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Contributions by Aart Blokhuis to finite geometry, discrete mathematics, and combinatorics

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The present Special Issue of Designs, Codes and Cryptography is dedicated to Aart Blokhuis, one of the leading mathematicians in finite geometry and combinatorics. In this preface we will survey some of his important results. Needless to say, this selection represents the interest of the editors. According to MathSciNet, Aart Blokhuis published over 130 papers which have received more than 1000 citations. His most frequent coauthors are Andries Brouwer, Henny Wilbrink and also the guest editors of this issue: Simeon Ball and Tamás Szőnyi, with whom he collaborated for several years. The third editor of this special issue, Michel Lavrauw, was a PhD student of Aart between 1997 and 2001.

Aart started his career in Eindhoven as a PhD student of Jaap Seidel (PhD awarded in 1983) and retired from the Technical University of Eindhoven recently. In his early career he obtained important results about few distance sets; the best bound on a 2-distance set is still due to him. He ingeniously used linear algebra methods in the proof, which was to become a trademark of his work. In the 1980s he obtained several results motivated by extremal combinatorics and coding theory. He proved a conjecture of van Lint and MacWilliams about cliques of the Paley-graph of square order [3], a paper that van Lint called a "gem". Aart himself would later comment that these results convinced him that he was a mathematician. His interest focussed on finite geometry but he was always ready to think about problems coming from extremal combinatorics and graph theory. He wrote a series of important papers with Jef Thas and Aiden Bruen on arcs in higher dimensions [8, 15]. Their work on associating a hypersurface to an arc not just led to substantial improvement on previous bounds on the size of the second largest complete arc (particularly, in spaces of even order) but was also the starting point of later investigations of arcs in higher dimensions and a proof of the MDS conjecture in some cases.

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In most of his research, Aart would combine algebraic, geometric and combinatorial ideas in novel ways to come up with beautiful results. A typical example of this is the main result in his paper with Ákos Seress and Henny Wilbrink on complete exterior sets of conics [9]. He is one of the pioneers of the polynomial method which he employed in many of his works. The results on cliques of the Paley graph are an example of this and these are among the best known results obtained by Aart in the nineties. An important starting point of his work was László Rédei's book *Lacunary polynomials over finite fields*. He developed the theory further and applied it to substantially improve the number of directions determined by a set of q points in the affine plane $\mathrm{AG}(2, q)$. His most cited result, [4], proved by using lacunary polynomials, confirms an old conjecture for blocking sets in Galois planes of prime order: if a blocking set in $\mathrm{PG}(2, p)$, p prime does not contain a line, then it has at least $3(p+1)/2$ points. The result is sharp. Another challenging problem at the time was to prove the non-existence of maximal arcs in Galois planes of odd order; this was achieved by Aart and his coauthors Simeon Ball and Francesco Mazzocca in [1]. Together with Simeon Ball, Andries Brouwer, Leo Storme, and Tamás Szőnyi, Aart also published two influential papers [10, 11], in which the theory of Lacunary polynomials is applied to obtain results concerning the number of directions of a graph of a function over a finite field, and multiple blocking sets. In the same decade, together with Eric Moorhouse, he used the linear algebra method to determine the p-rank of orthogonal spaces in [7]. This implied improved bounds on the largest size of caps and also non-existence of ovoids in some finite orthogonal spaces.

He continued being very active as the new century began. He obtained important results on scattered subspaces with respect to a spread [5], linear sets which are multiple blocking sets [2], and the classification of semifields [12]. All these results form the starting points of research topics that are very much the centre of interest today. In particular, some of the results are related to maximum rank distance (MRD) codes. He obtained many other important results afterwards in many different fields, in topics ranging from general combinatorics to coding theory and, of course, many further results in finite geometry. Without detailing all of these results, we mention his work on the Kakeya problem with Francesco Mazzocca [6], embedding three-nets in projective planes [14] with Gábor Korchmáros and Francesco Mazzocca, q-analogues of classical extremal set theory problems, see [13].

We have indicated several works with coauthors and Aart is extremely good at working together with other people. He always has surprising, deep new ideas which shed light on an unexpected aspect of the problem. He is also exceptionally quick and helpful as a mathematician; he is always ready to share his ideas and problems with other people.

Besides being an excellent mathematician he is also sincere and a good friend. He has a wonderful sense of humour, has many stories to tell, and is invariably the life and soul of the party.

The present volume has 14 contributions and 33 contributors. The enthusiasm for contributing was exceptional and it resulted in a fine collection of papers. Many of these articles are directly related to Aart's work and we are sure he will find much of interest in these contributions. There are papers from many aspects of combinatorics, including a large subset concerning problems in finite geometry and spectral graph theory.

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