(\epsilon,\lambda)^2 + \frac{F}{B}\left(\frac{X}{n}\right)^2 \quad (3)$$

In this formulation I take the parameters D and E to be positive, and moreover take D, E and F together to be such that the marginal externality is always positive over the relevant range for x and, in line with typical papers on externalities, I take n to be sufficiently large that a person ignores the external effect when choosing x. Some words of justification are in order for the sign of E and F. It may be that there a diminishing or increasing returns to average mask wearing. Since X/n is fixed, then neither D nor F affect the main results of this paper, (see below) so I set the issue of the sign of F aside.[4] For E, the underlying idea is that it is better to allocate the marginal mask to someone with fewer masks. In other words the externality has the 'weaker link' property often proposed for health public goods such as quarantine measures, vaccination and basic sanitation (Cornes 1993; Sandler and Arce 2002).

The relevant demand for masks is the same as in the non-externality case. Meanwhile, the optimal allocation is, $x(\epsilon) = \bar{x} + \frac{\epsilon}{1+E}$. In other words, the nature of the externality means that the variance of x is smaller than in no-externality case. The modified loss equation for rationing is now:

$$L^r = \sum\sum \frac{1}{n}\left(\bar{x} - (\bar{x} + \frac{\epsilon}{1+E})\right)^2 = \frac{V(\epsilon)}{1+E},$$

Meanwhile, under the pricing system, the loss function becomes:

$$L^p = \sum\sum \frac{1}{n}\left(A + B\lambda\hat{p} + \epsilon - (\bar{x} + \frac{\epsilon}{1+E})\right)^2,$$

The difference in losses can be written as,

$$L^p - L^r = \sigma^2 - \frac{2V(\epsilon)}{1+E}, \quad (4)$$

For all strictly positive E, the presence of the externality raises the relative advantage of rationing versus the price system. In this way, the result resembles that by Rivera-Batiz (1981) who considered an extension of Weitzman (1977) that allowed for an asymmetric loss function. The current paper provides motivation for that asymmetry.

[4] Using first principles from the spread of aerosols, Brienen et al. (2010) suggests constant or diminishing returns from increases in the mean level of mask use, meaning that F is zero or negative. Tracht et al. (2010) and Eikenberry et al. (2020) model F as effectively zero.

A key feature of this expression is that it is the derivative of the marginal externality effect that is critical, not the value of D. In other words, the arguments for rationing depend on the cost of unequal access to masks, and not on the marginal benefit of another mask in total.

2.2 A Discrete Example

It seems that many of the issues with masks arise from their use versus non-use. To focus, suppose that each person consumes a discrete number of masks, from the set $\{0, 1, 2\}$. Moreover, let the external benefit from a first mask be $E_1 > 0$, but assume that there is no marginal external benefit from a second mask.[5] Preferences are as before, so a person consumes zero masks when

$$\frac{(A+\epsilon)}{B} - \frac{1}{2B} - \lambda p \leq 0 \tag{5}$$

which produces a critical value of p below which a person opts to have no mask. Let $p(2)$ be the market price when $X/n = 2$. For simplicity I assume that at $p(2)$ everyone would buy at least one mask. The cut offs: for one versus zero masks, $0 = 2\epsilon - 1 + 2A - 2Bp\lambda$ and for two versus one mask, $0 = 2\epsilon - 3 + 2A - 2Bp\lambda$. Let $\lambda_1(\epsilon)$ be the value of λ that solves the first of these equations, for a given ϵ, while $\lambda_2(\epsilon)$ solves the second. In addition, I let the distributions of ϵ and λ be independent and uniform with support $[\underline{\epsilon}, \bar{\epsilon}]$ and $[\underline{\lambda}, \bar{\lambda}]$ respectively, with $\Delta_\lambda = (\bar{\lambda} - \underline{\lambda})$ and $\Delta_\epsilon = (\bar{\epsilon} - \underline{\epsilon})$.

Let us consider two cases.[6]

2.3 Case 1. No Maskless in Market Equilibrium

In this case, there is no external benefit from rationing. There may still be a need based argument. The loss function under rationing is,

$$L^r = 2n\left(\frac{X}{n} - 1\right)\left(2 - \frac{X}{n}\right), \tag{6}$$

Meanwhile in the efficient allocation, all agents with $\epsilon \geq \hat{\epsilon} = \bar{\epsilon} - (X/n - 1)(\bar{\epsilon} - \underline{\epsilon})$ receive two masks.

When masks are distributed by the market, someone may receive one mask who should receive two, and someone else should receive one, but actually receives two. Hence:

$$L^p = \frac{n}{\Delta_\lambda \Delta_\epsilon}\left(\int_{\underline{\epsilon}}^{\hat{\epsilon}}\int_{\underline{\lambda}}^{\lambda_2(\epsilon)} d\lambda d\epsilon + \int_{\hat{\epsilon}}^{\bar{\epsilon}}\int_{\lambda_2(\epsilon)}^{\bar{\lambda}} d\lambda d\epsilon\right) \tag{7}$$

[5] Again, to emphasize, a second mask means simply that the lifetime of the mask is shorter, or perhaps the effectiveness of mask use is higher.

[6] For brevity, I omit consideration of the situation where the number of masks is less than the number of individuals. If the largest number of masks owned by an individual is one, then in general, rationing will be inferior to pricing in such a case. However, if variations in λ and ϵ are large enough, such that some individuals have two masks in the market outcome, then as in the cases discussed below, rationing can be optimal.

Which becomes,

$$L^p = \frac{n\left[(\bar{\epsilon} - \hat{\epsilon})\left(2Bp\bar{\lambda} + 3 - 2A - (\bar{\epsilon} + \hat{\epsilon})\right) + (\hat{\epsilon} - \underline{\epsilon})\left(2A - 2Bp\underline{\lambda} - 3 + (\underline{\epsilon} + \hat{\epsilon})\right)\right]}{2\Delta_\lambda \Delta_\epsilon Bp}$$

We note that the equilibrium price solves the equation:

$$\frac{X}{n} - 1 = \frac{\beta - 2Bp\underline{\lambda}}{2\Delta_\lambda Bp} \tag{8}$$

where $\beta = \left(2A - 3 + (\underline{\epsilon} + \bar{\epsilon})\right)$. This relationship can also be written as, $2 - \frac{X}{n} = \frac{2Bp\bar{\lambda} - \beta}{2\Delta_\lambda Bp}$, giving,

$$L^p = 2n\left(\frac{X}{n} - 1\right)\left(2 - \frac{X}{n}\right) - \frac{n(\bar{\epsilon} - \hat{\epsilon})(\hat{\epsilon} - \underline{\epsilon})}{\Delta_\lambda \Delta_\epsilon Bp} = L^r - \frac{n(\bar{\epsilon} - \hat{\epsilon})(\hat{\epsilon} - \underline{\epsilon})}{\Delta_\lambda \Delta_\epsilon Bp} \tag{9}$$

This means that rationing is worse than pricing.

2.4 Case 2. $X > n$; Maskless in Market Equilibrium

In this case, I assume that in the efficient allocation everyone has at least one mask. The loss under rationing is the same as in the previous case. The loss function for pricing includes the sub-optimal externality and four terms connected to the distribution of masks in equality: people who receive 0 masks in the market but should get 1 mask, people who get 0 masks but should get two, people who get one mask but should get two, people who get two masks and should get one.

$$\begin{aligned} L^p = \frac{n}{\Delta_\lambda \Delta_\epsilon}&\left(\int_{\underline{\epsilon}}^{\hat{\epsilon}} \int_{\lambda_1(\epsilon)}^{\bar{\lambda}} d\lambda d\epsilon + 4\int_{\hat{\epsilon}}^{\bar{\epsilon}} \int_{\lambda_1(\epsilon)}^{\bar{\lambda}} d\lambda d\epsilon + \int_{\underline{\epsilon}}^{\hat{\epsilon}} \int_{\underline{\lambda}}^{\lambda_2(\epsilon)} d\lambda d\epsilon + \int_{\hat{\epsilon}}^{\bar{\epsilon}} \int_{\lambda_2(\epsilon)}^{\lambda_1(\epsilon)} d\lambda d\epsilon\right) \\ &+ nE_1 \int_{\underline{\epsilon}}^{\bar{\epsilon}} \int_{\lambda_1(\epsilon)}^{\bar{\lambda}} d\lambda d\epsilon \end{aligned} \tag{10}$$

or,

$$L^p = \frac{n}{\Delta_\lambda \Delta_\epsilon}\left(4\int_{\hat{\epsilon}}^{\bar{\epsilon}} \int_{\lambda_1(\epsilon)}^{\bar{\lambda}} d\lambda d\epsilon + \left(1 - \int_{\underline{\epsilon}}^{\hat{\epsilon}} \int_{\lambda_2(\epsilon)}^{\lambda_1(\epsilon)} d\lambda d\epsilon\right) + \int_{\hat{\epsilon}}^{\bar{\epsilon}} \int_{\lambda_2(\epsilon)}^{\lambda_1(\epsilon)} d\lambda d\epsilon\right) + nE_1\delta$$

The right hand side becomes,

$$\frac{4n(\bar{\epsilon} - \hat{\epsilon})\left(2Bp\bar{\lambda} - 2 - \beta\right)}{2\Delta_\lambda \Delta_\epsilon Bp} + \frac{n(\hat{\epsilon} - \underline{\epsilon})}{\Delta_\epsilon}\left(1 - \frac{\left(\lambda_1 - \lambda_2\right)}{\Delta_\lambda}\right) + \frac{n(\bar{\epsilon} - \hat{\epsilon})\left(\lambda_1 - \lambda_2\right)}{\Delta_\lambda \Delta_\epsilon} + nE_1\delta$$

Now, let the proportion of people with no mask be δ. So,

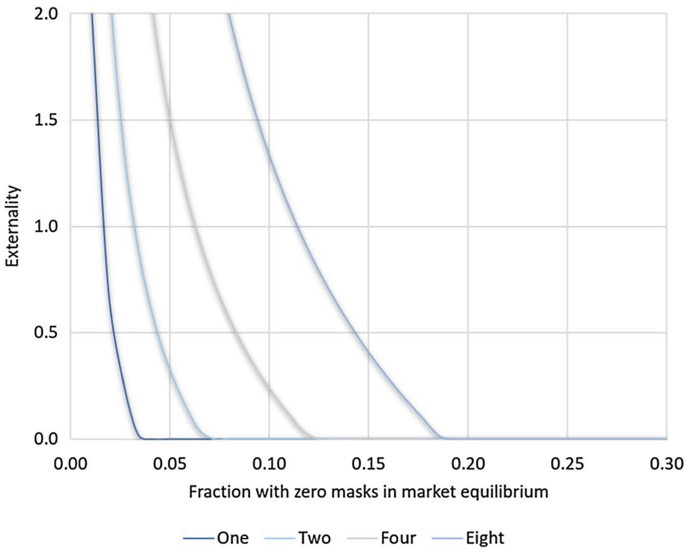

Fig. 1 Markets versus Rationing for different levels of scarcity. Notes. Areas to the right of curves show regions where rationing dominates. 'One', 'Two' etc. refer to the ratio $\Delta_\epsilon/\Delta_\lambda$

$$\delta = \int_{\underline{\epsilon}}^{\bar{\epsilon}} \int_{\lambda_1(\epsilon)}^{\bar{\lambda}} d\lambda d\epsilon = \frac{-\beta - 2 + 2Bp\bar{\lambda}}{2\Delta_\lambda Bp} \tag{11}$$

Meanwhile, $2(1-\delta) - \frac{X}{n} = \frac{2}{2\Delta_\lambda Bp}$ which implies that,

$$L^p = n\left(\left(2 - \frac{X}{n}\right)\left(\frac{X}{n} - 1 + 2\delta\right) + \left(\frac{X}{n} - 1\right)\left(2(1-\delta) - \frac{X}{n}\right)\right.$$
$$\left. + 4\delta\left(\frac{X}{n} - 1\right) - \frac{4(\bar{\epsilon} - \hat{\epsilon})(\underline{\epsilon} - \hat{\epsilon})}{2Bp\Delta_\epsilon\Delta_\lambda}\right) + nE_1\delta$$

Or, using the fact that $\frac{(\hat{\epsilon} - \underline{\epsilon})}{\Delta_\epsilon} = \left(\frac{X}{n} - 1\right)$,

$$L^p = L^r + n\left(2\delta - \frac{\left(\frac{X}{n} - 1\right)\left(2 - \frac{X}{n}\right)\Delta_\epsilon}{Bp\Delta_\lambda}\right) + nE_1\delta \tag{12}$$

Note that when there are no maskless, this expression reduces to (9), but in general, it has two extra terms, both involving δ (see "Appendix" for the formula for δ). The first term represents the inefficiency loss of having some individuals without masks, while the final term is the externality associated with unequal access. These two terms are positive and therefore potentially make rationing preferable to using the pricing mechanism. This is more likely to be case when the market leaves a significant proportion of consumers without a mask.

As supply improves (i.e. as X increases) then δ and p fall (see "Appendix"). Consequently, as supply gets larger and larger, it always becomes more efficient to use the price mechanism eventually. Moreover, because the second term inside the large parentheses is negative, pricing will be superior to rationing even when there are some maskless individuals in the market equilibrium. In other words, maskless individuls are a necessary but not sufficient condition for rationing to be optimal.

Figure 1 provides some results from simulations. Obviously, data for calibration is significantly lacking so these are largely illustrative. The horizontal axis shows the fraction of the population without masks in the market equilibrium. In these examples, for instance, 30% corresponds to an aggregate supply of 1.06 masks per person, while zero maskless is achieved when supply is 1.55 masks per person. Systematic evidence on supply shortages during the early stages of the pandemic is difficult to come by. OECD (2020) suggests global supply for masks (including surgical quality masks) was about half of demand in March 2020, but that is based on individuals using two masks per day. Meanwhile, Smith (2020) reports on self-reported mask wearing in public for March, when the figures were 70–80% in Japan and 80–90% in Taiwan, China and Korea. These numbers provide an upper bound on the truly maskless, since some part of the population may be refusing to use face covering.

The vertical axis is a measure of the external cost of unequal access: E_1, normalised by dividing by the utility of a single mask for an average individual. A value higher than one indicates that the utility of everyone else wearing one mask is greater than the utility obtained by the individual wearing a mask. As a benchmark, Eikenberry et al. (2020) for example discuss evidence on the 'outward efficiency' (i.e. protection of others) to the 'inward efficiency' (protection of self) of non-surgical masks and suggest a range of 0.6 to 4. If we assume that expected utility is linear in the efficiency, then this range provides some guidance on the relevant range on the vertical axis of Fig. 1.

The curves in the figure show the boundaries between rationing being optimal (to the right) and pricing being optimal (to the left) for various ratios of $\Delta_\epsilon/\Delta_\lambda$, with $[\underline{\lambda}, \bar{\lambda}]$ set to $[0.5, 1.5]$. Larger values of this ratio mean that the variation in need is high compared to variations in the marginal utility of income.

A few basic points are clear from the figure. First, as the number of maskless in the market equilibrium falls, the advantages of the market increase. And, as noted above, when there are no maskless, then there is no case for rationing here. Second, higher levels of variation in need raise the case for the market, because rationing is such a blunt instrument and ends up giving masks to individuals with low or high need randomly. Third, the boundaries between the market and rationing regions is relatively insensitive to the the size of E, but the sensitivity is larger for higher values of the $\Delta_\epsilon/\Delta_\lambda$ ratio. This last effect arises because when the variance in tastes is larger, then the market equilibrium yields a higher proportion of people without masks. The E_1 term is multiplicative with δ and so variations in E have a bigger impact.

3 Discussion

In a simple model based upon the idea that there are normative justifications for distributing goods on the basis of need, Weitzman (1977) showed that rationing maybe a superior policy in situations where a good in fixed supply must be distributed, but the value placed on the good does not correlate perfectly with the marginal utility of income. Here, I show that when applied to distribution of masks during an epidemic, there is a similar argument for rationing, which is augmented by the extra positive externality achieved when consumption is spread more evenly. While I have focused on the mask example, the same arguments apply to other discretely-consumed goods in short supply (such as screens in retail outlets or hand sanitizer) where use provides an external effect. Rationing may be desirable even if only a small percentage of the population lack access to masks in the market, provided the external benefit of masks is of the order of the personal benefit and the variation in needs across the population is relatively small.

The model suggests that in times of extreme shortage, there is an advantage to the kind of rationing system employed by the South Korean and Taiwanese governments (Kim 2020; Everington 2020). While this advantage is likely to be only for the period before supplies can be ramped up, it may be a valuable intervention in the context of a rapidly spreading pandemic. On the other hand, the model gives no support to the kind of policy pursued in Japan, where government-provided masks were supplemental and only slowly distributed.

Even when the existence of the externality gives a potential advantage to rationing, we must note that equal distribution may not yield an external benefit if a significant proportion of individuals refuse to use their masks (i.e. because of low or even negative ϵ). In such a case, other tools may be required to achieve efficiency.

Acknowledgements Japan Society for the Promotion of Science (Grant No. 17H02498.)

Compliance with Ethical Standards

Conflict of interest The author declares that he has no conflict of interest.

Appendix

To get the price, set out the equation for supply equals demand.

$$X = \frac{n}{\Delta_\lambda \Delta_\epsilon}\left(\int_{\underline{\epsilon}}^{\bar{\epsilon}} \int_{\lambda_2(\epsilon)}^{\lambda_1(\epsilon)} d\lambda d\epsilon + 2 \int_{\underline{\epsilon}}^{\bar{\epsilon}} \int_{\underline{\lambda}}^{\lambda_2(\epsilon)} d\lambda d\epsilon \right) \tag{13}$$

From this we get,

$$p = \frac{n(1+\beta)}{B\left(X\Delta_\lambda + 2n\underline{\lambda}\right)} \tag{14}$$

To get δ, go through a similar argument:

$$1 - \delta = \frac{(2+\beta)}{2Bp\Delta_\lambda} - \frac{\underline{\lambda}}{\Delta_\lambda} \tag{15}$$

When the right hand side of this equation is greater than 1, then $\delta = 0$. It is then clear that $\frac{\partial p}{\partial X} < 0$ and $\frac{\partial \delta}{\partial X} \leq 0$.

Given that at $X/n = 1$, $\delta > 0$ and that at $X/n = 2$, $\delta = 0$ (the latter by assumption), then it can also be shown that there is exactly one value of X/n below which rationing is optimal and above which the market is optimal. The function $L^p - L^r$ is continuous and strictly convex. It is clear from (12) that at $X/n = 1$ then rationing is optimal, i.e. $L^p > L^r$, while at $X/n = 2$, the market is superior ($L^p < L^r$). Thus there is exactly one value of X/n in the open interval (1, 2) at which $L^p = L^r$.

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